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3D polarization speckle as a demonstration of tensor version of the van Cittert-Zernike theorem for stochastic electromagnetic beams

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ABSTRACT

Laser speckle has received extensive studies of its basic properties and associated applications. In the majority of research on speckle phenomena, the random optical field has been treated as a scalar optical field, and the main interest has been concentrated on their statistical properties and applications of its intensity distribution. Recently, statistical properties of random electric vector fields referred to as *Polarization Speckle* have come to attract new interest because of their importance in a variety of areas with practical applications such as biomedical optics and optical metrology. Statistical phenomena of random electric vector fields have close relevance to the theories of speckles, polarization and coherence theory.

In this paper, we investigate the correlation tensor for stochastic electromagnetic fields modulated by a depolarizer consisting of a rough-surfaced retardation plate. Under the assumption that the microstructure of the scattering surface on the depolarizer is as fine as to be unresolvable in our observation region, we have derived a relationship between the polarization matrix/coherency matrix for the modulated electric fields behind the rough-surfaced retardation plate and the coherence matrix under the free space geometry. This relation is regarded as entirely analogous to the van Cittert-Zernike theorem of classical coherence theory. Within the paraxial approximation as represented by the ABCD-matrix formalism, the three-dimensional structure of the generated polarization speckle is investigated based on the correlation tensor, indicating a typical carrot structure with a much longer axial dimension than the extent in its transverse dimension.

Keywords: coherence, polarization, polarization speckle, coherence tensor

1. INTRODUCTION

Polarization and coherence are usually considered as the most important statistical properties of an optical field and have been studied intensively in the past decades, especially after the invention of the laser light source [1-5]. In recent years, optical beams with non-uniform distributed polarization states have attracted more and more attention. Therefore, a concept named polarization speckle has been introduced [6,7] to distinguish it from the conventional scalar laser speckle field with a uniform polarization. At the same time, the vectorial electromagnetic field description in terms of a 2×2 beam coherence-polarization matrix [8,9] has shown to be a valuable tool in order to investigate the evolution of polarization and coherence states during propagation together with their spatial distribution[10-15], especially after the setup of the unified theory of coherence and polarization[16,17].

Relevant researches like recent hot spot of the vectorial extension of conventional Van Cittert-Zernike theorem[18,19] also raise demands of describing this incoherent field with spatially non-uniformly distributed polarization state. The matrix form, tensor version extension of the conventional Van Cittert-Zernike theorem is first described by Gori[8,20] in the temporal domain, and then similar work in the spectral domain by Ostrovsky [19]. Further research work about the variation of the degree of coherence and degree of polarization within this field was developed both theoretically[18,21,22] and experimentally[23] soon after.

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In this paper, we will analyse the three-dimensional (3-D) statistics of the polarization speckle generated from an extremely rough-surfaced retardation plate, by which random de-correlation to all the field components together with randomly spatial depolarization was achieved. Particularly, the modulated field next to the plate becomes spatial incoherent and partially polarized, and behaves like an ideal secondary source for evaluating the tensor version of the van Cittert-Zernike theorem. Within the framework of complex ABCD theory[24,25], the analytical description of the modulated field's propagation will be derived in order to illustrate the generalized tensor version of Van Cittert-Zernike theorem for an arbitrary optical system under the paraxial approximation.

2. GENERATION OF A SPATIALLY INCOHERENT POLARIZATION SPECKLE BY A SPATIALLY INDEPENDENT ROUGH-SURFACED RETARDATION PLATE

Similar to the system once applied to examine the coherence and polarization properties of the electric field modulated by a rough-surfaced retardation plate [26], the optical propagation system shown in Fig 1 is utilized to assess the speckle modulation and propagation.

