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Summary (max 2000 characters):
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A primary motivation for—and part of—this work is the creation of a standard for uncertainty estimation and reporting, which is known as the IEC 61400-15. The author is actively contributing to this emerging standard, and the work herein thus far constitutes (most of) the vertical extrapolation portion of the IEC 61400-15 draft.
Preface

A primary motivation for this work was to help create a new standard for uncertainty estimation and reporting, which is known as the IEC 61400-15. The author is actively contributing to this emerging standard, and most of the work herein (sections 1-3) has already been written into the 61400-15 draft for vertical extrapolation uncertainty by the author. No conflict of interest is existing between the author’s DTU affiliation (with e.g. commercial product WAsP) and his IEC role, foremost because the WAsP uncertainty work had been completed in 2012-14 and subsequently published. Further, comparisons between uncertainties estimated for shear-extrapolation and WAsP did not happen until after completion of the respective theoretical work on each; section 4 was added later, after sections 1-3 had been derived and proposed in the IEC subgroup.

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Summary

This report provides formulations for estimation of uncertainties involved in vertical extrapolation of winds, as well as the total uncertainty incurred when winds observed at one height are extrapolated to turbine hub height for wind resource assessment. This includes new derivations for uncertainties inherent in determination of (wind) shear exponents, and subsequent vertical extrapolation of wind speeds. The report further outlines application of the theory and results of Kelly & Troen (2014-6) for gauging the uncertainty inherent in use of the European Wind Atlas (EWA) / WAsP method for vertical extrapolation. Lastly, a section has been added that compares the uncertainty in the two aforementioned methods. The independently-derived forms corresponding to each vertical extrapolation method give uncertainty estimates that are essentially the same for small vertical extrapolations \((z_{\text{pred}}/z_{\text{obs}})\); for larger extrapolations, WAsP-based extrapolation leads to smaller estimated uncertainties than the shear-extrapolation method.

A primary motivation for—and part of—this work is the creation of a standard for uncertainty estimation and reporting, which is known as the IEC 61400-15. The author is actively contributing to this emerging standard, and the work herein thus far constitutes (most of) the vertical extrapolation portion of the IEC 61400-15 draft.
1. Methodology to calculate vertical extrapolation uncertainty

The present methodology to calculate vertical extrapolation uncertainty is limited to:

- Vertical extrapolation of wind speed (e.g. mean wind speed, Weibull-$A$ parameter or reference wind speed) from one height above ground level to another. Vertical extrapolation of distribution shape (Weibull-$k$), wind direction, and turbulence intensity are not considered here. For vertical profiles of Weibull-$k$ parameter, the reader is directed to Kelly et al. (2014).

The methodology assumes statistical independence of vertical extrapolation uncertainty from other uncertainties, i.e. no correlation with horizontal extrapolation, long-term corrections, etc. It can also be assumed that vertical extrapolation uncertainty is normally distributed (Gaussian), allowing combination with other uncertainties and consistent with the Central Limit Theorem.

Three methods of vertical extrapolation are commonly used in wind energy (at present), via application of:

- power-law profile modelling;
- surface-based profile and/or linearized flow modelling, including surface roughness; and
- CFD wind flow modelling.

The first two are the most common, and the uncertainty inherent in their use is described in the subsections below. Regardless of the method chosen, a general expression for vertical extrapolation uncertainty should be compatible with the general form

$$\sigma_{\tilde{u},\text{ve}}^2 \approx \sigma_{\tilde{u},\text{obs}}^2 + k_{\text{sens}} \sigma_{\text{model}}^2 \quad (1)$$

where $\sigma_{\tilde{u},\text{ve}}$ is the total dimensionless uncertainty associated with vertical extrapolation of wind speed from one height above ground level (observation height) to another (prediction height); the tildes denote dimensionless values (expressible e.g. in %). On the right-hand side, $\sigma_{\tilde{u},\text{obs}}$ is the wind speed measurement uncertainty, $\sigma_{\text{model}}$ is the model uncertainty, and $k_{\text{sens}}$ is a coefficient based on the model sensitivity to one or more input parameters. Generally the model uncertainty can be roughly decomposed as $\sigma_{\text{model}}^2 = (\sigma_{\text{fit}}^2 + \sigma_{\text{rep}}^2)$, where $\sigma_{\text{fit}}$ is the uncertainty of fitting the model to data used for vertical extrapolation (including that due to vertical sampling); and $\sigma_{\text{rep}}$ is the uncertainty due to model representativeness, i.e. how well the model matches nature between observation height(s) and prediction height.
2. Power-law profile modelling

