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# Reconfigurable Control of Input Affine Nonlinear Systems under Actuator Fault

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**Abstract:** This paper proposes a fault tolerant control method for input-affine nonlinear systems using a nonlinear reconfiguration block (RB). The basic idea of the method is to insert the RB between the plant and the nominal controller such that fault tolerance is achieved without re-designing the nominal controller. The role of the RB is twofold: on one hand it transforms the output of the faulty system such that its behaviour is similar to that of the nominal one from the controller's viewpoint; on the other hand it modifies the control input to the faulty system such that the stability of the reconfigured loop is preserved. The RB is realized by a virtual actuator and a reference model. Using notions of incremental and input-to-state stability (ISS), it is shown that ISS of the closed-loop reconfigured system can be achieved by the separate design of the virtual actuator. The proposed method does not need any knowledge of the nominal controller and only assumes that the nominal closed-loop system is ISS. The method is demonstrated on a dynamic positioning system for an offshore supply vessel, where the virtual actuator is designed using backstepping.



## 1. INTRODUCTION

With the ever-increasing requirement on safety, reliability, availability, and performance of industrial systems, it is essential to design controllers that can tolerate occurrence of some faults without interrupting the operation while preserving the system stability, functionality, and simultaneously providing acceptable performance. Such controllers are called fault-tolerant. The area of fault-tolerant control (FTC) has attracted a lot of attention in the past two decades, see e.g. the review papers (Blanke et al., 1997; Patton, 1997; Jiang, 2005) and books (Isermann, 2006; Blanke et al., 2006).

FTC methods can broadly be divided into two classes: passive (PFTC) and active (AFTC). In PFTC, the structure and the parameters of the controller are fixed and pre-designed such that during operations the system is robust towards occurrence of a given set of faults. Hence, a PFTC solution is a common solution to the control problem for the nominal and the given set of faulty systems. However a common solution may not always exist, specially if severe faults are considered; when it exists it may be too conservative and result in low performance for the nominal operation. On the other hand, in AFTC the controller is not fixed and reacts to the occurrence of faults during operations by adjusting its parameters or structure. A fault detection and identification (FDI) module is designed to detect and identify the occurred fault and then based on this information, the controller is modified for the identified faulty system. Consequently, AFTC can usually provide better performance.

Most of the AFTC methods available in the literature rely on a batch of controllers designed for each considered faulty case. When the occurred fault is identified the nominal controller is replaced by the controller specifically designed for this faulty scenario.

The idea proposed in this paper is to keep the nominal controller in the loop and design a reconfiguration block, which is inserted between the faulty system and the nominal controller to guarantee the stability of the reconfigured closed-loop system. This idea, depicted in Fig. 1, is known as reconfiguration through fault-hiding. The goal of the reconfiguration block is twofold: it transforms the output of the faulty system such that from the viewpoint of the controller it has a similar behaviour to that of the nominal system; it changes the input from the nominal controller such that the stability of the reconfigured loop is guaranteed. The reconfiguration block is respectively realized by a virtual sensor, a virtual actuator, or a series connection of both of them in case of a sensor fault, an actuator fault, or a simultaneous sensor and actuator fault.

The idea of control reconfiguration using virtual sensors and actuators was proposed by Steffen (2005) for linear systems. Lunze and Steffen (2006) showed that control reconfiguration of a linear system after an actuator fault is equivalent to disturbance decoupling. Control reconfiguration methods using virtual actuators and sensors for piecewise affine systems and Hammerstein-Wiener systems were proposed in Richter et al. (2008), Richter and Lunze (2008), Richter et al. (2011) and Richter (2011).

AFTC for Lur'e systems with Lipschitz continuous nonlinearity subject to actuator fault using a virtual actuator was presented in Richter et al. (2012), where it was as-


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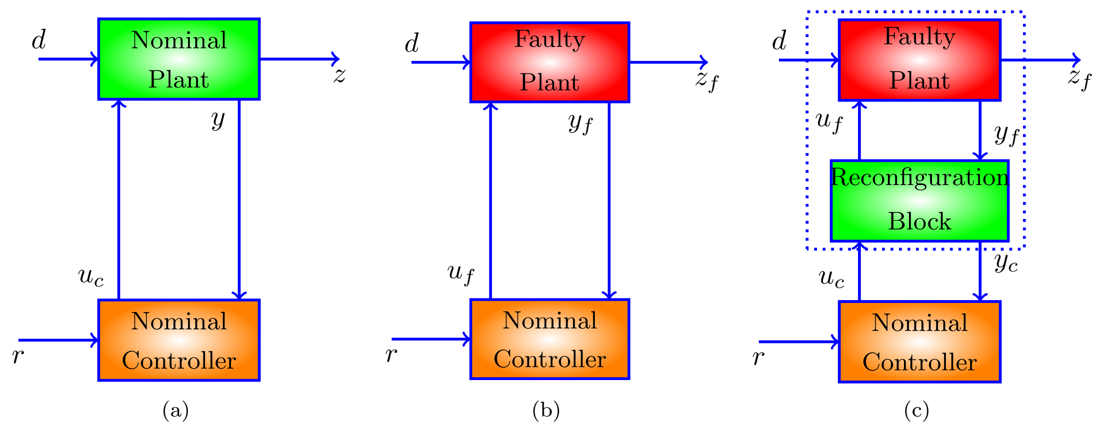


Fig. 1. Fault-tolerant control using a reconfiguration block: (a) nominal loop, (b) faulty plant with nominal controller, (c) reconfigured plant with nominal controller

sumed that the state of the faulty system is measurable. AFTC for a system with additive Lipschitz nonlinearity subject to actuator faults using a virtual actuator was presented in Khosrowjerdi and Barzegary (2013). Pedersen et al. (2014) proposed a new design method for the virtual actuator based on absolute stability theory, which was tested for the reconfiguration of power systems subject to faults in local controllers in emergency situations.

Fault tolerant control of polytopic linear parameter varying (LPV) systems subject to sensor faults using virtual sensor was proposed in de Oca and Puig (2010), where the structure of the nominal controller was assumed to be known. It was further assumed that the nominal controller consists of a state feedback combined with an LPV observer. Tabatabaeipour et al. (2012) considered the problem of control reconfiguration for continuous-time LPV systems with both sensor and actuator faults and without any assumptions about the structure of the nominal controller. In this context input-to-state stability properties of the reconfigured system were investigated. In Tabatabaeipour et al. (2014) the control reconfiguration for discrete-time LPV systems with both sensor and actuator faults were considered and both stability and performance of the reconfiguration block was investigated.

In this paper we extend the idea of reconfigurable control design using a reconfiguration block to input-affine nonlinear dynamical systems. Only actuator faults are considered, and the reconfiguration block is realized by a nonlinear virtual actuator. Using incremental stability properties, it is shown how to design the nonlinear virtual actuator independent of the nominal controller to achieve ISS of the reconfigured closed-loop system. The main contributions are given in Theorems 12, 13 and Corollary 14. The proposed method does not require any information about the nominal controller and only assumes that the nominal closed-loop system is ISS. The design of a fault-tolerant dynamic positioning system for an offshore supply vessel is utilised as case study. The design of the nonlinear virtual actuator is demonstrated using backstepping control. The simulation results show the effectiveness of the proposed method.

## 2. PRELIMINARIES

The field of real numbers and the set of nonnegative reals are respectively denoted by $\mathbb{R}$, and $\mathbb{R}_{\geq 0}$. For any vector $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{x}^{\mathrm{T}}$ stands for its transpose and $\|\mathbf{x}\| = \sqrt{\mathbf{x}^{\mathrm{T}}\mathbf{x}}$ denotes its Euclidean norm. Also, the $i$-th entry of $\mathbf{x}$ is denoted by $x_i$. The infinity norm of $\mathbf{x}$ denoted by $\|\mathbf{x}\|_\infty$ is given by $\max_i |x_i|$. Given a measurable function $\mathbf{u} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, its (essential) supremum is denoted by $\|\mathbf{u}\|_\infty$ which is defined as: $\|\mathbf{u}\|_\infty := \text{(ess)sup}\{\|\mathbf{u}(t)\|, t \geq 0\}$. The function $\mathbf{u}$ is essentially bounded if $\|\mathbf{u}\|_\infty < \infty$.