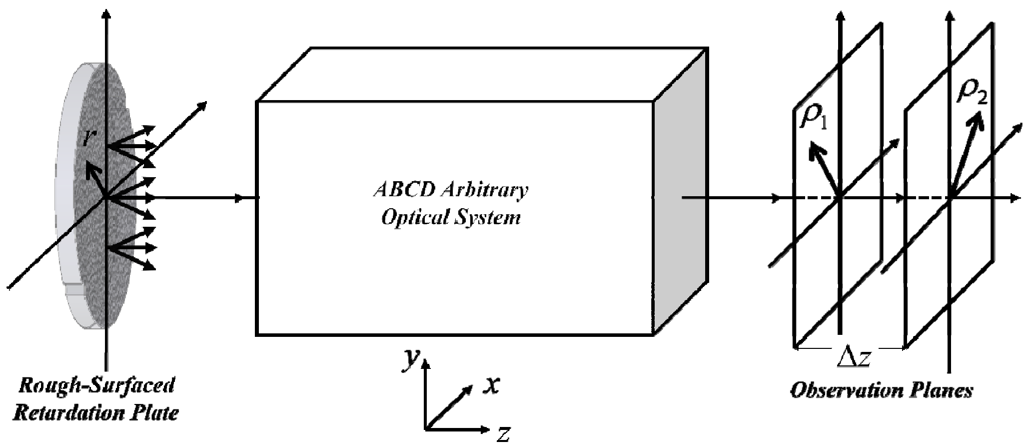

Figure 1. Schematic of the setup for obtaining the degree of polarization and coherence of field propagating through extremely rough-surfaced retardation plate and ABCD optical systems.

As a kind of polarization-dependent phase-modulating optical device possessing different refractive indices for incident light with different polarizations and propagation directions, the rough-surfaced retardation plate causes phase offset between two pointwise orthogonally polarized components of the incident field, and this phase offset is proportional to the stochastic thickness. Therefore, the spatially random depolarization is introduced, as well as the decorrelation effect. The field vector $\mathbf{E}(\mathbf{r},t)=\{E_x(\mathbf{r}),E_y(\mathbf{r})\}$ denotes electric field, and its statistical properties is specified via a 2×2 coherence matrix[2,17]

$$\mathbf{W}(\mathbf{r}_1,\mathbf{r}_2)=\begin{pmatrix}\langle E_x^*(\mathbf{r}_1)E_x(\mathbf{r}_2)\rangle & \langle E_x^*(\mathbf{r}_1)E_y(\mathbf{r}_2)\rangle \\ \langle E_y^*(\mathbf{r}_1)E_x(\mathbf{r}_2)\rangle & \langle E_y^*(\mathbf{r}_1)E_y(\mathbf{r}_2)\rangle\end{pmatrix} \tag{1}$$

where the asterisk * means complex conjugate, and angular brackets $\langle\ldots\rangle$ denote ensemble average. At the same time, the degree of coherence η and the degree of polarization P are extracted from these matrix elements[17]:

$$\eta(\mathbf{r}_1,\mathbf{r}_2)=\frac{tr\mathbf{W}(\mathbf{r}_1,\mathbf{r}_2)}{\sqrt{tr\mathbf{W}(\mathbf{r}_1,\mathbf{r}_1)tr\mathbf{W}(\mathbf{r}_2,\mathbf{r}_2)}} \tag{2}$$

and

$$P(\mathbf{r})=\left\{1-\frac{4\det\mathbf{W}(\mathbf{r},\mathbf{r})}{\left[tr\mathbf{W}(\mathbf{r},\mathbf{r})\right]^2}\right\}^{1/2}. \tag{3}$$

In the above equations, tr and $\det$ indicate the trace and determinant of the matrix, respectively.

Here, we define a vector $\mathbf{E}^i(\mathbf{r})=\{E^i{}_x(\mathbf{r}),E^i{}_y(\mathbf{r})\}$ for the incident field before, and $\mathbf{E}^t(\mathbf{r})=\{E^t{}_x(\mathbf{r}),E^t{}_y(\mathbf{r})\}$ for the transmission field after the depolarizer presented by the superscripts i or t respectively. The depolarizer plate is perpendicular to the propagation axis z, and is assumed to hold a constant amplitude transmittance equal to unity. Different effective phase delays $\varphi_m(\mathbf{r})=d(\mathbf{r})k(n_m-1)$, $(m=x,y)$ will be introduced to the $\hat{x}$ and $\hat{y}$ components of the incident fields, due to the refractive indices n_m $(m=x,y)$ assumed different for the orthogonally polarized wave components. While the local thickness $d(\mathbf{r})$ is varying across the plate, the relative phase shift between two orthogonal components $\Delta=k(n_y-n_x)d(\mathbf{r})$ will also fluctuate randomly. Although the discussion presented here is for a transparent structure, it will also be applicable for reflective models.

