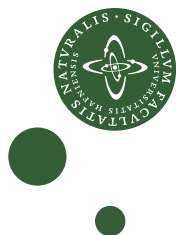


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**Abstracts**



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# On the Multivariate Extension of Countermonotonicity and its Statistical Application

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## Abstract

We study multivariate extension of countermonotonicity. For the bivariate case, the Frechet-Hoeffding upper and lower bounds define comonotonicity and countermonotonicity respectively, see for example [1]. Similarly in the multivariate case, one can define comonotonicity using the Frechet-Hoeffding upper bound. However since the multivariate Frechet-Hoeffding lower bound is not a distribution function any more, there is no obvious extension of countermonotonicity to multivariate dimensions. In this project, we first provide the equivalent condition for the countermonotonicity in bivariate random variables. Using this equivalent condition, we extend the countermonotonicity to multivariate case. This extension might be interesting to someone, because countermonotonic multivariate random variables attain the lower bounds of some well-known concordance measures, for example, multivariate Kendall's tau as defined in [2].

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# Multivariate Tweedie Lifetimes: The Impact of Dependence

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## Abstract

Systematic improvements in mortality increases dependence in the survival distributions of insured lives. This is not accounted for in standard life tables and actuarial models used for annuity pricing and reserving. Furthermore, systematic longevity risk undermines the law of large numbers; a law that is relied on in the risk management of life insurance and annuity portfolios. This paper applies a multivariate Tweedie distribution to incorporate dependence, which it induces through a common shock component. Model parameter estimation is developed based on the method of moments and generalized to allow for truncated observations. The estimation procedure is explicitly developed for various important distributions belonging to the Tweedie family, and finally assessed using simulation.

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# Foreign-currency interest-rate swaps in asset-liability management for insurers

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## Abstract

We consider an insurer with purely domestic business whose liabilities towards its policy holders have long durations. The relative shortage of domestic government bonds with long maturities makes the insurer's net asset value sensitive to fluctuations in the zero rates used for liability valuation. Therefore, in order to increase the duration of the insurer's assets, it is common practice for insurers to take a position as the fixed-rate receiver in an interest-rate swap. We assume that this is not possible in the domestic currency but in a foreign currency supporting a larger market of interest-rate swaps. Monthly data over 16 years are used as the basis for investigating the risks to the future net asset value of the insurer from using foreign-currency interest-rate swaps as a proxy for domestic ones in asset-liability management. We find that although a suitable position in swaps may reduce the standard deviation of the future net asset value it may significantly increase the exposure to tail risk that has a substantial effect on the estimation of the solvency capital requirements.

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# Using MADM Techniques in Selecting Media for Life Insurance Marketing in Developing Countries (case study: I.R.IRAN)

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## **Abstract**

Considering the cultural elements in developing countries, lack of insurance in household portfolio plus insurance aversion in most of these countries, indicating qualitative and quantitative factors in representing a better pattern for selecting the media type in insurance marketing by dimensionless these factors and attain an integrated pattern. This paper intends by using MADM techniques illustrates a mathematical approach in selecting media for an effective promotion.

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# Generalized Autoregressive Conditional Duration Model: A new model for claims reserving in Non life insurance

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## Abstract

In this paper, we have proposed a Generalized Autoregressive Conditional duration models. they extend the Generalized linear model to incorporate temporal dependence in the claims amounts of the insurance company. Models are applied to predict reserves. Results show that taking into account the temporal dependence between losses improve the evaluation of reserves in the future.

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# Optimal Reinsurance in the Presence of Counterparty Default Risk

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## Abstract

The optimal reinsurance arrangement is identified whenever the reinsurer counterparty default risk is incorporated in a one-period model. Our default risk model allows the possibility for the reinsurer to fail paying in full the promised indemnity, whenever it exceeds the level of regulatory capital. We also investigate the change in the optimal solution if the reinsurance premium recognises or not the default in payment. Closed form solutions are elaborated when the insurer's objective function is set via some well-known risk measure.

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# Annuity Valuation with Health State Dependence Utility Function

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## Abstract

In a post retirement phase, annuities are known as a product that provide a steady stream of income to retirees. Prior studies have shown that consumer who had an access to actuarially fair annuity product would always find a full annuitisation strategy as optimal. Most of these findings are consistent for a traditional, plain vanilla annuity even when the value for money varies for different product. However, the rate of annuitisation in many countries is still low and one of the issues with annuities is lack of liquidity to cover for any unexpected medical expenses for retirees. Recent development on annuity product design in Malaysia includes additional benefits in the form of lump sum payment for unfortunate events such as total permanent disability, critical illnesses and death. Such benefits could enhance the value of the product and it is important especially for potential annuity buyers, to see how the value for money of annuities changes with these additional benefits. This paper extends the Annuity Equivalent Wealth (AEW) model used by economists to value annuity product by incorporating several health states into the model. The model is developed using a discrete-time Multi State Markovian framework where national data is required to estimate the transition probabilities for each health state. Instead of using deterministic medical expenditure associated with different health state, we apply the health state dependence utility function which gives a different utility value of a unit of consumption depending on the state of health of the consumer.

Keywords: annuities, Annuity Equivalent Wealth, Muti State Markovian framework, health state dependence utility.

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# Relating deterministic and stochastic approaches to estimating the log-likelihood of the Markov-switching GARCH model

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## Abstract

The Markov-switching GARCH model allows for a GARCH structure with time-varying parameters. Its introduction in the 1990s was motivated by the inability of the standard GARCH model to reflect structural changes in financial time series which are usually found to occur over many years of data. Estimating this path dependent model is a challenging task because exact computation of the likelihood is infeasible in practice. This difficulty led to computationally intensive estimation procedures or to simpler techniques based on a crude simplification of the model. A novel approach is developed to perform maximum likelihood inference in the Markov-switching GARCH model. The proposed estimation procedure is related to a deterministic sequential Monte Carlo method and to particle filtering which enables the study of its convergence. The main advantage of the technique introduced is that it allows for an accurate and fast estimation of the Markov-switching GARCH model. The effectiveness of the algorithm is demonstrated in an empirical study with S&P 500 data.

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# Optimal dividend payment and regime switching in a compound Poisson risk model

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## Abstract

We study a mixed singular control/optimal stopping problem for an insurance company. The manager has the possibility of switching among several regimes, in each of the regimes the uncontrolled surplus of the company evolves as a different compound Poisson process with drift.

Switching among regimes could produce an instantaneous transition cost or benefit. The manager pays dividends to the shareholders and the goal is to find the dividend payment policy and the switches among regimes (times and destinations) which maximize the expected cumulative discounted dividend pay-outs until the ruin time.

We address both the cases of irreversible and reversible switching. These problems can be seen as obstacle problems. We characterize the optimal value function as the smallest viscosity solution of the associated Hamilton-Jacobi-Bellman equation. We prove that there exists an optimal dividend and switching strategy and that this strategy is stationary with a band structure. We find a verification result to check optimality even in the case where the optimal value function is not differentiable. We present numerical examples of irreversible switching (optimal time for acquisition of another company and optimal time for disinvestment of a branch of the company) as well as an example of reversible switching. In some of these examples the value function is neither concave nor differentiable.

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# Multivariate Risk Models and Queues Part II: Extensions to multi-dimensional ruin models and coupled premium income rates

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## Abstract

In Part I, we considered the joint ruin probability for a two-dimensional risk model with simultaneous arrivals, along with the dual bivariate queueing model. In this talk we build upon the previous concepts, and we show how one can use a simple recursion argument to generalize the model to an arbitrary multivariate ruin model.

We also consider models with coupled premium income rates, i.e. as long as the reserve process in one of the insurance lines is running at a maximum, the income in the other lines is increased. Some relations with the dual queueing models with coupled processors are also outlined. We study the multivariate ruin probability function by using Laplace transform methods and asymptotic methods.