The function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is called a class $\mathcal{K}$ function denoted by $\alpha \in \mathcal{K}$ if it is continuous, strictly increasing, unbounded and satisfies $\alpha(0) = 0$. The function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is called a class $\mathcal{KL}$ function denoted by $\beta \in \mathcal{KL}$ if $\beta(\cdot, t) \in \mathcal{K}$ and $\beta(r, t) \to 0$ as $t \to \infty$.

Consider the following nonlinear system

$$\Sigma : \begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)), \quad \mathbf{x}(0) = \mathbf{x}_0, \\ \mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t)), \end{cases} \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the state, $\mathbf{u}(t) \in \mathbb{R}^m$ is the input, $\mathbf{y}(t) \in \mathbb{R}^q$ is the output. We use the following stability definitions.

*Definition 1.* **0-global asymptotic stability** (Sontag (2008)) The system (1) with $\mathbf{u}(t) = 0$, $\forall t \in \mathbb{R}_{\geq 0}$ is called 0-globally asymptotically stable (0-GAS) if there exists a function $\beta \in \mathcal{KL}$ such that for all $t_0$ and $\mathbf{x}(t_0)$, the solution of the system satisfies

$$\|\mathbf{x}(t)\| \leq \beta(\|\mathbf{x}(t_0)\|, t). \tag{2}$$

*Definition 2.* **Input-to-state stability** (Sontag (2008)) The system (1) is called input-to-state stable (ISS) with respect to (w.r.t.) the input $\mathbf{u}(t)$ if there exist some $\beta \in \mathcal{KL}$ and some $\gamma \in \mathcal{K}$ such that for all $t_0$ and $\mathbf{x}(t_0)$ and all inputs $\mathbf{u}(t)$, all solutions of the system satisfy

$$\|\mathbf{x}(t)\| \leq \beta(\|\mathbf{x}(t_0)\|, t) + \gamma(\|\mathbf{u}(t)\|_\infty). \tag{3}$$

*Definition 3.* **Input-to-output stability** (Sontag (2008)) The system (1) is called input-to-ouput stable (IOS) w.r.t. the input $\mathbf{u}(t)$ and the output $\mathbf{y}(t)$ if there exist some $\beta \in \mathcal{KL}$ and some $\gamma \in \mathcal{K}$ such that for all $t_0$ and $\mathbf{x}(t_0)$ and all inputs $\mathbf{u}(t)$, the output of the system satisfies

$$\|\mathbf{y}(t)\| \leq \beta(\|\mathbf{x}(t_0)\|, t) + \gamma(\|\mathbf{u}(t)\|_\infty). \tag{4}$$

*Theorem 4.* **IOS of interconnected systems** (Jiang et al. (1994)) Consider the following interconnected systems:

$$\begin{cases} \dot{\mathbf{x}}_2 = \mathbf{f}_2(\mathbf{x}_2(t), \mathbf{y}_1(t), \mathbf{u}(t)) \\ \mathbf{y}_2 = \mathbf{h}_2(\mathbf{x}_2(t), \mathbf{y}_1(t), \mathbf{u}(t)) \\ \dot{\mathbf{x}}_1 = \mathbf{f}_1(\mathbf{x}_1(t), \mathbf{u}(t)) \\ \mathbf{y}_1 = \mathbf{h}_1(\mathbf{x}_1(t), \mathbf{u}(t)) \end{cases} \tag{5}$$

Assume that the first system is IOS w.r.t. the input $\mathbf{u}$ and the output $\mathbf{y}_1$ and the second system is IOS w.r.t. the input $(\mathbf{y}_1, \mathbf{u})$ and output $\mathbf{y}_2$. Then the interconnected system is IOS w.r.t. the input $\mathbf{u}$ and outputs $(\mathbf{y}_1, \mathbf{y}_2)$.

Next, we recall the definition of incremental stability. Incremental stability considers the stability and convergence of the trajectories with respect to each other rather than to an equilibrium point.

*Definition 5.* [Zamani and Tabuada (2011)] The nonlinear system (1) is incrementally globally asymptotically stable ($\delta$-GAS) if there exist a metric $d$ and a class $\mathcal{KL}$ function $\beta$ such that for all locally essentially bounded $\mathbf{u}$, all initial conditions $\boldsymbol{\xi}_0, \boldsymbol{\eta}_0 \in \mathbb{R}^n$ and all $t \geq 0$ the following inequality is satisfied

$$d(\mathbf{x}(t, \boldsymbol{\xi}_0, \mathbf{u}), \mathbf{x}(t, \boldsymbol{\eta}_0, \mathbf{u})) \leq \beta(d(\boldsymbol{\xi}_0, \boldsymbol{\eta}_0), t) \tag{6}$$

If the origin is an equilibrium point for (1), then $\delta$-GAS implies 0-GAS.

*Definition 6.* [Zamani and Tabuada (2011)] The nonlinear system (1) is incrementally input-to-state stable ($\delta$-ISS) if there exist a metric $d$, a class $\mathcal{KL}$ function $\beta$ and a class $\mathcal{K}_\infty$ function $\gamma$ such that for all inputs $\mathbf{u}_1, \mathbf{u}_2$, all initial conditions $\boldsymbol{\xi}_0, \boldsymbol{\eta}_0 \in \mathbb{R}^n$ and all $t \geq 0$ the following inequality holds true

$$d(\mathbf{x}(t, \boldsymbol{\xi}_0, \mathbf{u}_1), \mathbf{x}(t, \boldsymbol{\eta}_0, \mathbf{u}_2)) \leq \beta(d(\boldsymbol{\xi}_0, \boldsymbol{\eta}_0), t) + \gamma(\|\mathbf{u}_1 - \mathbf{u}_2\|_\infty). \tag{7}$$

From (6) and (7) it is straightforward to see that $\delta$-ISS implies $\delta$-GAS, but the converse is not true in general. Moreover it is concluded that if the origin is an equilibrium point for the system (1) then $\delta$-ISS implies ISS.

*Remark 7.* Definitions 5 and 6 are invariant under changes of coordinates because they are based on the existence of a generic metric $d$, not necessarily Euclidean. This coordinate invariance was not included in the former definitions provided by Angeli (2002) where only the Euclidean metric was considered.

Lyapunov characterizations of $\delta$-GAS and $\delta$-ISS were first presented in (Angeli, 2002), however those were not invariant under change of coordinates. Later, Zamani and Majumdar (2011) proposed the following Lyapunov characterizations that are coordinate independent.

*Theorem 8.* (Zamani and Majumdar (2011)). The nonlinear system (1) is $\delta$-GAS if there exist a function $V(\mathbf{x}_1, \mathbf{x}_2)$, a metric $d$, and class $\mathcal{K}_\infty$ functions $\alpha_1$ and $\alpha_2$ such that

$$\alpha_1(d(\mathbf{x}_1, \mathbf{x}_2)) \leq V(\mathbf{x}_1, \mathbf{x}_2) \leq \alpha_2(d(\mathbf{x}_1, \mathbf{x}_2)), \tag{8}$$

and for any $\mathbf{x}_1$ and $\mathbf{x}_2 \in \mathbb{R}^n$ and any $\mathbf{u} \in \mathcal{U}$ it is hold that

$$\frac{\partial V}{\partial \mathbf{x}_1}\mathbf{f}(\mathbf{x}_1, \mathbf{u}) + \frac{\partial V}{\partial \mathbf{x}_2}\mathbf{f}(\mathbf{x}_2, \mathbf{u}) \leq -\kappa V(\mathbf{x}_1, \mathbf{x}_2), \tag{9}$$

with $\kappa > 0$.

In case $V$ is a quadratic function, the system is called quadratically incrementally stable ($\delta$-QS). Similarly for $\delta$-ISS we have the following Lyapunov characterization.

