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A Dantzig-Wolfe Decomposition Algorithm for Linear Economic MPC of a Power Plant Portfolio

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Abstract: Future power systems will consist of a large number of decentralized power producers and a large number of controllable power consumers in addition to stochastic power producers such as wind turbines and solar power plants. Control of such large scale systems requires new control algorithms. In this paper, we formulate the control of such a system as an Economic Model Predictive Control (MPC) problem. When the power producers and controllable power consumers have linear dynamics, the Economic MPC may be expressed as a linear program and we apply Dantzig-Wolfe decomposition for solution of this linear program. The Dantzig-Wolfe decomposition algorithm for Economic MPC is tested on a simulated case study with a large number of power producers. The Dantzig-Wolfe algorithm is compared to a standard linear programming (LP) solver for the Economic MPC. Simulation results reveal that the Dantzig-Wolfe algorithm is faster than the standard LP solver and enables solution of larger problems.

Keywords: Economic Model Predictive Control, Linear Programming, Distributed Optimization, Power Systems

1. INTRODUCTION

Increasing prices of fossil fuels and climate concerns related to CO_2 have stimulated an increased political and technical interest in power systems that are not based on fossil fuels (Gore, 2006, 2009; Friedman, 2009; Danish Energy Agency, 2010; Danish Commission on Climate Change Policy, 2010; European Technology Platform SmartGrids, 2012). The predominant renewable energy sources in such a system are wind and solar energy. The power productions from sources such as the sun and the wind are stochastic. Inclusion of large shares of stochastic power producers in the power system requires that the existing power system is restructured such that they can quickly compensate for variations in energy production from the stochastic generators. Consequently, future power systems must include a large number of decentralized agile controllable power producers and consumers to compensate for the stochastic power production from wind turbines and solar plants. Such an integration of a large number of new power producers and consumers in the power system requires new control algorithms for balancing their power production and consumption.

In this paper, we present an optimization based controller for balancing the power production and consumption in an economic efficient way. The optimization based controller is obtained by formulating the power balancing problem as an Economic Model Predictive Control (MPC) problem. Many energy system components can be approximated well by linear models (Edlund et al., 2009). Accordingly, the Economic MPC for power systems with a large number of linear components results in large scale linear programs that must be solved efficiently and reliable in real time. Due to the decoupled dynamics of the energy components, the linear program representing the Economic MPC has a block angular structure that is utilized in the Dantzig-Wolfe algorithm. The key contributions of this paper is a Dantzig-Wolfe decomposition algorithm for Economic MPC of linear systems and demonstration of this Economic MPC to power systems with many power producers.

Previously, Economic MPC has been applied to smart energy systems such as refrigeration systems (Hovgaard et al., 2010, 2011, 2012a,b), heat pumps for residential buildings (Halvgaard et al., 2012c), solar heated water tanks (Halvgaard et al., 2012a), and batteries in electrical vehicles (Halvgaard et al., 2012b). Scattolini (2009) reviewed model predictive control for distributed systems. Using the terminology in Scattolini (2009), the Dantzig-Wolfe decomposition method presented in this paper is a distributed optimization method for solution of the centralized MPC. Other well-known techniques for distributed optimization that have been applied for MPC are Lagrange dual decomposition (Rantzer, 2009) and Bender's decomposition (Morsan et al., 2011a). Dantzig and Wolfe (1960, 1961) introduced a decomposition algorithm for large linear programs. This decomposition algorithm is known as the Dantzig-Wolge algorithm. Like Lagrange dual decomposition, the Dantzig-Wolfe algorithm uses Lagrange relaxation to decompose the large scale linear program into smaller linear programs. However, in Dantzig-Wolfe decomposition another linear program (the master problem) is used to compute the Lagrange multipliers, while

Lagrange dual decomposition uses heuristics for updating the Lagrange multipliers. Lasdon (1970), Chvatal (1983), Nazareth (1987), Dantzig and Thapa (2003), and Conejo et al. (2006) discuss decomposition algorithms for large scale linear programs in a tutorial manner. Recently, the Dantzig-Wolfe algorithm has been used in a number MPC-like applications. Cheng et al. (2007, 2008) used Dantzig-Wolfe decomposition in the steady-state target LP for a two-layer MPC algorithm. Gunnerud and Foss (2010) and Gunnerud et al. (2010) applied a Dantzig-Wolfe algorithm for production optimization in an oil field. Morsan et al. (2011b) studied a building temperature control problem and solved the MPC for that problem using a Dantzig-Wolfe decomposition algorithm. Based on an ℓ_1-penalty function, Edlund et al. (2011) formulated the power balancing problem as an MPC and solved the resulting large scale linear program using a Dantzig-Wolfe algorithm. Bürger et al. (2012) proposed an iterative algorithm related to the Dantzig-Wolfe algorithm that do not need a master problem.

This paper is organized as follows. Section 2 introduces Economic MPC for linear stochastic systems and show that such problems can be solved by solution of linear programs. Section 4 describes the Dantzig-Wolfe decomposition algorithm for linear programs with a block-angular structure. A power plant case study is introduced in Section 5 to illustrate the Economic MPC based on Dantzig-Wolfe decomposition. Conclusions are provided in Section 6.

2. ECONOMIC MPC FOR LINEAR SYSTEMS

This section is about the Economic Model Predictive Control (MPC) stated for linear programs, where the optimal solution is found minimizing an economic cost. In this paper, the control problem is stated as a linear program (Hovgaard et al., 2010). The slack variables are introduced in the optimization problem to adjust in case that the portfolio output fails in following the reference.

