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Transmission Expansion in an Oligopoly Considering Generation Investment Equilibrium

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Abstract

Transmission expansion planning (TEP) is a sophisticated decision-making problem, especially in an oligopolistic electricity market in which a number of strategic (price-maker) producers compete together. A transmission system planner, who is in charge of making TEP decisions, requires considering the future generation investment actions. However, in such an oligopolistic market, each producer makes its own strategic generation investment decisions. This motivates the transmission system planner to consider the generation investment decision-making problem of all producers within its TEP model. The strategic generation investment problem of each producer can be represented by a complementarity bi-level model. The joint consideration of all bi-level models, one per producer, characterizes the generation investment equilibrium that identifies the future evolution of generation investment in the market. This paper proposes a tri-level TEP decision-making model to be solved by the transmission system planner, whose objective is to maximize the social welfare of the market minus the expansion costs, and whose constraints are the transmission expansion limits as well as the generation investment equilibrium problem. This model is then recast as a mixed-integer linear programming problem and solved. Numerical results from an illustrative example and a case study based on the IEEE 14-bus test system demonstrate the usefulness of the proposed approach.

Keywords: Transmission expansion planning (TEP), generation investment equilibrium, strategic producers, mathematical program with equilibrium constraints (MPEC), equilibrium problem with equilibrium constraints (EPEC), oligopoly.

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1. Introduction

1.1. Background and Aim

In traditional vertically integrated power systems, a single entity, i.e., system operator, is in charge of making both transmission and generation expansion decisions. For this purpose, it solves a single optimization problem seeking to minimize the total expansion and operational costs [1]-[4], referring to a *centralized* planning decision-making framework. However, in competitive electricity markets, each producer makes its own generation investment decisions, while another entity, i.e., transmission system planner, makes transmission expansion decisions. This *decentralized* planning decision-making framework within electricity markets makes the expansion problems complex to model and solve [5]-[7]. This complexity is even worse in *oligopolistic* electricity markets in which a number of *strategic (price-maker)* producers compete together. Each strategic producer is able to alter the market-clearing outcomes to its own benefit through exercising market power [8]. The decision-making problem of each strategic producer can be represented by a complementarity bi-level model [8]-[9], aiming to maximize its profit through making most beneficial generation capacity investment (planning) and price offering (operation) decisions. However, the strategic decisions of various producers are interrelated through market-clearing conditions, forming a generation investment equilibrium problem [10]-[12]. Note that the transmission system planner requires considering the future evolution of generation investment in the market to solve its transmission expansion planning (TEP) decision-making problem. This motivates the consideration of TEP problem on top of the generation investment equilibrium problem, i.e., the TEP problem is constrained by the generation investment equilibrium problem [13]-[19]. This allows the transmission system planner to make informed decisions on new lines to be built that optimally facilitate the power transaction among different buses considering all existing and future generating units. In this structure, the generation investment decisions of producers are not exogenous (fixed) within the TEP problem because the new transmission lines to be built may change the generation investment decisions of the producers. According to this structure, we propose a complementarity tri-level TEP model to be solved by the transmission system planner, whose objective is to maximize the social welfare (SW) of the market minus the expansion costs. This model is constrained by the transmission expansion limits as well as the generation investment equilibrium problem.

1.2. Literature Review and Contributions

There are several papers in the literature addressing the market-based TEP problem considering the future generation investment decisions of the producers. For example, [6] proposes an iterative algorithm for coordinated generation and transmission capacity expansion in a perfectly competitive market. In the recent literature, e.g., [13]-[18], it is common to use the game-theoretic complementarity models to address the TEP decision-making problems while anticipating the generation investment decisions of the producers, referring to "proactive TEP". Pioneering works [13] and [14] propose Nash-Cournot equilibrium models to analyze how the strategic behavior of producers in an oligopoly affects the equilibrium between generation and transmission investments. However, the solution methodology relies on an iterative algorithm, and does not determine the optimal TEP decisions, but evaluates the impacts of predetermined TEP projects on SW of the market. The models proposed in [13] and [14] have been improved in [15] and [16] by proposing a tri-level model, whose lower level represents the market clearing, while the intermediate level represents a Nash equilibrium in generation capacity expansion. In addition, the upper level makes the TEP decisions considering the continuous expansion alternatives. This model is finally recast as a mixed-integer linear programming (MILP) problem. However, [15] and [16] assume a perfectly competitive market in the sense that the producers offer their productions at fixed prices, which are equal to their actual production costs. In a similar line, [17] proposes a tri-level TEP model in an oligopoly considering the strategic offer prices, and then solves this model using an iterative diagonalization algorithm. In addition, a bi-level model is proposed in [18] to derive the best TEP decision under different incentives while endogenously anticipating the generation expansion outcomes of a perfectly competitive market. For a similar purpose but using a non-complementarity model, [19] proposes a four-level TEP model considering the strategic behavior of producers in generation investment, and solves it using an agent-based model as well as a search-based technique to avoid computing equilibrium. In addition, [20] considers an oligopoly and evaluates the effects of a transmission augmentation on both market power and strategic generation investment decisions using a heuristic method. Finally, [21] proposes a bi-level model to address the renewable generation investment problem as well as the transmission system reinforcement. The modeling features and assumptions of the aforementioned papers are compared in [17].

The main contribution of the current paper is to propose a tri-level complementarity model to determine the optimal TEP decisions in an oligopoly, while endogenously considering the generation investment equilibrium. The solution technique is non-iterative, which relies on recasting the proposed model as a MILP.

Compared to [13] and [14] that consider the predetermined TEP projects, this paper determines the optimal TEP decisions in an oligopoly using a non-iterative game-theoretic technique. Compared to [15] that assumes a perfect competition with price-taker producers, we consider an oligopoly including a set of strategic producers, who are able to exercise market power not only in long run by strategic investment decisions but also in short run by strategic price offers. Similarly, compared to [16], [18] and [21] that assume a perfect competition including price-taker producers, we consider an oligopoly in which the producers offer at strategic prices. Compared to [17], [19] and [20], we propose a non-iterative solution technique based on complementarity modeling, which is generally more efficient to solve with respect to iterative diagonalization techniques.

2. Modeling Assumptions and Features

2.1. Modeling Assumptions

In this study, an hourly basis day-ahead electricity pool is considered. A market operator clears the pool considering a lossless DC representation of transmission network. This derives a locational marginal price (LMP) for each bus as a dual variable associated with the power balance equality at that bus.