*Theorem 9.* (Zamani and Majumdar (2011)). The nonlinear system (1) is said to be $\delta$-ISS if there exist a function $V(\mathbf{x}_1, \mathbf{x}_2)$, a metric $d$ and class $\mathcal{K}_\infty$ functions $\alpha_1$, $\alpha_2$, and $\rho$ such that

$$\alpha_1(d(\mathbf{x}_1, \mathbf{x}_2)) \leq V(\mathbf{x}_1, \mathbf{x}_2) \leq \alpha_2(d(\mathbf{x}_1, \mathbf{x}_2)), \tag{10}$$

and for any $\mathbf{u}_1, \mathbf{u}_2 \in \mathcal{U}$ and $\mathbf{x}_1, \mathbf{x}_2 \in \mathbb{R}^n$:

$$\frac{\partial V}{\partial \mathbf{x}_1}\mathbf{f}(\mathbf{x}_1, \mathbf{u}_1) + \frac{\partial V}{\partial \mathbf{x}_2}\mathbf{f}(\mathbf{x}_2, \mathbf{u}_2) \leq -\kappa V(\mathbf{x}_1, \mathbf{x}_2) + \rho(\|\mathbf{u}_1 - \mathbf{u}_2\|), \tag{11}$$

with $\kappa > 0$.

## 3. NOMINAL AND FAULTY NONLINEAR SYSTEM

Consider the fault-free plant $\Sigma_P$

$$\Sigma_P : \begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{B}\mathbf{u}_c + \mathbf{B}_d\mathbf{d}, \quad \mathbf{x}(t_0) = \mathbf{x}_0 \\ \mathbf{y} = \mathbf{C}\mathbf{x} \\ \mathbf{z} = \mathbf{C}_z\mathbf{x} \end{cases} \tag{12}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state, $\mathbf{u}_c \in \mathcal{U} \subset \mathbb{R}^m$ is the control input, $\mathbf{d} \in \mathcal{D} \subset \mathbb{R}^k$ is the input disturbance, $\mathbf{y} \in \mathbb{R}^q$ is the measured output, and $\mathbf{z} \in \mathbb{R}^p$ is the controlled output.

The vector field $\mathbf{f}(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ is continuously differentiable and locally Lipschitz in a domain $\mathcal{X} \subset \mathbb{R}^n$. This guarantees existence and uniqueness of the solution $\mathbf{x}(t, \mathbf{x}_0, \mathbf{u}_c, \mathbf{d})$ for any initial condition $\mathbf{x}_0$ and for all $t \geq t_0$. The state of the system (12) is assumed to be fully accessible to the measurement; hence the output matrix $\mathbf{C} = \mathbf{I}_n$, where $\mathbf{I}_n$ is the $n$-dimensional identity matrix.

The nominal nonlinear dynamical state feedback controller $\Sigma_C$ is

$$\Sigma_C : \begin{cases} \dot{\mathbf{x}}_c = \mathbf{f}_c(\mathbf{x}_c, \mathbf{x}), \quad \mathbf{x}_c(t_0) = \mathbf{x}_{c,0}, \\ \mathbf{u}_c = \boldsymbol{\alpha}(\mathbf{x}_c, \mathbf{x}) \end{cases} \tag{13}$$

where $\mathbf{x}_c \in \mathbf{R}^n$ is the controller state, $\mathbf{f}_c(\cdot,\cdot) : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is locally Lipschitz, and $\boldsymbol{\alpha}(\cdot,\cdot) : \mathbb{R}^n \times \mathbb{R}^n \to \mathcal{U} \subset \mathbb{R}^m$ is a smooth mapping.

*Assumption 10.* **ISS of the nominal closed-loop system:** Let $\mathbf{d}(t)$ be a bounded input disturbance, i.e. $\|\mathbf{d}(t)\|_\infty \leq \bar{d}$. The nominal fault-free closed-loop system $\Sigma_L = (\Sigma_P, \Sigma_C)$ composed by the fault-free nonlinear plant (12) and the nonlinear controller (13) is input-to-state stable with respect to the disturbance $\mathbf{d}(t)$.

Let now assume that at time $t_f > t_0$ an actuator fault occurs in the nominal nonlinear plant (12). As a consequence the input matrix $\mathbf{B}$ changes to

$$\mathbf{B}_f = \mathbf{B}\boldsymbol{\theta}, \tag{14}$$

where $\boldsymbol{\theta} = \mathrm{diag}(\theta_1, \theta_2, \ldots, \theta_m)$ with $\theta_i \in (0, 1]$ being an unknown parameter giving the level of control authority of each actuator after a fault has occurred.

The faulty nonlinear plant dynamics $\Sigma_{P_f}$ initialized at $\mathbf{x}_f(t_f) = \mathbf{x}(t_f)$ is given by

$$\Sigma_{P_f} : \begin{cases} \dot{\mathbf{x}}_f = \mathbf{f}(\mathbf{x}_f) + \mathbf{B}_f\mathbf{u}_f + \mathbf{B}_d\mathbf{d} \\ \mathbf{y}_f = \mathbf{x}_f \\ \mathbf{z}_f = \mathbf{C}_z\mathbf{x}_f \end{cases} \tag{15}$$

where $\mathbf{x}_f \in \mathbb{R}^n$ is the state of the faulty system, $\mathbf{u}_f \in \mathcal{U} \subset \mathbb{R}^m$ is the faulty control input, $\mathbf{y}_f \in \mathbb{R}^q$ and $\mathbf{z}_f \in \mathbb{R}^p$ are the measured and controlled outputs of the faulty system, respectively.

## 4. RECONFIGURATION PROBLEM

After occurrence it is assumed that the actuator fault is detected, isolated and its magnitude estimated within a time $t_d$ by an FDI unit. Once detection and isolation is achieved, in the proposed method for AFTC the nominal controller is kept in the loop and a reconfiguration block is inserted between it and the faulty plant. The RB is a dynamical system that receives the output of the faulty plant $\mathbf{y}_f$ and the output of the nominal controller $\mathbf{u}_c$ as its input, and produces the input to the faulty system $\mathbf{u}_f$ and the input to the nominal controller $\mathbf{y}_c$, see Fig. 1.

The nonlinear dynamics of the reconfiguration block can in general be represented as

$$\Sigma_R : \begin{cases} \dot{\mathbf{x}}_r = \mathbf{f}(\mathbf{x}_r, \mathbf{u}_c, \mathbf{y}_f), \\ \mathbf{u}_f = \mathbf{h}_r(\mathbf{x}_r, \mathbf{u}_c), \\ \mathbf{y}_c = \mathbf{h}_{ry}(\mathbf{z}, \mathbf{y}_f). \end{cases} \tag{16}$$

where $\mathbf{x}_r$ is an internal state. The RB must be designed such that the overall closed-loop system $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ is stable, and its performance fulfils some requirements. Different goals can be considered in the design of the reconfiguration block. In this paper, the focus is on the *stability recovery problem*.

*Problem 11.* Consider the nominal nonlinear system $\Sigma_P$ and the faulty nonlinear system $\Sigma_{P_f}$. Design, if possible, a reconfiguration block $\Sigma_R$ such that for all nominal controllers $\Sigma_C$ that render $\Sigma_L$ ISS w.r.t. $\mathbf{d}$, the closed-loop reconfigured system $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ is ISS w.r.t. $\mathbf{d}$.

## 5. STABILITY RECOVERY THROUGH FAULT HIDING

In this work, the RB is realized by a virtual actuator. After a fault has occurred the nominal nonlinear controller (13) may not be able to guarantee closed-loop stability and/or performance, hence a reconfiguration of the controller is needed. The reconfiguration is based on the design of a *virtual actuator*, i.e. an intermediate system that interfaces both with the faulty plant $\Sigma_{P_f}$ and the nominal controller $\Sigma_C$ such that control system keeps seeing the output of the nominal fault-free plant $\Sigma_P$, and the input to the faulty plant is compensated for the presence of the fault. The structure of the reconfigured loop with the virtual actuator is depicted in Fig. 2.

The nonlinear virtual actuator is given by

$$\Sigma_A : \begin{cases} \dot{\tilde{\mathbf{x}}} = \mathbf{f}(\tilde{\mathbf{x}}) + \mathbf{B}\mathbf{u}_c, \quad \tilde{\mathbf{x}}(t_0) = \mathbf{x}_0 \\ \mathbf{u}_f = \boldsymbol{\alpha}_\Delta(\tilde{\mathbf{x}}) - \boldsymbol{\alpha}_\Delta(\mathbf{x}_f) + \mathbf{N}\mathbf{u}_c \\ \mathbf{y}_c = \mathbf{x}_c \end{cases} \tag{17}$$

where $\tilde{\mathbf{x}} \in \mathbb{R}^n$ is the state of a reference model providing the trajectory of the fault-free plant $\Sigma_P$ in the absence of input disturbances, $\mathbf{d}(t) = 0$, and $\mathbf{N}$ is a gain matrix that feed-forwards the control input $\mathbf{u}_c$ to the plant input $\mathbf{u}_f$.

