DTU Library

Application of Transactive Control for Integrating Distributed Energy Resources into an Enhanced Distribution System

Dalsgaard, Mads Thelin; Yang, Guangya; Hu, Junjie

Publication date:
2016

Document Version
Publisher's PDF, also known as Version of record

Application of Transactive Control for Integrating Distributed Energy Resources Into an Enhanced Distribution System

Mads Thelin Dalsgaard   Guangya Yang   Junjie Hu

DTU Electrical Engineering
The Technical University of Denmark
23 June, 2016

DTU Electrical Engineering
Department of Electrical Engineering

Contents

1 | Introduction

The conversion to be independent of fossil fuels have gave e.g. electric vehicles (EV) and plug-in hybrid electric vehicles (PHEV) renewed attention. This focus is not only related to reduce fossil fuels, since they are considered as an asset in the future that potentially can provide auxiliary services in the power system. There are various of aspects proposed, while most of the aspects includes both charging and discharging of EV's, which contributes to avoid against load peaks in the network. This is also known as the vehicle-to-grid [1].

The number of EV's the past years have been increasing. As an example, the Transmission System Operator in Denmark (Energinet.dk) expects 400.000 electric vehicles in 2035. In 2013 this number was only around 1390 [2]. By this increasing amount of electric vehicles, the distribution - and low-voltage network will experience a higher loading level, since the energy consumed by the technologies comes entirely from the network. This may not be the whole truth in accordance to PHEV's, since other sources can be utilized as well. However, this report only deals with fully electric vehicles technologies, where all the energy is consumed from the network.

Assuming the EV's are unmanaged in the network through their charging periods, the increased amount of energy consumed will questioning the capacity in distribution - and low voltage networks, since the purpose at the time networks were built was different. Thus, the increasing energy consumed can end up being relatively high, which both leads to congestion problems, while critical low voltages in the networks might be experienced as well. This is mainly due to that EV's will react to the whole sale market, meaning that having low prices will lead to new load peaks by the EV's. These "low-price" periods will challenge the need of new capacity, since these peaks will consists of uncertainties.
To avoid against overloaded equipments, the most simple solution will be to rebuild the network with larger cables etc., and thereby increase the capacity. However, this might in the same time be the most expensive solution. In addition, it may also be the solution which ends up with the lowest utilization rate, since forecasts of EV's have uncertainties, as well as the future consumption pattern (including EV's) has. This questioning the future need of capacity in the networks, and thereby the future investments from utilities perspectives.

This report focuses on utilizing the already existing network, and thereby the already existing assets in operation, where potentially congestion and voltage problems, in reference to a high penetration of electric vehicles, are avoided. This is performed by directly controlling the charging of electric vehicles, where interactions between the Distribution System Operator (DSO) and the EV aggregators are introduced. These interactions are carried out by a price coordinator. This approach ensures against congestion and voltage problems to a certain level, and is also known as *Network-Constrained Transactive Control.*

2 | Transactive Control

The *Transactive Control* (TE) approach is also known as a market-based control approach, where the overall objective is to obtain equilibriums among generation, consumption and network constraints in the power system. This is done through exchanging of informations, where economic signalling is utilized to solve complex problems within power systems. Thus, TE represents a framework where interactions between different actors occurs through an economic signal. These interactions have one common goal: *optimize the utilization and allocation of resources*. Therefore, TE is a combination of both a control - and a economic mechanism, which guarantees a balance between demand and supply in the power system, where *value* is the important parameter.

Applications of TE in power systems are already in use by the market mechanism in Nord Pool spot (wholesale market), which deals with a trading - and clearing mechanism that ensures balance between demand and supply [3]. Several studies of TE as an application in power systems have been performed, which have resulted in several different aspects. This includes [3]:

1. Congestion and Voltage Management in Distribution Systems.
2. Frequency Control in Power Systems via Secondary Control.
3. Frequency Regulation via Tertiary Control.

Notice that secondary - and tertiary control refers to Load Frequency Control (LFC), which will not be further treated through this report.

Nonetheless, TE is still not well-known at the distribution level and in the retail market, while this report deals with how the concept of Transactive Control can integrate distributed energy resources (DER) in distribution systems with focus on:

- Minimization of charging costs for DER.
- Avoiding violation of network constraints, which includes transformer capacity and voltage level.
- Minimization of network losses.

In this context DER can represent electric vehicles, heat pumps, flexible loads, storage, distributed generation (photovoltaic cell etc.). However, through this report DER only refers to electric vehicles.
The mentioned objectives for the optimization through this report relates to a control approach, where congestion and voltage management are in focus. Furthermore, the network losses are included, since it receives increased attention from utilities due to the relation between losses and operational conditions in the power system.

2.1 The Transactive Control Framework

A graphically overview of Network-Constrained Transactive Control is represented in Figure 2.1.

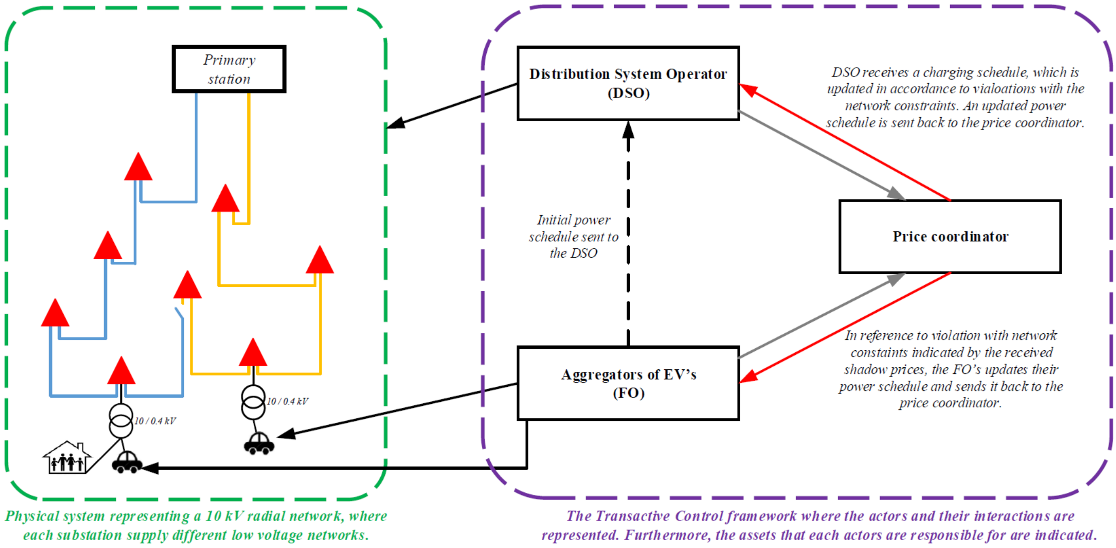

Figure 2.1: Overview of the Network-Constrained Transactive Control framework for a 10 kV radial in a distribution network. Two radials exists indicated by the different colours. The red triangles represents substations, where transformation from 10 kV to 0.4 kV occurs. Each substation is assumed to supply their own low voltage network.

The DSO have the responsibility of the network, while several aggregators can exists and be responsible for a certain number of EV's. Aggregators are also known as fleet operators (FO). It is assumed that each EV subscribe to one FO, which practically can be made through a contract etc. In the presented case in Figure 2.1, it is assumed that two aggregators are represented, named FO1 and FO2. FO1 is responsible for all EV's connected to the blue radial, while FO2 is responsible for all EV's connected to the yellow radial, assuming that the two radials represents two different areas.

When dealing with market-based coordination strategies of EV's with respect to congestion and voltage management systems, there are several ways it can be executed. Two common strategies are also known as distribution grid capacity market and dynamical grid tariff [4]. Through this report the *distribution grid capacity market strategy* is used, which includes the following control system:

1. Each aggregator (FO1 and FO2) in the network submits their charging schedule for each node in the network. Thus, for each node the aggregated capacity is informed for the whole scheduling period.

2. Through the price coordinator the DSO and aggregators interacts with each other. As a response to the submitted power schedule from the aggregators, they receives (based on calculations from the DSO) a price for each node. These prices reflects violation with the network constraints, if it exists in the network. If violations exists, the aggregators are requested to update their power charging schedules for the whole period. This updating process takes place until violations with the network constraints are avoided, where the process terminates. The market mechanism and

thereby the price, can be done in many ways. Through this report a shadow price based mechanism are chosen [5].

As already observed the three actors in this concept of Transactive Control have different purposes. Their individually primary focus is as follows:

- *Aggregator (FO)*
  Initially the aggregators performs the optimal power charging schedules for a given period. This generation of optimal power charging schedules focuses on minimizing the charging costs, and in the same time comply with the physical limits of the batteries. The initial optimal power charging schedules are sent to the DSO, wherein the further communication is through the price coordinator. Furthermore, the aggregators aims to update the schedules with respect to minimize the deviation from the optimal charging schedules in the continuous updating of the charging schedules.

- *Distribution System Operator (DSO)*
  The main objective from the DSO's point of view is to ensure safe and reliable operation of the network. This includes the absence of congestion and under-voltage problems through this report. This is ensured through the exchanging of bus informations to the price coordinator and aggregators. Furthermore, the DSO uses the initial optimal charging schedules as a reference in the optimization. This implies the final charging schedules to be as close as possible to the optimal charging schedules. The DSO does not have any incitements to have other references, since the DSO only needs to ensure reliable operational conditions in this report.

- *Price coordinator*
  The price coordinator represents a third party with a focus of manage the congestion prices, which occurs through interactions between the DSO and the aggregators. The responsibility of these interactions are located at the price coordinator.

Having obtained an equilibrium, where congestions and voltage violations are avoided, the aggregators enters the day-ahead market together with the power charging schedules. This analysis and optimizing process assumes that the prices from the day-ahead market are known. In other words; the forecast provided by the aggregators of the day-ahead prices is spot on. The same is true with the aggregators forecasts of the energy consumed by each EV. This will not be the case for each hour, while it might sometimes be the case that the aggregators enters the intra day market due to imbalance. In accordance to forecasts, their might be many different strategies, in which potentially can optimize the revenues for the aggregators. However, these concerns are related to risk management, which is not further considered through this report.

Additionally, it has to be noticed that potentially negotiations between the DSO and aggregators, due to violations with the network constraints, are only intended to occur at few nodes in the network. Negotiations are further not expected to occur in each period, while other incitements might arise to solve the capacity problem in the network. Hence, this framework should be viewed as a smart grid implementation to improve the utilization of the already existing assets in networks. Thereby, it can to some extent avoid problems such as congestion and voltage violations. This will lead to postponed investments from a utility company perspective, which is attractive due to the future uncertainties in the EV's consumption pattern, number of EV's, their locations etc. Nevertheless, it should be noticed that the possible future need of capacity in networks were not the basis, when the networks were build. Thus, capacity upgrading might also be a necessity at some point in the future.

2.2 Mathematical Modelling of Network-Constrained Transactive Control

The mathematically modelling of Network-Constrained Transactive Control can be divided into two parts:

- Generation of optimal power charging schedules for the EV's.
- The optimization between the aggregators and DSO through the price coordinator. This is also known as the main Transactive Control modelling part.

The two parts will be analyzed individually, and in the same sequence as stated above.

2.2.1 Generation of Optimal Power Charging Schedules

The basis for the optimization process is the initial generation of the optimal power charging schedules provided by the aggregators. As already mentioned, these optimal charging schedules needs to minimize the charging costs with respect to the electricity spot prices. In addition, it may also ensure sufficient energy for the required trips in the EV's, however this part is not further considered through this report. By assuming knowledge about the spot prices and the driving requirements of the EV's, the charging optimization of EV's can be formulated by linear programming [6]. Nonetheless, it should be noticed that the linear programming does not fully describe the charging process of EV's. This is mainly due to the physical charging behaviour of batteries, while uncertainties of the driving requirements also plays a role.

The objective function for the charging process can be described as given in equation (2.1).

$$
\begin{aligned}
&minimize \quad \sum_{i=1}^{N_k^{EV}} \sum_{j=1}^{N_T} \Phi_{i,j} \cdot P_{i,j} \cdot t \\
&subjected\ to: \\
&SOC_{0,i} \cdot E_{capacity,i} + \sum_{j=1}^{N^T} P_{i,j} \cdot t_{i,j} = SOC_{max,i} \cdot E_{capacity,i} \\
&0 \leq P_{i,j} \leq P_{max,i}
\end{aligned}
\tag{2.1}
$$

Through the mathematically description of the optimization problem, the indices i and j will remain the same. The index of time slots in a given scheduling period will be given by j ($j = 1, 2, ..., N_T$). Additionally, the number of EV's for each aggregator (k) are indexed by i ($i = 1, 2, ..., N_k^{EV}$). Thus, N_T and N_k^{EV} represents the total number of time slots in a given scheduling period and the total number of EV's for each aggregator, respectively.

The day-ahead electricity market prices are denoted $\Phi_{i,j}$, and given by a vector. This price should be viewed as a predicted price given by a forecast, however, it is assumed to be known through this analysis. $P_{i,j}$ is denoted as the decision variable given by a vector, where the physical comprehension is to make a decision whether a given time slot should be utilized for charging or not. This variable represents the i^{th} EV's charging power at time slot j, while t represents the length of each time slot.