The mean wind shear exponent ($\alpha$), defined through the power law wind profile

$$ U(z) = U(z_r) \cdot \left( \frac{z}{z_r} \right)^{\alpha}, \quad (2) $$

can be related most directly as

$$ \alpha = \frac{dU}{dz} \frac{U}{U(z)} \quad (3) $$

Note a mean $\alpha$ is specified, to vertically extrapolate mean wind speed $U$; this mean is recommended to be per directional sector, allowing for probability-weighted means (i.e. ‘frequency’ of occurrence per direction). We begin by assuming measurements covering an integer number of years, whereas a modification is possibly needed if using monthly means. Starting for the simplest case of two measurement heights ($z_1, z_2$), the centered, theoretically ‘exact’ formulation (3) is compatible with the commonly-used practical form of calculation,

$$ \alpha = \frac{\ln(U_2/U_1)}{\ln(z_2/z_1)} \quad (4) $$

Wind shear exponents are assumed to be calculated via (4), using wind speeds averaged over a fixed time interval (standard is 10 minutes; up to 30-minutes is allowable). These $\alpha$ will tend to follow a particular distribution $P(\alpha|U)$ conditional on wind speed itself, and depending on the height above ground relative to the effective surface roughness (i.e. including terrain complexity; c.f. Kelly et al., 2014a). Regarding the heights used for measurements, it is recommended that they are located within the same atmospheric regime (e.g. above the surface layer at night/in shallow ABLs, or within the surface-layer for daytime/unstable conditions); for subdivision of data into winter/night-time, the heights should all be above ~30 m. The heights used should also be larger than the local scale of terrain elevation changes.

In this section we do not consider changes in the shape of the wind distribution with height (Weibull-$k$ profile) when doing shear-extrapolation; unfortunately, though it is not correct, this reflects common wind industry practice. Effects of the $k$-profile can be seen in e.g. Kelly et al. (2014b).

The uncertainty in modelling the (mean) wind shear exponent $\alpha$ at heights above measurements may be simply expressed as

$$ \sigma_{\alpha}^2 = \sigma_{\text{fit}}^2 + \sigma_{\text{rep}}^2 \quad (5) $$

The right side contains terms that handle the “measurement” of mean wind shear exponent $\alpha$ via fitting, and representativity of the power law profile, respectively. Eq. 5 includes the assumption that the fit does not impact the representativity, i.e. these two sources of uncertainty are independent of each other. More specifically,

- $\sigma_{\text{fit}}^2$ is the uncertainty in obtaining $\alpha$ from measurements. For the basic case of measurements at 2 heights, $\sigma_{\text{fit}}^2$ includes the effect of the distance between anemometers...
o+n calculating $\alpha$. For the case of measurements at 3 or more heights, this term has a reduced value, which decreases with increasing number of anemometers\(^1\).