To express the incident and transmission fields' relationship $\mathbf{E}^t=\mathbf{E}^i\mathbf{T}$, the transmission matrix $\mathbf{T}$ is cited:

$$\mathbf{T}(\mathbf{r})=\begin{pmatrix} e^{-j\varphi_x(\mathbf{r})} & 0 \\ 0 & e^{-j\varphi_y(\mathbf{r})}\end{pmatrix}=\begin{pmatrix} e^{-jd(\mathbf{r})k(n_x-1)} & 0 \\ 0 & e^{-jd(\mathbf{r})k(n_y-1)}\end{pmatrix}, \tag{4}$$

and thus, the statistical connection between the coherence matrices of the incident field $\mathbf{W}^i(\mathbf{r_1},\mathbf{r_2})$ and modulated field $\mathbf{W}^t(\mathbf{r_1},\mathbf{r_2})$ is demonstrated by[15,27]:

$$\mathbf{W}^t(\mathbf{r}_1,\mathbf{r}_2)=\left\langle \mathbf{T}^\dagger(\mathbf{r}_1)\mathbf{W}^i(\mathbf{r}_1,\mathbf{r}_2)\mathbf{T}(\mathbf{r}_2)\right\rangle, \tag{5}$$

where $\dagger$ denotes the Hermitian conjugate.

By utilizing the mutual independence of the incident field's coherence property and the depolarizer's correlation property, the mutual coherence matrix $\mathbf{W}^t(\mathbf{r}_1,\mathbf{r}_2)$ in Eq.(5) could be rewritten as shown below to illustrate the statistical relation between the modulated field $\mathbf{E}^t$ at points $\mathbf{r}_1$ and $\mathbf{r}_2$[2,27]:

$$\begin{aligned}\mathbf{W}^t(\mathbf{r}_1,\mathbf{r}_2)&=\begin{pmatrix}\left\langle E^{t*}_x(\mathbf{r}_1)E^t{}_x(\mathbf{r}_2)\right\rangle & \left\langle E^{t*}_x(\mathbf{r}_1)E^t{}_y(\mathbf{r}_2)\right\rangle \\ \left\langle E^{t*}_y(\mathbf{r}_1)E^t{}_x(\mathbf{r}_2)\right\rangle & \left\langle E^{t*}_y(\mathbf{r}_1)E^t{}_y(\mathbf{r}_2)\right\rangle\end{pmatrix}\\ &=\begin{pmatrix} W^i_{xx}(\mathbf{r}_1,\mathbf{r}_2)\left\langle e^{j\Delta\varphi_{xx}(\mathbf{r}_1,\mathbf{r}_2)}\right\rangle & W^i_{xy}(\mathbf{r}_1,\mathbf{r}_2)\left\langle e^{j\Delta\varphi_{xy}(\mathbf{r}_1,\mathbf{r}_2)}\right\rangle \\ W^i_{yx}(\mathbf{r}_1,\mathbf{r}_2)\left\langle e^{j\Delta\varphi_{yx}(\mathbf{r}_1,\mathbf{r}_2)}\right\rangle & W^i_{yy}(\mathbf{r}_1,\mathbf{r}_2)\left\langle e^{j\Delta\varphi_{yy}(\mathbf{r}_1,\mathbf{r}_2)}\right\rangle\end{pmatrix}.\end{aligned} \tag{6}$$

