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AN ACENTRIC ROTATION OF HELICAL VORTEX PAIR

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The aim of this presentation is to test a possibility of a secondary solution for an acentric equilibrium rotation of the helical vortex pairs with the same pitch, sign and strength. These symmetrical conditions using for classical pairs of rectilinear vortices with a centric equilibrium rotation were expanded for the vortex pair having the helical symmetry (Delbende et al., 2015). Their consideration was limited by the solitary case of the symmetrical rotations of two identical co-rotating helical vortices (figure 1, a) because the 2-D vortex pair as a prototype of the helical pair deals with the unique solution only. However each helical vortex is capable of rotating by itself. This additional self-induced motion can provide a secondary solution of the helical vortex pair for which an acentric equilibrium positions becomes possible (figure 1, b).

![Figure 1](image1.png)

**Figure 1.** Two possibilities for equilibrium of helical vortex pairs: (a) centric equilibrium configuration; (b) acentric equilibrium configuration.

![Figure 2](image2.png)

**Figure 2.** Cross-(a) and profile-(b) sections of the streamlines for the second solution with the acentric helical vortex pair.

The current investigation addresses to the three-dimensional dynamics of the thin vortex filaments in an ideal fluids for which the self-induced vortex-tube motion expresses only by a bi-normal component in a natural coordinate system. This fact results from the intrinsic equations of...
the vortex filament motion which were strictly derived in early of the last century when a conception of the self-induced motion was established (Ricca, 1996). In contrast to the different wrong approximations of helical vortices (see e.g. Fuentes, 2015) the current study is based on the Kawada-Hardin’s solution for an infinite thin helical vortex filament (Fukumoto et al., 2015), which is rewritten in the light of modern development of the helical vortex theory (Ricca, 1994; Kuibin and Okulov, 1998; Boersma and Wood, 1999) and adapted to finite core case by a procedure, proposed in (Okulov, 2004), and after then it is summed together with the induction velocity of the second vortex filament of the vortex pair at the same fixed coordinate system. As a result of this investigation the secondary equilibrium solution for the helical pair with acentric vortex positions is found (figure 2).

### Appendix

**Typical mistakes in an estimation of the self-induced velocity**

17th August of 2015 many scientists concerning of helical vortex dynamics have received an ambitious letter (Velasco Fuentes, 2015) about his derivation of a new formula for the self-induced motion of helical vortices which was submitted for publication in European Journal of Mechanics B/Fluids. It should be mention that the proposed derivation takes typical mistakes which can unfortunately be tracked in many applications of helical vortex dynamics. This appendix, based on this fresh illustration, has been written to guarantee against repetition of these mistakes in future.

The first mistake appears when the researches try to use solutions for infinite thin vortex filament to describe a motion of the filament with a finite vortex core. The difference between both velocity fields is well-known and it was stated and described in early classical works on vortex dynamics (see refs in Kuibin and Okulov, 1998). Nevertheless the attempts (including the last one by Velasco Fuentes, 2015) of the wrong using of the first solution for the description of the second task take place up to now. Below we attract your attention to this problem again.

We will remain the main points about a valuation of the self-induced motion of any slender vortices in the natural coordinate system based on (Batchelor, 1967; Kuibin and Okulov, 1998). The unit vectors of the natural coordinate system directed along the tangent, the principal normal and binormal to the filament at some point $O$. In this system the velocity field induced by a curved vortex filament may be asymptotically presented at a small distance $\sigma$ from a filament as the sum of the pole, the logarithmic singularity and the regular term (Batchelor, 1967). The first of them describes the circulatory motion around the vortex axis and does not cause its displacement. The next two summands should describe the motion of the vortex filament in a direction of the binormal. The dimensionless form scaled to $\Gamma \kappa / 4\pi$ of the binormal velocity is

$$\hat{\mathbf{w}}_b^{(\text{Asympt})} = -\frac{2 \cos \theta}{\kappa \sigma} + \ln \frac{1}{\kappa \sigma} + C^{(\text{Asympt})}$$  \hspace{1cm} (1)

where $\sigma, \theta$ are the polar coordinates of the natural system; $\kappa$ is the vortex curvature.