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# Comparison Between Two Kind of Third Party Insurance Premium's Modeling

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## Abstract

Ratemaking is one of the main mathematical problems faced by actuaries. They must first estimate how much each insurance contract is expected to cost. This conditional expected claim amount is called the pure premium and it is the basis of the gross premium charged to the insured. This expected value is conditioned on information available about the insured and about the contract, which we call the input profile. On the other hand, the factors that affect on the automobile premium are divided into 3 groups:

1. Driver characteristics
2. Automobile characteristics
3. Usage of automobile

In this article, age, gender, marital status, type of driving license, age of driving license and nationality have used from driver characteristics; model of car, number of cylinder, car group, loading capacity and car capacity have used from automobile characteristics and finally usage, city and state have used from usage of automobile for finding third party insurance premium with data mining.

It should be noted that Generalized Linear Models (GLMs) have become popular and proven techniques for ratemaking and actuarial work over the past decade. For finding pure premium, two methods have used:

1. Decomposing claims into claim frequency and claim severity and finding the best models for claim frequency and claim severity in property damage claims and body injury claims. The standard GLM tariff analysis is to do separate analyses for claim frequency and claim severity, and then relativities for the pure premium are found by multiplying the results. The results show that the best distribution for claim severity is Gamma and the best model for claim frequency is Negative Binomial (with log link function for both distribution for both claims).
2. The other way for finding pure premium is modeling claims with a Tweedie model for  $1 < p < 2$  (the Tweedie distribution is a compound Poisson-Gamma mixture distribution for these  $p$ 's), to analyze the pure premium directly for two-part data. Two-part data, where there is a mass at zero and a continuous component. For insurance claims data, the zero represents no claim and the continuous component represents the amount of a claim. By this way we can analyze the pure premium directly.

Comparison between separation into two GLMs and using Tweedie distribution has shown that:

- Claim frequency is usually much more stable than claim severity and often much of the power of rating factors is related to claim frequency, these factors can then be estimated with greater accuracy.
- A separate analysis gives more insight into how a rating factor affects the pure premium.
- On the other hand assuming that  $N$  has Poisson distribution in Tweedie model is a wrong assumption and as we see in this article Negative Binomial distribution is better than Poisson distribution so totally results have shown that decomposing the claims has got better results for finding third party insurance premium.

Keyword: third party insurance premium, GLM, data mining, claim severity, claim frequency, Poisson Distribution, Negative Binomial Distribution, Gamma Distribution and Tweedie distribution.

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# Arbitrage absence and projective limits of martingale measures

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## Abstract

This paper addresses the equivalence between the absence of arbitrage and the existence of equivalent martingale measures. The equivalence will be established under quite weak assumptions since there are no conditions on the set of trading dates (it may be finite or infinite, with bounded or unbounded horizon) and the trajectories of the price process only have to be right-continuous.

Besides we will deal with arbitrage portfolios rather than free-lunches. The concept of arbitrage is much more intuitive than the concept of free lunch and has clearer economic interpretation. In order to overcome the usual mathematical difficulties arising when dealing with arbitrage strategies, the set of states of nature will be widened by drawing on projective systems of Radon probability measures, whose projective limit will be the martingale measure. The existence of densities between the “real” probabilities and the “risk-neutral” probabilities will be guaranteed by introducing the concept of “projective equivalence”. Hence some classical counter-examples will be solved and a complete characterization of the absence of arbitrage will be provided in a very general framework.

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# Benchmarks, CAPM-like formulae and good deal absence with ambiguity and coherent risk

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## Abstract

Risk measures beyond the variance have shown theoretical advantages when addressing some classical problems of Financial Economics, at least if asymmetries and/or heavy tails are involved. Nevertheless, in portfolio selection, these risk measures have generated several caveats such as the existence of good deals in most of the arbitrage free pricing models. Models such as Black and Scholes or Heston allow investors to build sequences of strategies whose expected return tends to infinite and whose risk remains bounded or tends to minus infinite. This paper studies whether this drawback still holds if the investor is facing the presence of multiple priors, as well as the properties of optimal portfolios in a good deal free framework.

With respect to the first objective, we find four possible results. If the investor uncertainty (set of priors) is too high he/she has no incentives to buy risky assets. As the uncertainty decreases the interest in risky securities increases. If her/his uncertainty becomes too low then two types of good deal may arise. Consequently, there is a very important difference between the ambiguous (with uncertainty) and the non ambiguous (uncertainty free) setting. Under ambiguity the investor uncertainty may increase in such a manner that the model becomes good deal free. Hence, ambiguity may help to overcome some meaningless findings in asset pricing.

With respect to our second objective, good deal free ambiguous models imply the existence of a benchmark generating a robust capital market line. The robust (worst-case) coherent risk of every strategy may be divided into systemic and specific, and no robust return is paid by the specific coherent robust risk. Extensions of the *CAPM* most important formulas will be proved.

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## Coherent pricing

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### Abstract

Risk measures beyond the variance are becoming more and more used by practitioners, regulators and researchers. However, in portfolio selection these risk measures have generated several drawbacks. Models such as Black and Scholes or Heston allow investors to build sequences of strategies whose expected return tends to infinite and whose risk remains bounded or tends to minus infinite.

This paper proposes a theoretical solution to the caveat above. Thus, we consider a pricing model and a coherent risk measure leading to an unbounded portfolio choice problem and then we construct a new pricing model satisfying two main properties; 1), the portfolio choice problem becomes bounded and the optimal risk/return ratio equals a target value previously estimated. 2), once a new method to measure the distance between two pricing models has been created, if  $d$  is the distance between the initial pricing model and the modified one, and  $\tilde{d}$  is the distance with other pricing model satisfying 1), then  $\tilde{d} \geq d$ .

Numerical and illustrative examples will be provided.

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## Comparison results for Garch processes

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### Abstract

We consider the general problem of stochastic comparison of Garch processes, when the distributions of the innovations  $\varepsilon_n$  or the value of the defining parameters are changed. We consider two very general classes of Garch processes: in the first one the recursive dynamic is specified in terms of the volatility, while in the second one it is specified in terms of the variance. The usual Garch (1,1) model belongs to the intersection of both classes.

Applying standard techniques for the comparison of iterative models, we identify several stochastic orders that are "propagated" from the innovations  $\varepsilon_n$  to the Garch process  $X_n$  itself. In particular, we show that if either  $\varepsilon_n \leq_{cx} \tilde{\varepsilon}_n$ ,  $\varepsilon_n^2 \leq_{st} \tilde{\varepsilon}_n^2$ , or  $\varepsilon_n^2 \leq_{icx} \tilde{\varepsilon}_n^2$ , then correspondingly  $X_n \leq_{cx} \tilde{X}_n$ ,  $X_n^2 \leq_{st} \tilde{X}_n^2$ ,  $X_n^2 \leq_{icx} \tilde{X}_n^2$ . The  $\leq_{st}$  ordering and the  $\leq_{icx}$  ordering between squared variables arise naturally in the Garch context; in the case of symmetric innovations, the first relation is equivalent to the peakedness ordering, while the second can be interpreted as a kurtosis ordering.

Motivated by financial applications, we consider the stochastic comparison of the sums  $S_n = \sum_{k=1}^n X_k$ . The problem is not trivial since the variables  $X_k$  are not independent, so standard results cannot be applied. Our main result is to prove that in the case of symmetric innovations, if  $\varepsilon_n \leq_{cx} \tilde{\varepsilon}_n$ , then  $S_n \leq_{cx} \tilde{S}_n$ . We conjecture this implication to be true also for more general innovations.

We then move to multivariate comparisons (of which the comparison results for the sums are a special case). We prove that in the case of symmetric innovations, if  $\varepsilon_n \leq_{cx} \tilde{\varepsilon}_n$ , then

$$E[f(X_1, \dots, X_n)] \leq E[f(\tilde{X}_1, \dots, \tilde{X}_n)],$$

for each  $f : R^n \rightarrow R$  convex and supermodular.

Finally, we discuss comparison results for  $X_n$  and  $S_n$  for varying parameters  $\alpha_0$ ,  $\alpha_1$  and  $\beta_1$  in the case of usual Garch(1,1) models.