2.1 The Stochastic System

Consider the stochastic system

$$\boldsymbol{x}_{k+1} = A\boldsymbol{x}_k + B\boldsymbol{u}_k + G\boldsymbol{w}_k + E\boldsymbol{d}_k \tag{1a}$$

$$\boldsymbol{y}_k = C\boldsymbol{x}_k + \boldsymbol{v}_k \tag{1b}$$

$$\boldsymbol{z}_k = C_z\boldsymbol{x}_k \tag{1c}$$

The initial state is distributed as $\boldsymbol{x}_0 \sim N(\hat{x}_{0|-1}, P_{0|-1})$, the process noise is distributed as $\boldsymbol{w}_k \sim N_{iid}(0, R_{ww,k})$, and the measurement noise is distributed as $\boldsymbol{v}_k \sim N_{iid}(0, R_{vv})$. $\boldsymbol{x}_k$ denotes the states, $\boldsymbol{u}_k$ denotes the manipulated variables (MVs), $\boldsymbol{y}_k$ denotes the measurement used for feedback, and $\boldsymbol{z}_k$ denotes output variables. $\boldsymbol{d}_k \sim N(\bar{d}_k, R_{dd,k})$ denotes process noise that can be predicted by a prognosis system and are predicted independently of the measurements y. Accordingly, we consider a non-standard situation, in which the process disturbance $\boldsymbol{d}_k$ can be predicted by some realization I_k^d of a stochastic information vector $\mathcal{I}_k^d$. We assume that the conditional variable has the distribution

$$\boldsymbol{d}_{k+j|k} = (\boldsymbol{d}_{k+j}|\mathcal{I}_k^d = I_k^d) \sim N(\hat{d}_{k+j|k}, R_{dd,k+j|k}) \tag{2}$$

In many situations in smart energy systems, d involves variables such as wind speed, temperature and sun radiation. Accordingly, the forecast $\boldsymbol{d}_{k+j|k}$ is the result of a weather prognosis. We denote the mean of these forecasts as

$$\mathcal{D}_k = \left\{\hat{d}_{k+j|k}\right\}_{j=0}^{N-1} \tag{3}$$

The manipulated variable, $\boldsymbol{u}_k$, is a stochastic variable. For the systems we consider, it is given by a function of the form $\boldsymbol{u}_k = \mu(\hat{x}_{k|k}, u_{k-1}, \mathcal{D}_k, \mathcal{F}_k, \mathcal{R}_k)$ with $\hat{x}_{k|k}$ being a filtered state estimate depending on the current measurement $\boldsymbol{y}_k$ as well as the history of the system summarized by the previous filtered state estimate, $\hat{x}_{k-1|k-1}$, and its covariance, $P_{k-1|k-1}$. $\mathcal{F}_k$ and $\mathcal{R}_k$ are some forecasts to be defined later. The fact that $\boldsymbol{u}_k$ is a stochastic variable implies that it is a function $\boldsymbol{u}_k : \Omega \mapsto \mathbb{R}^{n_u}$, i.e. $u_k = \boldsymbol{u}_k(\omega)$ for $\omega \in \Omega$ and $(\Omega, \mathcal{G}, P)$ is an associated probability field (Billingsley, 1995). The manipulated variables are limited by bounds and rate-of-movement constraints

$$u_{\min} \leq \boldsymbol{u}_k \leq u_{\max} \tag{4a}$$

$$\Delta u_{\min} \leq \Delta \boldsymbol{u}_k \leq \Delta u_{\max} \tag{4b}$$

These constraints says, that $u_k = \boldsymbol{u}_k(\omega)$ must satisfy the constraints. Accordingly, $\boldsymbol{u}_k$ *cannot* be normally distributed as the tails are removed by the constraints. It should also be noticed that these constraints are different from similar mean-value constraints and probabilistic constraints.

The outputs, $\boldsymbol{z}_k$, should be in some interval $[\boldsymbol{r}_{\min,k},\ \boldsymbol{r}_{\max,k}]$ where $\boldsymbol{r}_{\min,k} \sim \mathbb{F}(\bar{r}_{\min,k}, R_{(r_{\min},r_{\min}),k})$ and $\boldsymbol{r}_{\max,k} \sim \mathbb{F}(\bar{r}_{\max,k}, R_{(r_{\max},r_{\max}),k})$ are stochastic variables stemming from some distribution. Forecasts, $\mathcal{R}_k$, of the interval $[\boldsymbol{r}_{\min,k},\ \boldsymbol{r}_{\max,k}]$ are available and used by the controller. Let

$$\begin{aligned}\boldsymbol{r}_{\min,k+j|k} &= (\boldsymbol{r}_{\min,k+j}|\mathcal{I}_k^r = I_k^r) \\ &\sim \mathbb{F}(\hat{r}_{\min,k+j|k}, R_{(r_{\min},r_{\min}),k+j|k})\end{aligned} \tag{5a}$$

$$\begin{aligned}\boldsymbol{r}_{\max,k+j|k} &= (\boldsymbol{r}_{\max,k+j}|\mathcal{I}_k^r = I_k^r) \\ &\sim \mathbb{F}(\hat{r}_{\max,k+j|k}, R_{(r_{\max},r_{\max}),k+j|k})\end{aligned} \tag{5b}$$

such that the mean of the forecast, $\mathcal{R}_k$, may be denoted as

$$\mathcal{R}_k = \left\{\hat{r}_{\min,k+j|k}, \hat{r}_{\max,k+j|k}\right\}_{j=1}^{N} \tag{6}$$

In energy systems, the interval $[\boldsymbol{r}_{\min,k},\ \boldsymbol{r}_{\max,k}]$ can be related to the power consumption, indoor temperature in a building, temperatures in a refrigeration system or some desired state-of-charge of a battery. For some scenarios or disturbances, it may be very expensive or even impossible to keep the outputs $\boldsymbol{z}_k$ in the interval $[\boldsymbol{r}_{\min,k},\ \boldsymbol{r}_{\max,k}]$. For such situations, we introduce slack variables defined by

$$\boldsymbol{s}_k = \max\left\{0, \boldsymbol{r}_{\min,k} - \boldsymbol{z}_k, \boldsymbol{z}_k - \boldsymbol{r}_{\max,k}\right\} \tag{7}$$

such that the possible interval for the outputs is expanded to

$$\boldsymbol{r}_{\min,k} - \boldsymbol{s}_k \leq \boldsymbol{z}_k \leq \boldsymbol{r}_{\max,k} + \boldsymbol{s}_k \tag{8}$$

with $\boldsymbol{s}_k \geq 0$. The slack variables, $\boldsymbol{s}_k$, may represent selling or buying power from the short-term market, violation of temperature limits, or violation of state-of-charge limits. Every time $\boldsymbol{s}_k$ is non-zero, a penalty cost, e.g. the cost of buying or selling power on the short-term market, must be paid.