- $\sigma_{\text{rep}}$ characterizes the effect of how well the power law profile—and its wind shear exponent derived from observations at some heights—is expected to represent the real wind profile at some prediction height $z_{\text{pred}}$ above the observation heights. For the basic two-height power law profile that excludes surface information (Eq. 2), this uncertainty depends primarily upon $(z_{\text{pred}}/z_{\text{obs}})$ and $(z_{\text{pred}}/z_{0,\text{eff}})$, where $z_{\text{obs}} = (z_1 z_2)^{1/2}$ is the geometric-mean observation height\(^2\) and $z_{0,\text{eff}}$ is the effective roughness. An estimate for $z_{0,\text{eff}}$ is $[z_0 (\sigma_z + z_0)^2]^{1/3}$, with $z_0$ being the background roughness and $\sigma_z$ the rms terrain elevation for 3km upwind.\(^3\) The latter can be easily computed by taking the standard deviation of terrain heights within 3km of a site (with further refinement possible via directional-weighting). Otherwise, a crude relation of the form $\text{RIX} \sim a_W \ln \left( \frac{\sigma_z}{\Delta z_{\text{ref}}} \right)$ can be employed to relate RIX (in %) to $\sigma_z$, where the constant $a_W \sim 30$ and $\Delta z_{\text{ref}} \sim 100$ m for a range of complex-to-moderate terrain types.

A preliminary, basic form for $\sigma_{\text{repr}}$ follows from the difference between $\alpha(z_{\text{obs}})$ and $\alpha(z)$ for generalized $\ln(z/z_{0,\text{eff}})$ wind-speed dependence above the surface layer (e.g. Kelly et al., 2014a). Using $\ln(z/z_{0,\text{eff}}) = \ln(z/z_{\text{obs}}) + \ln(z_{\text{obs}}/z_{0,\text{eff}}) = \ln(z/z_{\text{obs}}) + a_{\text{obs}}^{-1}$, one obtains
\[
\frac{a_{\text{obs}} - a_{\text{eff}}(x_{0,\text{eff}})}{a_{\text{eff}}(x_{0,\text{eff}})} = a_{\text{obs}} \ln \left( \frac{z_{\text{pred}}/z_{\text{obs}}}{z_{0,\text{eff}}} \right);
\]
this is a simple estimate for $\tilde{\sigma}_{\text{repr}}$. Then using a heuristic model to account for complex terrain, we have
\[
\tilde{\sigma}_{\text{repr}} \approx a \cdot c_{f} \cdot \frac{\ln(z_{\text{pred}}/z_{\text{obs}})}{\ln(z_{\text{pred}}/z_{0,\text{eff}})}.
\] (6)

The coefficient $c_f$ is expected to be of order 1; empirically, using $c_f \sim 3–4$ leads to values from (5) that are generally consistent with a number of studies; e.g. estimates for uncertainty magnitudes by AWS Truepower and LiDAR tests by Enercon, for cases dominated by $\sigma_{\text{repr}}$.

For measurements at 2 heights, an approximate dimensionless form can be derived for the ‘fitting’ (vertical sampling) uncertainty, following from the differential of (4): ignoring height uncertainty, $\delta \ln \alpha \approx \delta (U_2 - U_1) / [\alpha \ln(z_2/z_1)]$, which leads to
\[
\tilde{\sigma}_{\text{fit}} \approx c_f \frac{\sigma_{\text{Uobs}}}{\alpha \ln(z_2/z_1)}.
\] (7)

Here the tildes denote normalized/non-dimensional quantities (i.e. percentage divided by 100), and $\sigma_{\text{Uobs}}$ is the normalized uncertainty in observation (e.g. given, as percent divided by 100). The coefficient $c_f$ is monotonic in the correlation between observational uncertainty at the different anemometers, and can also be adjusted to account for special conditions where the mean wind profile is known to deviate significantly from logarithmic or neutral form.

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\(^1\) The effect of vertical sampling causes an effective reduction in profile-fitting uncertainty, for increasing number of heights $n$.

\(^2\) For $n$ heights, where $n>3$, the effective observation height can be approximated by $z_{\text{obs}} = (\prod z_j)^{1/n}$.

\(^3\) Following the effective roughness response scale in Fourier $z_0$-response modelling, with $\sigma_z$ the effective surrogate for the outer spectral scale (see e.g. Astrup & Larsen, 1999).
The ‘fitting’ uncertainty $\hat{\sigma}_{\text{fit}}$ can also be estimated when one has 3 measurement heights; a basic/crude adjustment for 3–5 heights is to multiply (7) by $\sqrt{2/n}$, where $n$ is the number of measurement heights.

