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The Influence of Brittle Daniels System Characteristics on the Value of Load Monitoring Information

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Abstract

This paper addresses the influence of deteriorating brittle Daniels system characteristics on the value of structural health monitoring (SHM). The value of SHM is quantified as the difference between the life cycle benefits with and without SHM. A value of SHM analysis is performed within the framework of the Bayesian pre-posterior decision theory and requires (1) structural performance modelling and prediction, (2) structural integrity management models, (3) the (pre-posterior) modelling of SHM and (4) the coupling of SHM and the structural performance models. The pre-posterior decision theoretical framework facilitates that the value of SHM can be quantified before the SHM system is quantified and before data are acquired. The results of this study support decisions to select structural systems for which the SHM strategy load monitoring is optimal.

1 INTRODUCTION

Deteriorating and redundant structural systems constitute a large part of the build environment. Considering the importance of the build environment and the sparse societal resources, it is essential to ensure a safe and an efficient structural integrity management. However, the value of SHM is sparsely quantified in this regard. This paper addresses thus decision support to select structural systems for which SHM is optimal in terms of expected cost and risk reduction.

This paper builds upon the recently developed framework for the quantification of the value of SHM (e.g. [1] and [2]) and focusses on the explication of the system and component performance influence on the value of SHM. The framework for the quantification of the value of SHM is outlined and the structural system performance model and the SHM information model are described in detail. With a generic case study, the influence of structural system characteristics on the value of SHM is quantified and discussed. The paper closes with the conclusions.

2 QUANTIFICATION OF THE VALUE OF SHM FOR STRUCTURAL SYSTEMS

The value of SHM can be calculated with a Value of Information analysis within the framework of the Bayesian pre-posterior decision theory through the difference between the expected value of the life cycle benefits B_1 utilizing SHM and the expected value of the life cycle benefits B_0 without SHM (Equ. (1)).

$$V = B_1 - B_0 \tag{1}$$

In the context of managing the structural integrity with risk based inspection planning and taking basis in [3], the expected value of the life cycle benefit B_0 for a fixed inspection strategy i_{ins} can be written as the maximization of the expected benefits b with the l_n uncertain inspection outcomes $\mathbf{Z}_{ins} = \left[Z_{ins,1} \ldots Z_{ins,l} \ldots Z_{ins,l_n} \right]^T$, the m_n (inspection) action choices $\mathbf{a}_{RBI} = \left[a_{RBI,1} \ldots a_{RBI,m} \ldots a_{RBI,m_n} \right]^T$ and the o_n structural performance uncertainties $\mathbf{X} = \left[X_1 \ldots X_o \ldots X_{o_n} \right]^T$:

$$B_0 = \max_{a_{RBI,m}} E''_{X_o} \left[b\left(i_{ins}, Z_{ins,l}, a_{RBI,m}, X_o \right) \right] \tag{2}$$

Utilizing SHM, the expected value of the life cycle benefit B_1 is calculated for a fixed SHM strategy s delivering the j_n uncertain SHM information $\mathbf{Z}_s = \left[Z_{s,j} \ldots Z_{ins,j} \ldots Z_{ins,j_n} \right]^T$ with Equ. (3) taking basis in Equ. (2), i.e. in the structural integrity management with risk based inspection planning.

$$B_1 = \max_{a_{RBI,m}} E''_{X_o} \left[b\left(s, Z_{s,j}, i_{ins}, Z_{ins,l}, a_{RBI,m}, X_o \right) \right] \tag{3}$$

The notations for B_0 and B_1 build upon the extensive form of a decision analysis, i.e. that the posterior expectation is taken in regard to the structural performance where the term “posterior” relates to the updating with inspection information. It is noted, that the calculation of the expected benefits necessitate a benefit, cost and consequence model which will be described in the further.

3 STRUCTURAL SYSTEM PERFORMANCE AND INTEGRITY MANAGEMENT MODEL

In this section, the approaches for the structural performance and integrity management of the structural system are described. The structural performance model predicts the component and system structural reliability throughout the service life and consists of a fatigue deteriorating structural system model subjected to extreme loads. Based on the structural reliability, the expected benefits of the structural integrity management and the structural risks are calculated $(b(i_{ins}, Z_{ins,l}, a_{RBI,m}, X_o))$ and maximized to calculate the life cycle benefits B_0 see Equ. (2).

3.1 Structural system performance model

The structural performance model constitutes a fatigue deteriorating redundant system subjected to extreme loads and is modelled as Daniels system with brittle component behavior, see (Figure 1) and [4].