The first constraint given in equation (2.1) represents the state of charging (SOC) for the batteries. This constraint ensures that the requested energy at the end of each charging period equals the sum of the energy to be charged. The initial state of charge level for each individual EV is given by $SOC_{0,i}$, while the maximum state of charge level for each individual EV is given by $SOC_{max,i}$ (= 1). Additionally, the second constraint given in equation (2.1) represents limiting of the charging rate for each EV charger. It limits the charging rate to be equal to the maximum power charging rate ($P_{max,i}$) or less.

By applying linear programming to the objective function and the subjected constraints given in equation (2.1), it is possible to obtain the optimal charging schedules from the aggregators point of view, which focus on minimizing the charging costs. In addition, for the interactions between the aggregators and DSO, the locations of the aggregated charging schedules are required. Thus, aggregated charging schedules for each bus in the network should be provided by the aggregators. Thereby, the decision variable introduced in equation (2.1) will be denoted as $P_{i,j,b}$, where b represents the bus index, and N_B represents the total number of busses in the network ($b = 1, 2, ..., N_B$).
Hence, through the report the aggregated power charging schedule at bus b and in time slot j is calculated for each aggregator. The total power requirement in the network by each aggregator at bus b and in time slot j is given by equation (2.2).

$$P^E_{k,j,b} = \sum_{i \to b} P_{i,j,b} \tag{2.2}$$

Where k denotes the index for the number of aggregators, while let N_{FO} represents the total number of aggregators ($k = 1, 2, ..., N_{FO}$). This representation allows the DSO to investigate whether congestion and voltage issues exists due to the different power charging schedules for each aggregator.

2.2.2 Transactive Control Modelling

Generally, the overall goal is to find a feasible solution for the power charging schedules, in which avoid network constraint violations. If network constraint violations occurs through the optimization, it is reflected in the price. However, if network constraint violations not occurs, the price will equals zero. The already explained objectives for the DSO and aggregators needs to be mathematically formulated. This is done individually for a clearer view.

The Aggregators objective

It is necessary to establish a cost function for the aggregators, which represents the deviation in power charging at bus b in time slot j. An increased difference will indicate that there is a large difference between the requested charging level from the aggregators, and the allowed capability in the part of the network, where constraint violations occurs. The cost function is given by equation (2.3).

$$\begin{aligned} &minimize \quad \mu_k = C_{k,j,b} \cdot \left(\tilde{P}_{k,j,b} - P^E_{k,j,b} \right)^2 \\ &subjected \;\; to: \\ &\sum_{j=1}^{N_T} \tilde{P}_{k,j,b} \cdot t_i = \sum_{i \to b} (SOC_{Max,i} - SOC_{0,i}) \cdot E_{capacity,i} \end{aligned} \tag{2.3}$$

Where $C_{k,j,b}$ represents a weighting factor, which impacts the optimization process when differences occurs. Thus, by increasing the weighting factor smaller differences will not be preferred. Through this report the weighting factors are different from each aggregator, while the factor does not change from each time slot j. Furthermore, $\tilde{P}_{k,j,b}$ represents a control variable, and is the allowable level of charging power in time slot j at bus b through the optimization process. This allowable level of charging power is desired by the DSO. Additionally, by the constraint it can be observed that the power charging level in each time slot j at each bus b should fulfill the requirements for the individual EV.

The DSO objective

Overall, the DSO is interested in the most reliable operational conditions in the network. Through this report these operational conditions are related to avoid congestion problems, where the thermal capacity of the distribution transformer is of interest. Furthermore, voltage violations are also in focus. As an add-on in this optimization process, network losses are also considered as a part of the objective function, which is given in equation (2.4).

$$\begin{aligned}
&minimize \quad a \cdot \sum_{j=1}^{N_T} \sum_{b=1}^{N_B} \left(P_{DSO}(j,b) - \sum_{k=1}^{n_{FO}} P_{k,j,b}^{E} \right)^2 + b \cdot P_{loss} \\
&subjected \;\; to: \\
&\sum_{b=1}^{N_B} P_{DSO}(j,b) \leq P_{transformer,j}^{Max} \\
&U_{0,j,b} - \Delta U_{j,b} \geq U_{min,j,b}
\end{aligned} \tag{2.4}$$

Please note that $P_{k,j,b}^{E}$ is included as the reference for the DSO. This is in line with the fact that the DSO tries to allow as much charging power as possible with respect to the optimal charging schedules received from the aggregators in the start of the process.

The objective function represented in equation (2.4) contains two constants denoted a and b, which are arbitrary weighting factors. The control variable, $P_{DSO}(j,b)$, expresses the allowable power at bus b in time slot j - also known as the allowable power desired by the DSO. Since this report only consider a single substation with one distribution transformer, one constraint exists that ensures the aggregated allowable power by the DSO is equal to or less than the maximum power capacity of the transformer at time slot j ($P_{transformer,j}^{Max}$). This transformer capacity is with reference to the aggregators, which is deduced by the DSO. As an example, it might be the case that the DSO wants the aggregated power from all EV's at each time slot j to maximum load 50 % of the transformer capacity (with respect to the the rated power of the transformer). Hence, $P_{transformer,j}^{Max}$ is set to 50 % of the transformers capacity. This ensures that the DSO knows how much the aggregated power from all EV's loads the transformer besides the conventional loads. Furthermore, n_{FO} is introduced, and represents the number of aggregators that have EV's connected to bus b.

Network losses
An approximation of network losses in the low voltage network is carried out. Normally, the reactive power (Q) is usually small when the voltage is close to nominal (1 pu), while it is a good approximation to neglect it. This also implies that the loss can be approximated by [7]:

$$P_{loss} = I^2 \cdot R = \frac{P_{line}^2 + Q_{line}^2}{U^2} \cdot R = \frac{P_{line}^2 + 0^2}{1^2} \cdot R = P_{line}^2 \cdot R \tag{2.5}$$

In order to convert the network losses into the concerned problem, it has to consider the losses at each time slot j. In addition, the losses are referred to the bus that causes the losses (due to the connected load). This can be mathematically described as follows:

$$P_{loss} = \sum_{j=1}^{N_T} \sum_{b=1}^{N_B} P_{line,j,b}^2 \cdot R_b \tag{2.6}$$

Where the $P_{line,j,b}$ is calculated as indicated in equation (2.7) [7].

$$P_{line,j,b} = (A \cdot A^T)^{-1} \cdot A \cdot (P_{0,j,b} + P_{DSO}(j,b)) \tag{2.7}$$

Where A indicates the incidence matrix, which represents the connectivity in the low voltage network. This connectivity is represented by 1 or -1, where 1 indicates a connection between two busses with the right flow direction, whereas -1 indicates the opposite. Finally, $P_{0,j,b}$ represents the existing load at bus b, which relates to the conventional loads from households etc. Please note that the control variable is included and added to the conventional load. By this implementation the charging schedules (and thereby the new loads in the network) are considered in the loss calculations as well.

Voltage limitation
The given voltage constraint in equation (2.4) implies that the voltage is above the minimum allowable voltage denoted by $U_{min,j,b}$, while $U_{0,j,b}$ represents the initial voltage level. The voltage difference from the initial case and the voltage in each time slot, where potential power charging can occur, is given by $\Delta U_{j,b}$. Due to the new loads introduced by the EV's, $\Delta U_{j,b}$ should be viewed as a "new" voltage drop in the network, while $\Delta U_{j,b}$ it is subtracted from $U_{0,j,b}$. The voltage difference can be approximated by considering the Jacobian matrix (J), which is well-known from load flow calculations using the method of Newton-Raphson etc.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = J \cdot \begin{bmatrix} \Delta \Theta \\ \Delta U \end{bmatrix} \Rightarrow \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \dfrac{\partial \Delta P}{\partial \Theta} & \dfrac{\partial \Delta P}{\partial U} \\ \dfrac{\partial \Delta Q}{\partial \Theta} & \dfrac{\partial \Delta Q}{\partial U} \end{bmatrix} \cdot \begin{bmatrix} \Delta \Theta \\ \Delta U \end{bmatrix} \tag{2.8}$$

By the already stated assumption about neglecting the reactive power (Q = 0), ΔU is calculated as follows:

$$\begin{bmatrix} \Delta \Theta \\ \Delta U \end{bmatrix} = J^{-1} \cdot \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \Rightarrow \begin{bmatrix} \Delta \Theta \\ \Delta U \end{bmatrix} = J^{-1} \cdot \begin{bmatrix} \Delta P \\ 0 \end{bmatrix} \Rightarrow \Delta U = J^{-1} \cdot \Delta P \tag{2.9}$$

For this analysis the angle between two busses given by Θ is not further considered. Hence, in order to calculate the difference in voltage, only a sub matrix of the full Jacobian matrix is needed. Thus, with respect to the Transactive Control concept, the difference in voltage is calculated as follows.

$$\Delta U_{j,b} = J_{21}^{-1} \cdot P_{DSO}(j,b) \tag{2.10}$$

Please notice that $P_{DSO}(j,b)$ still remains the total allowable power desired by the DSO - also known as the control variable through the optimization process.

The overall objective function

To obtain the overall objective function, in which both encounter the aggragators objective and the DSO's objective, it can from a social welfare maximization point of view be formulated as given in equation (2.11).

$$
\begin{aligned}
&minimize \quad \sum_{k=1}^{N_F}\sum_{j=1}^{N_T}\sum_{b=1}^{N_B} C_{k,j,b} \cdot \left(\tilde{P}_{k,j,b} - P^E_{k,j,b}\right)^2 \\
&+ a \cdot \sum_{j=1}^{N_T}\sum_{b=1}^{N_B} \left(P_{DSO}(j,b) - \sum_{k=1}^{n_{FO}} P^E_{k,j,b}\right)^2 + b \cdot P_{loss} \\
&subjected \ \ to: \\
&\sum_{k=1}^{n_{FO}} \tilde{P}_{k,j,b} \leq P_{DSO}(j,b) \\
&\sum_{b=1}^{N_B} P_{DSO}(j,b) \leq P^{Max}_{transformer,j} \\
&U_{0,j,b} - \Delta U_{j,b} \geq U_{min,j,b} \\
&\sum_{j=1}^{N_T} \tilde{P}_{k,j,b} \cdot t_i = \sum_{i \to b} (SOC_{Max,i} - SOC_{0,i}) \cdot E_{capacity,i}
\end{aligned}
\tag{2.11}
$$

As it is observed in equation (2.11), the same constraints exists as stated in both the objective function for the aggregators and the DSO. However, the first constraint in equation (2.11) is new, which guarantees that the aggregated power from all power charging schedules not exceeds the allowable power desired by the DSO.

The Lagrangian Function

The objective can be reformulated by utilization of the Lagrange method, where the Lagrange multiplier is represented for each time slot j and each bus b, $\lambda_{j,b}$. This is in line with the framework, where the aggregators will receive a price for each bus, which will indicate violations of the network constraints. These prices are the shadow prices and analogous to the Lagrange multiplier.

Let the Lagrange multiplier corresponds to the first constraint given in equation (2.11), thereby the Lagrange function can be formulated as follows.

$$
\begin{aligned}
L(\tilde{P}_{k,j,b}, \tilde{P}_{j,b}, \lambda_{j,b}) = &\sum_{k=1}^{N_F}\sum_{j=1}^{N_T}\sum_{b=1}^{N_B} C_{k,j,b} \cdot \left(\tilde{P}_{k,j,b} - P^E_{k,j,b}\right)^2 \\
&+ a \cdot \sum_{j=1}^{N_T}\sum_{b=1}^{N_B} \left(P_{DSO}(j,b) - \sum_{k=1}^{n_{FO}} P^E_{k,j,b}\right)^2 + b \cdot P_{loss} \\
&+ \sum_{j=1}^{N_T}\sum_{b=1}^{N_B} \lambda_{j,b} \cdot \left(\sum_{k=1}^{n_{FO}} \tilde{P}_{k,j,b} - P_{DSO}(j,b)\right)
\end{aligned}
\tag{2.12}
$$

In order to understand the physical meaning of the Lagrange multiplier in this case, the represented function given in equation (2.12) should be considered. It is well-known that the optimum minimum point for a Lagrange function occurs, when the partial derivative of one of the independent variables equals zero. By differentiate with e.g. $P_{DSO}(j,b)$, it can from equation (2.12) be observed that the aggregators objective are cancelled out, while the last term equals $-\lambda_{j,b}$. Thus, the Lagrange multiplier will be expressed in terms of the difference between the allowable power desired by the DSO and the optimal power charging

schedules (generated from the aggregators in the start of the process). This indicates that $\lambda_{j,b}$ represents how large the difference is between the allowable power and the optimal power charging schedules at time slot j. The larger $\lambda_{j,b}$ is (positive), the greater is the difference between the allowable power and the optimal power charging schedules. Hence, larger values of $\lambda_{j,b}$ indicates an inferior business case from the aggregators point of view, since the aggregators are forced to make larger changes in the charging schedules, which causes the charging costs to increase. Therefore, if power charging at a certain bus leads to violations of the network constraints, the Lagrange multiplier for the specific bus will increase. In addition, if there are any problems with the network constraints in a certain time slot j, the Lagrange multiplier will equals zero.
It is worth noticing that the increment in $\lambda_{j,b}$, due to violation with the constraints, acts as a *"congestion"* price for the aggregators, since they are forced to change their optimal charging schedules, and thereby not obtain the minimum charging cost. Whether it is the aggregators that have to pay the whole congestion price is not further considered.