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# Insurer's incomplete information in out-of-court motor injury settlements

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## Abstract

The Spier's multiperiod negotiation model [1] is extended in which information asymmetry is allowed to be dynamic over the negotiation process. We assume that a round of defendant's offer and, in case of rejection, a plaintiff's counter-demand of compensation is made in each negotiation period. We focus on the distribution function of settlements over time to test the hypothesis of dynamic pattern of information. Competing risk models are applied to a Spanish motor insurance database to estimate the distribution function of settlements over time and analyze which factors affect delay in motor injury settlements. Time-varying effects are observed for those explanatory factors related to victim's age and the severity of injuries. Results indicate that the effect of these factors on the claim duration is attenuated over time supporting the assumption that the insurer's incomplete information is enriched through the negotiation process.

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# A posteriori ratemaking using bivariate Poisson models

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## Abstract

In [1] and [2] different bivariate Poisson regression models were used to make an *a priori* ratemaking taking into account the dependence between two type of claims. A natural extension for these papers is to consider *a posteriori* ratemaking (i.e. bonus-malus system) that also relaxes the independence assumption.

We introduce here different bivariate bonus-malus systems that integrate the *a priori* ratemaking based on the bivariate Poisson regression models, extending the existing literature for the univariate case to the bivariate case. These bivariate bonus-malus systems are applied to the same automobile insurance claims data set as used in [1] and [2] and the consequences for posterior premiums when the independence assumption is relaxed are analysed.

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# Risk measurement and model uncertainty: a simulation study

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## Abstract

Academic literature on risk measures mainly relies on the assumption that the probability distribution of financial positions is known. In reality, this distribution is unobservable and must be estimated from a limited sample of past data leading to model uncertainty. In the present contribution the impact of model uncertainty on the calculation of risk measures such as VaR is assessed, via the notion of residual estimation risk. For a given set of candidate models, posterior model weights are obtained via a Bayesian approach. Then capital estimation methods are considered based on: (a) worst case scenario; (b) highest model posterior; (c) averaging capital requirements over models; and (d) averaging loss distributions over models. We study the performance of these methods on a test set of models, which only partially overlaps with the set of models used in estimation. A simulation study is conducted. It emerges that there is great sensitivity to the set of models used for calculating capital requirements. Depending on the model set used, increasing the size of the data set has an adverse effect on residual estimation risk. Furthermore, using models that are more heavy-tailed than the one from which data originate can reduce rather than increase the estimated capital.

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## A microscopic model for longevity purposes

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### Abstract

We propose to study an Individual Based Model, also called microscopic model, for longevity purposes. This model, inspired by [2] and [1] in the field of ecology, and the work of [3] for demographic purposes, allows to take into account individual characteristics (called "traits") and age. The individual characteristics that can be taken into account are for example socio-economic characteristics such as socio-professional categories, education level and marital status, which have a real impact on demographic behavior. The population is subject to multiple changes at the level of individuals, who have demographic intensities linked to birth, death, evolution of their traits and migration. The demographic intensities of the individuals can vary according to their specific age and characteristics. At the level of the whole population, we allow to model stochastic exogenous factors, which can be interpreted as macro-economic indicators, and also catastrophic events.

In this model, the population is described by a point process, which is the solution of a particular Stochastic Equation based on the individual demographic intensities. The point process describing the population can be perfectly simulated in order to obtain multiple scenarios of the evolution of the population and various statistics of interest. This microscopic model also provides coherent macroscopic results. In a large population framework, *i.e.* when the size of the population goes to infinity, the population density is described by an equation that extends the well known McKendrick-Von Foerster Partial Differential Equation.

For longevity issues in actuarial contexts, pension funds and insurers can identify extreme aging scenarios and use this model for solvency purposes. By taking specific individual information into account, this microscopic model is also a tool for giving accurate estimation of an insurer's portfolio basis risk. One can estimate the longevity risk associated to a particular life insurance product by using the specific information of the annuitants. Numerical results will be provided for this longevity issue. In this context, the aim of this study is also to discuss the calibration method and quantify the variability linked to exogenous factors and population behaviour.

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# On simple ruin expressions in dependent Sparre Andersen risk models

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## Abstract

We provide a simple alternative derivation of an explicit formula of Kwan and Yang [1] for the probability of ruin in a risk model with a certain dependence between general claim inter-occurrence times and subsequent claim sizes of conditionally exponential type. The approach puts the type of formula in a general context, illustrating the potential for similar simple ruin probability expressions in more general risk models with dependence.

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# Conditional copula simulation for systemic risk stress testing

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## Abstract

The notion of systemic relevance of financial institutions is central in the discussion about lessons from the financial crisis. Both the banking as well as the insurance industry are dealing with this issue, in an effort to identify systemically important institutions and reduce the systemic risk in the international financial market. In this talk, we analyze the interconnectedness of institutions, as a major determinant of systemic risk, from a statistical point of view.

Interconnectedness of institutions is often investigated in the context of correlations or, more generally, using elliptical, especially Gaussian, distributions. Dependence may however be characterized by features like asymmetry and tail dependence. Such features can be accounted for appropriately using copulas in contrast to Gaussian distributions. For the particular purpose of a copula-based network analysis of the international financial sector, we construct a hierarchical copula model, which is built up by flexible copulas for non-overlapping groups of variables in different hierarchical levels. To investigate contagion effects, we develop methods for stress testing, exploiting the underlying dependence structure. Strategies for conditional simulation of common copulas are discussed. In particular, new approaches for Archimedean copulas as well as for the proposed hierarchical construction are derived.

In a case study of 38 major international institutions, 20 insurers and 18 banks, we analyze interdependencies of credit default swap spreads and perform a systemic risk stress test. Geographical as well as sectorial dependencies among the institutions in the international financial sector are captured using the hierarchical copula model. The specified dependence model and the results from the stress test then provide new insights into the interconnectedness of banks and insurers. We find that the failure of a bank apparently constitutes a larger systemic risk than the failure of an insurer.

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# Policyholder behaviour: A model for dependent interest rate and surrender rate

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## Abstract

Modelling interest and transition rates as dependent affine processes provides certain mathematical tractability for valuation of life insurance liabilities. Within this setup, we propose a specific model for modelling dependent interest and surrender rates for valuation of life insurance liabilities. The interest and surrender rates are modelled jointly as a 2-dimensional affine process. Within this setup, we can find so-called generalised forward rates, and use these to value certain life insurance liabilities. We numerically consider a simple savings contract with a buy-back option, and calculate the liabilities and the Solvency II capital requirement, and examine the effect of correlation on these quantities. We also consider a static hedging strategy where the interest rate risk is hedged.

Keywords: Surrender modelling; Policyholder behaviour; Affine Processes; Doubly Stochastic Process; Solvency II

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# A risk-based premium: What does it mean for DB plan sponsors?

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## Abstract

This paper develops a risk-based premium calculation model for the insurance provided by the Pension Benefit Guaranty Corporation (PBGC). It takes account of the pension fund's and the plan sponsor's investment policy and extends [1] by considering distress termination triggered by the sponsor's underfunding. We empirically illustrate our theoretical pricing formula for the 100 biggest American DB sponsoring companies. Our result clearly casts doubt on the current practice where about 70% of the PBGC premiums charged are flat. We observe that the funding ratio, the leverage, and the asset volatility of the sponsoring companies are three key risk factors in a risk-based premium calculation.

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# Optimal Termination Rules for DB Pension Insurance

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## Abstract

Defined-benefit (DB) pension plans have a long history of underfunding in the U.S. and in other countries. The average funding ratio of a sample of U.S. DB plans was 65.5% in 1978 and 76.8% in 1981 (Ippolito (1986)[6]). In the UK, estimated aggregate deficits for the DB schemes of FTSE 350 companies were £61 billion in November 2012. DB plans sponsored by largely traded corporations in Canada have fallen into deficits since 2000 (Armstrong (2004)[2]). As Jametti (2008)[7] argues, underfunding of DB plans is an equilibrium outcome in the incomplete financial market and acts as a motivation for the federal pension insurance programs in some countries, such as the Pension Benefit Guaranty Corporation (PBGC) in the U.S. and the Pension Benefit Guatantee Fund (PBGF) in Ontario, Canada. Severe pension deficits have already driven federal pension insurance programs into a financial deficit position. PBGC has been on average in \$20.76 billion deficit in the past ten years. With an exception of 2006, PBGF has run a deficit every year from 2003 to 2011, on average \$128.11 millions.