To obtain the uncertainty in power law profile extrapolation of mean wind speed, the propagation of measurement uncertainty must be incorporated properly with the uncertainty of modelling the wind shear exponent. Applying (2) for extrapolating $U_{\text{obs}}$ to $U_{\text{pred}}$ and taking the differential, one obtains

$$\delta U_{\text{pred}} = \frac{U_{\text{pred}}}{U_{\text{obs}}} \delta U_{\text{obs}} + U_{\text{pred}} \ln \left( \frac{z_{\text{pred}}}{z_{\text{obs}}} \right) \delta \alpha - \frac{\alpha}{z_{\text{obs}}} U_{\text{pred}} \delta z_{\text{obs}} .$$

Rewriting this non-dimensionally as $\delta \ln U_{\text{pred}} = \delta \ln U_{\text{obs}} + \ln \left( \frac{z_{\text{pred}}}{z_{\text{obs}}} \right) \delta \alpha - \alpha \delta \ln z_{\text{obs}}$, then squaring and retaining lowest-order terms (i.e. excluding cross-correlation terms as well), leads to an expression for non-dimensional uncertainty ($\tilde{\sigma}$) in mean wind speed due to power law profile extrapolation:

$$\tilde{\sigma}^2_{U_{\text{pred}}} \approx \tilde{\sigma}^2_{U_{\text{obs}}} + \sigma_a^2 \left[ \ln \left( \frac{z_{\text{pred}}}{z_{\text{obs}}} \right) \right]^2 .$$

The first term on the right-hand side of (9) is the uncertainty in wind measurement (aggregate, i.e. from 2 or more heights), and again $\sigma_d^2 = \alpha^2 \delta_d^2 = \tilde{\sigma}^2_{\text{fit}} + \tilde{\sigma}^2_{\text{repr}}$ is the (dimensional square) uncertainty of obtaining the shear exponent following (4). Using (5) along with the form of (4) in (9), then from (7) $\tilde{\sigma}_{\text{fit}} = c_f \tilde{\sigma}_{U_{\text{obs}}} / \left[ \ln (U_2/U_1) \right]$ so that the total vertical extrapolation uncertainty can be expanded and re-written as:

$$\tilde{\sigma}^2_{U_{\text{pred}}} \approx \tilde{\sigma}^2_{U_{\text{obs}}} \left[ 1 + c_f \frac{\alpha \ln \left( \frac{z_{\text{pred}}}{z_{\text{obs}}} \right)}{\ln (U_2/U_1)} \right]^2 + \frac{\alpha c_r \ln \left( \frac{z_{\text{pred}}}{z_{\text{obs}}} \right)}{\ln \left( \frac{z_{\text{pred}}}{z_{\text{obs}}} \right)} \ln \left( \frac{z_{\text{pred}}}{z_{\text{obs}}} \right) .$$

for the case of 2 measurement heights, again with $c_r \sim 4$ from (6). A final term due to the uncertainty in measurement heights also exists in Eqs. 9-10, and can be expressed as $\alpha^2 \tilde{\sigma}^2_{z_{\text{obs}}}$; however, this is neglected because it is much smaller than the other terms, since $\alpha^2 \ll 1$ for conditions where shear-extrapolation can be used (and also because $\tilde{\sigma}_{z_{\text{obs}}} < 10\%$, rarely exceeding 1%).

The dimensionless form of Eqs. 9-10 allows the uncertainties to be cast fractionally (e.g. in terms of percentages), and is amenable to similar expressions for uncertainty components calculated elsewhere, such as $\tilde{\sigma}_{U_{\text{obs}}}$; dimensionless (normalized) quantities are denoted with tilde.

The approach can be applied per sector, summing frequency-weighted $\tilde{\sigma}^2_{U_{\text{pred}}}$. Further, frequency-weighted calculations can also be used on a seasonal or diurnal basis.