The Transactive Control Concept Expressed by the Subgradient Method
Based on the constructed Lagrange function given in equation (2.12), the optimization problem can be solved by considering dual decomposition using the subgradient method. The subgradient method can advantageously be used for minimizing non differentiable convex functions [8]. However, the subgradient will not further be documented in this report.

The given Lagrange function in equation (2.12) is separable, where the dual function consists of the aggregator and the DSO [9].

$$f(\lambda_{j,b}) = f_{FO}(\lambda_{j,b}) + f_{DSO}(\lambda_{j,b}) \tag{2.13}$$

Thereby, the objective function for the aggregators and DSO are optimized individually by the subgradient method, which is an iterative method indexed by m $(= \lambda_{m,j,b})$. The method initialize the dual variable (the Lagrange multiplier in this case) arbitrary, however, it should comply with $\lambda_{0,j,b} \geq 0$. By utilizing the subgradient method, the dual decomposition algorithm is as follows:

1. **The aggregators:** *Find $\tilde{P}_{k,j,b}$ and $\lambda_{j,b}$ that minimizes $f_{FO}(\lambda_{j,b})$.*

$$\begin{aligned} & f_{FO}(\lambda_{j,b}) = \sum_{k=1}^{N_F}\sum_{j=1}^{N_T}\sum_{b=1}^{N_B} C_{k,j,b} \cdot \left(\tilde{P}_{k,j,b} - P^E_{k,j,b}\right)^2 + \sum_{j=1}^{N_T}\sum_{b=1}^{N_B} \lambda_{j,b} \cdot \left(\sum_{k=1}^{n_{FO}} \tilde{P}_{k,j,b}\right) \\ & subjected\ \ to: \\ & \sum_{j=1}^{N_T} \tilde{P}_{k,j,b} \cdot t_i = \sum_{i \to b} (SOC_{Max,i} - SOC_{0,i}) \cdot E_{capacity,i} \end{aligned} \tag{2.14}$$

2. **The DSO:** *Find $P_{DSO}(j,b)$ and $\lambda_{j,b}$ that minimizes $f_{DSO}(\lambda_{j,b})$.*

$$\begin{aligned} & f_{DSO}(\lambda_{j,b}) = \\ & a \cdot \sum_{j=1}^{N_T}\sum_{b=1}^{N_B} \left(P_{DSO}(j,b) - \sum_{k=1}^{n_{FO}} P^E_{k,j,b}\right)^2 + b \cdot P_{loss} - \sum_{j=1}^{N_T}\sum_{b=1}^{N_B} \lambda_{j,b} \cdot P_{DSO}(j,b) \\ & subjected\ \ to: \\ & \sum_{b=1}^{N_B} P_{DSO}(j,b) \leq P^{Max}_{transformer,j} \\ & U_{0,j,b} - \Delta U_{j,b} \geq U_{min,j,b} \end{aligned} \tag{2.15}$$

3. **The price coordinator:** *Update the dual variable* $\lambda_{m,j,b}$

$$\lambda_{m+1,j,b} = \lambda_{m,j,b} + \alpha_m \cdot \left(\sum_{k=1}^{n_{FO}} \tilde{P}^*_{k,j,b} - P_{DSO}(j,b)^* \right) \tag{2.16}$$

Through the updating process of the dual variable, $\tilde{P}^*_{k,j,b}$ and $P_{DSO}(j,b)^*$ indicates the solution to equation (2.14) and (2.15), respectively. Furthermore, α_m represents the step size, where a positive constant ($\alpha_m = \alpha$) as step size has been utilized through this report.

The above dual decomposition algorithm is applied until the absolute value of the difference between the last two dual variables are less than an arbitrary chosen threshold value.

$$|\lambda_{m,j,b} - \lambda_{m-1,j,b}| < \gamma \tag{2.17}$$

Where γ represents the arbitrary chosen threshold value. Through the report the threshold value is chosen in the range of 0.01 - 0.0001.

The above algorithm represents a convex optimization problem, which is solved by YALMIP - an optimization package in MATLAB. Further introduction and documentation regarding convex optimization problems will not be included in this report.

Flowchart of the Transactive Control Framework

Based on the introduction to the TE framework, as well as the mathematical modelling of the framework, a flowchart of the optimization process is found in Figure 2.2. As it is observed the flowchart can be divided into three parts:

- *Data collection*
  This part concerns about collection of the driving requirements for the individual EV, as well as the forecast of electricity spot prices for the given charging period. The sequence in the collection of these data is not needed to be as indicated in Figure 2.2. Please notice that driving requirements for each individual EV are not further considered through this analysis. Hence, all EV's are assumed to be fully charged after the Transactive Control. However, this might not be needed for each individual EV with respect to the driving requirements for each EV. Considering the driving requirements for each EV can potentially save charging costs, since fully charged EV's might not be needed. Furthermore, these considerations will also contributes to reduce the load peak created by the EV's, since some EV's might only require to be charged 50 % of full capacity, and thereby postpone some charging periods to another days / periods.

- *Generation of optimal charging schedule*
  Based on the data collection, each aggregator generates their own optimal power charging schedules, which are sent to the DSO (indicated in Figure 2.1). The DSO checks whether the optimal power charging schedules leads to congestion - or voltage problems. If all network constraints are met, the DSO accepts the charging schedules. If it is not the case, the main Transactive Control session starts.

- *Transactive Control*
  Due to congestion and/or voltage problems, the aggregators are forced to change their charging schedules with a focus on minimizing the deviation from the optimal charging schedules. This ensures the aggregators to minimize the increment in

charging costs. The DSO utilizes the new schedules to analyze whether congestion and/or voltage problems still exists. If violations still exists, the DSO generates an updated schedule containing the allowable power at bus b in time slot j. Finally, the price coordinator updates the prices. If the price has converged, the DSO accepts the charging schedules. Thus, converged prices indicates that all network constraints are met. Otherwise, the procedure restarts until converged prices are obtained.

The given algorithm of Network-Constrained Transactive Control in Figure 2.2 are implemented in MATLAB using YALMIP.

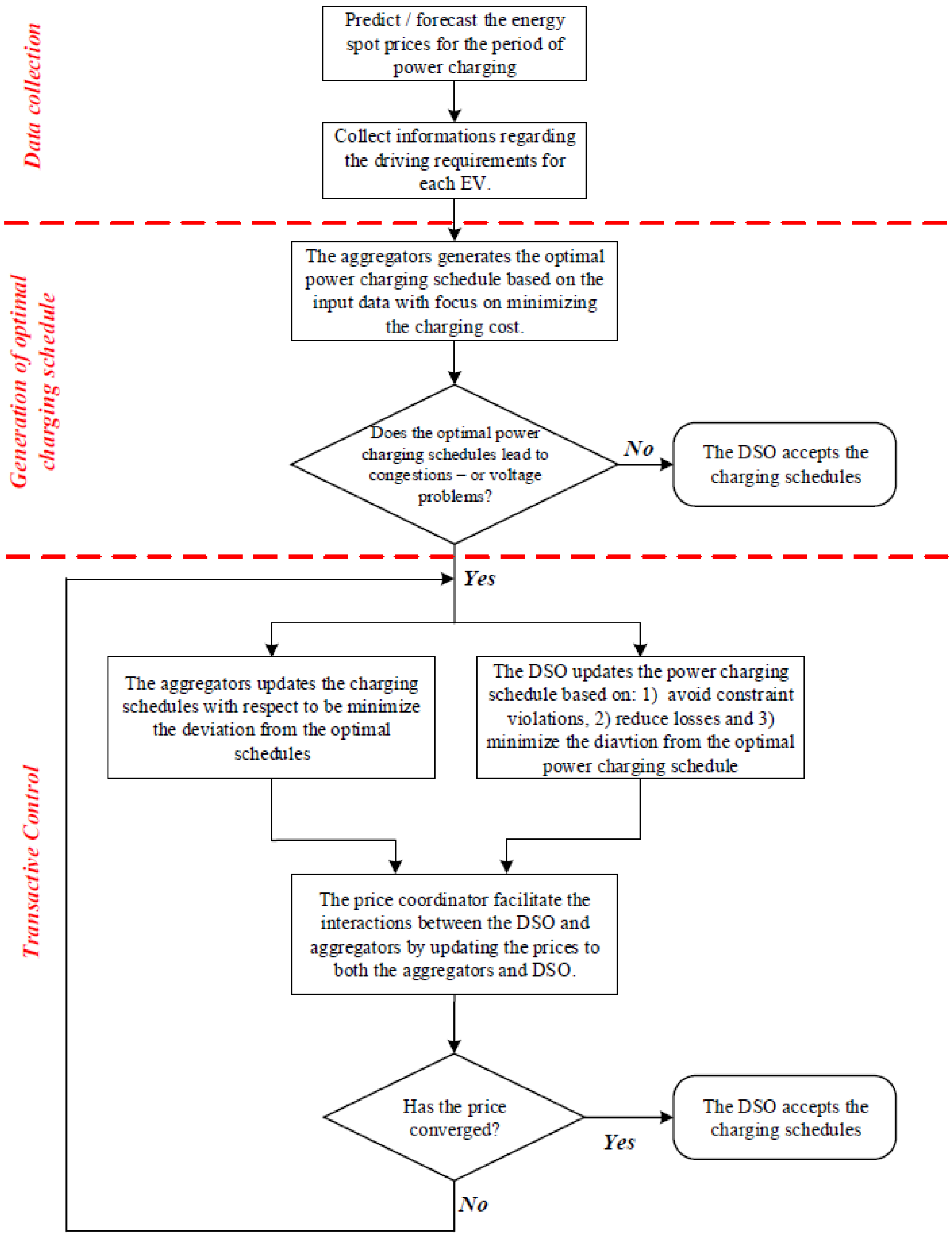

Figure 2.2: Flowchart of Network-Constrained Transactive Control.

3 | Transactive Control Using YALMIP

This part of the report introduces YALMIP, which is an optimization package added to MATLAB. This package allows users to advantageously solve convex - and nonconvex optimization problems, and can be used instead of the CVX-package (mathematically modelling of convex optimization) in MATLAB.
First, fundamentals about YALMIP will be introduced, which includes the choice of solvers and introduction to how objective functions, constraints etc. are handled by YALMIP. Secondly, the construction of the Transactive Control concept shown in Figure 2.2 by using YALMIP, will be explained.

3.1 Fundamentals About YALMIP

YALMIP is an advanced modelling language for convex - and nonconvex optimization problems, which can be implemented in MATLAB for free, where the same syntax as used in MATLAB is used in the YALMIP package. One of the benefits of using YALMIP is how fast the algorithms works; the YALMIP package has shown great performance through the analysis. YALMIP supports several problem classes, whereas linear-, quadratic- and second order cone programming are some of them which YALMIP supports.

Generally, YALMIP contains some internal solvers, which are used for global optimization etc. In addition, external solvers does also exists, which normally carries out the actual computational tasks, where the internal solves minor problems utilizing the external solvers [10].
For the concerned Transactive Control optimization problem, a solver for linear programming and convex optimization problems are needed. For linear programming, the solver called *linprog* is applied, while the solver called *SeDuMi* is applied for solving the convex optimization problem. *SeDuMi* is a widely used solver among YALMIP users, and is categorized as one of the second-order cone programming solvers. In this category the *SDPT3* solver also exists. The *SeDuMi* solver can advantageously be used for convex optimization problems, and has shown great performance through the analysis.

3.1.1 Construction of Optimization Problems in YALMIP

The most important parameter in the YALMIP package is the decision variable - also known as the parameter, in which are trying to be optimized. The decision variable is created by the command *sdpvar*, where a decision variable defined as a matrix $\mathbf{H}$ with m rows and n columns is defined as follows:

$$H = sdpvar(m, n) \tag{3.1}$$

The defined decision variable is used as a part of the objective function and in potential constraints subjected to the objective function. Having defined both the objective function and the constraints, it is possible to solve the problem by using the command *optimize*:

$$solution = optimize(Constraints, Objective, Options) \tag{3.2}$$

Where the *Constraints* exists as a vector with a certain number of equality and inequality constraints, while *Objective* denotes the objective function. *Options* is not required to be defined, while YALMIP will choose a solver that is already installed. Hence, by defining *Options*, it is possible to choose a specific solver for the optimization problem. Assuming the *SeDuMi* solver is chosen to solve the problem, *Options* can be defined as follows:

$$Options = sdpsettings('verbose', 0, 'solver', 'sedumi', 'allownonconvex', 0) \tag{3.3}$$

The above defined *Options* implies that all sub results through the process are hidden ($verbose = 0$), while the solver is chosen to be *SeDuMi*. Furthermore, it can be an advantage to include: $allownonconvex = 0$, which ensures that YALMIP not set up a nonconvex problem.