The average cost of operating the system in a period is the stochastic variable

$$\psi = \lim_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N} \boldsymbol{b}_k' \boldsymbol{z}_k + \boldsymbol{c}_k' \boldsymbol{u}_k + \boldsymbol{\rho}_k' \boldsymbol{s}_k \tag{9}$$

with $\boldsymbol{b}_k \sim \mathbb{F}(\bar{b}_k, R_{bb,k})$, $\boldsymbol{c}_k \sim \mathbb{F}(\bar{c}_k, R_{cc,k})$, and $\boldsymbol{\rho}_k \sim \mathbb{F}(\bar{\rho}_k, R_{\rho\rho,k})$ being unit costs. These unit costs are predicted by yet another forecasting system. The unit price forecasts are the conditional stochastic variables

$$\boldsymbol{b}_{k+j|k} = (\boldsymbol{b}_{k+j}|\mathcal{I}_k^p = I_k^p) \sim \mathbb{F}(\hat{b}_{k+j|k}, R_{bb,k+j|k}) \tag{10a}$$
$$\boldsymbol{c}_{k+j|k} = (\boldsymbol{c}_{k+j}|\mathcal{I}_k^p = I_k^p) \sim \mathbb{F}(\hat{c}_{k+j|k}, R_{cc,k+j|k}) \tag{10b}$$
$$\boldsymbol{\rho}_{k+j|k} = (\boldsymbol{\rho}_{k+j}|\mathcal{I}_k^p = I_k^p) \sim \mathbb{F}(\hat{\rho}_{k+j|k}, R_{\rho\rho,k+j|k}) \tag{10c}$$

and we denote the unit price forecast as

$$\mathcal{F}_k = \left\{ \hat{b}_{k+j+1|k}, \hat{c}_{k+j|k}, \hat{\rho}_{k+j+1|k} \right\}_{j=0}^{N-1} \tag{11}$$

2.2 Filtering and Prediction

The filtered estimate, $\hat{x}_{k|k} = E\{\boldsymbol{x}_k|\mathcal{Y}_k = Y_k\}$, of a system governed by (1) is computed using the Kalman filter (Jazwinski, 1970; Kailath et al., 2000; Jørgensen and Jørgensen, 2007; Jørgensen et al., 2011). The innovation is computed as

$$e_k = y_k - \hat{y}_{k|k-1} = y_k - C\hat{x}_{k|k-1} \tag{12}$$

The innovation covariance, $R_{e,k}$, the filter gain, $K_{fx,k}$, and the filtered state covariance, $P_{k|k}$, are computed as

$$R_{e,k} = R_{vv} + CP_{k|k-1}C' \tag{13a}$$
$$K_{fx,k} = P_{k|k-1}C'R_{e,k}^{-1} \tag{13b}$$
$$P_{k|k} = P_{k|k-1} - K_{fx,k}R_{e,k}K_{fx,k}' \tag{13c}$$

such that the filtered state can be computed by

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_{fx,k}e_k \tag{14}$$

Equations (12)-(14) are standard Kalman filter operations for the measurement update. The predictions are slightly different than the standard Kalman prediction due to the forecasts of d_k. Given the conditional predictions of the exogenous variables, $\hat{d}_{k+j|k}$, and the manipulated variables, $\hat{u}_{k+j|k}$, the conditional predictions of the states and the outputs are

$$\hat{x}_{k+1+j|k} = A\hat{x}_{k+j|k} + B\hat{u}_{k+j|k} + E\hat{d}_{k+j|k} \tag{15a}$$
$$\hat{z}_{k+j+1|k} = C_z\hat{x}_{k+1+j|k} \tag{15b}$$

for $j = 0, 1, \ldots, N-1$ and all $k \geq 0$. The corresponding covariances of the predicted states are

$$P_{k+j+1|k} = AP_{k+j|k}A' + GR_{ww,k+j}G' + ER_{dd,k+j|k}E' \tag{16}$$

2.3 A Certainty Equivalent Regulator for Economic MPC

By now we have defined the stochastic system and established the optimal filtering and prediction in this system. Next we will describe our method for computing the manipulated variables, u_k. We use a certainty equivalence assumption such that the regulator uses mean value predictions for all variables. Consequently, at time k, the predicted operating cost looking N periods ahead is

$$\phi = \sum_{j=0}^{N-1} \hat{b}_{k+j+1|k}' \hat{z}_{k+j+1|k} + \hat{c}_{k+j|k}' \hat{u}_{k+j|k} + \sum_{j=0}^{N-1} \hat{\rho}_{k+j+1|k}' \hat{s}_{k+j+1|k} \tag{17}$$

This cost function is linear in $\hat{z}_{k+j+1|k}$, $\hat{u}_{k+j|k}$, and $\hat{s}_{k+j+1|k}$. This objective function is not necessarily an exact penalty function that selects the slack variables as defined by (7). It will be exact if the prices, $\hat{\rho}_{k+j+1|k}$, are larger than the corresponding Lagrange-multipliers for output constraints of the form

$$r_{\min,k+j+1|k} \leq \hat{z}_{k+j+1|k} \leq r_{\max,k+j+1|k} \tag{18}$$

In the case with an exact penalty function for the output constraints (18), $\hat{s}_{k+j+1|k}$ will only be non-zero if (18) cannot be met.