It is easily noticed that the ensemble average terms in all matrix elements of Eq. (6) is determined by the corresponding characteristic function of the random variables $\Delta\varphi_{lm}(\mathbf{r}_1,\mathbf{r}_2)=\varphi_l(\mathbf{r}_1)-\varphi_m(\mathbf{r}_2)$, $(l,m=x,y)$ which are assumed to obey zero-mean Gaussian statistics and could thus be expressed by

$$\left\langle \exp\left\{j\Delta\varphi_{lm}(\mathbf{r}_1,\mathbf{r}_2)\right\}\right\rangle=\exp\left\{-\frac{k^2\left[(n_l-1)^2+(n_m-1)^2\right]\left\langle d^2(\mathbf{r})\right\rangle}{2}+k^2(n_l-1)(n_m-1)\left\langle d(\mathbf{r}_1)d(\mathbf{r}_2)\right\rangle\right\}, \tag{7}$$

as indicated in our previous analysis of polarization speckle generated by the rough-surfaced retardation plate depolarizer [26]. In addition, the zero mean surface thickness $\langle d(\mathbf{r})\rangle$ will be tacitly neglected in deriving Eq. (7), because it has no effect on the polarization state scrambling of the incident beam. We also take advantage of the isotropic Gaussian assumption of the surface thickness correlation function with surface thickness variance $\sigma_d{}^2$ and correlation length r_d :

$$\langle d(\mathbf{r}_1)d(\mathbf{r}_2)\rangle = \sigma_d{}^2 \exp\left\{-\left|\Delta\mathbf{r}\right|^2 / r_d{}^2\right\}, \tag{8}$$

where $\Delta\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$. In the case of large surface roughness and small lateral correlation length to obtain a phase difference greater than 2π, i.e., $(\mathrm{n}_l - 1)(n_m - 1)k^2\sigma_d^2 >> (2\pi)^2, (l, m = x, y)$, further progress can be made in a similar way used to describe the Gaussian rough-surfaced retardation plate [3,5,26]. Therefore, we have

$$\langle \exp\{j\Delta\varphi_{lm}(\mathbf{r}_1,\mathbf{r}_2)\}\rangle = \exp\left\{-\frac{k^2\sigma_d{}^2}{2}\left[(n_l - n_m)^2 + 2(n_l - 1)(n_m - 1)\frac{\left|\Delta\mathbf{r}\right|^2}{r_d{}^2}\right]\right. . \tag{9}$$

describing the correlation properties of the polarization speckle introduced by the surface features of the birefringent plate [26]. As a result, the corresponding degree of coherence η^t and degree of polarization $P^t(\mathbf{r},\mathbf{r})$ [17] of primary interest to evaluate the polarization and coherence modulation of the field can thus be obtained.

So far, we have derived the mathematical foundation for further discussion on incoherent polarization speckle's propagation through a complex ABCD optical system under the paraxial approximation. Particularly, for an extremely rough-surfaced retardation plate depolarizer discussed here, rapid varying thickness, small lateral correlation length r_d together with large fluctuation covariance σ_d generating fully developed speckle, a Dirac impulse delta approximation shown below will be introduced to the incoherent polarization speckle's mutual coherence matrix elements in Eq (9) to facilitate the evaluation of the Fresnel-integral for complex ABCD systems:

$$W_{lm}^t(\mathbf{r}_1,\mathbf{r}_2) \approx W_{lm}^i(\mathbf{r}_1,\mathbf{r}_2)\frac{\pi r_d{}^2\delta(\Delta\mathbf{r})}{k^2\sigma_d{}^2(n_l - 1)(n_m - 1)}\exp\left\{-\frac{k^2\sigma_d{}^2(n_l - n_m)^2}{2}\right\} \tag{10}$$

In derivation of the above equation, we took advantage of the fact that it is acceptable to regard a narrow Gaussian function as a Dirac delta function: $\lim_{\vartheta\to 0}\exp\{-|\Delta\mathbf{r}|^2/\vartheta^2\} \approx \pi\vartheta^2\delta(\Delta\mathbf{r})$, and thanks to this, further mathematical derivation for an analytical solution for the propagation of the coherence matrix through complex ABCD matrix becomes valid.