To analyse the stability of the reconfigured system we introduce the difference state $\mathbf{x}_\Delta = \tilde{\mathbf{x}} - \mathbf{x}_f$. The dynamics of the difference state is then given by

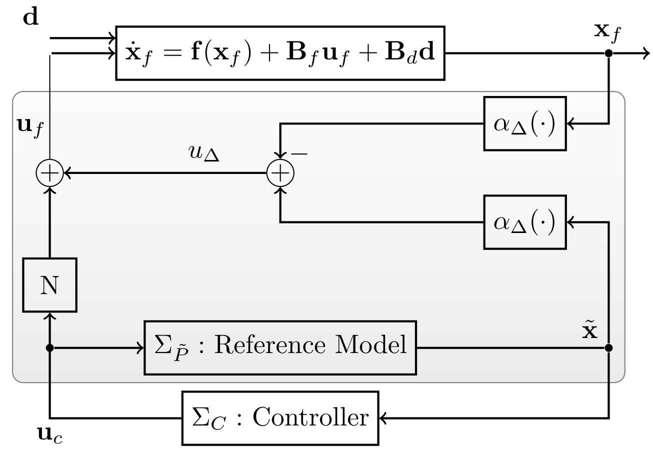


Fig. 2. Structure of the Reconfiguration Block

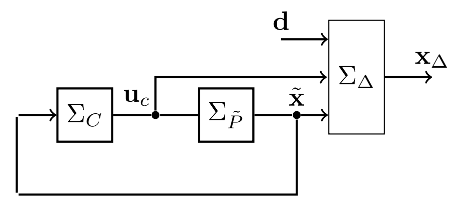


Fig. 3. Closed-loop reconfigured system as series interconnection of $(\Sigma_C, \Sigma_{\tilde{P}})$ and $\Sigma_\Delta$

$$\begin{aligned} \dot{\mathbf{x}}_\Delta &= \dot{\tilde{\mathbf{x}}} - \dot{\mathbf{x}}_f \\ &= \mathbf{f}(\tilde{\mathbf{x}}) + \mathbf{B}\mathbf{u}_c - [\mathbf{f}(\mathbf{x}_f) + \mathbf{B}_f\mathbf{u}_f + \mathbf{B}_d\mathbf{d}] \\ &= \mathbf{f}(\tilde{\mathbf{x}}) - \mathbf{B}_f\boldsymbol{\alpha}_\Delta(\tilde{\mathbf{x}}) - [\mathbf{f}(\mathbf{x}_f) - \mathbf{B}_f\boldsymbol{\alpha}_\Delta(\mathbf{x}_f)] \\ &\quad + (\mathbf{B} - \mathbf{B}_f\mathbf{N})\mathbf{u}_c - \mathbf{B}_d\mathbf{d} \\ &= \mathbf{f}(\tilde{\mathbf{x}}) - \mathbf{B}_f\boldsymbol{\alpha}_\Delta(\tilde{\mathbf{x}}) - [\mathbf{f}(\tilde{\mathbf{x}} - \mathbf{x}_\Delta) - \mathbf{B}_f\boldsymbol{\alpha}_\Delta(\tilde{\mathbf{x}} - \mathbf{x}_\Delta)] \\ &\quad + (\mathbf{B} - \mathbf{B}_f\mathbf{N})\mathbf{u}_c - \mathbf{B}_d\mathbf{d} \\ &= \boldsymbol{\kappa}_\Delta(\tilde{\mathbf{x}}) - \boldsymbol{\kappa}_\Delta(\tilde{\mathbf{x}} - \mathbf{x}_\Delta) + (\mathbf{B} - \mathbf{B}_f\mathbf{N})\mathbf{u}_c - \mathbf{B}_d\mathbf{d} \end{aligned} \tag{18}$$

where $\boldsymbol{\kappa}_\Delta(\boldsymbol{\xi}) \triangleq \mathbf{f}(\boldsymbol{\xi}) - \mathbf{B}_f\boldsymbol{\alpha}_\Delta(\boldsymbol{\xi})$.

In the following, we show the conditions for ISS of the difference system and we show that if the virtual actuator is designed independently such that the difference system is ISS, then the reconfigured closed-loop system is also ISS.

*Theorem 12.* (**Reconfigured system stability**) Consider the reconfigured closed-loop system $(\Sigma_{P_f}, \Sigma_A, \Sigma_C)$. If the nominal closed-loop system $\Sigma_L$ is ISS and the virtual actuator $\Sigma_A$ is designed such that the difference system $\Sigma_\Delta$ is ISS, then the reconfigured closed-loop system is ISS.

**Proof.** Introducing the new variable $\mathbf{x}_\Delta$ the dynamics of the closed-loop reconfigured system $(\Sigma_{P_f}, \Sigma_A, \Sigma_C)$ in new variables is re-written by:

$$\begin{aligned} &\Sigma_{\tilde{P}} : \begin{cases} \dot{\tilde{\mathbf{x}}} = \mathbf{f}(\tilde{\mathbf{x}}) + \mathbf{B}\mathbf{u}_c \\ \mathbf{y}_c = \mathbf{x}_c \end{cases} \\ &\Sigma_C : \begin{cases} \dot{\mathbf{x}}_c = \mathbf{f}_c(\mathbf{x}_c, \mathbf{y}_c) \\ \mathbf{u}_c = \boldsymbol{\alpha}(\mathbf{x}_c, \mathbf{y}_c) \end{cases} \\ &\Sigma_\Delta : \begin{cases} \dot{\mathbf{x}}_\Delta = \boldsymbol{\kappa}_\Delta(\tilde{\mathbf{x}}) - \boldsymbol{\kappa}_\Delta(\tilde{\mathbf{x}} - \mathbf{x}_\Delta) \\ \qquad\quad + (\mathbf{B} - \mathbf{B}_f\mathbf{N})\mathbf{u}_c - \mathbf{B}_d\mathbf{d} \end{cases} \end{aligned} \tag{19}$$

which is graphically depicted in Figure 3.

By Assumption 10 the nominal closed-loop system $\Sigma_L$ is ISS, hence also the closed-loop system $(\Sigma_{\tilde{P}}, \Sigma_C)$ is ISS since the reference model $\Sigma_{\tilde{P}}$ is a copy of the nominal open loop system $\Sigma_P$. By designing the virtual actuator $\Sigma_A$ such that it renders the dynamics of the difference system $\Sigma_\Delta$ ISS, then the reconfigured closed-loop system is ISS by Theorem 4.

### 5.1 Stability Analysis of the Difference System

The stability analysis of the difference system (18) is carried out within the framework of incremental stability theory.

Consider the system

$$\dot{\boldsymbol{\xi}} = \boldsymbol{\kappa}_\Delta(\boldsymbol{\xi}) + \mathbf{u}_1 = \mathbf{f}(\boldsymbol{\xi}) - \mathbf{B}_f\boldsymbol{\alpha}_\Delta(\boldsymbol{\xi}) + \mathbf{u}_1 \quad (20)$$

where $\boldsymbol{\xi} \in \mathbb{R}^n$ is the state, $\mathbf{f}(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ is the vector field defining the fault-free and the faulty plant, $\boldsymbol{\alpha}_\Delta(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear function of the state $\boldsymbol{\xi}$ to be designed, and $\mathbf{u}_1 = \vartheta[(\mathbf{B} - \mathbf{B}_f\mathbf{N})\mathbf{u}_c - \mathbf{B}_d\mathbf{d}]$ with $0 < \vartheta < 1$ .

*Theorem 13.* [**ISS of the difference system**] Consider the faulty nonlinear plant (15). If there exists a nonlinear stabilizing function $\boldsymbol{\alpha}_\Delta(\cdot)$ such that the $\boldsymbol{\xi}$-dynamics is $\delta$-ISS then the difference system (18) is ISS w.r.t. the nominal controller input $\mathbf{u}_c$ and the disturbance $\mathbf{d}$.