Given the mean value of the forecasts, i.e. $\mathcal{D}_k$, $\mathcal{R}_k$, and $\mathcal{F}_k$, the filtered state, $\hat{x}_{k|k}$, from (14), the previous input, u_{k-1}, as well as the predictions (15) and the objective function (17), the optimal trajectory of the predicted manipulated variables and slack variables, $\left\{ \hat{u}_{k+j|k}, \hat{s}_{k+j+1|k} \right\}_{j=0}^{N-1}$, may be computed by solution of the linear program

$$\min_{\hat{u},\hat{s}} \quad \phi = \phi(\left\{ \hat{u}_{k+j|k}, \hat{s}_{k+j+1|k} \right\}_{j=0}^{N-1}) \tag{19a}$$
$$s.t. \quad \hat{x}_{k+1+j|k} = A\hat{x}_{k+j|k} + B\hat{u}_{k+j|k} + E\hat{d}_{k+j|k} \tag{19b}$$
$$\hat{z}_{k+j+1|k} = C_z\hat{x}_{k+1+j|k} \tag{19c}$$
$$u_{\min} \leq \hat{u}_{k+j|k} \leq u_{\max} \tag{19d}$$
$$\Delta u_{\min} \leq \Delta\hat{u}_{k+j|k} \leq \Delta u_{\max} \tag{19e}$$
$$\hat{z}_{k+j+1|k} + \hat{s}_{k+j+1|k} \geq \hat{r}_{\min,k+j+1|k} \tag{19f}$$
$$\hat{z}_{k+j+1|k} - \hat{s}_{k+j+1|k} \leq \hat{r}_{\max,k+j+1|k} \tag{19g}$$
$$\hat{s}_{k+j+1|k} \geq 0 \tag{19h}$$

This linear program is based on the certainty equivalence assumption. Only the first input, $\hat{u}_{k|k}$, of this sequence is implemented. The function involving solution of (19) and selecting $\hat{u}_{k|k}$ is denoted as

$$u_k = \hat{u}_{k|k} = \mu(\hat{x}_{k|k}, u_{k-1}, \mathcal{D}_k, \mathcal{R}_k, \mathcal{F}_k) \tag{20}$$

2.4 Forecast based Certainty Equivalent MPC Algorithm

The certainty equivalent Economic MPC developed in this section is listed in Algorithm 1. It computes the manipulated variable, u_k, based on the current measurement, y_k, the previous input, u_{k-1}, the forecasts $(\mathcal{D}_k, \mathcal{R}_k, \mathcal{F}_k)$, and the smoothed mean-covariance estimate $(\hat{d}_{k-1|k}, R_{dd,k-1|k})$. The smoothed estimate, $(\hat{d}_{k-1|k}, R_{dd,k-1|k})$, is needed because we do the one-step prediction of the states, $\hat{x}_{k|k-1} = E\{\boldsymbol{x}_k|\mathcal{Y}_{k-1} = Y_{k-1}\}$ at time k when the information vector $\mathcal{I}_k^d = I_k^d$ has been realized and is known. These information availability considerations are the reason that the one-step predictions in Algorithm 1 must be expressed as (22a) and (24a).

The main computational load in Algorithm 1 is solution of the linear program (19).

3. DYNAMICALLY DECOUPLED SYSTEMS

In this section, we specialize the stochastic system (1) to a dynamically decoupled system. Such decoupled models are ubiquitous in energy systems. Furthermore, we demonstrate how the linear program (19) for dynamically decoupled systems have a block angular structure. This block-angular structure may be utilized for efficient solution of (19) using decomposition algorithms such as the Dantzig-Wolfe algorithm.

Algorithm 1 Certainty equivalent Economic MPC with external forecasts

Require:
Input: y_k, u_{k-1}
Memory: $\hat{x}_{k-1|k-1}$, $P_{k-1|k-1}$.
Forecasts:

$$\mathcal{D}_k = \left\{\hat{d}_{k+j|k}\right\}_{j=0}^{N-1}, \ (\hat{d}_{k-1|k}, R_{dd,k-1|k}) \tag{21a}$$

$$\mathcal{R}_k = \left\{\hat{r}_{\min,k+j+1|k}, \hat{r}_{\max,k+j+1|k}\right\}_{j=0}^{N-1} \tag{21b}$$

$$\mathcal{F}_k = \left\{\hat{b}_{k+j+1|k}, \hat{c}_{k+j|k}, \hat{\rho}_{k+j+1|k}\right\}_{j=0}^{N-1} \tag{21c}$$

One-step predictor and filter:
Compute the one-step prediction

$$\hat{x}_{k|k-1} = A\hat{x}_{k-1|k-1} + Bu_{k-1} + E\hat{d}_{k-1|k} \tag{22a}$$

$$\hat{y}_{k|k-1} = C\hat{x}_{k|k-1} \tag{22b}$$

Compute the innovation

$$e_k = y_k - \hat{y}_{k|k-1} \tag{23}$$

Compute

$$\begin{aligned} P_{k|k-1} = AP_{k-1|k-1}A' \\ + GR_{ww,k-1}G' + ER_{dd,k-1|k}E' \end{aligned} \tag{24a}$$

$$R_{e,k} = R_{vv} + CP_{k|k-1}C' \tag{24b}$$

$$K_{fx,k} = P_{k|k-1}C'R_{e,k}^{-1} \tag{24c}$$

$$P_{k|k} = P_{k|k-1} - K_{fx,k}R_{e,k}K'_{fx,k} \tag{24d}$$

Compute the filtered state

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_{fx,k}e_k \tag{25}$$

Regulator:
Compute $u_k = \mu(\hat{x}_{k|k}, u_{k-1}, \mathcal{D}_k, \mathcal{R}_k, \mathcal{F}_k)$ by solution of the linear program (19).
Return:
Manipulated variable: u_k
Update the memory with: $\hat{x}_{k|k}$, $P_{k|k}$