3. CHANGES IN THE DEGREE OF POLARIZATION AND THE DEGREE OF COHERENCE ON PROPAGATION

We now have the necessary tools to evaluate the propagation of the coherence matrix through an optical system. By the same time, the evolution of the coherence matrix for a spatially incoherent field with polarization variation becomes a general demonstration to provide physical insight into the vectorial Van Cittert-Zernike theorem. Under the paraxial approximation, the mutual coherence matrix $\mathbf{W}^o(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\Delta z)$ for the field at $\mathbf{p}_1$ in first observation plane after passing through a complex ABCD optical system and the field at $\mathbf{p}_2$ in another plane displaced by a distance Δz is connected to the mutual coherence matrix of the integral formulation [15,26]:

$$\mathbf{W}^o(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\Delta z) = \iint_{\pm\infty}\mathbf{W}^t(\mathbf{r}_1,\mathbf{r}_2)G_1^*(\boldsymbol{\rho}_1,\mathbf{r}_1)G_2(\boldsymbol{\rho}_2,\mathbf{r}_2)d\mathbf{r}_1 d\mathbf{r}_2, \tag{11}$$

where the Green's function is given by

$$G(\boldsymbol{\rho},\mathbf{r}) = -\frac{jk}{2\pi B}\exp\left\{-\frac{jk}{2B}\left(A\left|\mathbf{r}\right|^2 - 2\mathbf{r}\cdot\boldsymbol{\rho} + D\left|\boldsymbol{\rho}\right|^2\right)\right\}. \tag{12}$$

In the equation above, A, B, and D are the elements of the ABCD matrix $\mathbf{M}$ for the whole optical system under consideration, which is determined by the multiplication of the matrices for all the individual optical components, i.e., the lenses, free space propagations and aperture [24,25], and the mutual propagation distance is also characterized in this ABCD matrix elements. It is tacitly assumed in using Eq. (12) that the refractive indices in the input and output plane are identical. In addition, for the discussed system in this paper, we have the relationship between the ABCD matrices $\mathbf{M}_1$ for propagation to plane of $\boldsymbol{\rho}_1$ and $\mathbf{M}_2$ for propagation to plane of $\boldsymbol{\rho}_2$ as:

$$\mathbf{M}_2 = \begin{pmatrix} 1 & \Delta z \\ 0 & 1 \end{pmatrix} \mathbf{M}_1 . \tag{13}$$

Hence, we could rewrite the coherence matrix after travelling through the ABCD system by substitution of Eq.(10) into Eq. (11) and using the sifting property of the Dirac delta function[28,29]:

$$\mathbf{W}^o\left(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \Delta z\right) = \int_{\pm\infty} \mathbf{J}^t(\mathbf{r}) G_1^*\left(\boldsymbol{\rho}_1, \mathbf{r}\right) G_2\left(\boldsymbol{\rho}_2, \mathbf{r}\right) d\mathbf{r} \ , \tag{14}$$

where the modified polarization matrix $\mathbf{J}^t(\mathbf{r})$ for general incoherent source is defined as

$$\mathbf{J}^t(\mathbf{r}) = \frac{r_d{}^2 \pi}{k^2 \sigma_d{}^2 (n_x - 1)(n_y - 1)} \begin{pmatrix} W_{xx}^i(\mathbf{r},\mathbf{r}) \dfrac{(n_y - 1)}{(n_x - 1)} & W_{xy}^i(\mathbf{r},\mathbf{r}) \exp\left\{ -\dfrac{k^2 \sigma_d{}^2 (n_x - n_y)^2}{2} \right\} \\ W_{yx}^i(\mathbf{r},\mathbf{r}) \exp\left\{ -\dfrac{k^2 \sigma_d{}^2 (n_x - n_y)^2}{2} \right\} & W_{yy}^i(\mathbf{r},\mathbf{r}) \dfrac{(n_x - 1)}{(n_y - 1)} \end{pmatrix}, \tag{15}$$

and the relationship between the revised polarization matrix $\mathbf{J}^t$ and the coherence matrix $\mathbf{W}^t$ becomes $\mathbf{W}^t\left(\mathbf{r},\mathbf{r}\right) = \delta(\Delta\mathbf{r}) \mathbf{J}^t(\mathbf{r})$.