**Proof.** If the $\boldsymbol{\xi}$-dynamics is $\delta$-ISS then there exist $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}_\infty$ such that for any $t \geq 0$, any pair of initial conditions $(\boldsymbol{\xi}_0, \boldsymbol{\eta}_0)$, and any pair of input signals $(\mathbf{u}_1, \mathbf{u}_2)$ the following inequality is satisfied

$$\|\boldsymbol{\xi}(t, \boldsymbol{\xi}_0, u_1) - \boldsymbol{\eta}(t, \boldsymbol{\eta}_0, u_2)\| \leq \beta(\|\boldsymbol{\xi}_0 - \boldsymbol{\eta}_0\|, t) + \gamma(\|\mathbf{u}_1 - \mathbf{u}_2\|_\infty) \quad (21)$$

Let $\boldsymbol{\xi}_0 = \tilde{\mathbf{x}}_0$ and $\boldsymbol{\eta}_0 = \tilde{\mathbf{x}}_0 - \mathbf{x}_{\Delta,0}$. Then the $\mathbf{x}_\Delta$-dynamics (18) is given by the linear combination of the solutions of the $\boldsymbol{\xi}$-dynamics (20) for the two initial conditions $\boldsymbol{\xi}_0$, $\boldsymbol{\eta}_0$ and the two inputs $\mathbf{u}_1$, $\mathbf{u}_2$, with $\mathbf{u}_2 = (1 - \vartheta)[(\mathbf{B} - \mathbf{B}_f\mathbf{N})\mathbf{u}_c - \mathbf{B}_d\mathbf{d}]$. Therefore (21) can be rewritten as

$$\begin{aligned}\|\mathbf{x}_\Delta(t, \mathbf{x}_{\Delta,0})\| &= \|\boldsymbol{\xi}(t, \boldsymbol{\xi}_0, \mathbf{u}_1) - \boldsymbol{\eta}(t, \boldsymbol{\eta}_0, \mathbf{u}_2)\| \\ &\leq \beta(\|\boldsymbol{\xi}_0 - \boldsymbol{\eta}_0\|, t) + \gamma(\|\mathbf{u}_1 - \mathbf{u}_2\|_\infty) \\ &= \beta(\|\mathbf{x}_{\Delta,0}\|, t) + \gamma(\|\mathbf{u}\|_\infty) \quad (22)\end{aligned}$$

that is the $\mathbf{x}_\Delta$-dynamics is ISS with respect to $\mathbf{u} = (\mathbf{B} - \mathbf{B}_f\mathbf{N})\mathbf{u}_c - \mathbf{B}_d\mathbf{d}$.

*Corollary 14.* Consider the faulty nonlinear plant (15). If there exist a nonlinear function $\boldsymbol{\alpha}_\Delta(\cdot)$ such that the $\boldsymbol{\xi}$-dynamics with zero input, is $\delta$-QS then the difference system (18) is ISS w.r.t. the disturbance $\mathbf{d}$.

**Proof.** If the $\boldsymbol{\xi}$-dynamics is $\delta$-QS then there exist a quadratic function $V_\xi(\boldsymbol{\xi}, \boldsymbol{\eta})$ and $\mathcal{K}_\infty$ functions $\alpha_1$, $\alpha_2$, and $\kappa > 0$ such that for any pair of system trajectories $(\boldsymbol{\xi}, \boldsymbol{\eta})$

$$\alpha_1(\|\boldsymbol{\xi} - \boldsymbol{\eta}\|) \leq V_\xi(\boldsymbol{\xi}, \boldsymbol{\eta}) \leq \alpha_2(\|\boldsymbol{\xi} - \boldsymbol{\eta}\|) \quad (23)$$

$$\frac{\partial V_\xi}{\partial \boldsymbol{\xi}}\boldsymbol{\kappa}_\Delta(\boldsymbol{\xi}) + \frac{\partial V_\xi}{\partial \boldsymbol{\eta}}\boldsymbol{\kappa}_\Delta(\boldsymbol{\eta}) \leq -\kappa V(\boldsymbol{\xi}, \boldsymbol{\eta}) \quad (24)$$

Consider then the quadratic Lyapunov function

$$V_\Delta(\tilde{\mathbf{x}}, \tilde{\mathbf{x}} - \mathbf{x}_\Delta) = \mathbf{x}_\Delta^{\mathrm{T}}\mathbf{P}\mathbf{x}_\Delta = (\tilde{\mathbf{x}} - (\tilde{\mathbf{x}} - \mathbf{x}_\Delta))^{\mathrm{T}}\mathbf{P}(\tilde{\mathbf{x}} - (\tilde{\mathbf{x}} - \mathbf{x}_\Delta)), \quad (25)$$

with $\mathbf{P} = \mathbf{P}^T > 0$. Note that:

$$\frac{\partial V_\Delta}{\partial \tilde{\mathbf{x}}}\boldsymbol{\kappa}_\Delta(\tilde{\mathbf{x}}) + \frac{\partial V_\Delta}{\partial(\tilde{\mathbf{x}} - \mathbf{x}_\Delta)}\boldsymbol{\kappa}_\Delta(\tilde{\mathbf{x}} - \mathbf{x}_\Delta) = \mathbf{x}_\Delta^{\mathrm{T}}\mathbf{P}\left(\boldsymbol{\kappa}_\Delta(\tilde{\mathbf{x}}) - \boldsymbol{\kappa}_\Delta(\tilde{\mathbf{x}} - \mathbf{x}_\Delta)\right) \leq \kappa\mathbf{x}_\Delta^{\mathrm{T}}\mathbf{P}\mathbf{x}_\Delta \quad (26)$$

The derivative of $V_\Delta$ along the trajectories of the difference system (18) satisfies

$$\begin{aligned}\dot{V}_\Delta &= \mathbf{x}_\Delta^{\mathrm{T}}\mathbf{P}\left(\boldsymbol{\kappa}_\Delta(\tilde{\mathbf{x}}) - \boldsymbol{\kappa}_\Delta(\tilde{\mathbf{x}} - \mathbf{x}_\Delta)\right) + (\mathbf{B} - \mathbf{B}_f\mathbf{N})\mathbf{u}_c - \mathbf{B}_d\mathbf{d}) \\ &\leq -\kappa\mathbf{x}_\Delta^T\mathbf{P}\mathbf{x}_\Delta + \|\mathbf{x}_\Delta\|\|\mathbf{P}(\mathbf{B} - \mathbf{B}_f\mathbf{N})\|\|\mathbf{u}_c\| \\ &\quad + \|\mathbf{x}_\Delta\|\|\mathbf{P}\mathbf{B}_d\|\|\mathbf{d}\| \\ &\leq -b\|\mathbf{x}_\Delta\|^2 + \|\mathbf{x}_\Delta\|\|\mathbf{P}(\mathbf{B} - \mathbf{B}_f\mathbf{N})\|\|\mathbf{u}_c\| \\ &\quad + \|\mathbf{x}_\Delta\|\|\mathbf{P}\mathbf{B}_d\|\|\mathbf{d}\| \\ &\leq -(1 - \vartheta)b\|\mathbf{x}_\Delta\|^2 \quad (27)\end{aligned}$$

for $\|\mathbf{x}_\Delta\| \geq \frac{1}{\vartheta b}(\|\mathbf{P}(\mathbf{B} - \mathbf{B}_f\mathbf{N})\|\|\mathbf{u}_c\| + \|\mathbf{P}\mathbf{B}_d\|\|\mathbf{d}\|)$, where $b = \kappa\lambda_{\min}(\mathbf{P})$, which proves ISS of the system w.r.t. $\mathbf{d}$.

## 6. STUDY CASE - DYNAMIC POSITIONING SYSTEM

The effectiveness of the proposed nonlinear virtual actuator reconfiguration strategy is evaluated on the dynamic positioning (DP) system of an offshore supply vessel. DP systems are control systems that can maintain the position and orientation of a marine craft in the vicinity of an operating point exclusively by means of thrusters despite the presence of environmental disturbances such as wind, waves, and currents (DNV, 1990).