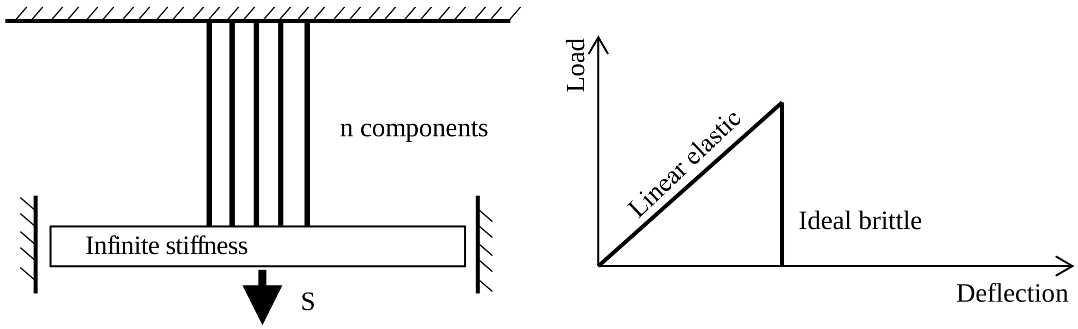

Figure 1: Daniels System

The probability of failure of a brittle Daniels system can be calculated with Equ. (4). It contains the product of the failure probabilities over n deteriorating components with time dependent and ordered realizations of component resistances $R_i(t)$ (see Equ. (5)), the model uncertainties $M_{R,i}$ and the system loading S multiplied with the loading model uncertainty M_S. The system loading is usually described with an extreme value distribution having a reference period of one year (see e.g. [5]).

$$P_{F_S} = P\left(M_{R,i} \cdot \max_{1\le i\le n}\left((n-i+1)\hat{R}_i(t)\right) - M_S S \le 0\right) \tag{4}$$

$$\hat{R}_1(t) \le \hat{R}_2(t) \le \ldots \le \hat{R}_n(t) \tag{5}$$

The deterioration induced resistance reduction is modelled with the reduction initial component resistance $R_{i,0}$ in dependency of a resistance reduction factor r_R multiplied with the crack size distribution $A_i(t)$ to wall thickness d_i ratio (Equ. (6)).

$$R_i(t) = R_{i,0}\left(1 - D_i(t)\right) = R_{i,0}\left(1 - r_R \frac{A_i(t)}{d_i}\right) \tag{6}$$

The crack size distribution is modelled with a fracture mechanics (FM) model which is calibrated to an SN fatigue model. The SN limit state function g_i^{SN} (Equ. (7)) for component i, i.e. hot spot, is formulated in dependency of fatigue capacity Δ, the annual number of stress cycles ν, the stress ranges $\Delta\sigma_i$ and the SN curve constants m and K. The expected value of the stress ranges $E\left[\Delta\sigma_i^m\right]$ is calculated with the model uncertainty M, the cut-off stress range s_0 and the Weibull scale parameter λ as well as the Weibull location parameter k.

$$g_i^{SN} = \Delta - \nu \cdot t \frac{E\left[\Delta\sigma_i^m\right]}{K} \text{ with } E\left[\Delta\sigma_i^m\right] = \left(Mk\right)^m \Gamma\left(1 + \frac{m}{\lambda}; \left(\frac{s_0}{k}\right)^{\lambda}\right) \tag{7}$$

The FM model is described with the limit state function g_i^{FM} (Equ. (8)) containing the critical crack depth $a_{i,c}$ and the crack depth distribution $a_i(t)$ at time t for the component i.

$$g_i^{FM} = a_{i,c} - a_i(t) \tag{8}$$

3.2 Structural system integrity management model

The structural system integrity management model builds upon the reliability based inspection and repair planning (see [6], [7], [8] and [9]) facilitating the maximization of the benefit throughout the life cycle, see Equ. (2), by identifying an optimal risk based inspection plan. The expected life cycle benefits B_0 are then calculated with Equ. (9) as the sum of the expected costs (negative expected benefits) of the componential structural integrity management, i.e. the expected costs for inspections $E\left[C_{i,Insp}\right]$ and repair $E\left[C_{i,R}\right]$, the risk of component fatigue failure $R_{i,D}$ and the risk for structural system failure R_{F_S}.

$$\begin{aligned} B_0 &= \max_{a_{RBI,m}} E''_{X_o}\left[b\left(i_{ins}, Z_{ins,l}, a_{RBI,m}, X_o\right)\right] = \\ &\max_{a_{RBI,m}} E''_{X_o}\left[-\left(\sum_{i=1}^{n}\left(E\left[C_{i,Insp}\right] + E\left[C_{i,R}\right] + R_{i,D}\right) + R_{F_S}\right)\right] \end{aligned} \tag{9}$$

It should be noted that the crack depth at year t, $A_t(t)$ see (Equ. (6)), is calculated conditional on the inspection outcomes which is calculated with the approach proposed in [10]. This algorithm can be interpreted as an enhancement of the classical rejection sampling algorithm for Bayesian updating and is here based on subset simulation ([11]).