Equation (14) is considered the generalized version of the tensor counterpart of the scalar Van Cittert-Zernike theorem for complex ABCD system under the paraxial approximation. It is evident that the mutual coherence of the resulting field is determined by the auto-coherence of the incoherent source field. This is consistent with some cases discussed before, like the free space systems [29,18] and the Fourier system [23], when we substitute the relevant ABCD parameters and polarization matrix $\mathbf{J}^t$ for given systems into Eq.(14).

Further deterministic analysis is impossible without knowing $\mathbf{J}^t$. For demonstration purpose only, and without loss of generality, we will confine the following discussion for a specific incident field that is a Gaussian beam just in front of the depolarizer, and linearly polarized by an angle θ with respect to the $\hat{x}$ axis. The mutual coherence matrix is given by

$$\mathbf{W}^i(\mathbf{r},\mathbf{r}) = I_o \exp\left\{ -\frac{2|\mathbf{r}|^2}{r_s^2} \right\} \begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix}, \tag{16}$$

where I_o is the on-axis intensity of the incident field. On substituting from Eq.(16) into Eq.(10), we obtain the expressions for the coherence matrix for the initially linearly polarized beam just behind the depolarizer:

$$\mathbf{W}^t(\mathbf{r}_1,\mathbf{r}_2) = \exp\left\{ -\frac{|\mathbf{r}_1|^2 + |\mathbf{r}_2|^2}{r_s^2} \right\} \frac{I_o r_d{}^2 \sin\theta\cos\theta\pi\delta\left(\Delta\mathbf{r}\right)}{k^2\sigma_d{}^2 (n_x - 1)(n_y - 1)} \times \begin{pmatrix} \dfrac{\cot\theta(n_y - 1)}{(n_x - 1)} & \exp\left\{ -\dfrac{k^2 \sigma_d{}^2 (n_x - n_y)^2}{2} \right\} \\ \exp\left\{ -\dfrac{k^2 \sigma_d{}^2 (n_x - n_y)^2}{2} \right\} & \dfrac{\tan\theta(n_x - 1)}{(n_y - 1)} \end{pmatrix}, \tag{17}$$

As a consequence of the analytical integral in Eq. (14) for $\mathbf{W}^t$ shown in Eq.(17), the propagated coherence matrix for fields in two observation planes with a displacement Δz is given by:

$$\mathbf{W}^{o}\left(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\Delta z\right)=\frac{I_o r_d^{\,2}\sin\theta\cos\theta\pi}{k^2\sigma_d^{\,2}(n_x-1)(n_y-1)}W\left(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\Delta z\right)\begin{pmatrix}\dfrac{\cot\theta(n_y-1)}{(n_x-1)} & \exp\left\{-\dfrac{k^2\sigma_d^{\,2}(n_x-n_y)^2}{2}\right\}\\ \exp\left\{-\dfrac{k^2\sigma_d^{\,2}(n_x-n_y)^2}{2}\right\} & \dfrac{\tan\theta(n_x-1)}{(n_y-1)}\end{pmatrix}, \tag{18}$$

where $W\left(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\Delta z\right)$ is:

$$\begin{aligned}W\left(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\Delta z\right)=&\frac{jk}{2\pi\left(-\Delta z\left(CB^*-DA^*\right)+2j\,\mathrm{Im}\left(BA^*\right)\right)}\exp\left(-\frac{jk}{2}\left[-\left|\boldsymbol{\rho}_1\right|^2\left(\frac{D}{B}\right)^*+\left|\boldsymbol{\rho}_2\right|^2\frac{D}{B+D\Delta z}\right]\right)\\&\exp\left(-\frac{jk\left(-2\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_2\left(B^*(B+D\Delta z)\right)+\left|\boldsymbol{\rho}_2\right|^2\left(B^*\right)^2+\left|\boldsymbol{\rho}_1\right|^2(B+D\Delta z)^2\right)}{2B^*(B+D\Delta z)\left(-\Delta z\left(CB^*-DA^*\right)+2j\,\mathrm{Im}\left(BA^*\right)\right)}\right).\end{aligned} \tag{19}$$

The influence of a given intensity radius and radius of curvature of the incident beam can easily be incorporated by including a limiting aperture and a lens, respectively, as the first element(s) in the optical train establishing the ABCD matrix. The incident power should be compensated as well, while it, as a constant, is important in the evaluation of most parameters [25].