In order to continuously monitor the optimization process, and whether the solution have converged or not, informations from the *optimize* commando can advantageously be extracted by the following statement:

```
if solution.problem = 0 then
    The process has converged, and the next step in the concerned optimization
    process can be executed;
else
    solution.info;
    yalmiperror(solution.problem);
end
```

Algorithm 1: If-statement to find errors in the optimization process in YALMIP.

The given if-statement in Algorithm 1 will indicate if convergence problems through the optimization process have occurred, and thereby protect against erroneous results.

3.2 Transactive Control Expressed by YALMIP

Based on the shown flowchart in Figure 2.2, several functions have been created, which relates to the different tasks given in the flowchart. The main script modelled in MATLAB follows the process shown in Algorithm 2.
Overall, the main script carries out the Transactive Control using a while-loop, where the optimal power charging schedules are calculated before the while-loop starts. If none constraint violations are observed, the prices will equals zero after the price update. Hence, the while-loop is stopped by the if-statement after the first iteration. Furthermore, it can be observed that four MATLAB functions are created and utilized:

- FOaggregationwithbus.
- Networkcalculation.
- FOscheduleadjustment.
- DSOnewoperation.

They all have different tasks within the Transactive Control concept, where YALMIP has been used as the programming language inside the functions. All functions will individually be explained.

Data: Collecting data about the spot prices and the driving requirements for the EV users;

for $k = 1 : N_{FO}$ **do**

  Calculate the optimal power charging schedules for aggregator k ($= P^E_{k,j,b}$) using the *FOaggregationwithbus* MATLAB function;

end

Calculate the aggregated charging power in the entire network by $\sum_{k=1}^{n_{FO}} \tilde{P}_{k,j,b}$;

Create the decision variable $P_{control}$ using the command *sdpvar*;

Calculate initial network conditions and create the objective function for P_{loss} using the *Networkcalculation* MATLAB function;

while $m < \beta$ **do**

  for $k = 1 : N_{FO}$ **do**

    Update the power charging schedules for aggregator k ($= \tilde{P}_{k,j,b}$) using the *FOscheduleadjustment* MATLAB function;

  end

  The DSO calculates the network conditions, and updates the allowable power charging schedule ($= P_{DSO}(j, b)$) using the *DSOnewoperation* MATLAB function;

  The price coordinator updates the prices (the Lagrange multiplier) in accordance to equation (2.16);

  if $|\lambda_{m,j,b} - \lambda_{m-1,j,b}| < \gamma$ **then**

    Break;

  end

end

Use the solution of $P_{control}$ to calculate P_{loss};

Algorithm 2: Overview of the Transactive Control setup in MATLAB.

The while-loop stops after an arbitrary chosen threshold value, β. Through the analysis the threshold value has been set to 100.

By using YALMIP a decision variable, also known as the control variable, has been created, $P_{control}$. This control variable indicates the allowable power desired by the DSO ($= P_{DSO}(j, b)$), and utilized by all functions except the *FOaggregationwithbus* function. The control variable exists as a matrix containing 52 rows and 33 columns. The number of rows equals the number of time periods through the optimization, hence 52 time periods with an interval of 15 minutes are considered (indexed by j). Furthermore, the considered low voltage network contains 33 busses (including the slack bus), while the columns represents the number of busses (indexed by b). The network will be introduced in the next chapter.

FOaggregationwithbus

The aim of the function is to create optimal charging schedules for each aggregator, which is utilized through the optimization, since it is the DSO reference through the process. As already explained, the generation of optimal charging schedules are performed by linear programming, where YALMIP is set to use the solver *linprog*.

It is assumed that two aggregators exists represented by *FO1* and *FO2*, where each aggregator has 18 EV's to control. Hence, two functions called *FO1aggregationwithbus* and *FO2aggregationwithbus* have been created, where the only difference is where the EV's are located and connected in the network. The number of EV's for each aggregator at each bus can be observed in Table 3.1, while a schematic overview of the network and loads are shown in the next chapter.

Bus	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
FO1	1	0	0	1	0	0	0	1	2	0	1	1	0	2	0	0	0	0	1	0	0	1	0	1	0	0	0	0	1	2	1	2
FO2	1	0	2	0	0	0	2	1	1	0	2	0	0	2	0	0	1	0	0	1	0	0	1	1	2	0	1	0	0	0	0	0

Table 3.1: Overview of how many EV's connected at each bus for each aggregator.

The MATLAB code for the two functions using YALMIP can be further investigated in Appendix **??** and **??**, where it is observable that two new decision variables are created for the linear programming task. These variables are created in accordance to equation (2.1), and represents $P_{i,j}$. Please notice that the decision variable *FOpower* represents the output of the function, and thereby the optimal charging schedules. Additionally, it can be observed that the constraints are analogous to the given constraints in equation (2.1).

Networkcalculation

The purpose of the function is to calculate the initial conditions in the network by using MATPOWER, where only the conventional loads (P_0) are represented. From these calculations the Jacobian matrix and initial voltages are extracted. The Jacobian matrix is utilized to obtain ΔU, while the initial voltages are used in the constraints subjected to the objective function for the DSO.

Furthermore, the objective function which describes the network losses, P_{loss}, is created. It has been found out that the calculation of equation (2.7), which includes the sparse matrix A, is a heavy computational task. Since the objective function for P_{loss} is the same through the process, it is not necessary to include the calculations in the while-loop. It can advantageously be executed before the while-loop, and thereby save computational time. The *Networkcalculation* function is represented in Appendix **??**. Please notice that the decision variable $P_{control}$ is included, which is in line with equation (2.7).

FOscheduleadjustment

Due to violations with the network constraints, the aggregators are forced to change the schedules. This updating of charging schedules are performed by the *FOscheduleadjustment* function, and can be investigated in Appendix **??** and **??**. Since two aggregators exists in this analysis, two functions called *FO1scheduleadjustment* and *FO2scheduleadjustment* have been created. They update the charging schedules for FO1 and FO2, respectively. In these functions the Lagrange multiplier is an important parameter, since it indicates which busses that leads to violations of network constraints. Hence, the planned charging load at these busses has to be regulated by the aggregators.

The creation of the objective function, as well as the constraints, are analogous to equation (2.3). Please note that a new decision variable called P_{agg} has been created, which represents the new charging schedules after adjustments - also denoted as $\tilde{P}_{k,j,b}$ in equation (2.3). Furthermore, it can be observed that the solver *SeDuMi* has been utilized to solve the convex optimization problem.

DSOnewoperation

Based on the optimal charging schedules and updated prices, the DSO tries to optimize the charging schedule in a way that removes network constraint violations. Updating of the allowable power at each bus from the DSO is carried out by the *DSOnewoperation* function. This function creates the optimization problem given in equation (2.4), and uses the solver *SeDuMi* to solve the convex optimization problem. The structure of the function and how YALMIP is applied to the optimization problem, can be seen in Appendix **??**. The function allows to consider whether voltage limits and network losses should be included in the optimization. The differences will be further investigated in the next chapter.

4 | Case Studies of Transactive Control

This chapter introduces the considered low voltage network, which is the basis for the analysis. After the introduction of the low voltage network, the Transactive Control concept is analyzed by two cases:

- **Case 1:** In this case the DSO does not consider either voltage limits or network losses. Hence, the network losses is excluded from the objective function, while all constraints regarding voltage limits are excluded.
- **Case 2:** In this case the DSO both consider voltage limits, as well as network losses. Therefore, the constraints and objective functions examined through the report at this point are applied.

In addition, a cost analysis in the second case will performed with the focus of analyzing the congestion price for the aggregators. Thirdly, a perspective of the Lagrange multiplier will be performed, where a new rule to the Transactive Control concept in Algorithm 2 is introduced. Finally, a perspective of the discrimination between the aggregators is examined.

The charging schedules (containing 52 periods) will be based on the electricity prices shown in Figure 4.1.

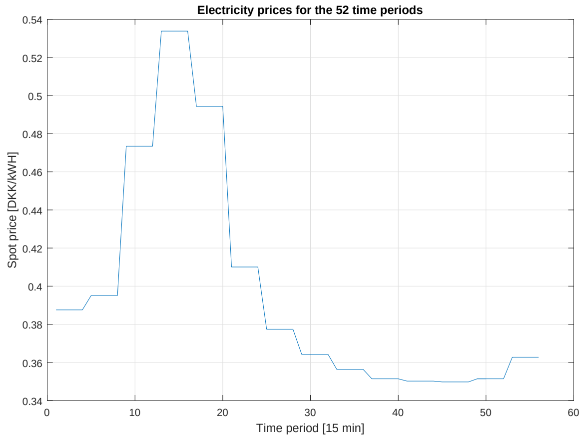

Figure 4.1: Electricity prices over 52 periods.

Over the last 20 periods the prices are lower compared to the first 20 periods (out of 52 periods), as it can be observed in Figure 4.1. This implies that the aggregators predominantly are planning to charge the EV's in the last 20 periods. Hence, congestion problems as well as voltage violations are expected in the low price periods. Therefore, it is expected that the aggregators tries to move their charging of power towards the first 30 periods, which implies the total charging costs to increase.

Technical Parameters
In order to compare the two cases, several technical parameters have been hold constant through the analysis. It is assumed that the DSO only allows the aggregated charging power to load the transformer with maximum 120 kW ($= P_{transformer}^{Max}$). Furthermore, the DSO only allows a minimum voltage of 0.90 pu, when the voltage limits are included in the constraints.
All the EV's connected to the network have a maximum power charging rate of 3.7 kW ($= P_{max}$), and a battery with a capacity of 24 kWh ($= E_{Capacity}$). The batteries for all the EV's are assumed to have an initial state of charge equal to 0.2 ($= SOC_0$). Please notice that the power charging rate is turned into kW15min by multiplying by 0.25 kWh in the *FOaggregationwithbus* function. This is necessary due to time slots with an interval of 15 min. Hence, the output from the functions are afterwards converted back to kW by dividing the output with 0.25 kWh.

Finally, the threshold value γ to ensure convergence in the optimization process is chosen to 0.001 through all analysis. Additionally, the step size in the updating process of the Lagrange multiplier is set to 0.1 ($= \alpha_m$).

4.1 The low voltage network

The low voltage network contains, as already explained, 33 busses including the slack bus (reference). The low voltage network is shown in Figure 4.2.

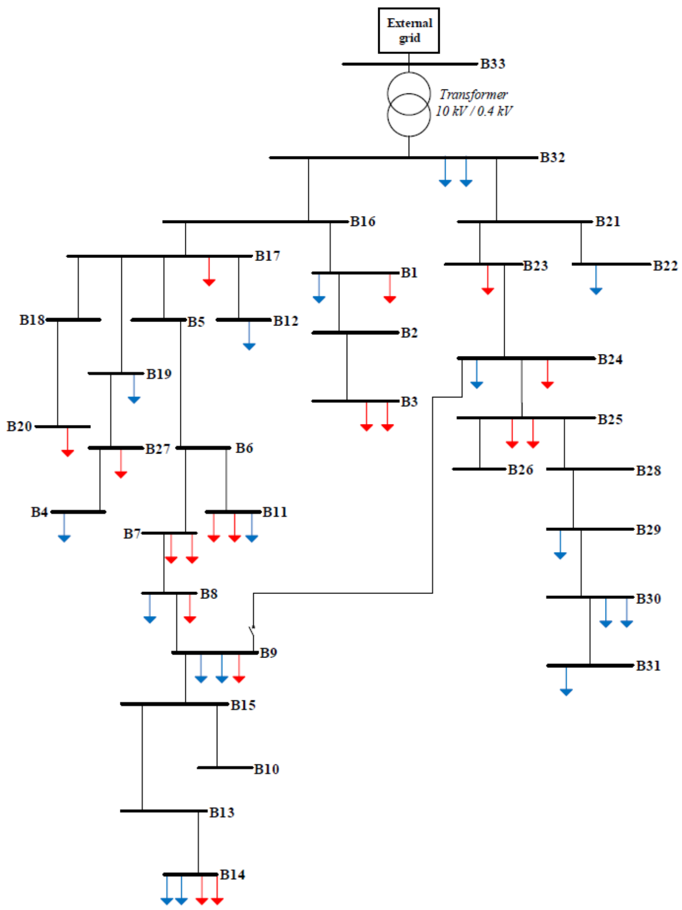

Figure 4.2: Overview of the low voltage network. Each load in the network represents one EV, where blue loads belongs to FO1, and red loads belongs to FO2.

The network loads indicated in Figure 4.2 represents only the EV's, while the conventional loads are not represented in the shown network. However, conventional loads exists at all busses (excluding bus 32 and 33). Please note that the line between bus 9 and 24 are out of service.