There are a variety of drivers for the severe deterioration of the financial condition of those federal insurance programs. Most of the academic literature thus far has focused on the ineffective premium calculation and instead proposed a risk-based premium, see Sharpe (1976)[10], Lewis and Pennachi (1994)[9], Brown (2008)[3] or Chen (2011)[4], just to name a few. However Kalra and Jain (1997)[8] argue that by law the federal insurance programs do not have the opportunity to control the riskiness of the pension funds and they are also restricted in adjusting premiums. Therefore the main way by which the PBGC or the PBGF can intervene is by prematurely terminating insured pension funds. This raises the question of whether the federal insurance programs should terminate severely underfunded pension funds and if they should then when exactly pension funds should be terminated.

Our paper gives insight into the two questions above by finding optimal termination ratios for an insured pension fund in a one-year expected power-type utility maximization problem framework. Our specified termination rule states that the pension plan will be terminated and taken over by the federal insurance program once its financial status goes down the predetermined termination ratio. In particular we are the first to take into account the risk aversion of the beneficiaries and more importantly we incorporate regulatory constraints as the shortfall probability and the Expected Shortfall (ES) constraint into the one-year expected utility maximisation problem. The inclusion of shortfall probability constraint into the expected utility maximisation originates from the annual solvency test of Dutch pension plans, which helps alleviate current insolvencies of the federal insurance program. The ES constraint focuses on restricting ongoing-concern liabilities of the federal insurance program. The second constraint has the advantage that it can assess the size of the losses of ongoing but not terminated pension plans and therefore it can better identify pension funds with the highest cost of insolvencies, see Doff (2008)[5].

We find that by considering the expected utility maximisation problem solely with the shortfall probability constraint a termination rule is only optimal for more risk averse

beneficiaries, but not for risk-neutral and less risk averse ones. This result justifies to some extent the passive behaviour of the PBGC. The adding of the ES constraint however justifies premature termination rules independent of the preferences of the beneficiaries. Moreover for realistic parameter values we obtain the result that only severely underfunded pension funds should be closed, which is in line with the findings of Acharya and Dreyfus (1989)[1] and Kalra and Jain (1997)[8]. In our model in the benchmark case the optimal termination ratio particularly depends on the risk aversion of the pension beneficiaries.

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# The Markov additive risk process under an Erlangized dividend barrier strategy

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## Abstract

In this talk, we consider a Markov additive insurance risk process under a randomized dividend strategy in the spirit of [1]. Decisions on whether to pay dividends are only made at a sequence of dividend decision time points whose intervals are Erlang( $n$ ) distributed. At a dividend decision time, if the surplus level is larger than a predetermined dividend barrier, then the excess is paid as a dividend as long as ruin has not occurred. In contrast to [1], it is assumed that the event of ruin is monitored continuously (see [2] and [3]), i.e. the surplus process is stopped immediately once it drops below zero. The quantities of our interest include the Gerber-Shiu expected discounted penalty function and the expected present value of dividends paid until ruin. Solutions are derived with the use of Markov renewal equations. Numerical examples are given, and the optimal dividend barrier is identified in some cases.

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# Mutual exclusivity revisited and an explicit lower bound on the Tail Value-at-Risk of a sum of risks

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## Abstract

Mutual exclusivity is an extreme negative dependence structure that was first proposed and studied systematically in [1]. In this article, we revisit this notion and characterize mutually exclusive random variables via their pairwise counter-monotonic property and minimality with respect to the convex order. These characterizations highlight the role of mutual exclusivity as the strongest negative dependence structure, generalizing counter-monotonicity to a multi-dimensional setting. Furthermore, we consider the problem of finding the lower bound on the Tail Value-at-Risk of a sum of risks with given non-negative marginal distributions. By utilizing an extremal representation of Tail Value-at-Risk, we derive a new lower bound which admits an explicit description and characterizes mutual exclusivity in the sense that it is attained when and only when the risks are mutually exclusive.

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# An efficient Monte Carlo algorithm for valuation of VA contracts with guaranteed minimum withdrawal benefits

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## Abstract

Variable annuities (VAs) with guarantees are popular insurance contracts. However, after the recent financial crisis, most insurers are not willing to sell products with traditional guarantees of living benefits such as GMMB or GMAB. These types of guarantees are too costly. VA contracts with guaranteed minimum withdrawal benefits (GMWB) are more suitable products for insurers to sell, because the cost of GMWB, compared to traditional guarantees, is much cheaper. Fair valuation and hedging strategies are crucial for the risk management of VA contracts with GMWB. GMWB gives the insured the possibility to withdraw a pre-specified amount annually, even if the sub-account value has fallen below this amount. With an immediate GMWB, a certain percentage ( $g$ ) of the single premium (the guaranteed amount  $w_0$ ) can be withdrawn annually from the policy for  $T$  years ( $gT$  is commonly set to 1). If, at maturity  $T$ , the sub-account value of the policy  $W_T$  is greater than  $gw_0$ , then the policy holder can receive  $W_T$ ; otherwise the insured can receive  $gw_0$ . The insurer also charge guarantee fee and management fee. These fees are deducted from the sub-account periodically. Most of the previous research use continuous withdrawal model and assume proportional guarantee fee and management fee, which makes process of the account value follows simple stochastic processes. However, for real contracts, the insured can only withdraw money at specific time points and guarantee fee and management fee are deducted periodically. Therefore, it is more appropriate to model the sub-account process based on discrete withdrawal model. Also, the previous research do not consider fixed management fee. Under discrete withdrawal model and the assumption of periodic fixed management fee, the sub-account process becomes path-dependent. This makes Monte Carlo methods a viable tool for valuation of such contracts. When using Monte Carlo methods to value such guarantees, risk neutral scenarios of the account value are simulated. However, for most of scenarios, GMWB turn out being out-of-money. This implies crude Monte Carlo method is inefficient. In this paper, we design an efficient Monte Carlo valuation algorithm by using the technique of Conditional Monte Carlo. We demonstrate the efficiency of the algorithm numerically and prove the algorithm is always more efficient over crude Monte Carlo in terms of variance of the point estimators.

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# The two-sided tradeoff insurance premium based on conditional percentile rank deviations

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## **Abstract**

This paper proposes the “tradeoff premium.” The tradeoff premium is a weighted average loss from an insurance contract. Higher weights are imposed for smaller and larger loss outcomes, reflecting aversion to over and underpricing respectively. Overpricing causes loss of business in a competitive market and underpricing depletes capital. The tradeoff premium is hence “two-sided”, and contrasts with “one sided” existing premiums where “large” loss outcomes and underpricing are the focus. The tradeoff premium is shown to comprise of the average loss, a loading to avoid underpricing and a discount to avoid overpricing. Hence avoiding both over and underpricing involves a tradeoff, and relative sizes of the loading and discount determine the overall magnitude of the tradeoff premium.

The tradeoff premium depends on a “reference point” dividing over and underpricing within the range of loss outcomes. Changing the reference point shifts the relative aversion between over and underpricing. For example increasing the reference point decreases the tradeoff premium by raising the relative aversion to overpricing. A zero reference point implies nil aversion to overpricing, and yields “one-sided” existing premiums.

Weights in the tradeoff premium are formed by imposing an increasing aversion function on the “distance” between loss outcomes and the reference point. This paper measures distance using “conditional percentile rank deviation”. Conditional percentile rank deviations are differences between percentile ranks of loss outcomes and the reference point, and scaled to fit the unit interval. Using this weight formulation, the tradeoff premium is the average loss under a density transformed using cumulative prospect theory, where cumulative probabilities of moderate and extreme outcomes are underweighted and overweighted, respectively. When the reference point is zero, the tradeoff premium reduces to distortion and spectral risk measures.