As it is common in the generation and transmission expansion studies, e.g., [10], [15], [22], [23], we use a static investment model considering a single future target year to make the optimal TEP decisions for that year. Another alternative is to use a multi-stage dynamic model [24]-[25] that makes the expansion decisions at different points in time. It is straightforward to upgrade the proposed static model to the multi-stage one, but at the cost of increased computational complexity. For such an extension, an index for time periods needs to be added within all variables. In addition, the annualized investment costs in the static model should be replaced by the actual investment costs while considering the amortization rates in different time stages. To avoid anticipating information, a set of non-anticipativity constraints is also required. Further information can be found in [7].

Similar to [10], [17], [19], [22], and pursuing simplicity, the potential sources of uncertainty such as future demand growth and variable generation (e.g., wind power) are not considered. However, if considered, the TEP problem can be modeled using a stochastic programming approach [26] or a robust optimization technique [27]-[29]. In addition, we do not consider the reliability indices and renewable targets [30], transmission network payment schemes [31], risk aversion [25], [32] and their impacts on TEP decisions.

In line with [10], [15] and [22], the load-duration curve of the market for the target year is approximated through a number of stepwise demand blocks. A weighting factor is associated with each demand block referring to the number of hours within the target year, whose demand levels are represented by that block. As another alternative, a set of operating conditions can be used to represent the various levels of demand and variable generation considering their potential correlation [26], [33]. In addition, we do not consider the intertemporal constraints, e.g., ramping limits of thermal generating units and reservoir capacity of hydro generating units; otherwise, the TEP problem may require clustering methods to find appropriate representative days within the target year [34]. We also ignore the binary variables indicating the commitment status of conventional generating units as well as their characteristics requiring binary variables, e.g., start-up costs.

Modeling Features: A Tri-level Model

The structure of the proposed tri-level TEP model is illustrated in Fig. 1 and explained below:

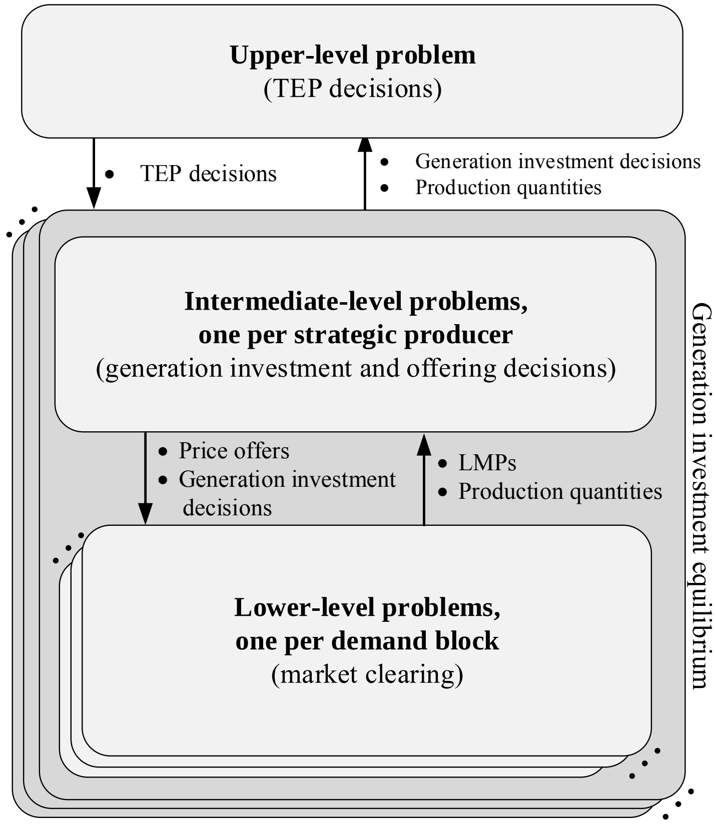

Figure 1: Tri-level structure of the proposed TEP model in an oligopoly

Lower-level problems:

The lower-level problems, one per demand block, clear the market, based on given transmission and generation investment as well as price offering decisions. The market-clearing outcomes are LMPs and production and consumption quantities.

Intermediate-level problems:

Each producer behaves strategically in terms of its capacity expansion and price offering decisions, while it seeks to maximize its profit. The strategic behavior of each producer is modeled using a complementarity bi-level model. Each intermediate-level problem, one per producer, is constrained by all lower-level problems, and determines the most beneficial i) price offers, and ii) generation investment actions of the corresponding strategic producer. Note that the TEP decisions to be made by the transmission system planner are fixed within the intermediate-level problems. The joint consideration of all intermediate-level problems characterizes the generation investment equilibrium.

Upper-level problem:

The transmission system planner makes the TEP decisions in the upper-level problem, whose objective is to maximize the SW of the market minus the transmission expansion costs. This problem is constrained by the transmission expansion limits and all intermediate-level problems, i.e., the generation investment equilibrium.

One important observation is that all lower-, intermediate- and upper-level problems are interrelated, and therefore, they cannot be separately solved. In one hand, as depicted in Fig. 1, the intermediate-level problems depend on LMPs and production quantities coming from the lower-level problems, and the lower-level problems are affected by offering and generation investment decisions determined in the intermediate-level problems. On the other hand, the TEP decisions to be made in the upper-level problem affect the strategic decisions of the producers in the intermediate-level and the market-clearing outcomes in the lower-level problems, while the outcomes of the lower- and the intermediate-level problems alter the value of SW of the market to be maximized in the upper-level problem.

To solve this tri-level problem, we use a methodology similar to the one proposed in [35] and [36]. First, the optimality conditions associated with each lower-level problem are derived using a primal-dual transformation that renders a set of mathematical programs with equilibrium constraints (MPECs), one per producer. The joint consideration of all MPECs characterizes an equilibrium problem with equilibrium constraints (EPEC), whose solution identifies the generation investment equilibrium. In order to solve the proposed TEP problem, we derive the Karush-Kuhn-Tucker (KKT) conditions of the EPEC, which provide its strong stationary conditions. This mathematical procedure transforms the original tri-level model into a single-level optimization problem that contains the upper-level problem constrained by the KKT conditions of the EPEC.

3. Formulation

The main symbols used in this paper are listed below:

Indices:

d	Index for demands
g	Index for strategic producers
i	Index for generating units
l	Index for demand blocks
n, m	Indices for buses

Sets:

D	Set of demands
G^{C}	Set of candidate generating units
G^{E}	Set of existing generating units
Ω_g	Set of generating units belonging to producer g
Φ_n^{C}	Set of candidate lines connected to bus n
Φ_n^{E}	Set of existing lines connected to bus n

The first three sets above include subscript n if referring to the set of demands/units located at bus n.