### 6.1 Vessel Model for DP Operations

For the problem at hand the analysis of the vessel motion is restricted to the horizontal plane neglecting the dynamics associated with the heave, roll and pitch motions. The interested reader can find details about notation and modelling of marine crafts in (Fossen, 2011).

Let $\boldsymbol{\eta} \triangleq [N, E, \psi]^{\mathrm{T}}$ be the position-orientation vector in the North-East-Down (NED) inertial frame, and $\boldsymbol{\nu} \triangleq [u, v, r]^{\mathrm{T}}$ be the velocity vector in the body frame. The vessel dynamics can be described by the following nonlinear model

$$\dot{\boldsymbol{\eta}} = \mathbf{R}(\psi)\boldsymbol{\nu} \quad (28)$$

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{N}(\boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{\tau} \quad (29)$$

where $\mathbf{M} = \mathbf{M}^{\mathrm{T}} > 0$ is the mass-inertia matrix that accounts for the rigid body and hydrodynamics effects; $\mathbf{N}(\boldsymbol{\nu})\boldsymbol{\nu} = \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\nu}$ accounts for rigid-body and hydrodynamic Coriolis-centripetal forces $(\mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu})$, and dissipative forces due to hull-water interaction $(\mathbf{D}(\boldsymbol{\nu})\boldsymbol{\nu})$. For low-speed operation, like dynamic positioning, the quadratic velocity terms due to nonlinear damping and fictitious forces can be neglected and only linear damping is considered $(\mathbf{N}(\boldsymbol{\nu})\boldsymbol{\nu} = \mathbf{D}\boldsymbol{\nu})$. $\mathbf{R}(\psi)$ is a rotational matrix function of the ship heading angle $\phi$, which transforms a vector from the body frame to the NED frame. The vector of generalized forces and moments $\boldsymbol{\tau} = \boldsymbol{\tau}_t + \boldsymbol{\tau}_w$ takes into account the actions of the thrusters $\boldsymbol{\tau}_t$, and the environmental disturbances such as wind $\boldsymbol{\tau}_w$.

The vessel is assumed to be equipped with two azimuth thrusters, one close to the bow and one close to the stern, and one tunnel thruster, positioned between the bow thruster and midship. Let $\mathbf{u}_c = [n_{az,1}, n_{tu}, n_{az,2}]^{\mathrm{T}}$ be the vector of thrusters' shaft speeds, then the considered actuators configuration gives rise to control forces and moments according to

$$\boldsymbol{\tau}_t = \mathbf{B}_1\mathbf{u}_c = \mathbf{K}\mathbf{T}(\boldsymbol{\varphi})\mathbf{u}_c \, . \quad (30)$$

$\mathbf{K} = \text{diag}\{K_1, K_2, K_3\}$ is the thrust coefficient matrix, and $\mathbf{T}(\boldsymbol{\varphi})$ is the actuator configuration matrix

$$\mathbf{T}(\boldsymbol{\varphi}) \triangleq \begin{bmatrix} \cos\varphi_1 & 0 & \cos\varphi_2 \\ \sin\varphi_1 & 1 & \sin\varphi_2 \\ l_1\sin\varphi_1 & l_2 & -l_3\sin\varphi_2 \end{bmatrix}$$

where $l_i$ ($i \in \{1,2,3\}$) are the moment arms in yaw with respect to the ship centre of gravity, and $\varphi_j$ ($j \in \{1,2\}$) are the angles of rotation of the azimuth thrusters w.r.t the fore-aft direction.

### 6.2 Nominal DP Controller

The general DP control objective is to track a time-varying reference trajectory $\boldsymbol{\eta}_d \in \mathcal{C}^2$, which is bounded ($\|\boldsymbol{\eta}_d\|_\infty \leq \bar{\eta}_d < \infty$) with bounded derivatives. If $\boldsymbol{\eta}_d$ is constant then dynamic positioning reduces to station keeping, which is a set-point regulation problem.

In order to solve the DP control problem a change of coordinates is introduced. Let $\tilde{\boldsymbol{\eta}} \triangleq \boldsymbol{\eta} - \boldsymbol{\eta}_d$ be the tracking error in the NED frame, and $\mathbf{s} \triangleq \dot{\tilde{\boldsymbol{\eta}}} - \boldsymbol{\Lambda}\tilde{\boldsymbol{\eta}}$ be an additional measure of tracking, with $\boldsymbol{\Lambda} > 0$ a diagonal design matrix. It can be shown (Fossen and Strand, 1999) that the vessel dynamics can be rewritten as

$$\dot{\boldsymbol{\eta}} = \mathbf{R}(\psi)\boldsymbol{\nu} - \dot{\boldsymbol{\eta}}_d \tag{31}$$

$$\begin{aligned}\mathbf{M}_\eta(\boldsymbol{\eta})\dot{\mathbf{s}} = &-\mathbf{D}_\eta(\boldsymbol{\eta})(\mathbf{s} + \dot{\boldsymbol{\eta}}_d - \boldsymbol{\Lambda}\tilde{\boldsymbol{\eta}}) + \mathbf{R}(\psi)(\mathbf{B}_1\mathbf{u}_c + \boldsymbol{\tau}_w) \\ &- \mathbf{M}_\eta(\boldsymbol{\eta})(\ddot{\boldsymbol{\eta}}_d - \boldsymbol{\Lambda}(\mathbf{R}(\psi)\boldsymbol{\nu} - \dot{\boldsymbol{\eta}}_d))\end{aligned} \tag{32}$$

Let $\mathbf{x} \triangleq [\tilde{\boldsymbol{\eta}}^{\mathrm{T}}, \mathbf{s}^{\mathrm{T}}]^{\mathrm{T}}$ be the state vector, $\mathbf{u}_c$ the control input, and $\mathbf{d} = \boldsymbol{\tau}_w$ the wind generated disturbance. Then the fault-free plant $\Sigma_P$ is given by

$$\Sigma_P : \begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{B}(\boldsymbol{\eta})\mathbf{u}_c + \mathbf{B}_d(\boldsymbol{\eta})\mathbf{d}\ ,\ \mathbf{x}(t_0) = \mathbf{x}_0 \\ \mathbf{y} = \mathbf{x} \\ \mathbf{z} = \mathbf{x}_1 \end{cases} \tag{33}$$

where

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} \mathbf{R}(\psi)\boldsymbol{\nu} - \dot{\boldsymbol{\eta}}_d \\ -\mathbf{M}_\eta^{-1}(\boldsymbol{\eta})\mathbf{D}_\eta(\boldsymbol{\eta})(\mathbf{s} + \dot{\boldsymbol{\eta}}_d - \boldsymbol{\Lambda}\tilde{\boldsymbol{\eta}}) - (\ddot{\boldsymbol{\eta}}_d - \boldsymbol{\Lambda}\dot{\tilde{\boldsymbol{\eta}}}) \end{bmatrix}$$

$$\mathbf{B}(\boldsymbol{\eta}) = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}_\eta^{-1}(\boldsymbol{\eta})\mathbf{R}(\psi)\mathbf{B}_1 \end{bmatrix}\ ,\ \mathbf{B}_d(\boldsymbol{\eta}) = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}_\eta^{-1}(\boldsymbol{\eta})\mathbf{R}(\psi) \end{bmatrix}$$

The nominal controller for the system (33) under the assumption that $\mathbf{d} = \mathbf{0}$ has been implemented based on the design by Fossen and Strand (1999), who have used the nonlinear MIMO backstepping technique to obtain a DP control system that guarantees global exponential stability of the tracking error dynamics.