4.2 Case 1: Exclusion of Voltage Limits and Network Losses

The aim of this case is to investigate how the network conditions will be affected, when fundamental network constraints, such as voltage limits, are excluded. Hence, all voltage constraints subjected to the objective functions are excluded in the optimization process. Furthermore, the network loss term in the objective function for the DSO is excluded.
The development in the prices (the Lagrange multiplier) and the allowable power desired by the DSO, can be observed in Figure 4.3.

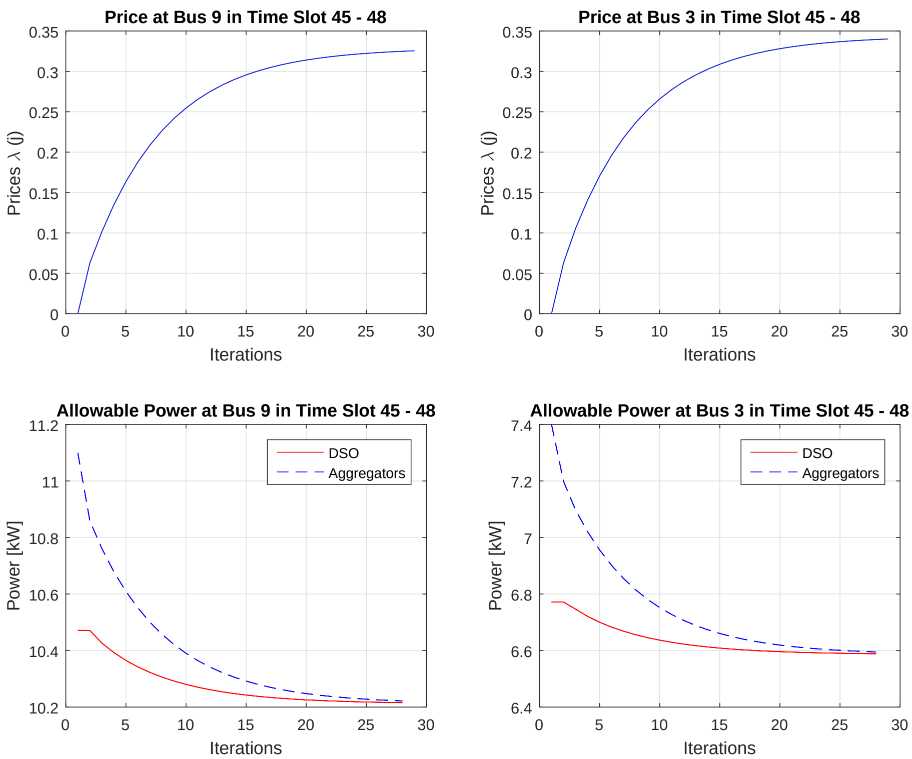

Figure 4.3: Development in prices and allowable power by the DSO at bus 3 and 9 in case 1. The dotted line indicates the aggregated power of FO1 and FO2.

As observed by Figure 4.3, bus 3 and 9 are considered. Those two busses will be the basis for both case 1 and 2. Furthermore, only time slot 45 to 48 are considered in both case 1 and 2. These time periods are particularly interesting due to the low prices in accordance to Figure 4.1.

By Figure 4.3 it can be observed that convergence is reached after 29 iterations in reference to the chosen threshold value γ. Positive prices are obtained at both bus 3 and 9, since the planned charging schedules exceeds the allowable power desired by the DSO (indicated by the two lower graphs). However, it can be observed that the aggregators and the DSO reach consensus about the allowable power, where 10.2 kW and 6.6 kW are allowed at bus 9 and 3, respectively. In accordance to the optimal charging schedules, it can be observed that the aggragators are forced to move around 1 kW at both bus 3 and 9, in order to comply with the requirements from the DSO. Hence, the 1 kW at both busses has been moved to order time periods, where higher prices exists, and thereby cause an increased charging cost for both aggregators - also known as the congestion price.

To further investigate how the charging schedules for both aggregators develops, Figure 4.4 should be considered.

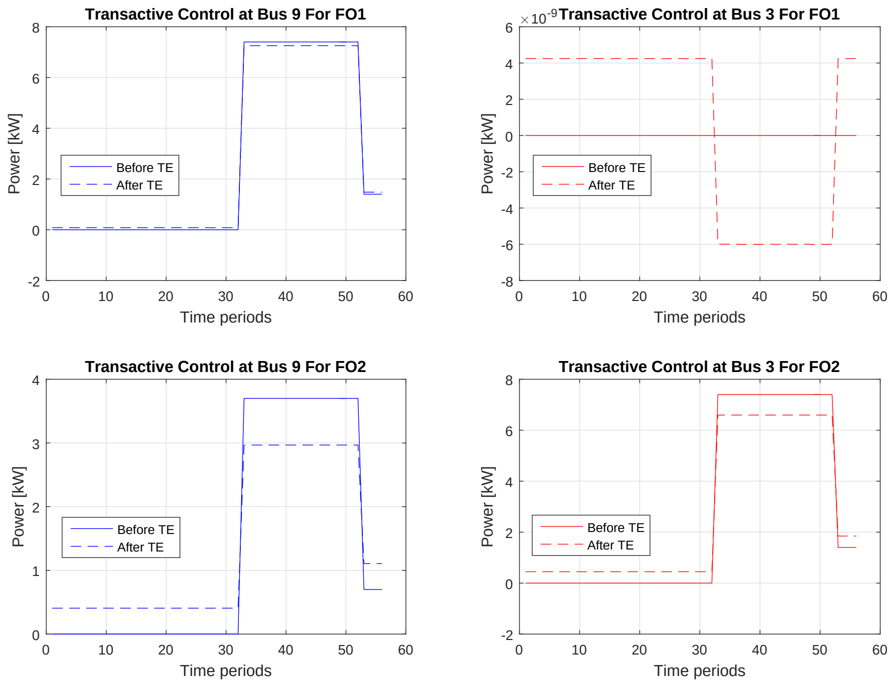

Figure 4.4: Development in the charging schedules for bus 3 and 9 for all 52 time periods in case 1.

Generally, the pattern in the charging schedules are not surprising, since it follows the low price periods in reference to Figure 4.1. As indicated by the low voltage network in Figure 4.2, FO1 does not control any EV's connected to bus 3, while the charging schedule given in Figure 4.4 for FO1 at bus 3 equals zero. Thus, it is only FO2 that is forced to reduce the charging power at bus 3. Since both FO1 and FO2 have EV's connected to bus 9, they are both forced to reduce the charging power. In this case it can be observed that FO2 ends up with the largest reduction in reference to the optimal schedule. However, it can also be observed that FO2 ends up by charging a small amount of power through the considered 52 time periods. By this charging pattern it requires that the EV is available through all 52 time periods. These considerations are not included through this analysis.

Finally, it can also be observed that around 20 periods of 15 mins are needed to fully charge the EV's in the optimal charging schedules, which equals to 5 hours.

Network Conditions

Even though the voltage limits and network losses are removed from the optimization, the constraint regarding the loading of the transformer by the aggregators still exists. The aggregated power from all connected EV's in the network before and after the optimization can be observed in Figure 4.5.

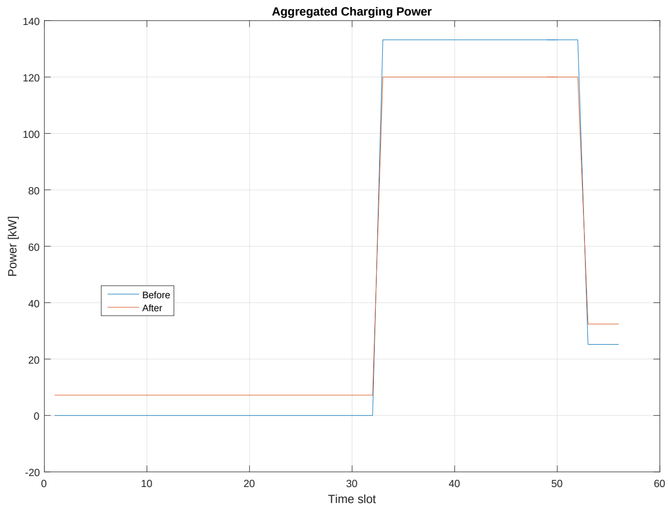

Figure 4.5: The aggregated power from all EV's before and after Transactive Control in case 1.

As it can be observed, the aggregated power in reference to the optimal charging schedules from FO1 and FO2 cause the transformer to be loaded by more than 120 kW as determined by the DSO. By applying Transactive Control it is observed that the aggregated power from the final charging schedules equals 120 kW, and thereby complies with the requirement from the DSO. The charging of EV's are moved to other time periods, which is observable in Figure 4.5, where a small amount of power is continuously charged besides the low price periods.

Additionally, the changed charging schedules will also affect the voltage conditions in the network to some extent. The development in the minimum voltage before and after the Transactive Control can be observed in Figure 4.6. As observed in Figure 4.6, the voltage after Transactive Control has increased. This is not surprising, since the Transactive Control forces the aggregators to change their schedules, and thereby distribute the charging of EV's over several periods instead of only in the low price periods. Hence, the loading level in low price time periods will decrease, which implies the voltage in the network to increase with respect to the case with optimal charging schedules (Before TE control). However, the improvements are not significant, since the voltage is increased by only 0.2 pu. This leads to a final minimum voltage of 0.863 pu, which not complies with DS/EN 50160. This shows the importance of considering voltage limits, since it can ensure a certain minimum voltage level, and thereby ensure the operational conditions. This contributes to that the DSO can postpone potentially investments regarding upgrading of e.g. the capacity in the network, in order to improve the voltage conditions.

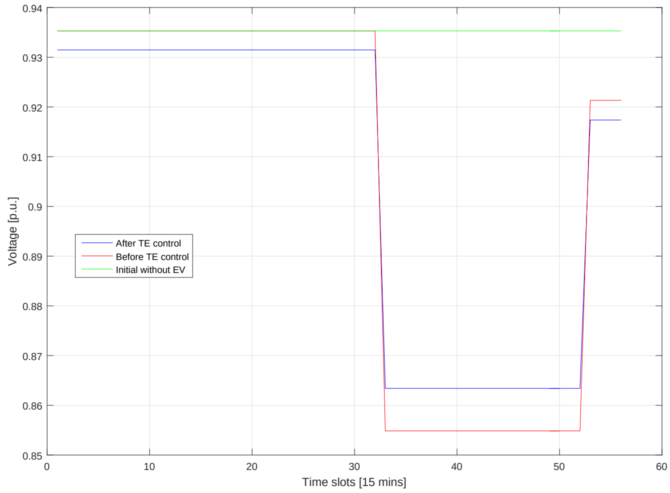

Figure 4.6: Development in the minimum voltage level in case 1.

4.3 Case 2: Including Voltage Limits and Network Losses

Compared to case 1 the purpose of case 2 is to investigate how the network conditions are improved by considering both voltage limits and network losses. Therefore, case 2 includes all constraints explained through the report, while the objective function applied in case 2 is analogous to equation (2.11).
For case 2 the development in prices and allowable power through the optimization process can be observed in Figure 4.7.

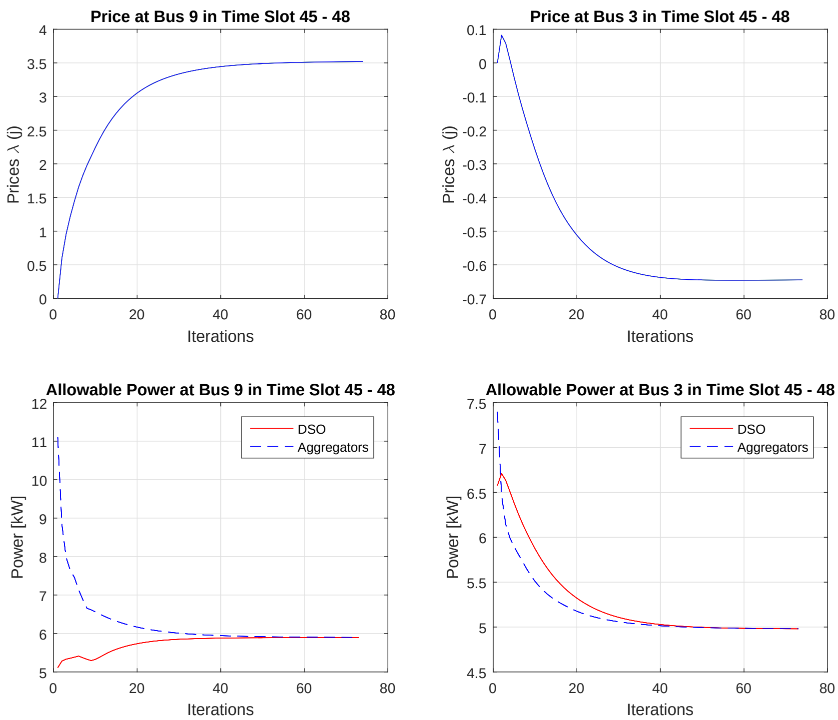

Figure 4.7: Development in prices and allowable power by the DSO at bus 3 and 9 in case 2. The dotted line indicates the aggregated power from FO1 and FO2.