Consider a set, $\mathcal{P} = \{1, \dots, P\}$, of dynamically decoupled systems

$$\boldsymbol{x}_{i,k+1} = A_i\boldsymbol{x}_{i,k} + B_i\boldsymbol{u}_{i,k} + G_i\boldsymbol{w}_k + E_i\boldsymbol{d}_k \quad i \in \mathcal{P} \tag{26a}$$

$$\boldsymbol{y}_{i,k} = C_i\boldsymbol{x}_{i,k} + \boldsymbol{v}_{i,k} \quad i \in \mathcal{P} \tag{26b}$$

$$\boldsymbol{z}_{i,k} = C_{z,i}\boldsymbol{x}_{i,k} \quad i \in \mathcal{P} \tag{26c}$$

that jointly create the following measurement, $\tilde{\boldsymbol{y}}_k$, and output, $\tilde{\boldsymbol{z}}_k$:

$$\tilde{\boldsymbol{y}}_k = \sum_{i=1}^{P} \tilde{C}_i\boldsymbol{x}_{i,k} + \tilde{\boldsymbol{v}}_k \tag{27a}$$

$$\tilde{\boldsymbol{z}}_k = \sum_{i=1}^{P} \tilde{C}_{z,i}\boldsymbol{x}_{i,k} \tag{27b}$$

The dynamically decoupled system (26)-(27) is a special case of (1) with the variables defined as

$$\begin{aligned} \boldsymbol{x}_k &= [\boldsymbol{x}_{1,k};\ \boldsymbol{x}_{2,k};\ \dots;\ \boldsymbol{x}_{P,k}] \\ \boldsymbol{u}_k &= [\boldsymbol{u}_{1,k};\ \boldsymbol{u}_{2,k};\ \dots;\ \boldsymbol{u}_{P,k}] \\ \boldsymbol{y}_k &= [\boldsymbol{y}_{1,k};\ \boldsymbol{y}_{2,k};\ \dots;\ \boldsymbol{y}_{P,k};\ \tilde{\boldsymbol{y}}_k] \\ \boldsymbol{z}_k &= [\boldsymbol{z}_{1,k};\ \boldsymbol{z}_{2,k};\ \dots;\ \boldsymbol{z}_{P,k};\ \tilde{\boldsymbol{z}}_k] \\ \boldsymbol{r}_{\min,k} &= [\boldsymbol{r}_{\min,1,k};\ \boldsymbol{r}_{\min,2,k};\ \dots;\ \boldsymbol{r}_{\min,P,k};\ \tilde{\boldsymbol{r}}_{\min,k}] \\ \boldsymbol{r}_{\max,k} &= [\boldsymbol{r}_{\max,1,k};\ \boldsymbol{r}_{\max,2,k};\ \dots;\ \boldsymbol{r}_{\max,P,k};\ \tilde{\boldsymbol{r}}_{\max,k}] \\ \boldsymbol{v}_k &= [\boldsymbol{v}_{1,k};\ \boldsymbol{v}_{2,k};\ \dots;\ \boldsymbol{v}_{P,k};\ \tilde{\boldsymbol{v}}_k] \end{aligned}$$

and the corresponding state space matrices defined as

$$\begin{aligned} A &= \text{block diag}([A_1\ A_2\ \dots\ A_P]) \\ B &= \text{block diag}([B_1\ B_2\ \dots\ B_P]) \\ G &= [G_1;\ G_2;\ \dots;\ G_P] \\ E &= [E_1;\ E_2;\ \dots;\ E_P] \end{aligned}$$

$$C = \begin{bmatrix} C_1 & & & \\ & C_2 & & \\ & & \ddots & \\ & & & C_P \\ \tilde{C}_1 & \tilde{C}_2 & \dots & \tilde{C}_P \end{bmatrix} \quad C_z = \begin{bmatrix} C_{z,1} & & & \\ & C_{z,2} & & \\ & & \ddots & \\ & & & C_{z,P} \\ \tilde{C}_{z,1} & \tilde{C}_{z,2} & \dots & \tilde{C}_{z,P} \end{bmatrix}$$

Eqs. (26)-(27) may be used to model the dynamics of a system of P individual linear plants with local measurements, $\boldsymbol{y}_{i,k}$, and outputs, $\boldsymbol{z}_{i,k}$. Collectively the system generate the output signal, $\tilde{\boldsymbol{z}}_k$, and the measurement, $\tilde{\boldsymbol{y}}_k$. For energy systems, the output signal $\tilde{\boldsymbol{z}}_k$ may represent the total net power generated by the P controllable plants.

For the dynamically decoupled system, the predicted cost at time k (17) may be specialized to

$$\begin{aligned} \phi_k &= \sum_{i=1}^{P}\sum_{j=0}^{N-1} \hat{b}'_{i,k+j+1|k}\hat{z}_{i,k+j+1|k} + \hat{c}'_{i,k+j|k}\hat{u}_{i,k+j|k} \\ &\quad + \sum_{i=1}^{P}\sum_{j=0}^{N-1} \hat{\rho}'_{i,k+j+1|k}\hat{s}_{i,k+j+1|k} \\ &\quad + \sum_{j=0}^{N-1} \hat{\tilde{\rho}}'_{k+j+1|k}\hat{\tilde{s}}_{k+j+1|k} \\ &= \sum_{i=1}^{P}\phi_{i,k} + \sum_{j=0}^{N-1} \hat{\tilde{\rho}}'_{k+j+1|k}\hat{\tilde{s}}_{k+j+1|k} \end{aligned} \tag{30}$$

with the local objective functions for $i \in \mathcal{P}$

$$\begin{aligned} \phi_{i,k} = \sum_{j=0}^{N-1} \hat{b}'_{i,k+j+1|k}\hat{z}_{i,k+j+1|k} + \hat{c}'_{i,k+j|k}\hat{u}_{i,k+j|k} \\ + \sum_{j=0}^{N-1} \hat{\rho}'_{i,k+j+1|k}\hat{s}_{i,k+j+1|k} \end{aligned} \tag{31}$$