Parameters:

C_i^{G}	Production cost of unit i [\$/MWh]
$K_g^{\max}$	Annualized investment budget of producer g [\$]
K_i	Annualized investment cost of candidate unit $i \in G^{\mathrm{C}}$ [\$/MW]
Q_i^{G}	Capacity of existing unit $i \in G^{\mathrm{E}}$ [MW]
Q_i^{Gmax}	Maximum capacity of candidate unit $i \in G^{\mathrm{C}}$ [MW]
$Q_{l,d}^{\mathrm{D}}$	Quantity bid of demand d in block l [MW]
$Q_{n,m}$	Transmission capacity of line (n, m) [MW]
S	A non-negative factor indicating supply security
$X_{n,m}$	Reactance of transmission line (n, m) [ohm]
$\Upsilon_{n,m}$	Annualized investment cost of candidate transmission line (n, m) [\$]
$\overline{\Gamma}$	Annualized investment budget for transmission expansion [\$]
ρ_l	Weighting factor of block l [hour]
$\beta_{l,d}^{\mathrm{D}}$	Price bid of demand d in block l [\$/MWh]

Variables:

$p_{l,n}$	LMP at bus n in block l [\$/MWh]
Q_i^{G}	Capacity of candidate unit $i \in G^{\mathrm{C}}$ [MW]
$q_{l,i}^{\mathrm{G}}$	Power production of unit i in block l [MW]
$q_{l,d}^{\mathrm{D}}$	Power consumption of demand d in block l [MW]

$u_{n,m}$	Binary decision variable to indicate if candidate transmission line (n, m) is built (equal to 1) or not
$\beta_{l,i}^{\mathrm{G}}$	Price offer of unit *i* in block *l* [$/MWh]
$\delta_{l,n}$	Voltage angle of bus *n* in block *l* [rad]

3.1. Lower-level Problems: Market Clearing for Each Demand Block

Given the offering and planning decisions, lower-level problem (1) clears the market for demand block l. The objective function (1a) maximizes the SW of the market, and is subject to market-clearing constraints (1b)-(1g). Note that (1) is a linear and convex problem, since the offering and planning decisions are treated as fixed parameters within the lower-level problems. The dual variables are indicated in the constraints following a colon:

$$\left\{ \underset{q_{l,i}^{\mathrm{G}},\, q_{l,d}^{\mathrm{D}},\, \delta_{l,n}}{\text{Maximize}} \left(\sum_{d \in D} \beta_{l,d}^{\mathrm{D}}\, q_{l,d}^{\mathrm{D}} - \sum_{i \in (G^{\mathrm{C}} \cup G^{\mathrm{E}})} \beta_{l,i}^{\mathrm{G}}\, q_{l,i}^{\mathrm{G}} \right) \right. \tag{1a}$$

subject to:

$$\sum_{d \epsilon D_n} q_{l,d}^{\mathrm{D}} - \sum_{i \in (G_n^{\mathrm{C}} \cup G_n^{\mathrm{E}})} q_{l,i}^{\mathrm{G}} + \sum_{m \epsilon (\Phi_n^{\mathrm{E}} \cup \Phi_n^{\mathrm{C}})} \frac{u_{n,m}}{X_{n,m}} \left(\delta_{l,n} - \delta_{l,m}\right) = 0 \;\; : p_{l,n} \qquad \forall n \tag{1b}$$

$$0 \le q_{l,i}^{\mathrm{G}} \le Q_i^{\mathrm{G}} \qquad : \underline{\sigma}_{l,i}^{\mathrm{G}}, \overline{\sigma}_{l,i}^{\mathrm{G}} \qquad \forall i \in (G^{\mathrm{C}} \cup G^{\mathrm{E}}) \tag{1c}$$

$$0 \le q_{l,d}^{\mathrm{D}} \le Q_{l,d}^{\mathrm{D}} \qquad : \underline{\sigma}_{l,d}^{\mathrm{D}}, \overline{\sigma}_{l,d}^{\mathrm{D}} \qquad \forall d \in D \tag{1d}$$

$$\frac{u_{n,m}}{X_{n,m}} \left(\delta_{l,n} - \delta_{l,m}\right) \le Q_{n,m} \;\; : \sigma_{l,n,m}^{\mathrm{L}} \qquad \forall n, \forall m \in (\Phi_n^{\mathrm{E}} \cup \Phi_n^{\mathrm{C}}) \tag{1e}$$

$$-\pi \le \delta_{l,n} \le \pi \qquad : \underline{\sigma}_{l,n}^{\delta}, \overline{\sigma}_{l,n}^{\delta} \qquad \forall n \tag{1f}$$

$$\left. \delta_{l,(n=1)} = 0 \qquad : \sigma_l^1 \qquad \right\} \; \forall l. \tag{1g}$$

Constraint (1b) enforces the power balance at each bus and for each demand block, whose dual variable ($p_{l,n}$) provides the LMP at that bus. Constraints (1c) and (1d) limit the production and consumption levels of units and demands, respectively. Constraint (1e) enforces the transmission capacity of all existing and candidate lines. Constraints (1f) restricts nodal voltage angles, whereas constraints (1g) introduces $n = 1$ as the voltage angle's reference bus.

3.2. Intermediate-level Problems: Generation Investment and Price Offering Decisions of Each Strategic Producer

For strategic producer g, the intermediate-level problem (2) maximizes its profit through making strategic price offering ($\beta_{l,i}^{\mathrm{G}}, \forall l, \forall i$) and strategic generation investment ($Q_i^{\mathrm{G}}, \forall i \in G^{\mathrm{C}}$) decisions:

$$\left\{ \underset{\Xi_g^{\mathrm{IL}}}{\text{Maximize}} \left(\sum_l \rho_l \left[\sum_{i \epsilon [(G^{\mathrm{C}} \cup G^{\mathrm{E}}) \cap \Omega_g]} q_{l,i}^{\mathrm{G}} \left(p_{l,[n:i \epsilon (G^{\mathrm{C}} \cup G^{\mathrm{E}})]} - C_i^{\mathrm{G}}\right) \right] - \sum_{i \in (G^{\mathrm{C}} \cap \Omega_g)} K_i\, Q_i^{\mathrm{G}} \right) \right. \tag{2a}$$

subject to:

$$0 \le Q_i^{\mathrm{G}} \le Q_i^{\mathrm{Gmax}} \qquad \forall i \in \left(G^{\mathrm{C}} \cap \Omega_g\right) \tag{2b}$$

$$\sum_{i \in (G^{\mathrm{C}} \cap \Omega_g)} K_i\, Q_i^{\mathrm{G}} \le K_g^{\max} \tag{2c}$$

$$\beta_{l,i}^{\mathrm{G}} \ge 0 \qquad \forall l, \forall i \in \left[(G^{\mathrm{C}} \cup G^{\mathrm{E}}) \cap \Omega_g\right] \tag{2d}$$

$$\sum_{i \in (G^{\mathrm{C}} \cup G^{\mathrm{E}})} Q_i^{\mathrm{G}} \geq S \sum_{d \in D} Q_{(l=1),d}^{\mathrm{D}} \tag{2e}$$

$$\left.\text{lower level problems (1)} \quad \forall l \quad \right\} \quad \forall g. \tag{2f}$$

The set of primal variables of problem (2), i.e., Ξ_g^{IL}, includes Q_i^{G} for candidate units of producer g, offer prices $\beta_{l,i}^{\mathrm{G}}$ for all units of producer g as well as all primal and dual variables of lower-level problems (1).