By using different aversion functions, tradeoff premiums are shown to be two-sided equivalents of existing one-sided premiums or risk measures, again emphasizing concerns to reflect both extremes of the loss distribution. For instance the “two-sided Value-at-Risk” is a weighted average of upper and lower extreme percentiles. The “two-sided Conditional-Tail-Expectation” is also a weighted average, of conditional expected losses in the upper and lower tails. In both examples, changing the reference point shifts the relative representation of “large” and “small” losses in the tradeoff premium.

Other properties of the tradeoff premium are explored. The tradeoff premium is “nearly coherent.” Only the subadditivity property of coherence is not satisfied since both the loading and discount in the tradeoff premium reduce when insurance contracts are aggregated, to reflect lower loss variability from diversification. The overall impact of aggregation hence involves a tradeoff.

Finally the tradeoff premium is “balanced” when it is equal to the selected reference point. If this occurs, conditional percentile rank deviations are linked to actual premium



surplus or shortfall. The “balanced premium” always exists and is unique. In addition it equalises expected premium shortfall and surplus under a transformed loss density, hence balancing conflicting objectives to avoid over and underpricing.

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# Prediction of regionalized car insurance risks based on control variates

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## Abstract

We show how regional prediction of car insurance risks can be improved by combining explanatory modeling with phenomenological models from industrial practice. Motivated by the control-variates technique, we propose a suitable combined predictor. We provide explicit conditions which imply that the mean squared error of the combined predictor is smaller than the mean squared error of the standard predictor currently used in industry and smaller than predictors from explanatory modeling. We also discuss how a non-parametric random forest approach may be used to practically compute such predictors and consider an application to German car insurance data.

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# Time Consistent Portfolio Selection under Short-selling Prohibition: from Discrete to Continuous Setting

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## Abstract

In this talk, I shall introduce the time consistent strategies in the mean-variance portfolio selection with short-selling prohibition in both discrete and continuous time settings. Recently, [2] considered the problem with state dependent risk aversion in the sense that the risk aversion is inversely proportional to the current wealth, they showed that the time consistent control is linear in wealth. Considering the discrete counterpart of their continuous time equilibrium control, the corresponding equilibrium wealth process can take negative values; and this negativity in wealth will lead the investor to a risk seeker which results in an unbounded value function that is economically unsounding. To deal with this limitation, we eliminate the chance of getting non-positive wealth by prohibiting short-selling. Using backward induction, the equilibrium control in discrete time setting is shown to be linear in wealth. An application of the extended Hamilton-Jacobi-Bellman equation (HJB) (see [1]) makes us to conclude that the continuous time equilibrium control is also linear in wealth, where investment-to-wealth ratio satisfying an integral equation, which can be solved uniquely. We also show that the discrete time equilibrium controls converge to that in continuous time setting. Finally, some numerical studies will be provided.

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# Rare Event Simulation for the Ruin Problem with Investments and Related Perpetuity Sequences

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## Abstract

In a number of applications, particularly in financial and insurance mathematics, it is of interest to characterize the tail distribution of a random variable  $V$  satisfying the distributional equation  $V \stackrel{\mathcal{D}}{=} f(V)$ . Standard examples include the problem of estimating ruin under stochastic investments, and the problem of estimating the tail probabilities of perpetuity sequences and of the ARCH(1) and GARCH(1,1) financial time series models. In each of these problems, it is known that asymptotically,

$$\mathbf{P}\{V > u\} \sim Cu^{-\xi} \quad \text{as } u \rightarrow \infty.$$

In this talk, we present a computational method based on importance sampling for evaluating these rare event probabilities exactly. We prove that our estimator is consistent, strongly efficient, and optimal within a wide class of importance sampling estimators. Moreover, borrowing methods from nonlinear renewal theory, we derive sharp asymptotics for the running time of the algorithm. Our algorithm is based on a novel *dual* change of measure over a random time interval, combined with ideas from Markov chain theory. We illustrate our methods with a variety of examples illustrating the ease and scope of the implementation.

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# Lessons learned form trying to calibrate a classical risk model to real data

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## Abstract

In this talk we would like to share some of the lessons learned when trying to implement risk theory models/principles to assess an insurance company's risk. Working with real data from a few lines of business, we have tried to calibrate some recent risk models to the given data, in an attempt to better model the risk-return dynamics of each line of business.

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# Computing finite-time survival probabilities using multinomial approximations of risk models

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## Abstract

We consider the problem of computing survival probabilities over a finite time horizon for different risk processes. We start by considering the classical risk model and then we turn to analyze the perturbed risk process. Both the models rely on the assumption that the firm's reserve does not produce investment incomes. Because this assumption is quite restrictive, it is reasonable to consider more general risk processes where the insurer receives interest from its surplus. We assume that in the market there is the opportunity of investing in a risky asset with dynamics described by a geometric Brownian motion so that a perturbed risk model with stochastic investment income can be considered.

The proposed approach to compute survival probabilities over a finite time horizon is based on the construction of a multinomial discrete-time process that approximates the risk processes above mentioned. We divide the finite time horizon into a number of time intervals of equal length and the approximating process is described by a reconnecting lattice with an absorbing barrier located just below zero, so that the process remains trapped there as it becomes negative. Once the multinomial lattice approximating the evolution of the risk process has been constructed, the survival probability is computed via a simple recursive scheme. In particular, the finite-time survival probability may be obtained as the probability that the reserve process remains nonnegative during the whole time horizon considered. Hence, at each time slice, the survival probability associated with a generic node is computed by summing up the transition probabilities of the predecessors corresponding to nonnegative values of the insurer's reserve. Once the conditional probabilities that the reserve remains nonnegative in correspondence with the final nodes of the lattice is obtained, the required survival probability is computed as the sum of all such probabilities. The reconnecting structure of the multinomial lattice and the presence of the absorbing barrier keep the computational cost of the proposed algorithm at a reasonable level.

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# Approximations for sums of dependent random variables using the checkerboard empirical copula

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## Abstract

We propose a method to estimate the distribution of a sum of dependent random variables. We suppose we have a small sample of observations of the dependence structure or copula but that the marginal variables are known or that at least more observations of them are given. The method consists of using the sample from the copula to create what we call a *checkerboard empirical copula*. From this new copula and the information on the marginal variables, estimations are done using usual Monte Carlo methods. In our work we don't try to estimate the copula but to optimally combine the available information on each marginal variable and copula to approximate the aggregation. This research is motivated from the fact that in some practical situations the marginal distributions are well known but the number of joint observations is no sufficient to accurately estimate the copula of a high dimensional random vector. Some applications on risk measures calculation are shown as well as comparisons with other methods.

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## Longevity risk hedging and basis risk

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### Abstract

The improvements of longevity are intensifying the need for capital markets to be used to manage and transfer the risk through longevity-linked securities. Nevertheless the difference between the reference population of the hedging instrument ("hedging population") and the population of members of a pension plan or the beneficiaries of an annuity portfolio ("exposed population") determines a significant heterogeneity which causes the so-called basis risk.

Many authors investigated the basis risk between populations associated with annuity portfolios and life insurance portfolios. For example, [2] found empirical evidence of a natural hedge operating between such portfolios, implying that the basis risk between them is relatively small; [1] provided a calculation of the risk reduction between hypothetical annuity and life insurance portfolios using historical mortality experience data: the results suggest significant benefits in terms of reduction in risk and economic capital; [5] developed a stochastic model for measuring population basis risk by considering several variants of the Lee-Carter model [4] for two populations.