By introducing the network loss term in the objective function, as well as the voltage constraints, convergence is now reached after 74 iterations. This should be compared to case 1, where only 29 iterations were needed. Furthermore, it can also be observed that positive prices exists at bus 9, while negative prices occurs at bus 3. At bus 9 the charging power by the aggregators exceeds the allowable power desired by the DSO, while positive prices occurs. This is the picture throughout the optimization process, while the aggregators and DSO reach consensus around 6 kW at bus 9. Furthermore, it can be observed that the price at bus 9 has increased compared to the final price at bus 9 in case 1. Since both the voltage constraints and network losses are included in the optimization, the allowable power desired by the DSO at bus 9 has decreased in case 2. Hence, the difference between the allowable power and charging power at bus 9 has increased in case 2, which is reflected by higher prices at bus 9 in case 2.

The same development is not observed at bus 3, where negative prices occurs. In the first iterations, it can be observed that the charging power exceeds the allowable power, while the price starts increasing ($\lambda_{j,3} > 0$). However, since FO2 changes the charging schedule to contain a lower requirement of power than allowed by the DSO at bus 3, the price becomes negative. Thus, the price ends up being negative, where the DSO and FO2 reach consensus at around 5 kW at bus 3. The development of negative prices are not surprising, when the updating of the Lagrange multiplier in equation (2.16) is considered. As it can be observed, the last term in equation (2.16) will become negative if the allowable power by the DSO is larger than the aggregators power charging schedules. Hence, if the allowable power desired by the DSO is larger than the power charging schedules through several iterations, the final price will likely become negative. Thereby, negative prices indicates to the aggregators that they consume less than allowed by the DSO at bus i.

The charging schedules after the optimization process has converged can be observed in Figure 4.8.

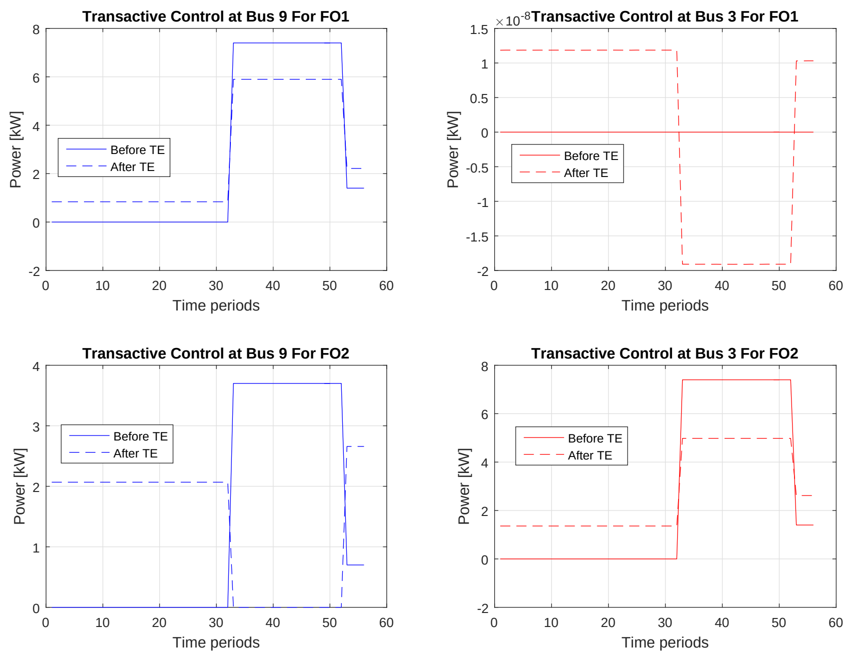

Figure 4.8: Development in the charging schedules for bus 3 and 9 for all 52 time periods in case 2.

Once again the charging schedule for FO1 at bus 3 equals zero, since FO1 not controls any EV's connected to bus 3. In addition, it can be observed that FO2 decreases the charging power at bus 3 even though the price ends up being negative. However, this shows that other time periods have been more suitable with respect to minimize the charging costs. Larger changes at bus 9 have also occurred, where FO2 once again is the one with the largest reduction in the charging schedule in reference to the optimal schedule. The charging peak observed in the charging schedule for FO2 at bus 9 has totally been removed compared to the final schedule for FO2 in case 1, while FO2 now charge a certain amount of power in each time period besides the low price periods. This is most likely due to the implementation of network losses and voltage constraints, since a charging peak from both FO1 and FO2 in the same time periods might cause too low voltages and too high losses. Hence, it is now only FO1 that causes a charging peak in the time periods 45 to 48. This discrimination between aggregators is further investigated later in the report.

Network Conditions

By the implementation of extra network constraints compared to case 1, the loading of the transformer due to charging of EV's, as well as the minimum voltages, is of interest. The aggregated power from all EV's is given by Figure 4.9.

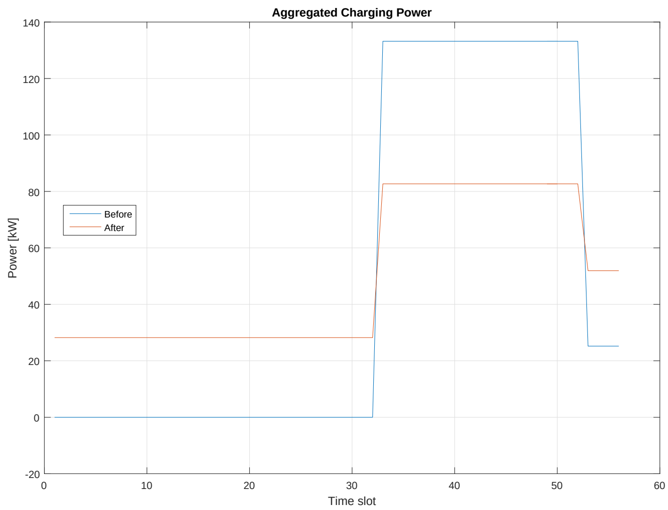

Figure 4.9: The aggregated power from all EV's before and after Transactive Control in case 2.

Compared to case 1, the aggregated power from all EV's is now reduced to below 120 kW. This is not surprising after voltage constraints and network losses are considering through the optimization. This will generally force the aggregators to distribute the charging of EV's even more compared to case 1, and thereby reduce the peak. This distribution of power charging can also be observed in Figure 4.9, where the aggregated power besides the low price periods has increased.

Additionally, the development in the minimum voltage can be observed in Figure 4.10.

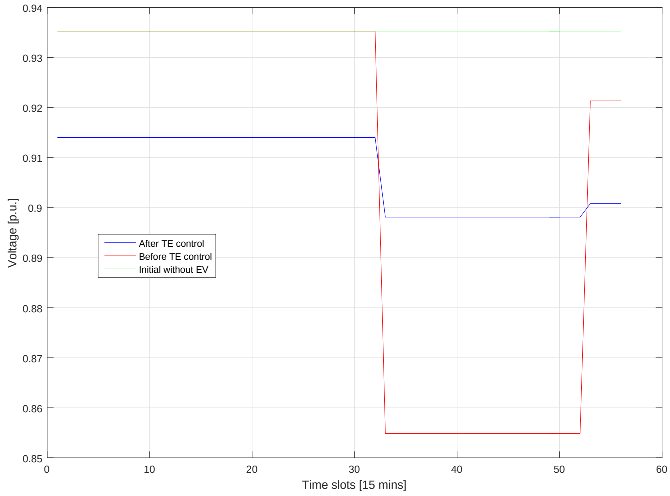

Figure 4.10: Development in the minimum voltage level in case 2.

As expected, the minimum voltage is close to 0.90 (= 0.8981) due to the settings in the constraints. This shows the opportunity the DSO possesses, since the voltage constraints can be utilized to ensure certain operational conditions. This helps the DSO to comply with the different network requirements (e.g. DS/EN 50160) even though penetration of EV's occurs in the future.

4.3.1 Comparison of case 1 and case 2

In order to compare the two cases, the network conditions have to be compared. The results from both cases can be observed in Table 4.1.

Case		***Network loss [MWh]***	***Energy [MWh]***	***Loss ratio [%]***	***Voltage [pu]***
Case 1	*Before TE*	0.1348	2.0699	6.51	0.8548
	After TE	0.1270	2.0611	6.16	0.8634
Case 2	*Before TE*	0.1348	2.0699	6.51	0.8548
	After TE	0.1106	2.0455	5.41	0.8981

Table 4.1: Network conditions in case 1 and case 2.

As indicated by Table 4.1, the losses ends up being higher when the objective function not includes losses (case 1). However, case 1 improves the network losses, energy through the transformer, loss ratio and the minimum voltage by applying Transactive Control. In this case the loss ratio is the ratio given by the network losses and the energy through the transformer. These improvements are mainly due to the fact that the charging of EV's have been distributed throughout several periods instead of only in the low price periods. Nevertheless, it can also be observed that further improvements are obtained for all parameters in Table 4.1, when voltage constraints and network losses are included in the optimization. Mainly the losses, and thereby the loss ratio, has decreased. This improvements implies better network conditions, which are reflected in the voltage improvements.

4.3.2 Cost Analysis of the Congestion Price

As already explained, the aggregators pays a congestion price, when the DSO not accepts the optimal charging schedules due to violations with the network constraints. Thus, the aggregators are forced to charge the EV's in periods with higher prices.
Based on case 2 the costs have been calculated for the aggregators at bus 3 and 9. The development in costs at bus 3 is shown in Figure 4.11

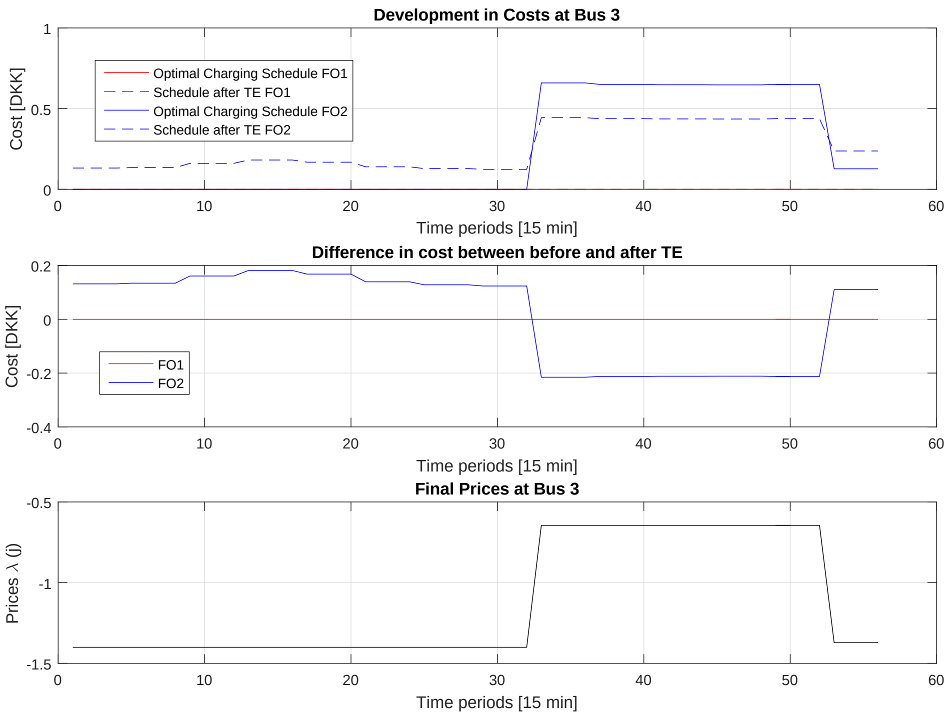

Figure 4.11: Development in costs for bus 3 in case 2.

As expected, the largest costs occurs in the low prices periods by using the optimal schedules. This will ultimately minimize the charging costs as much as possible. However, the final schedules reduces the costs in the low price periods, while new charging costs now occurs in all time periods. This is in line with the charging schedules shown in Figure 4.8, where it was observed that the final schedule from FO2 at bus 3 leads to a small amount of power to be charged throughout all time periods (besides the charging peak).

Additionally, it can be observed how the negative price at bus 3 increases towards being positive, when the charging peak from FO2 occurs in the time periods 32 to 52. This is not surprising since the difference between the allowable power desired by the DSO and the power charging schedule (from FO2 at bus 3) becomes smaller. Hence, the prices will move towards being positive in accordance to equation (2.16).

To compare the costs at bus 3, the development in costs at bus 9 can be observed in Figure 4.12.