It is known that the dimensionless self-induced velocity of a helical vortex with a finite-core size of $\varepsilon$ after an integrating of (1) is described by the formula

$$\hat{\mathbf{w}}_b^{(\text{Ind})} = \ln \frac{1}{\kappa \varepsilon} + C^{(\text{Ind})}.$$  \hspace{1cm} (2)

That is an analog of (1) without accounting the pole because the integral from this contribution vanishes due to symmetry. In (1) the quantity $C^{(\text{Asympt})}$ depends only on the vortex filament
geometry and in (2) the value of $C^{(Sind)}$ depends also on the same geometric parameters of vortex as well as on the vorticity distribution inside the vortex core. There is an interrelation between both quantities under the same vortex geometry which mainly depends on the vorticity distribution in the vortex core (Batchelor, 1967; Ricca, 1994; Kuibin and Okulov, 1998; Boersma and Wood, 1999; Okulov, 2004). For example for helical vortex with a uniform vorticity distribution in a small vortex core the difference is $1/4$ (Boersma and Wood, 1999), that gives

$$C^{(Sind)} = C^{(Asympt)} + \frac{1}{4}.$$  (3)

**Figure 3.** Self-induced rotation of the helical vortices with non-dimensional pitch $\tau$: solid line – results of Okulov (2004) for the helix with finite vortex core $\varepsilon = 0.1$ and dashed line – the incorrect solution of Velasco Fuentes (2015) by Kawada-Hardin solution for infinite thin helical filament. $\Omega$ is a non-dimensional angular velocity in the fixed coordinate system; $\Omega_b$ is an associative value in natural coordinate.

**Figure 4.** The entangled data of the self-induced motion in both fixed and natural coordinate systems from Fig. 5 of (Velasco Fuentes, 2015): “Linear and angular velocities, $U^*$ and $\Omega^*$, respectively, as functions of the vortex pitch ($\tau$) for the given vortex radius ($\varepsilon = 0.1$).” Figure 3 with the Okulov’s data on $\Omega$ in fixed coordinates repeats the curve “Okulov (2004)” for $\Omega^*$ here and his associative $\Omega_b$ in natural coordinates of solid line on figure 3 have a good correlation with data by Mezic et al. (1998) putted for $\Omega_b$ here.
The difference between the “C” terms in (3) can take different values depending on a form of the vortex, core and vorticity distributions, e.g. this value for a vortex ring was estimated as 3/4 in (Tung and Ting, 1967). Nevertheless sometime the wrong approximations of the self-induced vortex motion by the asymptotic velocity (1) of infinite thin vortex ring or helical filament without the necessary correction to the finite core can be found in several research reports and articles. For example the Kawada-Hardin’s solution (Fukumoto et al., 2015) corresponding to the approximation (1) for the infinite thin helical filament was only used in (Velasco Fuentes, 2015). The author did not include the correction (3) and he derived a wrong result. The black dashed line (figure 3) shows his wrong solution without the finite core correction (3) and green solid line coincides to the correct result by Okulov 2004 with the necessary correction by 1/4 (Okulov, 2004).

The second typical mistake concerns with an entanglement of the coordinate systems. For example in fig. 5 of (Velasco Fuentes, 2015), reproduced here in figure 4, all data were calculated in different coordinates and then all curves were wrongly put on the same plots without any selections. The correct curves of the Okulov’s data (2004) calculated in both fixed and natural coordinates were put separately in figure 3. His curve ($\Omega$) in the fixed coordinate was reproduced on the plots (figure 4) with entangled data of (Velasco Fuentes, 2015) and the same curve in the natural coordinate ($\Omega^*$) can be found in a good correlation with Mezic et al. (1998) in figure 4. This correct comparison supports both simulations of the self-induced velocity made by two different methods (Mezic et al., 1998) and (Okulov, 2004) and simultaneously both ones turn down the incorrect curves of (Velasco Fuentes, 2015).

References
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