The DP backstepping nominal control law is given by (Fossen and Strand, 1999)

$$\boldsymbol{\phi} = \mathbf{R}(\psi)\boldsymbol{\nu} \triangleq -\boldsymbol{\Lambda}\tilde{\boldsymbol{\eta}} + \dot{\boldsymbol{\eta}}_d \tag{34}$$

$$\begin{aligned}\mathbf{u}_c \triangleq \mathbf{B}_1^{-1}\mathbf{R}^{\mathrm{T}}(\psi)\big[&\mathbf{D}_\eta(\boldsymbol{\eta})(\dot{\boldsymbol{\eta}}_d - \boldsymbol{\Lambda}\tilde{\boldsymbol{\eta}}) - \mathbf{K}_p\tilde{\boldsymbol{\eta}} - \mathbf{K}_d\mathbf{s} \\ &+ \mathbf{M}_\eta(\boldsymbol{\eta})(\ddot{\boldsymbol{\eta}}_d - \boldsymbol{\Lambda}(\mathbf{R}(\psi)\boldsymbol{\nu} - \dot{\boldsymbol{\eta}}_d))\big]\end{aligned} \tag{35}$$

where $\boldsymbol{\phi}$ is the virtual control that stabilizes the $\tilde{\boldsymbol{\eta}}$ dynamics in the first step; $\mathbf{K}_p > 0$, and $\mathbf{K}_d > 0$ are diagonal design matrices. The nominal closed-loop dynamics reads

$$\dot{\tilde{\boldsymbol{\eta}}} = -\boldsymbol{\Lambda}\tilde{\boldsymbol{\eta}} + \mathbf{s} \tag{36}$$

$$\dot{\mathbf{s}} = -\mathbf{M}_\eta^{-1}(\boldsymbol{\eta})\mathbf{K}_p\tilde{\boldsymbol{\eta}} - \mathbf{M}_\eta^{-1}(\boldsymbol{\eta})(\mathbf{D}_\eta(\boldsymbol{\eta}) + \mathbf{K}_d)\mathbf{s} \tag{37}$$

For a detailed overview of the design strategy and of the stability properties of the origin of (36)-(37) the reader is addressed to (Fossen and Strand, 1999).

The control law (34)-(35) has been selected because backstepping controllers are known to guarantee ISS with respect to input disturbances (Krstic et al., 1995).

### 6.3 Virtual Actuator DP Controller

Actuator faults in the system (33) appears as the reduction of one or more coefficients of the matrix $\mathbf{K}$ from the nominal values. Therefore the faulty input matrix is given by $\mathbf{B}_{1,f} \triangleq \mathbf{K}_f\mathbf{T}(\boldsymbol{\varphi})$, where $\mathbf{K}_f = \text{diag}\{\theta_1K_1, \theta_2K_2, \theta_3K_3\}$. In this study case we focus on partial reduction of the thrust coefficients, that is the scaling factors $\theta_i \in (0,1]$ ($i \in \{1,2,3\}$) specifically cannot assume value zero.

The virtual actuator DP control law is

$$\boldsymbol{\alpha}_\Delta(\mathbf{z}) : \begin{cases} \boldsymbol{\phi}_\Delta = \mathbf{R}(\psi)\boldsymbol{\nu} \triangleq -\boldsymbol{\Lambda}\mathbf{z}_1 + \dot{\boldsymbol{\eta}}_d \\ \mathbf{u}_\Delta \triangleq \mathbf{B}_{1,f}^{-1}\mathbf{R}^{\mathrm{T}}(\psi)\big[\mathbf{D}_\eta(\boldsymbol{\eta})(\dot{\boldsymbol{\eta}}_d - \boldsymbol{\Lambda}\mathbf{z}_1) \\ \qquad + \mathbf{M}_\eta(\boldsymbol{\eta})(\ddot{\boldsymbol{\eta}}_d - \boldsymbol{\Lambda}(\mathbf{R}(\psi)\boldsymbol{\nu} - \dot{\boldsymbol{\eta}}_d)) \\ \qquad - \mathbf{K}_p\mathbf{z}_1 - \mathbf{K}_d\mathbf{z}_2\big] \end{cases} \tag{38}$$

where $\mathbf{z} = [\mathbf{z}_1^{\mathrm{T}}, \mathbf{z}_2^{\mathrm{T}}]^{\mathrm{T}}$ is a dummy vector representing either the state of the faulty system or the state of the reference model. The virtual actuator exploits the knowledge of the magnitude of the fault through the input matrix $\mathbf{B}_{1,f}$.

Let $\boldsymbol{\xi} = [\tilde{\boldsymbol{\eta}}, \mathbf{s}]^{\mathrm{T}}$ be the state of the closed-loop system, then its dynamics reads

$$\dot{\boldsymbol{\xi}} = \mathbf{A}(\boldsymbol{\eta})\boldsymbol{\xi} + \mathbf{B}_d(\boldsymbol{\eta})\mathbf{d} \tag{39}$$

where

$$\mathbf{A}(\boldsymbol{\eta}) = \begin{bmatrix} -\boldsymbol{\Lambda} & \mathbf{I} \\ -\mathbf{M}_\eta^{-1}(\boldsymbol{\eta})\mathbf{K}_p & -\mathbf{M}_\eta^{-1}(\boldsymbol{\eta})(\mathbf{D}_\eta(\boldsymbol{\eta}) + \mathbf{K}_d) \end{bmatrix}$$

Consider the Lyapunov function candidate

$$V(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2) = (\boldsymbol{\xi}_1 - \boldsymbol{\xi}_2)^{\mathrm{T}}\mathbf{P}(\boldsymbol{\eta})(\boldsymbol{\xi}_1 - \boldsymbol{\xi}_2) \tag{40}$$

where

$$\mathbf{P}(\boldsymbol{\eta}) = \begin{bmatrix} \mathbf{K}_p & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_\eta(\boldsymbol{\eta}) \end{bmatrix} = \mathbf{P}^{\mathrm{T}}(\boldsymbol{\eta}) > 0$$

which satisfies that

$$\begin{aligned}\frac{\partial V}{\partial \boldsymbol{\xi}_1}\dot{\boldsymbol{\xi}}_1 + \frac{\partial V}{\partial \boldsymbol{\xi}_2}\dot{\boldsymbol{\xi}}_2 &= (\boldsymbol{\xi}_1 - \boldsymbol{\xi}_2)^{\mathrm{T}}(\mathbf{P}(\boldsymbol{\eta})\mathbf{A}(\boldsymbol{\eta}) \\ &\quad + \mathbf{A}^{\mathrm{T}}(\boldsymbol{\eta})\mathbf{P}(\boldsymbol{\eta}))(\boldsymbol{\xi}_1 - \boldsymbol{\xi}_2) \\ &\quad + (\boldsymbol{\xi}_1 - \boldsymbol{\xi}_2)^{\mathrm{T}}\mathbf{P}(\boldsymbol{\eta})\mathbf{B}_d(\boldsymbol{\eta})(\mathbf{d}_1 - \mathbf{d}_2) \\ &= -(\boldsymbol{\xi}_1 - \boldsymbol{\xi}_2)^{\mathrm{T}}\mathbf{Q}(\boldsymbol{\eta})(\boldsymbol{\xi}_1 - \boldsymbol{\xi}_2) \\ &\quad + (\boldsymbol{\xi}_1 - \boldsymbol{\xi}_2)^{\mathrm{T}}\mathbf{P}(\boldsymbol{\eta})\mathbf{B}_d(\boldsymbol{\eta})(\mathbf{d}_1 - \mathbf{d}_2) \\ &\leq \kappa V(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2) + \gamma(\|\mathbf{d}_1 - \mathbf{d}_2\|)\end{aligned} \tag{41}$$

where

$$\mathbf{Q}(\boldsymbol{\eta}) = \begin{bmatrix} \mathbf{K}_p\boldsymbol{\Lambda} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_\eta(\boldsymbol{\eta}) + \mathbf{K}_d \end{bmatrix} = \mathbf{Q}^{\mathrm{T}}(\boldsymbol{\eta}) > 0\ .$$

Therefore according to Theorem 9 the closed-loop system (39) is $\delta$-ISS.

### 6.4 Simulation Results

The DP backstepping controller with virtual actuator has been tested on a model of an offshore supply vessel subject to wind disturbances. The numerical values of the parameters of the ship and of the nominal controller are given in Table 1.