4 STRUCTURAL SYSTEM PERFORMANCE AND INTEGRITY MANAGEMENT MODEL WITH SHM

The SHM system and their data are modelled in the framework of the Bayesian pre-posterior decision theory taking basis in characteristics of model uncertainties. Model uncertainties apply to almost all models utilized in engineering such as analytical, empirical or semi-empirical models and may be determined by means of measurements (see e.g. [5]). This implies that measurements, i.e. SHM data, contain information about the model uncertainties which can be exploited in the probabilistic modelling. In this way, yet unknown SHM data can be modeled.

SHM for fatigue provides thus information about the model uncertainties in the SN fatigue model for the individual components. Then the expected stress ranges for fatigue are calculated in dependency of realizations of the model uncertainties $\hat{M}$ (Equ. (10)) accounting for the SHM uncertainty U.

$$E\left[\Delta\sigma_i \mid \hat{M}\right]=\left(\hat{M}Uk\right)^m \Gamma\left(1+\frac{m}{\lambda};\left(\frac{s_0}{k}\right)^\lambda\right) \tag{10}$$

On structural system level, the SHM information may lead also to knowledge of the associated loading model uncertainties, i.e. when long term load monitoring in conjunction with an extreme value analysis has been performed. Then, the system failure probability is calculated by utilizing the realizations of the loading model uncertainty considering the loading measurement uncertainties U_S (Equ. (11)). Note, that the realizations of the resistance are required to facilitate the solution of the brittle Daniels system formulation and do not interfere with the SHM modelling.

$$P_{F_S}=P\left(M_{R,i}\cdot\max_{1\le i\le n}\left(\left(n-i+1\right)\hat{R}_i\left(t\right)\right)-\hat{M}_S U_S S\le 0\right) \tag{11}$$

The expected value of the life cycle benefit utilizing SHM B_1 is calculated with the expected value of the costs for the componential structural integrity management $E\left[C_{i,Insp}^{s,\mathbf{Z}}\right]$ and $E\left[C_{i,R}^{s,\mathbf{Z}}\right]$, for SHM $E\left[C_{i,SHM}\right]$, for the risks of component fatigue failure $R_{i,D}^{s,\mathbf{Z}}$ and system failure $R_{F_S}^{s,\mathbf{Z}}$ which are influenced by the SHM information ($s,\mathbf{Z}$, see Equ. (12)).

$$\begin{aligned} B_1 &= \max_{a_{RBI,m}} E''_{X_o}\left[b\left(s,Z_{s,j},i_{ins},Z_{ins,l},a_{RBI,m},X_o\right)\right]= \\ &\max_{a_{RBI,m}} E''_{X_o}\left[-\left(\sum_{i=1}^{n}\left(E\left[C_{i,Insp}^{s,\mathbf{Z}}\right]+E\left[C_{i,R}^{s,\mathbf{Z}}\right]+E\left[C_{i,SHM}\right]+R_{i,D}^{s,\mathbf{Z}}\right)+R_{F_S}^{s,\mathbf{Z}}\right)\right]\end{aligned} \tag{12}$$

5 CASE STUDY

5.1 Structural system performance and integrity management model

A brittle Daniels system consisting of $n=5$ hot spots with the fatigue design factors of 2.0 (three hot spots) and 3.0 (two hot spots) is analyzed. The system loading S is resisted by the 5 components with the initial resistance $R_{0,i}$. The mean of the initial resistance is calibrated to the initial component probability failure $P\left(F_{c,i}\right)=1.0\cdot10^{-2}$ when not varied, see TABLE 1. The loading of the Daniels system and the resistance of the components are Log-Normal and Weibull distributed with a standard deviation of 0.1. The probabilistic models for the model uncertainties M_R and M_S are determined in accordance with [5].

The FM model constitutes a 2D and single slope Paris' law crack growth model taking basis in [12]. The initial crack size is modelled Exponentially distributed in line with [13]. The expected values of the crack growth parameter and of the stress intensity factor model uncertainty are calibrated to the SN model.