The paper focuses on the longevity risk management by securitization, providing a framework for measuring the basis risk impact on the hedging strategies. To this aim we propose a model that measure the population basis risk involved in a longevity hedge, in the functional demographic model (FDM) setting. [3] suggests that the FDM forecast accuracy is arguably connected to the model structure, combining functional data analysis, non-parametric smoothing and robust statistics. In particular, the decomposition of the fitted curve via basis functions represents an advantage, since they capture the variability of the mortality trend, by separating out the effects of several orthogonal components. In order to quantify the basis risk, we define a stochastic mortality model for two populations based on the FDM framework. We consider both an independent FDM (the hedging population is independent from the exposed population) and a joint FDM (both populations are jointly driven by a single index of mortality over time). Under the proposed mortality model, we build a longevity hedging strategy involving a portfolio of q-forwards calibrated through the key-q-duration (KQD), i.e. the annuity portfolios price sensitivity to a shift in a key mortality rate (see [6] for further information about KQD). The shifts are firstly modelled as in [6] and then adjusted with the standard deviation of the exposed population mortality in order to realise a more effective hedge. In order to analyse the hedge effectiveness we consider the present value of both unexpected cash flows of the insurance portfolio and payoffs from the q-forwards involved in the hedging portfolio. The KQD



of these quantities as well as an adjustment factor depending on the specified mortality model allow to find the required notional amount of the q-forwards in presence of basis risk.

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# Optimal investment under uncertain lifetime with stochastic mortality and stochastic interest rates

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## Abstract

This paper derives closed-form expressions for the optimal consumption and portfolio allocation of an agent who aims to maximize utility from wealth at his/her retirement date and utility from intermediate consumption, while being exposed to mortality risk and stochastic interest rates. The agent is allowed to invest in a complete market in which mortality risk is traded via a longevity swap. The mortality of the agent is modelled by a doubly stochastic Poisson process. The mortality rate and the short rate are assumed to follow (independent) Cox-Ingersoll-Ross processes. Conditions for existence of an optimal solution are provided explicitly in terms of the model parameters.

Papers which are close to this work are Menoncin [4], Deelstra et al. [1] and Kraft [2, 3]. Menoncin [4] considers the problem of an investor maximizing utility from consumption on an uncertain horizon where the investor is exposed to mortality risk and the (stochastic) mortality rate is traded via a longevity bond. Our paper essentially is a special case of Menoncin [4] where both the short rate and the mortality rate follow a Cox-Ingersoll-Ross process. For this particular case we extend the results in Menoncin [4] by proving existence of a solution and providing an analytic characterization. Deelstra et al. [1] and Kraft [2, 3] derive the optimal strategy for an agent maximizing power utility from terminal wealth by investing in a market with a stochastic short-rate following a Cox-Ingersoll-Ross process. Whereas Deelstra et al. [1] use the martingale approach, Kraft [2], [3] solves the corresponding HJB equation and proves a verification theorem. Furthermore, Kraft [2], [3] provides candidate solutions for the case where the agent also derives utility from consumption. Our paper extends Deelstra et al. [1] by proving existence of a solution in the case of utility from terminal wealth and consumption in a complete market, provided that certain conditions on the model parameters are met.

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# Variance reduction techniques for estimating Limited Value-at-Risk and Limited Conditional Tail Expectation

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## Abstract

We introduce Limited Value-at-Risk and Limited Conditional Tail Expectation risk measures. We propose some variance reduction techniques for estimating Limited Value-at-Risk and Limited Conditional Tail Expectation in case it is not possible to obtain analytical formulas. The aggregate loss corresponding to the Stop-Loss reinsurance model is derived, using Limited Value-at-Risk and Limited Conditional Tail Expectation measures. We consider various restricted optimal retention problems in Stop-Loss reinsurance and obtain necessary and sufficient conditions for the existence of the optimal solution. Computational results are provided using simulations.

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# Instantaneous mean-variance hedging and instantaneous Sharpe ratio pricing in a regime-switching financial model, with applications to equity-linked insurance claims

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## Abstract

We study hedging and pricing of unattainable contingent claims in a non-Markovian regime-switching financial model. Our financial market consists of a bank account and a risky asset whose dynamics are driven by a Brownian motion and a multivariate counting process with stochastic intensities. The interest rate, drift, volatility and intensities fluctuate over time and, in particular, they depend on the state (regime) of the economy which is modelled by the multivariate counting process. Hence, we can allow for stressed market conditions. We assume that the trajectory of the risky asset is continuous between the transition times for the states of the economy and that the value of the risky asset jumps at the time of the transition. We find the hedging strategy which minimizes the instantaneous mean-variance risk of the hedger's surplus and we set the price so that the instantaneous Sharpe ratio of the hedger's surplus equals a predefined target. We use Backward Stochastic Differential Equations. Interestingly, the instantaneous mean-variance hedging and instantaneous Sharpe ratio pricing can be related to no-good-deal pricing and robust pricing and hedging under model ambiguity. We discuss key properties of the optimal price and the optimal hedging strategy. We also use our results to price and hedge mortality-contingent claims with financial components (equity-linked insurance claims) in a combined insurance and regime-switching financial model.

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# Closed form solutions for cross correlations in level crossings of multiple correlated Gaussian processes

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## Abstract

A variety of problems in physical and biological sciences can be mathematically understood by considering statistical properties of level crossings of random Gaussian processes (see [3]). Prominent problems involving level crossings of multiple Gaussian variables include a diverse set such as spike coordination of neurons in the brain, but also insurance risk assessment or stock price movements. Indeed, different conceivable applications for our considered model could include risk correlations between two companies that share some of their supplier chain [1], probability calculations of coincident break down of compliant offshore structures in random sea waves [2] or other scenarios where coincidences of level crossings are of interest.

Notably, a growing number of these problems demands a consideration of level crossings emerging from at least two latent correlated Gaussian processes. Here, we address this problem by providing a complete closed form correlation function for two temporally correlated Gaussian processes using Hermite polynomials theory. Furthermore using the Wiener Chaos theory we provide a bivariate Central Limit Theorem for the up-crossing of two temporally correlated Gaussian processes. Because such problems often involve the action of at least two interacting level crossing sets we expect that closed form solutions derived in this article can help drive advances numerous fields.

Different applications are presented in our study and different numerical simulations are provided in order to illustrate our theoretical results.

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# Retirement options: alternatives to life annuities and traditional investment planning

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## Abstract

The current trend in several countries is for individuals to plan and save for their own retirement. Faced with an ageing population, governments have been recoiling from a modern utopia of a relatively high income guaranteed for all citizens in their old age. Employers are closing down their defined-benefit pension schemes, schemes which had offered retirement income security to many people, and replacing them with defined-contribution pension schemes, which have no such guarantees.

As part of their retirement planning, an individual must decide how they deal with risk, such as mortality risk and investment risk. Should they bear it all, or transfer it to a financial institution for a fee? For mortality risk, the benefit of the latter choice is the pooling of risk. That is, with enough individuals, the idiosyncratic mortality risk can be eliminated.

However, is the fee offered for the complete removal of the mortality risk too high? An alternative is for an individual to pool directly their mortality risk with others, through a vehicle called a pooled annuity fund. Such a structure enables individuals to reduce their idiosyncratic mortality risk, although it still exposes them to systematic mortality risk.

We analyze a pooled annuity fund from a participant's perspective by comparing it to a mortality-linked fund, a type of variable payout life annuity, which gives a return linked to the force of mortality but subject to a cost. The cost is a proxy for the fee offered by an insurer for the complete removal of an individual's mortality risk. Fixing the instantaneous volatility of return on wealth, we find that the expected return on the pooled annuity fund is higher except when the costs are very low in the mortality-linked fund. Similar results are obtained when maximizing the expected lifetime utility of consumption, assuming a constant relative risk aversion utility function. In both settings, our results indicate that a participant may be willing to accept the mortality risk of the pooled annuity fund, even when only 100 individuals are pooling their mortality in the pooled annuity fund.

In conjunction with the above decision, an individual must also determine how to invest their savings over their lifetime. We introduce some recent work in this direction, which takes a new perspective on the framing of the investment decision to the individual.

The work presented is based on [1] and on-going work.

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## Chain-Ladder with bivariate development factors

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### Abstract

In this paper we generalize the model of Th. Mack [1] for the well known Chain-Ladder method in IBNR calculations by coping with dependent accident years, i.e. the conditional expectation of the cumulated claim payment for accident year  $i$  and development year  $j$  depends linearly on the payments of the previous accident year **and** the one of the previous development year:

$$\mathbf{E} [C_{i,j} | \sigma \{C_{m,n}, m+n \leq i+j-1\}] = \alpha_{i-1} \cdot C_{i-1,j} + \beta_{j-1} \cdot C_{i,j-1}$$

for  $1 < i < I, 1 < j < J$ . The pair  $(\alpha, \beta) := (\alpha_1, \dots, \alpha_{I-1}, \beta_1, \dots, \beta_{J-1})$  are the *bivariate development factors (BDF)*.