Table 1. Vessel & Controller Parameters

| Quantity | Symbol | Value |
|---|---|---|
| Length overall | $L_{OA}$ | 76.2 [m] |
| Beam | $B$ | 18 [m] |
| Centre of gravity | $[x_G, y_G, z_G]^{\mathrm{T}}$ | $[42, 0, 0]^{\mathrm{T}}$ [m] |
| Moment arms | $[l_1, l_2, l_3]^{\mathrm{T}}$ | $[27.4, 17.2, -10.5]^{\mathrm{T}}$ [m] |
| Angle of rotations | $[\varphi_1, \varphi_2]^{\mathrm{T}}$ | $[30, 30]^{\mathrm{T}}$ [deg] |
| Mass-inertia matrix | $\mathbf{M}$ | $\mathrm{diag}\,\{5.3e6, 8.3e6, 3.7e9\}$ |
| Damping matrix | $\mathbf{D}$ | $\begin{bmatrix} 5.0e4 & 0 & 0 \\ 0 & 2.7e5 & -4.4e6 \\ 0 & -4.4e6 & 4.2e8 \end{bmatrix}$ |
| Thrust coeff. matrix | $\mathbf{K}$ | $\mathrm{diag}\,\{1.4e5, 1.4e5, 1.4e5\}$ |
| Proportional gain | $\mathbf{K}_p$ | $\mathrm{diag}\,\{2.5, 2.5, 2.5\}$ |
| Derivative gain | $\mathbf{K}_d$ | $\mathrm{diag}\,\{2.5, 2.5, 2.5\}$ |
| | $\mathbf{\Lambda}$ | $\mathrm{diag}\,\{1, 1, 1\}$ |
| Time constants | $\mathbf{T}_a$ | $\mathrm{diag}\,\{25, 25, 25\}$ [sec] |

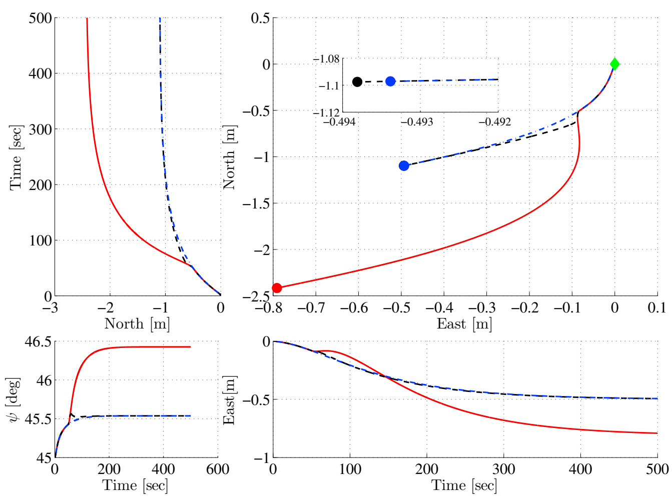


Fig. 4. North-East trajectory of the supply vessel affected by faults in both azimuth thrusters while subject to a constant wind. The green diamond is the desired operating point, and the dots represent the end position of the vessel. The blue trajectory shows the behaviour of the fault-free vessel; the red trajectory shows the faulty ship without reconfiguration, while the dashed black trajectories represent the system after being reconfigured.

In order to obtain realistic time responses to step changes of the set-point or of the disturbance the thrusters have been model as first order systems with rate and magnitude saturation

$$\mathbf{T}_a \dot{\mathbf{u}}_e = \mathbf{u}_c - \mathbf{u}_e \tag{42}$$

where $\mathbf{u}_c$ is the vector of commanded shafts speed by the control law, $\mathbf{u}_e$ is the vector of delivered shafts speed, and $\mathbf{T}_a = \mathrm{diag}\,\{\tau_{az,1}, \tau_{tu}, \tau_{az,2}\}$ is the actuator time constant matrix. This has requested to extend the nominal control law (34)-(35) by backstepping once more through the actuator dynamics. The implemented solution is based on Fossen and Berge (1997).

The vessel is subject to a constant wind disturbance with speed $V_w = 20$ m/s and direction $\beta_w = 30$ degrees with respect to the North. At time $t_f = 50$ seconds both azimuth thrusters are subject to faults of equal magnitude which reduce the respective thrust coefficients of 50%, i.e. $\mathbf{K}_f = \mathrm{diag}\,\{0.5K_1, K_2, 0.5K_3\}\ \ \forall\, t \geq t_f$. It is assumed

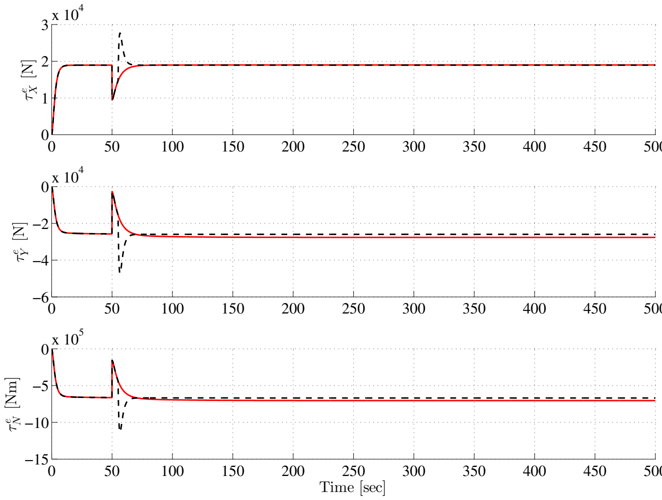


Fig. 5. Forces and moments in surge ($\tau_X$), sway ($\tau_Y$), and yaw ($\tau_N$) without (red lines) and with (black dashed lines) reconfiguration.

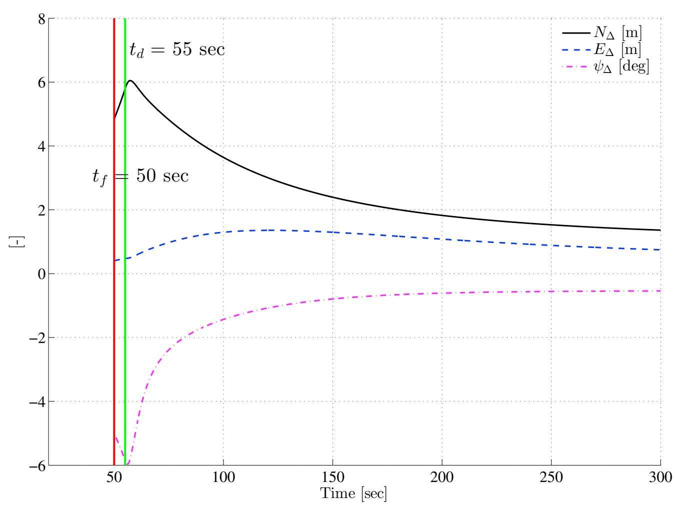


Fig. 6. Behaviour of the position-orientation components of the difference state $\mathbf{x}_\Delta$: after the reconfiguration of the control system at $t = t_d$ with the virtual actuator the components converge to bounded values close to zero.

that within 5 seconds from faults occurrence an FDI module has detected and isolated the faults, and that the reconfiguration of the control system has taken place.

Figures 4-6 show the performance of the DP control system without and with the virtual actuator reconfiguration. Although stability of the closed-loop system is not compromised it is evident the beneficial action of the control system reconfiguration. The presence of the virtual actuator allows the reconfigured vessel (black dashed line) to remain in very close proximity of the desired operating point (green diamond at $(N, E) = (0, 0)$ in Fig. 4), with performance extremely close to those of the nominal system (blue dashed-dotted line). Conversely the faulty ship (red line) shows deviations from the desired operating point approximately 50% larger in both direction compared to the reconfigured system. Figure 6 clearly shows the input-to-state stable behaviour of the difference state

$\mathbf{x}_\Delta$: after the reconfiguration has taken place at $t = t_d$ its components converge to a neighbourhood of the origin, whose size is obviously a function of the magnitude of the disturbance.

## 7. CONCLUSIONS

In this paper a new method for FTC of nonlinear systems subject to actuator faults using a nonlinear reconfiguration block was proposed. The main idea of the method is to achieve fault-tolerance without re-designing the nominal controller by inserting the reconfiguration block between the faulty system and the nominal controller. The proposed method does not need any knowledge of the nominal controller and it is only assumed that the nominal closed-loop system is input-to-state stable. It was shown that if the virtual actuator is designed separately such that the difference system is $\delta$-ISS, then the reconfigured closed-loop system is ISS. The effectiveness of the method is shown on a case study of dynamic positioning system of an offshore supply vessel, where the virtual actuator is designed using the backstepping control technique.

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