Consequently, the linear program (19) may be formulated as the following linear program

$$\min \quad \phi_k = \sum_{i=1}^{P}\phi_{i,k} + \sum_{j=0}^{N-1} \hat{\tilde{\rho}}'_{k+j+1|k}\hat{\tilde{s}}_{k+j+1|k} \tag{32}$$

subject to the local constraints $\forall i \in \mathcal{P}$ and $\forall j \in \mathcal{N}$

$$\hat{x}_{i,k+j+1|k} = A_i\hat{x}_{i,k+j|k} + B_i\hat{u}_{i,k+j|k} + E_i\hat{d}_{k+j|k} \quad (33a)$$
$$\hat{z}_{i,k+j+1|k} = C_{z,i}\hat{x}_{i,k+j+1|k} \quad (33b)$$
$$u_{\min,i} \leq \hat{u}_{i,k+j|k} \leq u_{\max,i} \quad (33c)$$
$$\Delta u_{\min,i} \leq \Delta\hat{u}_{i,k+j|k} \leq \Delta u_{\max,i} \quad (33d)$$
$$\hat{z}_{i,k+j+1|k} + \hat{s}_{i,k+j+1|k} \geq \hat{r}_{\min,i,k+j+1|k} \quad (33e)$$
$$\hat{z}_{i,k+j+1|k} - \hat{s}_{i,k+j+1|k} \leq \hat{r}_{\max,i,k+j+1|k} \quad (33f)$$
$$\hat{s}_{i,k+j+1|k} \geq 0 \quad (33g)$$

as well as the connecting constraints $\forall j \in \mathcal{N}$

$$\hat{\tilde{z}}_{k+j+1|k} = \sum_{i=1}^{P} \tilde{C}_i \hat{x}_{i,k+j+1|k} \quad (34a)$$
$$\hat{\tilde{z}}_{k+j+1|k} + \hat{\tilde{s}}_{k+j+1|k} \geq \hat{\tilde{r}}_{\min,k+j+1|k} \quad (34b)$$
$$\hat{\tilde{z}}_{k+j+1|k} - \hat{\tilde{s}}_{k+j+1|k} \leq \hat{\tilde{r}}_{\max,k+j+1|k} \quad (34c)$$
$$\hat{\tilde{s}}_{k+j+1|k} \geq 0 \quad (34d)$$

The linear program (32)-(34) has a block-angular structure that may be used for its efficient solution using a Dantzig-Wolfe decomposition algorithm.

4. THE DANTZIG-WOLFE ALGORITHM

The Dantzig-Wolfe algorithm is a decomposition algorithm to solve large dimensions linear programming problems which have a block diagonal structure, (Dantzig and Wolfe, 1961). This decomposition technique breaks the problem into independent subproblems, which are coordinated by a master problem (MP). The units communicate only with the MP, exchanging Lagrange multipliers.

The optimization problem we investigate is a block-angular structured linear problem (35), (Hovgaard et al., 2010; Edlund et al., 2011), obtained from (32)-(33) and (34)

$$\min_{\{q_i\}_{i=1}^{P}, s} \quad \phi = \sum_{i=1}^{P} c_i' q_i + d' s \quad (35a)$$

$$s.t. \quad \begin{bmatrix} F_1 & F_2 & \dots & F_P & E \\ G_1 & & & & \\ & G_2 & & & \\ & & \ddots & & \\ & & & G_P & \\ & & & & I \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_P \\ s \end{bmatrix} \geq \begin{bmatrix} g \\ h_1 \\ h_2 \\ \vdots \\ h_P \\ 0 \end{bmatrix} \quad (35b)$$

where F_i stands for the connecting constraints and G_i for the decoupled constraints of each subsystem. The F_i are obtained introducing the impulse response matrices into (33).

Our optimization variables, q_i and s, are related to (32). It has to be noticed that while in the previous sections we consider P subsystems, here the slack variables are considered as an independent unit as well; therefore the Dantzig-Wolfe decomposition splits the control problem into $P + 1$ subsystems.

The key of this decomposition technique is the theorem of convex combinations.

Theorem 1. Let $\mathcal{Q} = \{q|\ Gq \geq h\}$ be nonempty, closed and bounded, i.e. a polytope. The extreme points of $\mathcal{Q}$ are denoted v^j with $j \in \{1, 2, ..., M\}$. Then any point q in the polytopic set $\mathcal{Q}$ can be written as a convex combination of extreme points

$$q = \sum_{j=1}^{M} \lambda_j v^j \quad (36a)$$
$$s.t. \quad \lambda_j \geq 0, \qquad j = 1, 2, ..., M \quad (36b)$$
$$\sum_{j=1}^{M} \lambda_j = 1 \quad (36c)$$

Proof. See Dantzig and Thapa (2003).

As a decomposition algorithm, the first step is defining the Master Problem, (Ho and Loute, 1981). Using the theorem of convex combinations, the polytopes are defined by (37) where v_i^j are the vertices of $\mathcal{Q}_i$, (Dantzig and Wolfe, 1961).

$$\mathcal{Q}_i = \{q_i|\ G_i q_i \geq h_i\} \quad (37a)$$
$$q_i = \sum_{j=1}^{M_i} \lambda_{ij} v_i^j \quad (37b)$$
$$\sum_{j=1}^{M_i} \lambda_{ij} = 1 \quad (37c)$$
$$\lambda_{ij} \geq 0 \qquad j = 1, 2, \dots, M_i \quad (37d)$$