Objective function (2a) maximizes the profit of strategic producer g, containing its operational profit over the target year minus the annualized generation investment cost. Constraint (2b) limits the capacity of candidate units to be built by producer g. Constraint (2c) enforces the annualized financial budget of producer g for investing in new units. Constraint (2d) enforces the offer prices to be non-negative. Although constraints (2b)-(2d) are included within the formulation; however, one can remove them if they are not relevant for a particular market or a producer. For example, constraint (2c) could be relaxed if producer g does not suffer from the availability of financial resources for building new units. In addition, constraint (2e) represents a regulatory condition for a market in which the corresponding market regulator imposes such a condition to guarantee the supply security and to avoid generation capacity deficit. Finally, constraint (2f) contains all lower-level problems (1) representing the market clearing for all demand blocks.

Each intermediate-level problem (2), one per strategic producer, is in fact a bi-level problem. This can be transformed into an MPEC through replacing each lower-level problem (1) included in (2f) with its primal-dual optimality conditions. This transformation is provided in appendix A, yielding MPECs (5), one per producer. The collection of all those MPECs results in the EPEC.

3.3. Upper-level Problem: TEP Decisions

The transmission system planner solves the upper-level problem (3) below and makes the TEP decisions through a trade-off between the expansion costs and the SW of the market:

$$\underset{\Xi^{\mathrm{UL}}}{\text{Minimize}} \left\{ \sum_{n,(m \epsilon \Phi_n^{\mathrm{C}})} u_{n,m}\ \gamma_{n,m} + \sum_{i \in G^{\mathrm{C}}} K_i\ Q_i^{\mathrm{G}} - \sum_{l} \rho_l \left[\sum_{d \in D} \beta_{l,d}^{\mathrm{D}}\ q_{l,d}^{\mathrm{D}} - \sum_{i \in (G^{\mathrm{C}} \cup G^{\mathrm{E}})} C_i^{\mathrm{G}}\ q_{l,i}^{\mathrm{G}} \right] \right\} \tag{3a}$$

subject to:

$$\sum_{n,(m \epsilon \Phi_n^{\mathrm{C}})} u_{n,m}\ \gamma_{n,m} \leq \overline{\Gamma} \tag{3b}$$

$$u_{n,m} = 1 \qquad \forall n, \forall m \in \Phi_n^{\mathrm{E}} \tag{3c}$$

$$u_{n,m} \in \{0,1\} \qquad \forall n, \forall m \in \Phi_n^{\mathrm{C}} \tag{3d}$$

$$\text{MPECs (5)} \quad \forall g. \tag{3e}$$

The set of primal variables of problem (3), i.e., Ξ^{UL}, includes the binary decision variables $u_{n,m}$ as well as all primal and dual variables associated with the EPEC, i.e., MPECs (5), included in (3e).

Objective function (3a) minimizes the annualized transmission and generation expansion costs minus the annual SW of the market. Unlike the lower-level objective function (1a), the *true* SW of the market is considered in (3a) including the actual production costs of all existing and newly built generating units (and not their strategic offers). There are two reasons: first, the transmission system planner generally invests in the candidate lines which are the best options for the social welfare of the competitive (and not the oligopolistic) market. This may end up to an increase in the market competitiveness. The second reason is to maintain the linearity of the model, since the true SW is a linear expression while the one with the strategic offers is nonlinear.

Constraint (3b) enforces the annualized financial budget limit for building new lines. Constraints (3c) and (3d) distinct the existing and candidate lines. Finally, constraint (3e) includes all strategic producers' MPECs rendering the EPEC.

In order to solve (3), we first replace the EPEC in (3e) by its KKT conditions. However, each MPEC (5) in (3e), one per strategic producer, is nonlinear and nonconvex, whose KKTs provide strong stationary conditions, which are not necessarily the optimality conditions. Therefore, the KKTs of the EPEC may yield no, or single, or multiple optimal solutions (equilibrium points), or it may even end up to local solutions or saddle points [36].

The KKT conditions associated with the EPEC are derived in Appendix B, including three sets of constraints, i.e., Γ_1, Γ_2 and Γ_3. Note that these conditions include several non-linearities, however, their mixed-integer linear equivalences are provided in Appendix C. Accordingly, the TEP problem (3) is transformed into a MILP problem as follows:

$$\underset{\Xi^{\mathrm{UL}}}{\text{Minimize}} \quad (3\text{a}) \tag{4a}$$

subject to:

$$(3\text{b}) - (3\text{d}) \tag{4b}$$

$$\text{Mixed integer linear form of sets } \Gamma_1 - \Gamma_3. \tag{4c}$$

In this paper, we use an *ex-post* validation technique [10] to check whether each solution obtained from (4) is in fact a Nash equilibrium point. In such a point, no market party can improve its objective (profit in case of producers, and SW in case of transmission system planner) by unilaterally changing its strategies (either long-term investment decisions or short-term offering decisions). Similar to Illustrative Example and Case Study in Section 4, we consider a duopoly with strategic producers P1 and P2. The steps of the ex-post validation technique developed are as follows:

1. Consider a solution obtained for the proposed model (4). Let us denote the optimal strategies obtained for producers P1, P2 and transmission system planner as X^{P1}, X^{P2} and X^{T}, respectively.
2. Solve the strategic generation investment problem for producer P1, while the strategies of producer P2 and transmission system planner are fixed to X^{P2} and X^{T}. This problem is an MPEC, similar to (2), which can be readily linearized. We refer to strategies of producer P1 obtained from this step as Y^{P1}.
3. Solve a similar problem but for producer P2. It is an MPEC in which the strategies of producer P1 and transmission system planner are fixed to X^{P1} and X^{T}. We refer to strategies of producer P2 obtained from this step as Y^{P2}.
4. Solve another MPEC in which the long- and short-term strategic decisions of producers P1 and P2 are fixed to X^{P1} and X^{P2}, and determine the optimal transmission expansion decisions Y^{T}.
5. Check whether $\mathrm{X}^{\mathrm{P1}} = \mathrm{Y}^{\mathrm{P1}}$, $\mathrm{X}^{\mathrm{P2}} = \mathrm{Y}^{\mathrm{P2}}$ and $\mathrm{X}^{\mathrm{T}} = \mathrm{Y}^{\mathrm{T}}$. If so, the solution obtained is in fact a Nash equilibrium because no market party motivates to deviate from its strategy unilaterally.