Similar ideas can be found in [2]. The case  $\alpha \equiv 0$  corresponds to Mack's model.

We show how to estimate the BDFs. The estimator turns out to be implicitly unbiased. This leads to estimations of the yearly and total provisions. We also show how a smooth transition from Mack's model to ours can be established. Examples are given.

Keywords: *Chain Ladder with dependent accident years, dependent bivariate development factors, IBNR-provision.*

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# Maximising Exponential Utility of Restricted Dividend Payments in a Brownian Risk Model

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## Abstract

We consider a diffusion approximation to the classical risk model. In this setting we study the following optimization problem:

$$\max_C \mathbb{E} \left[ U \left( \int_0^\infty e^{-\delta t} dC_t \right) \right],$$

where  $U(x) = (1 - \exp(-\gamma x))/\gamma$  denotes the utility function with risk aversion parameter  $\gamma$ ,  $C$  denotes the accumulated dividend process, and  $\delta$  a discounting factor. We only allow dividend strategies with restricted payments.

The problem with unrestricted payments has been already considered before by Schachermayer et al. [1]. We will see that the introduction of a boundary for the dividend payments can simplify the considered problem. In some special cases we will be able to give a closed expression for the value function and for the optimal strategy.

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# What Can The English Longitudinal Study of Ageing Tell About Care Home Entry and The Length of Stay in Care Homes?

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## Abstract

**Background:** The funding of long-term care for older people is a challenge facing ageing populations worldwide. Additionally, one can argue that the conditions that give rise to long term care needs may reduce remaining life expectancy. Thence, for governments to design achievable financing strategy of long term care both entry to care home, and the length of stay in care homes are two substantial pieces of information that is used to predict costs associated with ageing.

**Objectives:** The overall aim is to identify predictors of entry to care home and length of stay in care homes in England and hence being able to provide appropriate predictions of costs of financing long-term care for older people.

**Methods:** We undertake new analysis of existing data from the English Longitudinal Study on Ageing [1], to investigate the factors affecting the entry to care homes and their effects on the length of stay in care homes and their relationship with ageing in England.

**Results:** A representative sample of 11,448 persons living in households in England aged 50 and older are studied to identify predictors of entry to care homes. Entry to care home was identified by outcome of the interview, where persons were not interviewed if they left the household to a care home anytime between interviews. 153 elders, 1.33% of the original cohort entered a care home during the eight years from 2002 to 2010. Using Cox proportional hazard regression, we examined the effect of 11 variables: age, cohort, gender, marital status, living with others, education, social class, number of bedrooms in the household, smoking, drinking and the need for help in ADL (Activities of Daily Living). All factors were tested for being significant predictors of death and/or entry to care home. Proxy interviews were added to the analysis of entry to care home, where drinking and smoking were omitted from the analysis as they were not asked in proxy interviews. From 2008 there were care home interviews, that is reflected in the increase of proxy interview from 163 interviews on 2002 to 515 interviews on 2010. Our results were compared for consistency with other similar studies, for example [2], [3], [4] and [5]

**Conclusion:** We aim to use existing evidence and extend the new analysis to other sample datasets to assess and model the contribution entry and staying in care home might make to the costs of old age for different population groups. This could have an impact on the future system of care and services for the elderly in the UK.

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# A PDE approach to pricing contingent claims under liquidity risk

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## Abstract

The ongoing financial crisis has highlighted that liquidity risk is a major concern for financial institutions. Especially insurance companies as managers of large portfolios must have robust yet tractable models to assess the impact of illiquidity in markets and derive appropriate management actions. The methods used in our work can also be applied to the valuation of market reserves of life insurance contracts that depend on capital markets under liquidity risk.

We present a partial differential equation (PDE) approach for the pricing of contingent claims in a liquidity risk and price impacts model. Option prices under liquidity risk are shown to be solutions of a class of semilinear degenerate parabolic PDEs on bounded domains. We prove the existence and uniqueness of weak solutions of this type of equation. The resulting derivative prices and their gradient are smooth on certain domains. We give a natural decomposition of derivative prices into a "classical" part (without trade impact and liquidity costs) plus an error term reflecting trade impact and liquidity costs.

In this work, we build on the liquidity model of Jarrow and Roch [2] (a version of the model of [1]), and assume that prices are affected by trades through a change in the risk-return premium. We develop the model in probabilistic terms and define the wealth process resulting from this construction. We then define the replication problem as the solution of a backward stochastic differential equation (BSDE), to which we associate a PDE.

We show the existence and uniqueness of this PDE and prove that its solution  $u$  is also the solution of the associated BSDE. We choose to work with the concept of weak solutions [3] as opposed to viscosity solutions for the simple reason that it allows us to obtain information about the growth of the gradient  $Du$  in an  $L^2$ -space, which would be impossible to do in the viscosity framework. Indeed, an important feature of a PDE describing a liquidity setting is that the replication strategy, given by the derivative of the option price with respect to the underlying, converges to the replication strategy in a frictionless setting when the number of options replicated is small.

The detailed analysis leads to explicit and effective (i.e., computable) bounds on the  $L^2$ -norms for the derivative price and its delta. Due to the analysis being highly explicit, it serves as a basis for numerical approaches to the PDE. The high regularity of the option prices suggests that numerical approaches will have a fast convergence rate.

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## Pricing annuity guarantees under a double regime-switching model

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### Abstract

This paper is concerned with the valuation of equity-linked annuities and variable annuities with mortality risk under a double regime-switching model, which provide a way to endogenously determine the regime-switching risk. The model parameters and the reference asset price level are modulated by a continuous-time, finite-time, observable Markov chain. In particular, the risk-free interest rate, the appreciation rate, the volatility and the jump martingale describing the jump component of the reference investment fund are related to the modulating Markov chain. A generalized version of the regime-switching Esscher transform is applied to select an equivalent martingale measure in the incomplete financial market. Under the equivalent martingale measure, analytical pricing formulas for the embedded options in these products are derived by the inverse Fourier transform. Fast Fourier transform approach is adopted to discretize the integral pricing formulas and calculate the prices of the embedded options even in a multi-regime case. Numerical examples are provided to illustrate the practical implementation of our valuation process.

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# The moments of the discounted loss and the discounted dividends for spectrally negative risk process

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## Abstract

We consider a spectrally negative risk where upon ruin event the deficit is immediately paid, and the process restarts from level 0. We consider risk processes with and without dividend barrier. In a risk process with a dividend barrier  $b$ , all the surplus above  $b$  is paid as dividend. Applying theory of exit times for spectrally negative Lévy process and its reflection at the maximum and at the minimum we obtain recursive formulas for:

1. The moments of the expected discounted loss until the process reaches  $b$ . This is equivalent to obtain the moments of the discounted dividends in the dual model.
2. The moment of discounted loss for model with and without dividend barrier for the infinite horizon, with and without dividend barrier.
3. The moments of the discounted dividends for the infinite horizon.
4. The expected profit (dividends payments minus recovery payments ) per time unit.

Obtaining the moment of the expected loss in this model is equivalent to obtaining the moments of the discounted dividend until ruin or in the infinite horizons for the dual model.

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# Consistent loss prediction for a portfolio and its subportfolios

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## Abstract

For a portfolio of risks consisting of two subportfolios, we study the problem of whether or not the predictors based on the subportfolios are consistent with those based on the full portfolio. We study this aggregation problem for the additive (or incremental loss ratio) method [2] and compare the results with those obtained earlier [4] for the chain–ladder method; see also [1] and [3]. It turns out that due to the use of volume measures the results for the additive method are particularly simple. We illustrate our results for a portfolio of risks and its subportfolios consisting of small or large claims, respectively.