The linear block angular problem (35) can be then rewritten as the equivalent Master Problem (38) for $P + 1$ subproblems

$$\min_{\lambda} \quad \phi = \sum_{i=1}^{P+1} \sum_{j=1}^{M_i} f_{ij} \lambda_{ij} \quad (38a)$$
$$s.t. \quad \sum_{i=1}^{P+1} \sum_{j=1}^{M_i} p_{ij} \lambda_{ij} \geq g \quad (38b)$$
$$\sum_{j=1}^{M_i} \lambda_{ij} = 1, \qquad i = 1, 2, ..., P+1 \quad (38c)$$
$$\lambda_{ij} \geq 0, \qquad i = 1, 2, ..., P+1;\ j = 1, 2, \dots, M_i \quad (38d)$$

f_{ij} and p_{ij} are defined as

$$f_{ij} = c_i' v_i^j \quad (39a)$$
$$p_{ij} = F_i v_i^j \quad (39b)$$

The (38) defines as well the Lagrange multipliers π from the coupling constraints (38b), ρ for (38c) and κ_{ij} from (38d). The Master Problem (38) has fewer constraints than the original optimization problem (35), but more variables as the vertices of each polytope are included. For this reason the Reduced Master Problem is introduced as a MP but with l number of vertices, where $l \leq M_i$:

$$\min_{\lambda} \quad \phi = \sum_{i=1}^{P+1} \sum_{j=1}^{l} f_{ij} \lambda_{ij} \tag{40a}$$

$$s.t. \quad \sum_{i=1}^{P+1} \sum_{j=1}^{l} p_{ij} \lambda_{ij} \geq g \tag{40b}$$

$$\sum_{j=1}^{l} \lambda_{ij} = 1, \qquad i = 1, 2, ..., P+1 \tag{40c}$$

$$\lambda_{ij} \geq 0, \qquad i = 1, 2, ..., P+1; \; j = 1, 2, \dots, l \tag{40d}$$

in which $l \leq M_i$ for all $i \in \{1, 2, \dots, P+1\}$. Obviously, the Reduced Master Problem can be regarded as the Master Problem with $\lambda_{i,j} = 0$ for $j = l+1, l+2, \dots, M_i$ and all $i \in \{1, 2, \dots, P+1\}$. Initially, a feasible extreme point to the Master Problem (38) is needed. (4.1) addresses this topic . We assume now that a feasible extreme point has been computed. We can use this feasible extreme point to form a Reduced Master Problem with $l = 1$.
We denote the solution to the Reduced Master Problem (40) as λ_{ij}^{RMP} such that a feasible solution to Master Problem (38) is

$$\lambda_{ij} = \lambda_{ij}^{RMP} \qquad i = 1, 2, \dots, P; \; j = 1, 2, \dots, l \tag{41a}$$

$$\lambda_{ij} = 0 \qquad i = 1, 2, \dots, P; j = l+1, l+2, \dots, M_i \tag{41b}$$

Analysing the Karush-Kuhn-Tucker conditions for the (38), these are stated as

$$\sum_{i=1}^{P+1} \sum_{j=1}^{M_i} p_{ij} \lambda_{ij} - g \geq 0 \quad \perp \quad \pi \geq 0 \tag{42a}$$

$$\sum_{j=1}^{M_i} \lambda_{ij} - 1 = 0 \tag{42b}$$

$$\lambda_{ij} \geq 0 \quad \perp \quad \kappa_{ij} = \left[c_i - F_i' \pi\right]' v_i^j - \rho_i \geq 0 \tag{42c}$$

They are already satisfied for $i = 1, 2, \dots, P+1$ and $j = 1, 2, \dots, l$. For all $i = 1, 2, \dots, P+1$ and $j = l+1, l+2, \dots, M_i$ instead, they are satisfied if $\min_i \psi_i - \rho_i \geq 0$ where

$$\psi_i = \min_{v_i^j} \left[c_i - F_i' \pi\right]' v_i^j \qquad i = 1, 2, \dots, P+1 \tag{43}$$

v_i^j is an extreme point of the polytope $\mathcal{Q}_i = \{q_i : G_i q_i \geq h_i\}$. Therefore, using the Simplex Algorithm we may compute the solution of (43) as the solution of the linear program

$$\psi_i = \min_{q_i} \quad \phi = \left[c_i - F_i' \pi\right]' q_i \tag{44a}$$

$$s.t. \quad G_i q_i \geq h_i \tag{44b}$$

for $i = 1, 2, \dots, P+1$. The programs (44) are called subproblems.

If $\psi_i - \rho_i \geq 0$ for all $i = 1, 2, \dots, P+1$, the solution generated by the Reduced Master Problem is optimal. We can compute the solution to original problem (35) by

$$q_i^* = \sum_{j=1}^{l} v_i^j \lambda_{ij} \qquad i = 1, 2, \dots, P+1 \tag{45}$$

Instead if $\psi_i - \rho_i < 0$ for some $i \in \{1, 2, \dots, P+1\}$ then the KKT conditions are not satisfied and the solution generated by the Reduced Master Problem is not a solution to the Master Problem. In this case, we augment the Reduced Master Problem with the new extreme points, v_i^{l+1}, obtained by solution of the subproblems (44).

The next iteration of the algorithm starts with the solution of the new Reduced Master Problem. The algorithm terminates in a finite number of iterations as there is a finite number of extreme points in a polytope.

Algorithm 2 Dantzig-Wolfe

Compute the initial feasible vertex for the Master Problem (38).
If any points is found then stop.
l=1. Converged=false
while $Converged == false$ **do**
Solve the $l-th$ RMP (40)
Solve all the subproblems ($i = 1, 2, ..., P+1$) (44) considering the π from (40b) and ρ_i from (40c).
  if $\phi_i - \rho_i \geq 0 \quad \forall i \in \{1, 2, ..., P+1\}$ **then**
    Coverged = true
    The optimal solution is given by (45)
  else
    Compute the coefficients for the new columns in the RMP

$$f_{i,l+1} = c_i' v_i^{l+1}$$
$$p_{i,l+1} = F_i v_i^{l+1}$$

    $l = l+1$
  end if
end while

4.1 Initial feasible vertex

In (Dantzig and Thapa, 2003) the initial feasible solution for the Master Problem is obtained by Phase I procedure. A feasible vertex of the block angular linear program (35) is identical to a feasible vertex of the Master Problem (38) as these two linear programs are different representations of the same problem. The initial feasible vertex of the Master Problem (38) may be computed solving the following linear program

$$\min_{\alpha, \{q_i, \beta_i\}_{i=1}^{P+1}} \quad \phi_I = \mathbf{e}_\alpha' \alpha + \sum_{i=1}^{P+1} \mathbf{e}_{\beta_i}' \beta_i \tag{46a}$$

$$s.t. \quad \sum_{i=1}^{P+1} F_i q_i + R\alpha \geq g \tag{46b}$$

$$G_i q_i + S_i \beta_i \geq h_i \qquad i = 1, 2, \dots, P+1 \tag{46c}$$

$$0 \leq \alpha \leq |g| \tag{46d}$$

$$0 \leq \beta_i \leq |h_i| \qquad i = 1, 2, \dots, P+1 \tag{46e}$$

with R and S diagonal matrices defined for $i = j$ and $p = q$ as

$$R_{ij} = \begin{cases} 1 & g_i \geq 0 \\ -1 & g_i < 0 \end{cases} \quad (S_i)_{p,q} = \begin{cases} 1 & (h_i)_p \geq 0 \\ -1 & (h_i)_p < 0 \end{cases}$$