Note that the similar ex-post steps can be considered in case the number of producers is more than two.

4. Numerical Results

This section verifies the well-functioning of the proposed tri-level TEP model through an Illustrative Example and a Case Study based on the IEEE 14-bus test system [37]. The computational issues for both Illustrative Example and Case Study are discussed in Appendix D.

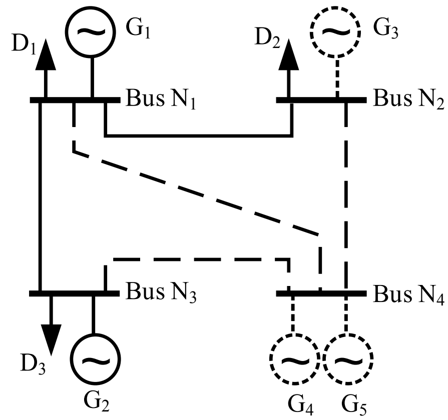

Figure 2: Network of Illustrative Example

4.1. Illustrative Example

In this section, we consider a four-bus power system as depicted in Fig. 2. Two existing generating units (G_1 and G_2) and three candidate ones (G_3, G_4 and G_5) are considered. In addition, we take into account transmission lines N_1-N_2 and N_1-N_3 as existing lines, whereas lines N_1-N_4, N_2-N_4 and N_3-N_4 are candidates to be built. In Fig. 2, the candidate units and lines are highlighted by dash-line. Three demands D_1, D_2 and D_3 are also connected to the system. The susceptance of each line, either existing or candidate, is 500 S. The capacity of each existing line is 350 MW. The capacity and the annualized investment cost of each candidate line are presented in Table 1. The annualized investment budget of the transmission system planner is \$10 million.

Table 1: Data for the candidate transmission lines (Illustrative Example)

Candidate line	Capacity [MW]	Annualized investment cost [million \$]
N_1-N_4	200	3
N_2-N_4	500	5
N_3-N_4	300	4

Table 2: Data for the existing generating units (Illustrative Example)

Generating unit	Capacity [MW]	Production cost [\$/MWh]
G_1	380	10
G_2	220	14

Table 3: Data for the candidate generating units (Illustrative Example)

Generating unit	Maximum capacity to be built [MW]	Production cost [\$/MWh]	Annualized investment cost [\$/MW]
G_3 and G_5	500	18	17500
G_4	750	12	42000

Table 4: Demand data across different blocks (Illustrative Example)

	Demand block 1		Demand block 2		Demand block 3	
	Quantity bid [MW]	Price bid [\$/MWh]	Quantity bid [MW]	Price bid [\$/MWh]	Quantity bid [MW]	Price bid [\$/MWh]
D_1	320	35	290	32	250	30
D_2	350	33	310	30	280	28
D_3	330	31	300	28	270	26

Tables 2 and 3 give the data for existing and candidate generating units, respectively. According to Table 3, two different technologies are considered for generation investment: (i) the base-load technology (unit G_4) with a comparatively high investment cost, but a comparatively low production cost, and (ii) peaking generation technology (units G_3 and G_5) whose investment costs are comparatively low, and whose production costs are comparatively high.

Regarding the demand level in the future target year, three demand blocks are considered, whose weighting factors are 1280, 3250 and 4230, respectively. Note that the summation of all these weighting factors is 8760, i.e., the number of hours in a year. For each block, Table 4 provides the quantity and price bid of each demand. In addition, the supply security factor (S) is assumed to be 1.05.

We obtain the investment results for three different cases as follows:

- Case 1) a single strategic producer owns all existing and future generation portfolio of the market. This case refers to a monopoly. The annualized investment budget of the producer is $27 million.
- Case 2) two strategic producers (P_1 and P_2) are considered creating a duopoly. Producer P_1 owns existing unit G_1, whereas existing unit G_2 belongs to producer P_2. Both producers can invest in both generation technologies considered, i.e., base-load and peaking. The annualized investment budget of producers P_1 and P_2 are $15 million and $12 million, respectively.
- Case 3) this case is similar to Case 2, but producer P1 behaves in a non-strategic manner, i.e., it offers the production of each unit within its portfolio at a price identical to the production cost of that unit.

Table 5: Transmission and generation investment results obtained for Cases 1 to 3 (illustrative example)

	Case 1	Case 2	Case 3
New lines to be built	N_3-N_4	N_1-N_4	N_1-N_4
Generation investment decisions of each producer	G_3 (150 MW) G_4 (300 MW)	G_3 (250 MW) by P_1 G_4 (200 MW) by P_1	G_3 (281.63 MW) by P_2 G_4 (168.37 MW) by P_2
Annualized transmission expansion cost (million $)	4	3	3
Annualized generation investment cost of each producer (million $)	15.22	12.78 (P_1) 0 (P_2)	0 (P_1) 12.00 (P_2)
Annual profit of each producer (million $)	106.15	79.07 (P_1) 25.96 (P_2)	58.16 (P_1) 45.99 (P_2)
Total Annualized profit of producers (million $)	106.15	105.03	104.15
Annual true SW of the market (million $)	64.58	80.81	126.03

Table 5 presents the transmission and generation investment results obtained for Cases 1 to 3. As expected, the single producer in the monopolistic case, i.e., Case 1, gains the highest profit with respect to the total profit of the producers in duopolistic Cases 1 and 2. Therefore, that producer spends more money to invest in both base-load and peaking units. Accordingly, the transmission system planner decides to build line N_3-N_4, whose transmission capacity is higher than that of line N_1-N_4. Note that the annual SW of the market in this monopolistic case is comparatively low ($64.58 million) due to the lack of competition.

Compared to Case 1, the strategic producers in Case 2 spend less money to invest in new units since more peaking units are built. Note that in this case, both new generating units are invested by producer P_1. However, as discussed in [38], several generation investment results can be obtained, in which the *total* invested capacity in each generation technology and the locations of those units are identical, but their ownership distribution between two producers is different. In other words, the only difference among potential solutions is the investors of the newly built units. For example, another generation investment solution is to build 250-MW (as unit G_3) and 200-MW (as unit G_4) by producers P_1 and P_2, respectively. This implies that from the transmission system planner's point of view, the future generation fleet is the same among all solutions. Therefore, the transmission expansion decision is unique and consistent for all potential generation investment solution, which is to build line N_1-N_4. Note that this line is decided to be built since a comparatively lower generation capacity is invested at bus N_4. In Case 2, the annual SW of the market is increased with respect to that in Case 1 due to a higher level of competition.