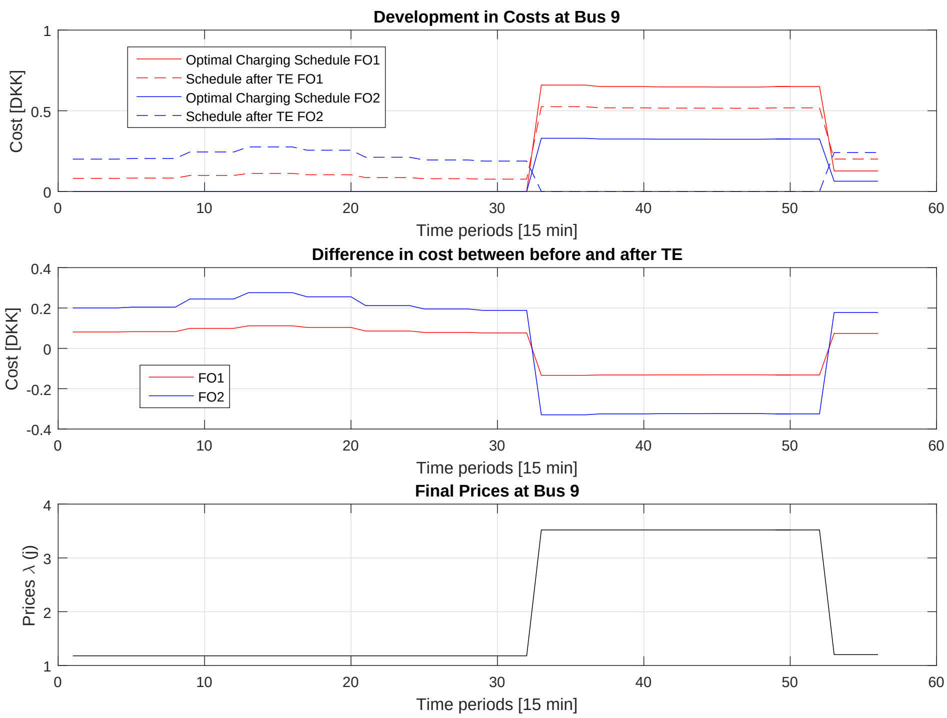

Figure 4.12: Development in costs for bus 9 in case 2.

From both FO1's and FO2's point of view, they experience a reduction in their costs in the low price periods compared to their optimal charging schedules. However, new costs occurs both for FO1 and FO2 at other time periods due to the changed schedules. As expected, the reduction of costs relates to the time periods with high prices, since these time periods have indicated that the charging schedules have caused violations of the network constraints.

By considering the final charging schedules for FO1 and FO2, the total congestion price for FO1 and FO2 is given in Table 4.2.

Aggregator	***Total costs before Transactive Control***	***Total costs after Transactive control***	***Total congestion price***
FO1	121.7217 DKK	125.9135 DKK	4.1918 DKK
FO2	121.7217 DKK	135.3535 DKK	13.6318 DKK

Table 4.2: Overview of total charging costs and congestion price.

As it can be observed, both FO1 and FO2 have to pay a congestion price due to violations of the network constraints. It can also be observed that FO2 has the highest congestion price to pay. This might depends among others on the location of where the EV's are connected in the network, which is further investigated in section 4.5.

4.4 Perspectives of the Lagrange Multiplier

One of the most essential parameters in the Transactive Control concept is the price, also known as $\lambda_{j,b}$ through the analysis. The interpretation of the price can be done many in ways, however it has been clear through the analysis that is expresses the difference between the allowable power and the power charging schedules to some extent. It has also been found out that negative prices in time slot j indicates that the power charging schedule is less compared to the allowable power desired by the DSO at time slot j. In other words; there is no problem regarding the network constraints.

In continuation of a deeper understanding of $\lambda_{j,b}$, a new rule has been implemented in the while-loop given in Algorithm 2. After having updated $\lambda_{j,b}$ it is checked whether negative prices occurs. If this is the case the negative prices are set to zero. A negative price and the price zero may have two different significances. Negative prices indicates that more power is allowed to be charged at bus b in time slot j, while the price zero indicates that the charging power at bus b in time slot j equals the allowed power. Hence, by changing negative prices to zero will indicate that the maximum level of charging power has already been reached. This may force the aggregators to change their schedules in another way, compared to cases where negative prices are allowed. Nonetheless, negative and zero prices have one thing in common: they both indicates that there is no problem regarding the network constraints. Thereby, negative prices may be treated as being zero.

Based on the same assumptions, constraints and objective functions as applied in case 2, the development in allowable power and prices at bus 3 and 9 can be observed in Figure 4.13.

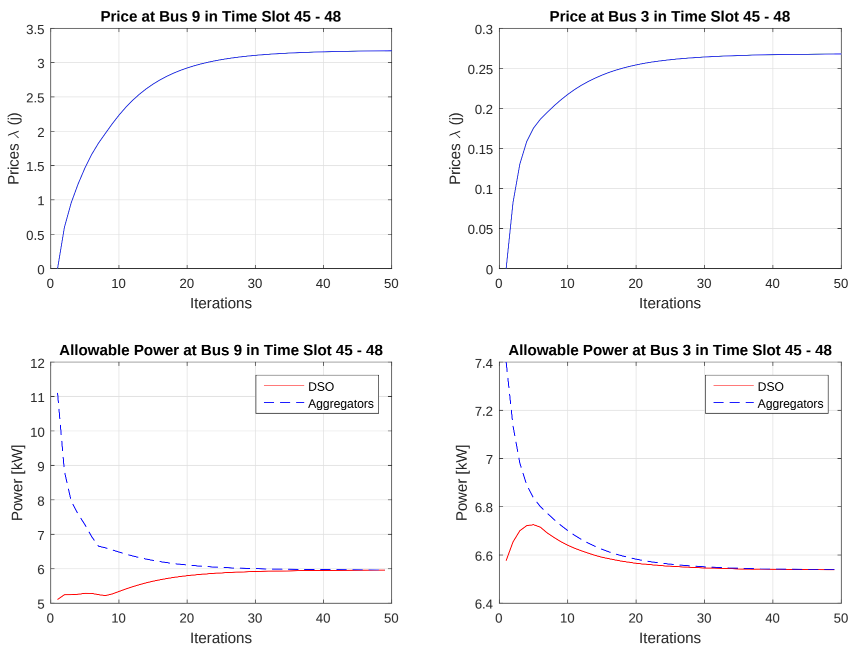

Figure 4.13: Development in allowable power and prices at bus 3 and 9 without having negative prices

Compared to the allowable power and prices in case 2 given in Figure 4.7, only small differences have occurred at bus 9. However, the negative prices at bus 3 has turned into being positive. Furthermore, the allowable power has change from around 5 kW to around

6.5 kW. This might be due to the fact that all negative prices now equals zero, hence the optimization process now tries to find the time slots, where the smallest violations of network constraints happens. This is found by distribute the charging of EV's even further compared to case 2, while the allowable power desired by the DSO has increased. These changes have resulted in the charging schedules at bus 3 and bus 9 as indicated in Figure 4.14.

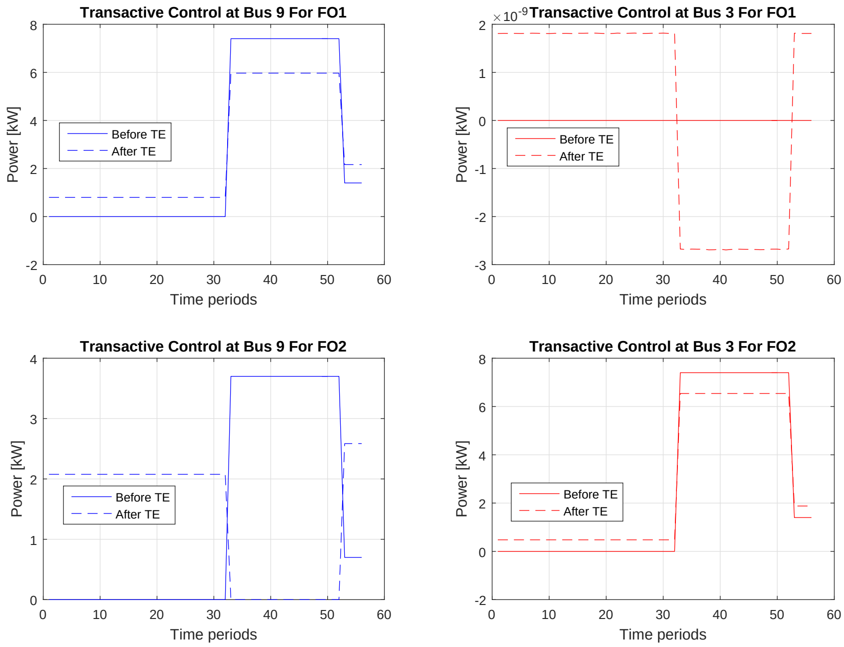

Figure 4.14: Development in the charging schedules for bus 3 and 9 for all 52 time periods without negative prices.

As indicated in Figure 4.14 the largest differences have occurred at bus 3, where the final schedule from FO2 has become closer to the optimal schedules. This will decrease the costs at bus 3. However, FO2 is still forced to charge the vehicle at bus 9 in the high price periods.

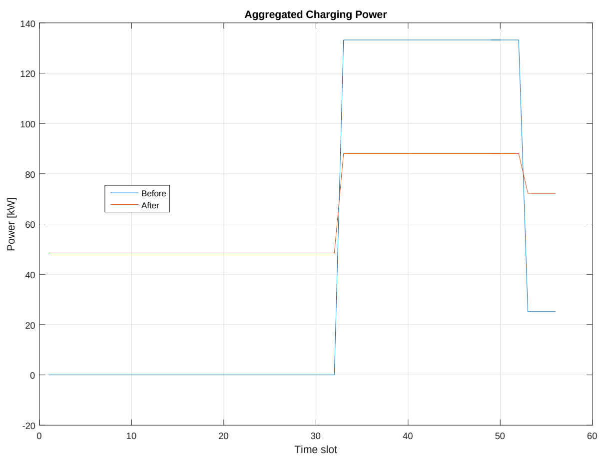

Figure 4.15: The aggregated power from all EV's before and after Transactive Control without negative prices.

In addition, the aggregated power of all EV's through all 52 time periods can be observed in Figure 4.15. The distribution of charging power compared to case 2 has clearly changed by observing Figure 4.15. The aggregated power from all EV's has gained an offset compared to the optimal charging schedules, which indicates that the charging of power has been distributed even further compared to case 2. Hence, by setting negative prices equal to zero leads the loading level in the network to increase, which has a negative effect on the network losses. An overview of the network conditions are shown in Table 4.3.

Case		*Network loss [MWh]*	*Energy [MWh]*	*Loss ratio [%]*	*Voltage [pu]*
Case without negative prices	*Before TE*	0.1348	2.0699	6.51	0.8548
	After TE	0.1191	2.2634	5.26	0.8980

Table 4.3: Network conditions by setting negative prices to zero.

As it can be observed by Table 4.3 the network losses have increased compared to case 2, while the voltage is close to the required minimum voltage (= 0.90 pu). Due to the increased losses it is not surprising that the energy through the transformer has increased as well.

As a perspective to the offset of the aggregated charging power indicated by Figure 4.15, the costs for bus 3 are shown in Figure 4.16.

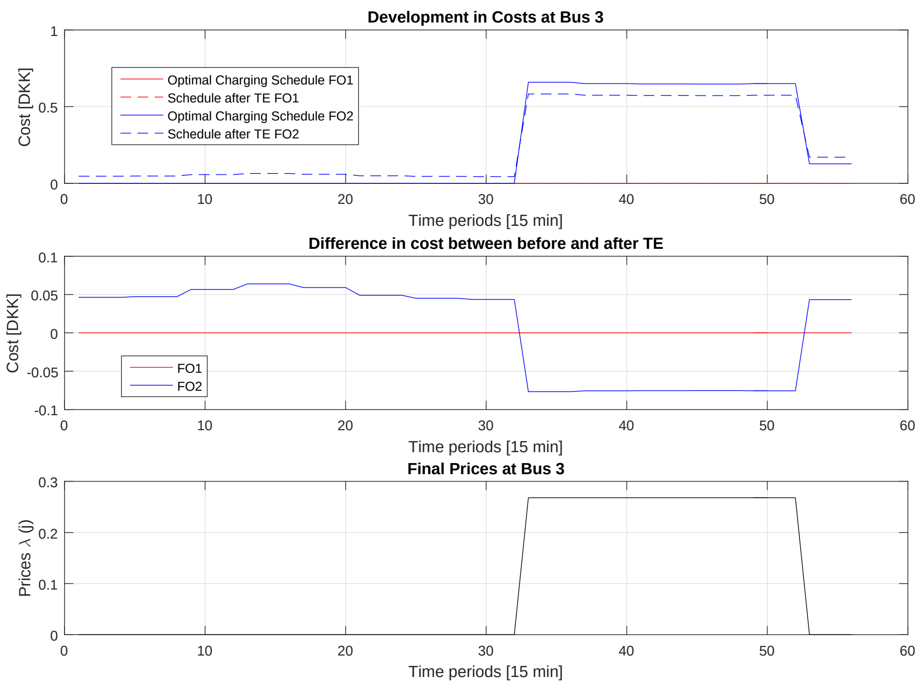

Figure 4.16: Development in cost for bus 3 without negative prices.

Compared to case 2 the costs for the final schedule in the low price period has increased, which is caused by the increased allowable power by the DSO. Hence, by changing negative prices to zero has moved the final charging schedule closer to the optimal schedule.