It should be noticed that the computation of a feasible vertex of (38), i.e. solution of (46) by the Dantzig-Wolfe algorithm, is of approximately the same computational complexity as the computation of the optimal solution when a feasible vertex is available. This means that we can utilise the block-angular structure efficiently in the computation of a feasible vertex. It also means that just finding a feasible vertex may be just as expensive as computing the optimal solution. Therefore, if a feasible vertex is readily available, it should be used directly instead of applying a phase I simplex procedure.
A feasible initial vertex for our problem (35) may be defined as

$$\{q_i^0 = q_{i,min}\}_{i=1}^{P} \quad s = max\left\{g - \sum_{i=1}^{P} F_i q_i^0, 0\right\} \tag{47}$$

The Dantzig-Wolfe algorithm is a part of a MPC controller, so the previous solution is always available and it can be used to compute the initial vertex as well.

5. RESULTS

In this section we provide an example of a controller which implements the Dantzig-Wolfe algorithm for energy systems.
The algorithm developed is compared to a centralized MPC controller. We consider a scenario of distributed energy system (DES) with several power generators.

5.1 Closed-loop simulations

We implement the Dantzig-Wolfe decomposition in solving the linear program (19) as described in Algorithm (1). The simulation runs over an horizon of 100 time steps. Here the benchmark is an energy system with two power plants, where both process noise and measurements noise are affecting the system. In this case Figure (1) demonstrates that the total portfolio output follows the reference.

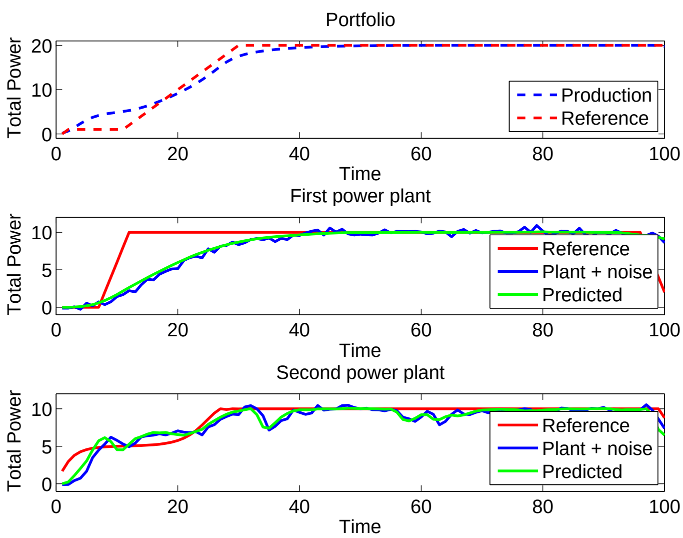

Fig. 1. Closed loop simulation.

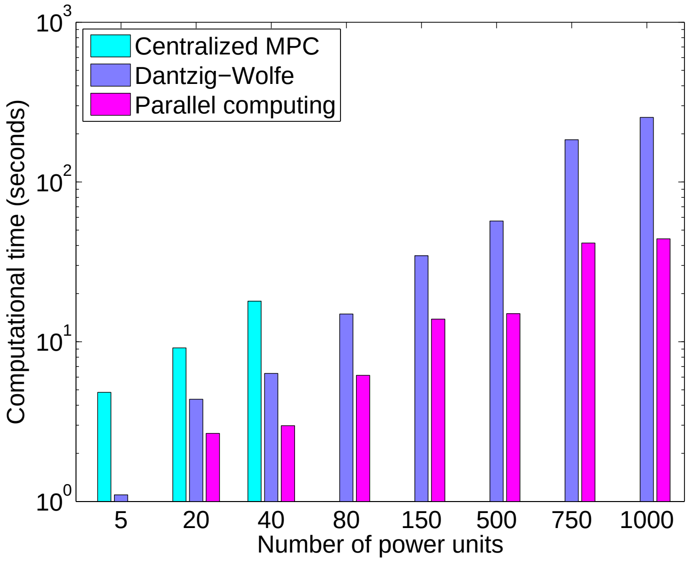

Fig. 2. Centralized MPC and Dantzig-Wolfe computational times.

5.2 Computational time

To investigate how the Dantzig-Wolfe perform in controlling large-scale energy systems, we compare it to a centralized MPC controller in open loop simulations. The latter fails in solving the control problem where the number of power units is high, i.e. more than 60 power generators due to the large size of the problem, as depicted in Figure 2. It appears that implementing the Dantzig-Wolfe algorithm, solves quicker the control problem compared to a centralized MPC controller. Furthermore in the Dantzig-Wolfe decomposition, the subproblems (44) can be solved in parallel; such way of computing reduces the computational time as Figure 2 demonstrates.

6. CONCLUSIONS

In this paper we have developed a controller for large scale energy systems. All the power units are dynamically decoupled. In this way, the control problem shows a block-angular structure which allows the implementation of decomposition techniques.
The controller obtained solves the control problem (35) implementing the Dantzig-Wolfe decomposition algorithm. Under such control action, the manipulated output follows the reference even when noises are affecting the system. This approach has potential in large-scale systems, as the computational time taken is lower compared to a centralized MPC controller. Furthermore the Dantzig-Wolfe algorithm allows parallel computing which improves speed of the algorithm.

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