In Case 3, the profit of producer P_1 is considerably decreased because it behaves in a non-strategic manner. This brings an opportunity for strategic producer P_2 to spend the entire of its investment budget to build new units. In this case, the TEP decision remains unchanged with respect to that in Case 2, however, the SW of the market is significantly increased because one of the producers behaves competitively.

The ex-post checking procedure explained in Section 3 confirms that all results reported for Cases 1 to 3 of Illustrative Example refer to Nash equilibrium points.

4.2. Case Study: The IEEE 14-bus Test System

This section considers the IEEE 14-bus test system [37] including eleven loads, five generating units (G_1 to G_5) and twenty lines as existing facilities, as depicted in Fig. 3. Six demand blocks with identical weighting factors (i.e., 1460) are considered. The load level in the first demand block is identical to that in the original paper [37] raised by 50%. The bid price of each demand in the first block is given in Table 6. In the next five demand blocks, the demand level of each load is identical to that in [37] multiplied by 1.3, 1.2, 1.1, 1.0, and 0.9, respectively. In addition, their bid prices over the second to sixth blocks are identical to those in Table 6 multiplied by 0.95, 0.90, 0.85, 0.80, and 0.75, respectively. Table 7 includes the data for existing units as well as candidate units, i.e., G_6 and G_7. In addition, Table 8 contains the data for four candidate transmission lines. The transmission capacity of each existing line is identical to that in [37] raised by 40%. Similar to Illustrative Example, two strategic producers, i.e., P_1 and P_2, are considered creating a duopoly. Producer P_1 owns the existing units G_1, G_3, and G_5, while the rest belong to P_2. It is assumed that both producers can invest in all candidate units. The transmission and generation investment budgets are unlimited, and the supply security factor S is assumed to be 1.1.

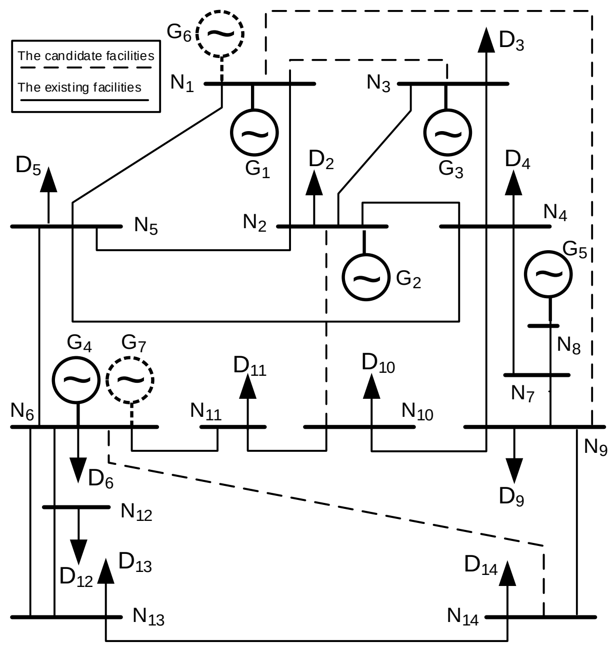

Figure 3: Network of the IEEE 14-Bus test system (Case Study)

Table 6: Bid price data in the first demand block (Case Study)

Load	D_2	D_3	D_4	D_5	D_6	D_9	D_{10}	D_{11}	D_{12}	D_{13}	D_{14}
Bid price [$/MWh]	35	35	35	35	40	38	40	38	40	38	38

Table 7: Data for the existing and candidate generating units (Case Study)

Generating units	G_1	G_2	G_3	G_4	G_5	G_6	G_7
Capacity [MW]	200	150	130	150	100	Max: 200	Max: 200
Production cost [$/MWh]	20	22	16	30	25	19	29
Annualized investment cost [$/MW]	-	-	-	-	-	55000	22000

Table 8: Data for the candidate transmission lines (Case Study)

Candidate line	N_1-N_3	N_1-N_9	N_2-N_{10}	N_6-N_{14}
Capacity [MW]	168	168	168	168
Annualized cost [million $]	2.2	3.0	2.0	2.5
Susceptance [S]	500	768	521	909

As the generation investment results, Producer P_1 invests in the base-load technology (i.e., unit G_6) with the capacity of 116.03-MW, while Producer P_2 decides to build a 122.52-MW peaking unit (i.e., unit G_7). According to these generation investment decisions, the transmission system planner decides to expand the transmission system through investing in two new lines, i.e., N_1-N_3 and N_1-N_9. Based on these planning results, the annual profit of producers P_1 and P_2 are $37.92 million and $16.18 million, respectively, while the annual true

SW of the market is expected to be \$70.32 million. The ex-post procedure explained in Section 3 validates that the result obtained for Case Study is in fact a Nash equilibrium point.

5. Conclusions

This paper proposes a complementarity tri-level model for determining the optimal transmission expansion decisions in an oligopoly considering the generation investment equilibrium. This model is recast as a MILP and applied to an illustrative example and a case study based on the IEEE 14-bus test system. The results obtained verify the well-functioning of the proposed model.

The main advantage of the proposed model is that it assists the transmission system planner to appropriately model the strategic behavior of all producers in terms of their generation capacity expansion and price offering decisions. Therefore, it makes more informed transmission expansion decisions aiming to maximize the social welfare of the market.

The main challenge of the proposed model is the computational issues, especially in case of large-scale systems with hundreds of buses, lines and generating units. Accordingly, our future work will focus on improving the solution methodology based on decomposition and/or parallelization techniques. In addition, a similar study considering stochastic generation sources is a part of our future work, which requires modeling their generation uncertainty.

Appendix A: *MPEC Formulation*

This appendix provides the formulation of the MPEC corresponding to bi-level model (2), representing the strategic decision-making of producer g. The dual variables associated with producer g are indicated in each constraint following a colon:

$$\left\{ \underset{\Xi_g^{\mathrm{IL}}}{\text{Maximize}}\ (2\mathrm{a}) \right. \tag{5a}$$

subject to:

$$0 \le Q_i^{\mathrm{G}} \le Q_i^{\mathrm{Gmax}} \qquad : \underline{\mu}_{g,i}^{\mathrm{inv}}, \overline{\mu}_{g,i}^{\mathrm{inv}} \quad \forall i \in \left(G^{\mathrm{C}} \cap \Omega_g\right) \tag{5b}$$

$$\sum_{i \in (G^{\mathrm{C}} \cap \Omega_g)} K_i Q_i^{\mathrm{G}} \le K_g^{\max} \quad : \mu_g^{\mathrm{Budget}} \tag{5c}$$