To compare whether the final schedules in general have been moved closer to the optimal schedules, the charging costs in Table 4.3 should be considered.

Aggregator	***Total costs before Transactive Control***	***Total costs after Transactive control***	***Total congestion price***
FO1	121.7217 DKK	125.0603 DKK	3.3386 DKK
FO2	121.7217 DKK	134.5310 DKK	12.8093 DKK

Table 4.4: Overview of total charging costs and congestion price without negative prices.

As it can be observed by Table 4.4, the aggregators ends up with a lower congestion price by setting negative prices equal to zero through the optimization compared to case 2. This indicates that several charging schedules have moved closer to the optimal schedule, and thereby minimized the increment in the costs.

Therefore, by forcing negative prices to zero through the optimization process seems to being an advantage from the aggregators point of view. The same is partly true for the DSO, since all network constraints are met. However, the loading level will in general increase, which may have negative impacts in the future, since capacity investments might approaching faster than expected for the DSO.

4.5 Perspective about Discrimination Between Aggregators

As it has been observed, the FO2 has been penalized more through the optimization process in terms of higher congestion prices. As already indicated, this is properly due to the location of where the EV's are connected in the network. However, the discrimination of the two aggregators will be examined by two cases to analyze the causes of the discrimination.

4.5.1 Impacts by the Same Location of EV's Between Aggregators

Generally, it is important that the optimization process not discriminates the aggregators, if they have the same number of EV's, and the EV's are connected at the same locations. To analyze whether this is the case, the location of FO2's EV's have been changed to the same locations as FO1. Furthermore, the constant $C_{k,j,b}$ for each aggregator in equation (2.3) is set at the same value. By including all constraints with respect to the DSO, the final charging schedules for FO1 and FO2 are shown in Figure 4.17.

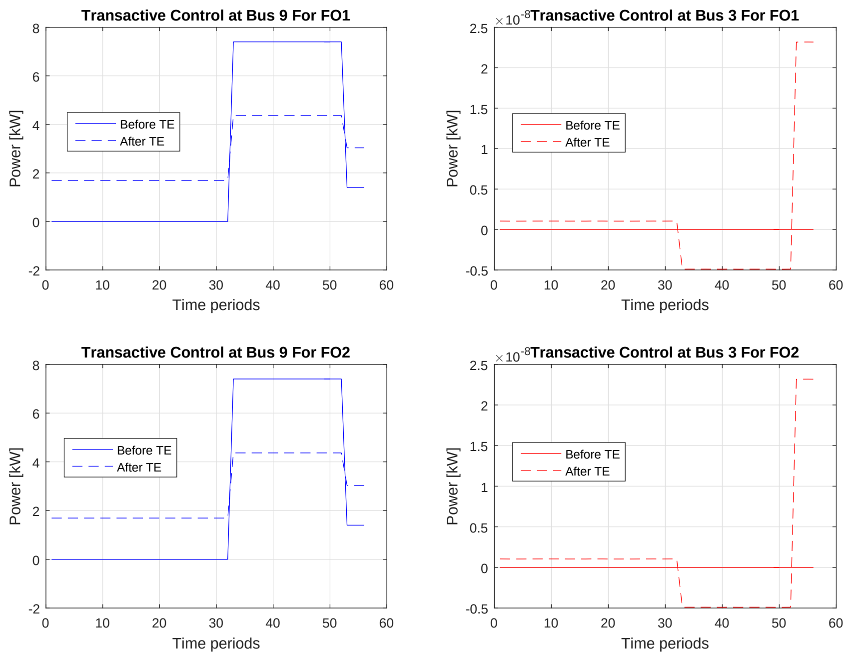

Figure 4.17: Development in the charging schedules for bus 3 and 9 for all 52 time periods with changed locations of EV's for FO2.

As it can be observed in Figure 4.17, both FO1 and FO2 have the same charging schedules, while the discrimination between the two aggregators has been removed. In addition, if different constants are assumed ($C_{1,j,b} = 0.5$ and $C_{2,j,b} = 0.1$) FO2 are penalized more than FO1. The costs for the two cases can be observed in Table 4.5.

Case	*Aggregator*	*Total costs before Transactive Control*	*Total costs after Transactive Control*	*Total Congestion price*
Equal constants	*FO1*	121.7217 DKK	129.5638 DKK	7.8421 DKK
	FO2	121.7217 DKK	129.5638 DKK	7.8421 DKK
Different constants	*FO1*	121.7217 DKK	124.5676 DKK	2.8459 DKK
	FO2	121.7217 DKK	135.7244 DKK	14.0028 DKK

Table 4.5: Influence in the costs for FO1 and FO2 in reference to the constant in the objective function for the aggregators.

As indicated by Table 4.5, FO1 and FO2 ends up with the same final costs of 129.5638 DKK with equal constants, which leads to a congestion price of 7.8421 DKK for both aggregators. In addition, the difference in the congestion price with different constants is significant. This confirms that the discrimination between the aggregators shown through the previously analysis are impacted by the constant $C_{k,j,b}$ to some extent. Therefore, the choice of constants should be carefully chosen in order avoid larger discriminations between aggregators.

4.5.2 Impacts of Changing the Locations of EV's for FO2

As already mentioned, the location of where the EV's are connected in the network affects the final charging schedules, since the network can have limitations in some parts of the network. Limitations can be related to the size of cables etc. These limitations can exists for one aggregator due to the location of EV's, while another aggregators can be fortunate and avoid them due to the location of their EV's.

By considering the shown low voltage network in Figure 4.2, four EV's controlled by FO2 have been relocated. The relocation is given in Table 4.6.

EV nr.	***Bus location before***	***Bus location after***
6	8	21
7	9	16
10	14	32
11	14	32

Table 4.6: Relocation of EV's for FO2.

By investigating the relocation of the four represented EV's in Table 4.6, the loads have in general been moved closer to the distribution transformer. This with the conviction of that it will give FO2 less challenges with network limitations, and thereby minimize the congestion price. Furthermore, the relocation leads to that only FO1 have EV's located at the ends of the network, which might contribute to an increased congestion price for FO1.

The costs for FO1 and FO2 after having relocated four EV's controlled by FO2 can be observed in Table 4.7.

Aggregator	***Total costs before Transactive Control***	***Total costs after Transactive control***	***Total congestion price***
FO1	121.7217 DKK	126.2831 DKK	4.5614 DKK
FO2	121.7217 DKK	130.6027 DKK	8.8810 DKK

Table 4.7: Overview of total charging costs and congestion price by changing the location of four EV's controlled by FO2.

From Table 4.7 the dependency of where the EV's are located can be observed. As expected the congestion price for FO2 has decreased in reference to case 2, while the opposite is the case for FO1 in reference to case 2. Thereby, the discrimination among aggregators in Transactive Control is highly affected by the locaiton of where the EV's are connected in the network. This is an important factor from aggregators point of view, since they will prefer to control EV's located in the parts of networks, where less limitations exists. This will in total lead to a minimized congestion price, which contributes to be a better business case at the end.

5 | Conclusion

This report has covered the Network-Constrained Transactive Control framework (TE), including modelling of the framework using YALMIP, which is an optimization package in MATLAB. The purpose of TE is to handle future congestion - and voltage problems, in which is caused by e.g. electric vehicles (EV). It is the expectations that the amount of electric vehicles are increasing, which questioning the future capacity in distribution networks. Thereby, TE can be used as an application which tries to optimize the charging schedules from the EV's in a way that complies with the network constraints. This will contribute to improve the utilization of the already existing assets, and thereby postpone investments regarding the future need of capacity in networks. This is an advantage due to the fact that uncertainties regarding number of EV's in the future, energy requirements from the EV's etc. exists.

Generally, the Transactive Control concept examined through this report have included three actors: the DSO, the aggregators and the price coordinator. The DSO and the aggregators utilize the price coordinator as communication channel, where this communication is through an economic signal, also known as the shadow prices. Through these interactions the price coordinator is responsible for updating the prices to the DSO and the aggregators, in order to reach consensus about the final charging schedules. Through this process the DSO and aggregators have different purposes.
The DSO has had one main interest through this analysis, which concerns about complying with the network constraints. The network constraints in this report have included requirements about the minimum voltage level, as well as how much the aggregated power from the electric vehicles loads the distribution transformer. Finally, the network losses have been included in the objective function, with the aim of reducing the network losses through the optimization. In addition, the DSO tries to allow as much power as possible at each bus in reference to the optimal charging schedules provided by the aggregators.
The aggregators, also known as the fleet operators, are responsible for a certain number of EV's. In this report two aggregators have existed, where both aggregators have been responsible for 18 EV's. The overall goal for the aggregators is to minimize the total charging costs with respect to the electricity spot prices. Through this optimization the aggregators provides the optimal charging schedules, which reflects the charging of EV's in time periods with low prices. These optimal schedules have been possible to establish by using linear programming. The optimal schedules will be used by both the DSO and the aggregators through the optimization process, where the aggregators tries to minimize the deviation from the optimal schedules and the final schedules. This implies to minimize the increment in charging costs.

The framework has been possible to model as a convex optimization problem, where dual decomposition using the subgradient method have been used as algorithm to find the optimal solution. This approach has shown great performance, where convergence has been obtained through the analysis.

To analyze the fundamentals of TE two cases have been established. One case did not consider minimum voltage requirements as well as network losses. The second case considered all network constraints and losses. It was found that both cases optimized the voltages and losses in the network, since the aggregators were forced to distribute the charging of EV's to other periods than the low price periods. This distribution was mainly caused by the constraint regarding the loading of the distribution transformer. However, by implementing network constraints in the second case showed the opportunities the DSO possesses, since the minimum voltage levels and losses were improved even more compared to case 1. This contributes to help the DSO comply with network requirements given by e.g. EN/DS 50160.

Through the analysis it was found that aggregator 2 was more penalized in terms of costs compared to aggregator 1. In connection to this discrimination it was found that the location of EV's in the network highly affect the final charging schedules, and thereby the total costs. Furthermore, the objective function for the aggregators contains constants, which have to be chosen carefully to not increase the discrimination between aggregators.
Finally, the interpretation of the prices received by the price coordinator have been examined. It was found that positive prices indicates that the aggregated charging schedules at bus i exceeds the allowable power desired by the DSO. Thus, positive prices indicates to the aggregators that they need to reduce the level of charging power at bus i. Additionally, negative prices have been observed, which indicates to the aggregators that they consume less power than allowed by the DSO. In other words: there is no problem regarding network constraints. Hence, it was analyzed whether negative prices could be treated as prices with a value of zero, since a price of zero also indicates that no problems exists in reference to the network constraints. Through this analysis it was found that all requirements were still met by having zero prices, however the loading level in the network was in general increased besides the low price periods. Furthermore, the total charging costs for both aggregators were decreased compared to the case where negative prices were allowed. Hence, aggregators will prefer to treat negative prices as zero, where the opposite may be the case for the DSO. This is due to the fact that the increased loading level leads to investments regarding the capacity in the network to approach faster than expected. This may lead to expenses for the DSO, in which was expected to occur later.

Overall, the TE framework has shown great possibilities to optimize the utilization of the already existing assets in networks, and thereby solve complex problems within networks when penetration as EV's increases. This contributes to development the future distribution - and low voltage networks in the most reliable way, and thereby obtain the most reliable operation of networks in the future.

Bibliography

[1] Olle Sundström and Carl Binding, "Flexible Charging Optimization for Electric Vehicles Considering Distribution Grid Constraints", in IEEE Transactions on Smart Grid, Vol. 3, No. 1, 2012, pp. 26-37.

[2] Mads Thelin Dalsgaard, "Reconfiguration of Power Distribution Networks by Evolutionary Algorithm for Reliability Improvement", Technical report, special course, DTU, 2016.

[3] Junjie Hu et al., "Transactive Control: A Framework for Operating Power Systems Characterized by a High Penetration of Distributed Energy Resources", J. Mod. Power Syst. Clean Energy, MPCE, 2016.

[4] Junjie Hu et al., "Coordinated Charging of Electric Vehicles for Congestion Prevention in the Distribution Grid", in IEEE Transactions on Smart Grid, Vol. 5, No. 2, 2014, pp. 703-711.

[5] Guangya Yang (March 2015), "Economic Dispatch", [PPT in course Power System Operation at DTU].

[6] Olle Sundström and Carl Binding, "Planning Electric-Drive Vehicle Charging under Constrained Grid Conditions", IEEE International Conference on Power System Technology, 2010.

[7] Guangya Yang et al., "Application of Network-Constrained Transactive Control To Electric Vehicle Charging for Secure Grid Operation", IEEE, 2016.

[8] Stephen Boyd and Almir Mutapcic (2008, April 13), "Subgradient Methods" [Online]. Available: https://see.stanford.edu/materials/lsocoee364b/02-subgrad_method_notes.pdf.

[9] Stephen Boyd et al., "Notes on Decomposition Methods" [Online], Available: https://see.stanford.edu/materials/lsocoee364b/08-decomposition_notes.pdf.

[10] Johan Löfberg, "YALMIP" [Online], Available: http://users.isy.liu.se/johanl /yalmip/pmwiki.php.