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4 One can also include the Weibull-weighted effect of the mean wind-speed dependence that arises from the width of $P(\alpha|U)$ being inversely proportional to $U$ [Kelly et al., 2014a].
3. Logarithmic-based profile

Extrapolation of mean wind speed via stability-perturbed logarithmic profile, such as used by “WAsP” software and other methods based on the European Wind Atlas (i.e. from the perturbed geostrophic drag law, as in Troen et al. 1987), relies on use of the surface roughness length \( z_0 \). Characterization of the uncertainty for perturbed-log-law extrapolation is found in Kelly & Troen (2016), for standard (i.e. default) WAsP application to vertical extrapolation; the expression therein is translatable to dimensionless uncertainty in vertical extrapolation, which can be re-written as

\[
\tilde{u}_{\text{pred}} = \tilde{u}_{\text{obs}} + c_w \cdot \ln \left( \frac{z_{\text{pred}}}{z_{\text{obs}}} \right) \tag{11}
\]

where the empirically-determined constant is \( c_w \approx 0.02 \) (i.e. 2%). Note that (11) is recommended only for moderate extrapolations with \( \frac{z_{\text{pred}}}{z_{\text{obs}}} < 2 \), with an additional linear term needed for more extreme extrapolation; we forego such at present, recommending extrapolations up to twice the measurement height.

A smaller roughness uncertainty component also exists for the WAsP extrapolation, and has been analysed in a more recent work (c.f. Kelly & Jørgensen, VindkraftNet meeting, Feb.2015). This component is implicit in the last term of (11), and becomes negligible for the case with no horizontal extrapolation (prediction from/to a single mast). At this time the minor dependence on roughness uncertainty is suppressed, and left for future publication.

4. Comparison of uncertainty estimates for vertical extrapolation, via shear-exponent and WAsP methods

The behaviour of the uncertainty estimation formulations can be compared, i.e. for both shear-exponent and EWA-based (WAsP) vertical extrapolations. Figure 1 shows the estimated uncertainty for both formulations, including two cases for shear-exponent extrapolation. For the latter, results for two different fitting uncertainty values are plotted, and the lowest measurement height is taken to be 55m while \( c_f \) is estimated as 0.5; the equivalent upper-measuring heights are shown in the figure legend for the case of shear from 2 observation heights. The WAsP-default method (green) is shown for the case of any typical observation height; however here (for both methods) the observations are assumed to not be taken over forest.
Figure 1. Uncertainty in vertical extrapolation of mean wind speed (minus measurement uncertainty $\sigma_{\text{obs}}$), estimated for shear-exponent method (via Eqns.4-10), and for EWA/perturbed-log (WAsP) method (via Eqn.11). For shear ($\alpha$)-extrapolation, lower-measurement height taken to be 55m in this plot. Blue: shear-extrapolation, with 38% fitting uncertainty (equivalent to upper height of 80m); gold: shear-extrapolation, for $z_2=100$m; green: shear-extrapolation with $z_2=100$m but $\alpha = 0.3$; $\sigma_{\text{fit}}$ values via Eq. 7. Red: EWA/perturbed log-law (WAsP default) method.

From the figure one can see that for limited extrapolation distances, the estimated uncertainty in vertical extrapolation is comparable for both WAsP-based extrapolation and shear-based extrapolation in typical conditions, particularly for smaller $\sigma_{\text{fit}}$; however, $\alpha$-based $\sigma_{\text{pred}}^2$ can be smaller than the default-valued WAsP extrapolation uncertainty, for prediction heights within ~30% of measurement height, especially for measurement heights that are not too close to each other. At larger relative extrapolations ($z_{\text{pred}}/z_{\text{obs}}$), WAsP appears to give lower uncertainty in mean wind speeds, with shear-extrapolation giving dramatically higher uncertainty for larger height ratios.

However, for annual energy production this dramatic difference could be reduced: the systematic error in shear-based extrapolation is potentially offset by neglect of the vertical variation of Weibull-$k$ (such neglect is not uncommon in industrial practice); however for some heights this difference can also be amplified. These two errors, and the subsequently modified uncertainty (error) in AEP prediction, are a current subject of study, with ongoing validation. Further, validation of the shear-extrapolation uncertainty estimate-model is currently underway and pending publication.

References


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