The analytic expressions in Eqs. (18) and (19) thus facilitate the calculation of the spatial coherence and degree of polarization for a given depolarizer illuminated with linearly polarized light.

A representative example for a simple field propagation system, here a free space propagation system with a preceding aperture of size r_s modelling the illuminating spot size as shown in Fig. 1 will be employed. The point $\boldsymbol{\rho}_1$ is in the first observation plane fixed with a propagation distance z to the rough-surfaced retardation plate, and the corresponding ABCD matrix $\mathbf{M}_1$ is given by:

$$\mathbf{M}_1=\begin{pmatrix}1-jz/z_R & z\\ -j/z_R & 1\end{pmatrix}, \tag{20}$$

where z_R is the Rayleigh range $z_R=kr_s^{\,2}/2$ for an aperture of size r_s. At the same time, the point $\boldsymbol{\rho}_2$ is in another plane moving along the propagation axis, and its distance to the first observation plane is denoted as Δz.

By substituting the ABCD matrix elements into Eq. (18) and (19), the propagated coherence matrix for such system could be calculated to assess the evolution of the degree of polarization and the degree of coherence during free space propagation. Based on this, we obtain the expression for the degree of coherence and degree of polarization for the resultant field:

$$\eta^{o}\left(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\Delta z'\right)=\frac{\exp\left\{-\dfrac{j}{z'}\left[\left|\boldsymbol{\rho}_1'\right|^2-\dfrac{\left|\boldsymbol{\rho}_2'\right|^2}{\Delta z'+1}\right]-\dfrac{\left|\dfrac{\boldsymbol{\rho}_2'}{\Delta z'+1}-\boldsymbol{\rho}_1'\right|^2}{2z'^2\left[1-\dfrac{\Delta z' j}{2\left(\Delta z'+1\right)z'}\right]}\right\}}{1-\dfrac{\Delta z' j}{2\left(\Delta z'+1\right)z'}}; \tag{21}$$

$$P^o(\boldsymbol{\rho}') = \left\{ 1 - 4\tan^2\theta \frac{1-\exp\left\{-k^2\sigma_d{}^2(n_x-n_y)^2\right\}}{\left[\frac{n_y-1}{n_x-1} + \tan^2\theta\frac{n_x-1}{n_y-1}\right]^2} \right\}^{1/2} . \tag{22}$$

For convenience of illustration, the variables in the above equations are normalized as: $z' = z/z_R$, $\Delta z' = \Delta z/z$ and $\boldsymbol{\rho}' = \boldsymbol{\rho}/r_s$. As a result, a partially coherent and polarized field is generated from an incoherent source after propagation as implied in Eq. (21) and (22). The degree of polarization of this propagated field is independent of the location $\boldsymbol{\rho}'$ and invariable during propagation, and this property of consistent polarization state is in accordance with earlier discussions for free space propagation [18,19]. Especially, in the case of $z \to \infty$, denoting a far propagation distance, $\left|\eta^o\left(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \Delta z\right)\right| \to 1$ results in a coherent field, which is consistent with the prediction of the vectorial van Cittert-Zernike theorem.