The SN fatigue model takes basis in [14]. The model uncertainties M for the fatigue loading (Equ. (7)) are subdivided into the model uncertainties for the load calculation M_L, for the nominal stress calculation M_σ, for the hot spot stress calculation M_{HS} and for the weld quality M_Q. The location parameter k of the long-term stress distribution is adjusted to reach an accumulated fatigue damage of 1.0 after $t = FDF \cdot t_{SL}$ years using the characteristic value for K.

TABLE 1: PROBABILISTIC STRUCTURAL PERFORMANCE MODEL

Var.	Dim.	Dist.	Exp. value	Std. dev.	Var.	Dim.	Dist.	Exp. value	Std. dev.
M_R	-	LN	1.0	0.05	υ	yr^{-1}	Det.	$3.0x10^6$	
$R_{0,i}$	-	LN	Cal.	0.1	t_{SL}	yr	Det.	20.0	
M_S	-	LN	1.0	0.1	M_L		LN	0.89	0.27
S	1/y	WBL	3.5	0.1	M_σ		LN	1.01	0.12
r_R	-	Det.	0.6	-	M_{HS}		LN	1.02	0.20
Δ	-	LN	1.0	0.3	M_Q		LN	1.02	0.20
lnK	-	N	28.995	0.572	d	mm	Det.	16	
m	-	Det.	3.0		a_c		Det.	16	
k	MPa	LN	Dep. on FDF	$0.2 \cdot \mu_k$	DoB		Det.	0.5	
$1/\lambda$	-	Det.	1.2		r_{aspect}		Det.	0.2	
s_0	MPa	Det.	0.0						

LN: Lognormal, N: Normal, EX: Exponential, Cal.: Calibrated, WBL: Weibull

The correlation model takes the correlation of the component fatigue deterioration, the component initial resistances and the component resistance model uncertainties into account. A correlation of the fatigue deterioration of $\rho\left(D_i, D_j\right) = 0.6$ is assumed (when not varied) following [15]. The component resistances are correlated with $\rho\left(R_{0,i}, R_{0,j}\right) = 0.5$ (when not varied). The component resistance model uncertainties are assumed to be correlated with $\rho\left(M_{R,i}, M_{R,j}\right) = 0.5$.

The structural integrity management model takes basis in the risk based inspection and repair planning on component, i.e. hot spot, level. That means that an optimal inspection plan for each of the hot spots is determined such that a given annual probability threshold for the fatigue failure Δp_D is not exceeded in the service life of 20 years. The inspection strategy is magnetic particle inspections (MPI) which is modelled with the parameters $\alpha = 0.63$ and $\beta = 1.16$ following e.g. [7].

The cost model for the service life integrity management and the calculation of risks takes basis in generic normalized costs for inspection $C_{i,Insp} = 1.0 \cdot 10^{-3}$, for repair $C_{i,R} = 1.0 \cdot 10^{-2}$ and for the consequences in case of hot spot fatigue failure

$C_{i,D} = 1.0$ and structural system failure $C_{F_S} = 100$, see [7] and [16]. The discounting rate is assumed to $r = 0.05$.

5.2 SHM strategy

The SHM strategy consists of monitoring the system loading (see Equ. (11)) and of hot spot fatigue stresses, i.e. the hot spot loading (Section 4), throughout the service life. The expected values of the stress ranges for the individual hot spots are modeled conditional on the realizations of the hot spot loading model uncertainty, i.e.:

$$E\left[\Delta\sigma_i \mid \hat{M}_L\right] = \left(\hat{M}_L M_\sigma M_{HS} M_Q U k\right)^m \Gamma\left(1 + \frac{m}{\lambda}; \left(\frac{s_0}{k}\right)^\lambda\right) \tag{13}$$

The model uncertainties for the nominal stress calculation, for the hot spot stress calculation and for the weld quality are assumed not to be determinable with this SHM strategy. The measurement uncertainty $U = U_S \sim N(1.0, 0.05)$ accounts for the uncertainties associated with the observations of the structural system and the hot spot loading building upon quantified measurement uncertainties in [17].

The costs for this SHM strategy consisting of 5 measurement channels comprise the investment ($1.33 \cdot 10^{-4}$ per channel), the installation ($1.33 \cdot 10^{-4}$ per channel) and the operation ($1.33 \cdot 10^{-4}$ per year) according to [18]. The SHM cost model is calibrated to the integrity management cost model (see previous Section).