$$\sum_{i \in (G^{\mathrm{C}} \cup G^{\mathrm{E}})} Q_i^{\mathrm{G}} \ge S \sum_{d \in D} Q_{(l=1),d}^{\mathrm{D}} \quad : \mu_g^{\mathrm{sec}} \tag{5d}$$

$$\beta_{l,i}^{\mathrm{G}} \ge 0 \quad : \mu_{g,l,i}^{\beta} \ \forall l, \forall i \in \left[(G^{\mathrm{C}} \cup G^{\mathrm{E}}) \cap \Omega_g\right] \tag{5e}$$

$$\sum_{d \in D_n} q_{l,d}^{\mathrm{D}} - \sum_{i \in (G_n^{\mathrm{C}} \cup G_n^{\mathrm{E}})} q_{l,i}^{\mathrm{G}} + \sum_{m \in (\Phi_n^{\mathrm{E}} \cup \Phi_n^{\mathrm{C}})} \frac{u_{n,m}}{X_{n,m}} \left(\delta_{l,n} - \delta_{l,m}\right) = 0 \quad : \mu_{g,l,n}^{\mathrm{bal}} \quad \forall l, \forall n \tag{5f}$$

$$0 \le q_{l,i}^{\mathrm{G}} \le Q_i^{\mathrm{G}} \quad : \underline{\mu}_{g,l,i}^{\mathrm{G}}, \overline{\mu}_{g,l,i}^{\mathrm{G}} \quad \forall l, \forall i \in (G^{\mathrm{C}} \cup G^{\mathrm{E}}) \tag{5g}$$

$$0 \le q_{l,d}^{\mathrm{D}} \le Q_{l,d}^{\mathrm{D}} \quad : \underline{\mu}_{g,l,d}^{\mathrm{D}}, \overline{\mu}_{g,l,d}^{\mathrm{D}} \quad \forall l, \forall d \in D \tag{5h}$$

$$\frac{u_{n,m}}{X_{n,m}} \left(\delta_{l,n} - \delta_{l,m}\right) \le Q_{n,m} : \mu_{g,l,n,m}^{\mathrm{L}} \quad \forall l, \forall n, m \in (\Phi_n^{\mathrm{E}} \cup \Phi_n^{\mathrm{C}}) \tag{5i}$$

$$-\pi \le \delta_{l,n} \le \pi \quad : \underline{\mu}_{g,l,n}^{\delta}, \overline{\mu}_{g,l,n}^{\delta} \quad \forall l, \forall n \tag{5j}$$

$$\delta_{l,(n=1)} = 0 \quad : \mu_{g,l}^{1} \qquad \forall l \tag{5k}$$

$$\sum_{i \in (G^{\mathrm{C}} \cup G^{\mathrm{E}})} \beta_{l,i}^{\mathrm{G}} q_{l,i}^{\mathrm{G}} - \sum_{d \in D} \beta_{l,d}^{\mathrm{D}} q_{l,d}^{\mathrm{D}} + \sum_{i \in (G^{\mathrm{C}} \cup G^{\mathrm{E}})} \overline{\sigma}_{l,i}^{\mathrm{G}} Q_i^{\mathrm{G}} + \sum_{d \in D} \overline{\sigma}_{l,d}^{\mathrm{D}} Q_{l,d}^{\mathrm{D}} + \sum_{n,m \in (\Phi_n^{\mathrm{E}} \cup \Phi_n^{\mathrm{C}})} \sigma_{l,n,m}^{\mathrm{L}} Q_{n,m}$$
$$+ \pi \sum_n \left(\overline{\sigma}_{l,n}^{\delta} + \underline{\sigma}_{l,n}^{\delta}\right) = 0 \ : \Psi_{g,l} \ \forall l \tag{5l}$$

$$\beta_{l,i}^{\mathrm{G}} - p_{l,[n:i \in (G_n^{\mathrm{C}} \cup G_n^{\mathrm{E}})]} - \underline{\sigma}_{l,i}^{\mathrm{G}} + \overline{\sigma}_{l,i}^{\mathrm{G}} = 0 \quad : \eta_{g,l,i}^{\mathrm{G}} \quad \forall l, \forall i \in (G^{\mathrm{C}} \cup G^{\mathrm{E}}) \tag{5m}$$

$$-\beta_{l,d}^{\mathrm{D}} + p_{l,(n:d \in D_n)} - \underline{\sigma}_{l,d}^{\mathrm{D}} + \overline{\sigma}_{l,d}^{\mathrm{D}} = 0 : \eta_{g,l,d}^{\mathrm{D}} \ \forall l, \forall d \tag{5n}$$

$$\sum_{m\epsilon(\Phi_n^{\mathrm{E}}\cup\Phi_n^{\mathrm{C}})}\frac{u_{n,m}}{X_{n,m}}\left(p_{l,n}-p_{l,m}+\sigma_{l,n,m}^{\mathrm{L}}-\sigma_{l,m,n}^{\mathrm{L}}\right)+\overline{\sigma}_{l,n}^{\delta}-\underline{\sigma}_{l,n}^{\delta}+(\sigma_l^{1})_{n=1}=0\ :\eta_{g,l,n}^{\delta}\ \ \forall l,\forall n \quad (5o)$$

$$\underline{\sigma}_{l,i}^{\mathrm{G}}\geq 0,\overline{\sigma}_{l,i}^{\mathrm{G}}\geq 0\quad :\underline{\xi}_{g,l,i}^{\mathrm{G}},\overline{\xi}_{g,l,i}^{\mathrm{G}}\quad \forall l,\forall i\in(G^{\mathrm{C}}\cup G^{\mathrm{E}}) \quad (5p)$$

$$\underline{\sigma}_{l,d}^{\mathrm{D}}\geq 0,\overline{\sigma}_{l,d}^{\mathrm{D}}\geq 0\quad :\underline{\xi}_{g,l,d}^{\mathrm{D}},\overline{\xi}_{g,l,d}^{\mathrm{D}}\quad \forall l,\forall d \quad (5q)$$

$$\sigma_{l,n,m}^{\mathrm{L}}\geq 0\quad :\xi_{g,l,n,m}^{\mathrm{L}}\quad \forall l,\forall n,\forall m\epsilon\left(\Phi_n^{\mathrm{E}}\cup\Phi_n^{\mathrm{C}}\right) \quad (5r)$$

$$\left.\underline{\sigma}_{l,n}^{\delta}\geq 0,\overline{\sigma}_{l,n}^{\delta}\geq 0\quad :\underline{\xi}_{g,l,n}^{\delta},\overline{\xi}_{g,l,n}^{\delta}\quad \forall l,\forall n\right\}\quad \forall g. \quad (5s)$$

Within each MPEC (5) above, one per producer, note that (5a)-(5e) are those included in (2a)-(2e), while conditions (5f)-(5s) are equivalent to (2f), i.e., lower-level problems (1). In details, conditions (5f)-(5k) are primal constraints in (1). Condition (5l) is the strong duality equality associated with (1) enforcing the equality of primal and dual objective functions at the optimal solution. Finally, conditions (5m)-(5s) are dual constraints of (1).