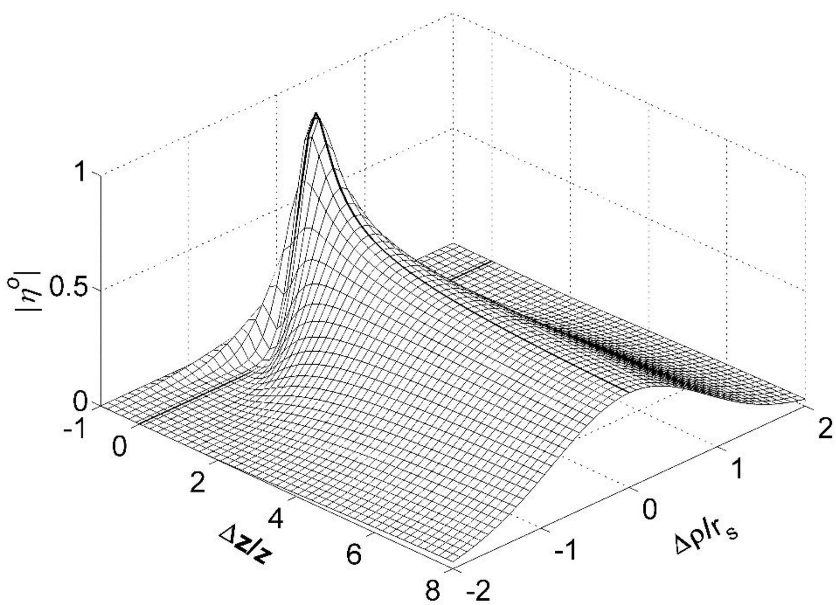

Figure 2. Three-dimensional degree of coherence distribution for polarization speckle as a function of the lateral position separation normalized by the spot size and the longitudinal displacement measured in units of propagation distance.

In metrology, the 3D speckle shape is usually the parameter in which we are interested. For the special case of a typical on axis speckle, its longitude and lateral sizes are respectively determined by the amplitude of degree of coherence for points $\boldsymbol{\rho}_1 = \boldsymbol{\rho}_2 = 0$ with displacement Δz:

$$\left|\eta^o\left(0, 0, \Delta z'\right)\right| = \frac{1}{\left(1 + \frac{\Delta z'^2}{4\left(\Delta z' + 1\right)^2 z'^2}\right)^{1/2}}, \tag{23}$$

and the amplitude of degree of coherence for points $\boldsymbol{\rho}_1 = -\boldsymbol{\rho}_2 = \Delta\boldsymbol{\rho}/2$ symmetrically located about the z-axis in the same plane:

$$\left|\eta^o\left(\frac{\Delta\boldsymbol{\rho}'}{2}, -\frac{\Delta\boldsymbol{\rho}'}{2}, 0\right)\right| = \exp\left\{-\frac{\left|\frac{\Delta\boldsymbol{\rho}'}{z'^2}\right|^2}{4}\right\} . \tag{24}$$

To provide the phenomena described above, we plotted the amplitude of $\eta^o(\Delta\boldsymbol{\rho}, -\Delta\boldsymbol{\rho}, \Delta z)$ as shown in figure 2, indicating a typical bullet-shaped structure from its contours. The lateral speckle size, i.e. the lateral coherence length of the resultant field can be extracted from the width of the Gaussian term in Eq. (24) as $\sqrt{2}zr_s / z_R$, which will increase with the propagation distance. The fact of an increasing longitudinal coherence length for further propagation is indicated in Eq. (23), as well. This significant dependence of the increasing speckle volume on the propagation distance is in accordance with the van Cittert-Zernike theorem. Finally, with the help of complex ABCD theory, we verified that the field's steady polarization property is also valid for more general propagation systems.

4. CONCLUSIONS

In this paper, we investigated the 3-D statistical properties of polarization speckle, especially the polarization and coherence of the electromagnetic fields modulated by a depolarizer consisting of a rough-surfaced retardation plate. The coherence matrix for the modulated electric fields has been derived sharing a formal analogy to the van Cittert-Zernike theorem for scalar optical fields. Within the framework of complex ABCD formalism, the propagation property of the mutual coherence matrix has been investigated to reveal the evolution of the polarization and coherence properties associated with the polarization speckle. Meanwhile, the three-dimensional structures of the generated polarization speckle have been studied from the mutual coherence matrix, indicating a typical carrot structure with a much longer axial dimension than the extent in its transverse dimension. Hence, this paper can provide a practical insight into a wide class of partially coherent electric fields with random polarization. For possible experimental study, a novel optical system combining the polarization imaging and speckle imaging can be used to verify the predicted phenomena on 3D polarization speckle.

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