5.3 Value of load monitoring in dependency of the system characteristics

Figure 2 depicts the value of SHM (Equ. (1)) for the strategy load monitoring throughout the service life in dependency of the component probability of failure and the correlations between the component resistances and deterioration. The value of SHM strategy load monitoring is calculated by quantifying the service life benefits B_1 utilizing SHM (Equ. (12)) and B_0 without SHM (Equ. (9)) with the fatigue failure thresholds $3.0 \cdot 10^{-3}$ representing the minimum fatigue deterioration level kept throughout the service life.

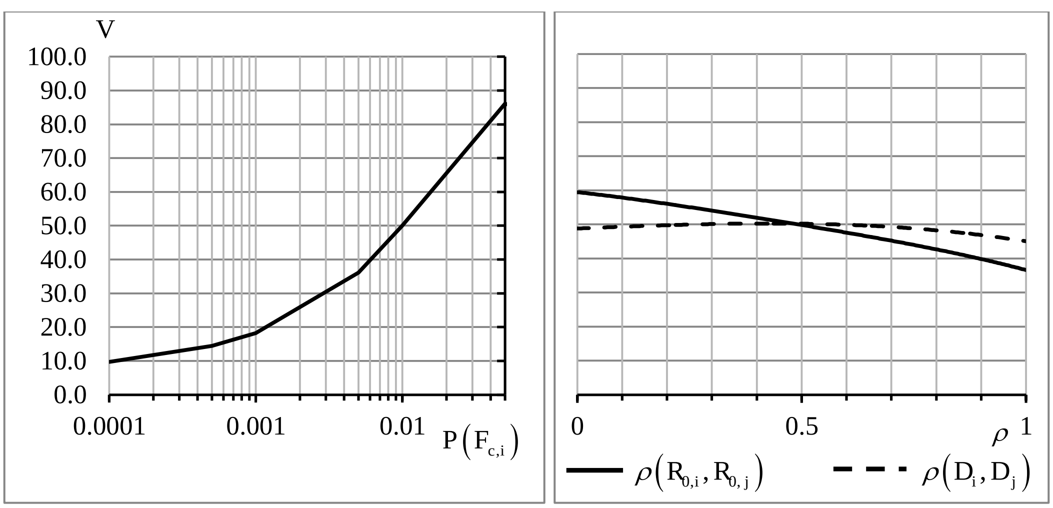

Figure 2: Value of SHM for a brittle Daniels system in dependency of the initial component failure probability (left), the correlation between the resistances and the deterioration (right) throughout the service life.

The value of load monitoring increases for an increase of the initial component probability of failure. This can be explained by (1) the reduction of the system resistance, (2) the constancy of the system and fatigue loading and (3) thus the constancy of the loading uncertainty reduction. As a consequence, the risk reduction throughout the service life increases non-linearly on system level leading to increasing value of load monitoring.

Considering the resistance correlation $\rho\left(R_{0,i}, R_{0,j}\right)$, the value of load monitoring decreases slightly with a concave curvature (in relation to the horizontal axis) with increasing resistance correlation. The reason for this behavior is the increase of the system reliability with increasing resistance correlation in combination with the constancy of the uncertainty and thus the risk reduction throughout the service life, as previously reasoned.

With increasing damage correlation $\rho\left(D_i, D_j\right)$, the value of load monitoring stays approximately constant. Here, it is also observed that the system reliability dependency on the damage correlation influences primarily the value of load monitoring.

6 CONCLUSIONS

This paper contains a study of the value of the SHM strategy load monitoring with a Value of Information analysis within the framework of the Bayesian pre-posterior decision theory. The framework of the Bayesian pre-posterior decision theory facilitates that the value of load monitoring can be quantified before load monitoring implementation and data acquisition. The value of load monitoring is quantified as the difference between the life cycle benefits with and without load monitoring and is analyzed for generic structural systems modelled as redundant systems with brittle component behavior. For this aim structural performance and prediction models including deterioration, structural integrity management models, pre-posterior load monitoring models coupled to the structural performance models are utilized.

This paper reveals that the highest value of load monitoring considering brittle Daniels systems is achieved for systems with a low structural reliability throughout the service life. This means that load monitoring may be optimal for deteriorated systems, for system where the loading has increased and/or for systems with a high correlation of the component resistance and deterioration mechanisms.

It can be concluded that the value of load monitoring is inversely proportional influenced by the structural system reliability during the service life caused by the constancy of the loading uncertainty reduction. The value of SHM is hereby strongly influenced by the component reliability, comparably moderately by the resistance correlation and slightly by the deterioration correlation.

The SHM strategy load monitoring can be modelled in the framework of the Bayesian pre-posterior decision theory by exploiting the characteristics of the model uncertainties in conjunction with measurements and with SHM uncertainties.

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