Appendix B: *KKT Conditions of the EPEC*

The KKT conditions of the EPEC consist of three sets of constraints as follows:

1) The equality conditions obtained from differentiating the Lagrangian associated with MPECs (5) with respect to variables in Ξ_g^{IL}. We refer to this set of conditions by Γ_1. An example of the members of this set is below:

$$K_i-\underline{\mu}_{g,i}^{\mathrm{inv}}+\overline{\mu}_{g,i}^{\mathrm{inv}}+\mu_g^{\mathrm{Budget}}K_i-\mu_g^{\mathrm{sec}}-\sum_l\overline{\mu}_{g,l,i}^{\mathrm{G}}+\sum_l\Psi_{g,l}\overline{\sigma}_{l,i}^{\mathrm{G}}$$
$$=0\ \ \forall g,\forall i\in\left(\mathrm{G^C}\cap\Omega_g\right) \quad (6a)$$

$$-\mu_g^{\mathrm{sec}}-\sum_l\overline{\mu}_{g,l,i}^{\mathrm{G}}+\sum_l\Psi_{g,l}\ \overline{\sigma}_{l,i}^{\mathrm{G}}=0\qquad \forall g,\forall i\in\mathrm{G^C},\forall i\notin\Omega_g \quad (6b)$$

Conditions (6) above are derived from differentiating the Lagrangian corresponding to each MPEC, presented in (5), with respect to generation investment decisions, i.e., $Q_i^{\mathrm{G}},\forall i\in G^{\mathrm{C}}$.

2) Complementarity conditions derived from the inequality constraints of MPECs (5). We refer to this set of conditions by Γ_2. An example of the members of this set is below:

$$0\leq Q_i^{\mathrm{G}}\perp\underline{\mu}_{g,i}^{\mathrm{inv}}\geq 0\qquad \forall g,\forall i\in(\mathrm{G^C}\cap\Omega_g) \quad (7)$$

which corresponds to inequality constraint (5b).

3) The equality constraints within MPECs (5) given by (8) below. We refer to this set of constraints by Γ_3.

(5f), (5k)-(5o) $\qquad \forall g.$ (8)

Appendix C: *Linearization Techniques*

There are four types of non-linearities within sets $\Gamma_1-\Gamma_3$ introduced in Appendix B:

- *Conditions* Γ_2: There are two techniques to linearize each complementarity condition of set Γ_2 without approximation, namely Big-M approach [39] and SOS1-based approach [40], as described below:

  The Big-M approach [39] linearizes each complementarity condition of the form $0\leq q\perp\mu\geq 0$ through the following set of equations: $q\geq 0$, $\mu\geq 0$, $\mu\leq a\times M$, and $q\leq(1-a)\times M$. Note that a is an auxiliary binary variable and M is a large enough positive constant. The use of this linearization technique is costly in the large-size problems due to the increased number of binary variables. The selection of appropriate values for M is also challenging, so that a too large value may result in complementarity not being satisfied, while a value that is too small may lead to numerical ill-conditioning.

  The SOS1-based approach [40] is another alterative, which adds special ordered set variables of type 1 (SOS1 variables) to the problem. This technique replaces each complementarity condition of the form $0\leq q\perp\mu\geq 0$ by the following set of equations: $q\geq 0$, $\mu\geq 0$, $q+\mu=s_1+s_2$, and $q-\mu=s_1-s_2$. Note that s_1 and s_2 are SOS1 variables, i.e., at most one of them can take a strictly positive (non-zero) value. This technique can also be costly in large-size problems due to the increased number of SOS1 variables.

The third option [40], but at the cost of approximation, is similar to the SOS1-based approach. In this option, the SOS1 variables s_1 and s_2 are relaxed to be non-negative variables. However, the objective function is penalized by adding a term of the form $-b \times (s_1 + s_2)$, where b is a small positive constant. This option is mathematically straightforward since it adds neither binary variables nor SOS1 variables. However, the value selection for parameter b is challenging, so that an inappropriate value can bring a significant approximation.

In this paper, we use all the three alternatives above, each applied to different complementarity conditions.

- *The bilinear terms within (5l) included in (8):* In a linear optimization problem, e.g., the lower-level problems (1), the strong duality equality is equivalent to the set of all complementarity conditions associated to the inequality constraints of that problem [36]. Therefore, we replace the strong duality equality (5l) by its equivalent set of complementarity conditions. Each complementarity condition can be then linearized using the Big-M, or the SOS1-based, or the penalization technique.
- *The bilinear terms including Ψ_{gt}:* The proposed model can be parameterized in dual variables Ψ_{gt} without incurring any approximation [36]. The reason is that the MPEC (5) is nonconvex, and therefore the values obtained for its dual variables are not unique. This redundancy brings some degree of freedom in the selection of values for those dual variables, and allows the parameterization of dual variables Ψ_{gt}.
- *The bilinear terms including binary variables u_{nm}:* A similar Big-M approach [40]-[41] is used. For example, consider a bilinear term of the form $u \times \delta$ where u and δ are binary and continuous variables, respectively. This bilinear term is replaced by an auxiliary continuous variable f, and then the following linear constraints including a big positive constant M are added: $-u \times M \leq f \leq u \times M$ and $-(1-u) \times M \leq (f - \delta) \leq (1-u) \times M$.

Appendix D: *Computational Issues*

The proposed MILP model (4) is solved using CPLEX under GAMS on an Intel® Core™ i7-2620M with 4 processors clocking at 2.7 GHz and 4 GB of RAM. The optimality gap is fixed to zero. For both Illustrative Example and Case Study, Table 9 gives the computational times and the number of variables and constraints.

Table 9: Computational time and number of variables/constraints

	Illustrative Example	Case Study
Computational time	19 seconds	18 hours
Number of continuous variables	1267	28536
Number of binary variables	782	1648
Number of SOS1 variables	0	3648
Number of equality constraints	456	6893
Number of inequality constraints	1428	1848

Note that we use the Big-M approach only in Illustrative Example to linearize all complementarity conditions, and therefore, no SOS1 variable appears. However, we use all Big-M, SOS1-based, and also penalization methods to linearize complementarity conditions in Case Study, and therefore, both binary and SOS1 variables exist.

According to Table 9, the computational time in Case Study is significantly higher with respect to that in Illustrative Example. This motivates the use of parallelization and/or decomposition techniques for the large-scale applications, which is in fact our future work.

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