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# Systematic Mortality Risk: An Analysis of Guaranteed Lifetime Withdrawal Benefits in Variable Annuities

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## Abstract

Guaranteed lifetime withdrawal benefits (GLWB) embedded in variable annuities provide a type of life annuity which addresses systematic mortality risk while indirectly protecting the policyholders from the downside risk of fund investment. Using tractable equity and stochastic mortality model, we evaluate GLWB in a continuous time framework. The paper investigates how different specifications of financial and demographic parameters affect the fair guarantee fee being charged, as well as their effects on the profit and loss distribution if the estimated parameters are different to the true parameters of the underlying dynamics. Parameter and model risks turn out to be significant since the guarantee is of a very long term nature. Systematic mortality risk underlying the guarantee is quantified using several risk measures. Static hedging of systematic mortality risk via S-forward is examined with respect to different levels of equity exposure.

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# Risk Minimization for Equity-Indexed Annuities

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## **Abstract**

In this talk, we consider the pricing of equity-indexed annuities using dynamic risk measures and present dynamic hedging strategies underlying these valuations. There are two common approaches to deal with equity-linked products: the financial and actuarial approaches. One of our goals is to integrate both approaches in order to protect insurance companies against unhedgeable risks. An optimal investment strategy, consisting of risk-free and risky assets, is sought such that the risk of future losses are minimized at the lowest cost for the issuer. The pool of risky assets, which includes the underlying index, is expanded to admit European type options. We show that our optimization problem could be solved using linear programming technics. The transaction costs play a crucial role when considering the derivative market. A detailed numerical analysis is performed for equity-indexed annuities including the transaction costs.

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# Modelling Cause-of-Death Mortality and the Impact of Cause-Elimination

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## Abstract

Changes in underlying mortality rates significantly impact insurance business as well as private and public pension systems. Individual mortality studies have data limitations; aggregate mortality studies omit many relevant details. The study of causal mortality represents the middle ground, where population data is used while cause-of-death information is retained. We use internationally classified cause-of-death categories and data obtained from the World Health Organization. We model causal mortality simultaneously in a multinomial logistic framework. Consequently, inherent dependence amongst the competing causes is accounted for; a detail often overlooked in previous approaches. Furthermore, this framework allows us to investigate the effects of improvements in, or the elimination of, cause-specific mortality in a sound probabilistic way. This is of particular interest for scenario-based forecasting purposes. We compare the subsequent impact on survival with previous causal models and show our approach to be more conservative in a mortality-risk sense. Finally, we quantify the impact of cause-elimination on aggregate mortality using residual life expectancy and apply our model to a French case study.

**Keywords:** Cause-of-Death Mortality, Multinomial Logistic Regression, Cause-Elimination, Life Expectancy, Forecasting

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# Innovative financial instruments for minimizing flood risks

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## Abstract

The overall objective is to minimize the flood risk in a region by transferring financial resources from *damage covering* to *damage preventing*, assuming that:

1. Some preventive measures exist whose cost is by far less than the cost of covering the damages they prevent.
2. These preventive measures require target investments (project costs), whose amount cannot be sustained by a Central Public Administration (CPA) normal economic programming.
3. The above CPA aversion to investment on preventive measures is even worsened (not diminished!) by the frequent occurrence of catastrophes. In many cases, in fact, a catastrophe occurrence triggers a *negative* financial flow, in the sense that resources previously allocated for prevention projects are diverted toward damage coverage.

Points 1) and 2) are the main motivation for issuing *project options* (which may be subscribed by local public administrations, LPA) to support and finance risk prevention measures.

Points 2) and 3) are the main motivations for accompanying the project options with a second financial instrument, similar to *catastrophe bonds* (which can be acquired by potential investors, PI), that: a) reinforces the remuneration in terms of gains (not simply loss reductions) subsequent to risk reduction; b) assures the CPA for the damage coverage in case of catastrophe, so that re-allocation of resources is not required.

Both typologies of contracts incorporate a trigger mechanism, independent from posterior economic optimization and objectively evaluated by a third party arbiter.

As a consequence, a dynamic interaction among CPA, LPA's and PI's takes place, which we represent by an evolutionary game modeled through a replicator dynamics.

Taking as a case study the flood risk in the area of Florence (in Tuscan region) we assume realistic ranges for the model parameters and analyze, on such basis, the possible evolution and bifurcations of the game so as to gather qualitative indicators concerning the efficiency and convenience of the proposed financial instruments.

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## Is the finite-time Gerber–Shiu function a risk measure?

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### Abstract

The Gerber–Shiu (G–S) function defined over a finite time horizon can be used to hedge insurance risk processes, see [1]–[3]. In this paper we study the properties of the G–S function, evaluated for a risk process over a short-term and compare them to those of well accepted risk measures.

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# A regression approach to risk quantification in economic capital

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## Abstract

Financial services firms, banking or insurance alike, require capital to sustain their businesses under adverse business conditions. The underlying unexpected risks are manifested in their capital requirement. Recent drive by the regulators and the firms to place businesses on a risk-based platform has necessitated the calculation of capital requirement based on the likelihood of unforeseen adverse circumstances, giving rise to the so-called economic capital.

In this paper, we address quantification of the exact contribution of each risk factor underlying the economic capital. Knowledge of actual risk components could answer many fundamental questions related to risk dynamics of a business and could aid in developing better risk management practices. For example, study of the dependency of risk capitals under unfavourable business situations can provide the firm an overview of the multivariate nature of the risks that the firm is running which is important for enterprise risk management.

Generally, risk capitals combine to form the economic capital. So either the risk capitals should be found out first and then obtain the economic capital or vice versa. The first approach, called the “bottom-up” approach, usually makes certain assumptions about the nature of individual risk distributions to compute the individual risk capitals. They are then aggregated to obtain the economic capital. Thereafter the economic capital is again split into the risk capitals by applying various techniques such that they sum to the economic capital (see [1]).

Here we have followed a “top-down” approach which calculates the economic capital first and hence to know the contribution from individual risk factors, the component of economic capital attributable to the individual risk factors should be found out. This requires dis-aggregation of economic capital into component risk capitals.

In this paper, we propose the disaggregation of economic capital into component risk capitals using regression approaches. We have taken the example of a banking product – Equity Guarantee Bond, and a pure endowment insurance product to explain the proposed methodology to split the economic capital required for each of them into corresponding risk components. We discuss the modelling, state the (dis)advantages of this approach and its applicability to practical purposes. We also include methods so as to make risk components add up to the economic capital, and highlight the concept of quantification of diversification benefits arising thereof.

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# Revisiting Rosenlund's Reserve by Detailed Conditioning method

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## Abstract

Rosenlund [1] introduces a method for claims reserving, called the Reserve by Detailed Conditioning (RDC) method, that is designed for individual claims instead of run-off triangles. The RDC method is a distribution-free reserving method which returns an estimate of the expected value of the outstanding reserve for an unsettled claim, given a set of claim characteristics. In particular, Rosenlund [1] uses the waiting-for-report period, the payments and the finalization period. As a first contribution we investigate the performance of the RDC method on a real liability insurance portfolio (for private individuals) from a European insurance company. We perform a back test to check the accuracy of the method and compare the results obtained with RDC to the results reported in Antonio & Plat [2] and Pigeon, Antonio & Denuit [3]. The development of a stochastic RDC method is the second contribution of this paper. By conditioning on a set of claim characteristics (as in Rosenlund) we project the development of individual claims by simulating from the empirically observed development of claims.

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# Transform Analysis and Insurance and Financial Mathematics for Lévy Processes: A Recursive Approach

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## Abstract

This paper proposes a recursive scheme to compute the Laplace transform (in time) of functionals of Lévy processes. Such functionals appear in many problems in insurance and financial mathematics and their accurate and efficient computation, which our method achieves, is of major importance. Examples include the computation of the finite time ruin probability in risk theory and that of the price of an Asian option. Our method is based on Feynman-Kac integration.

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# A penalized dividend optimization problem in a Brownian setting with finite time horizon

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## Abstract

In a recent paper [1] we have proved that the solution of the de Finetti problem of maximization of discounted dividend payments within a finite time horizon  $T$  in a diffusion approximation setting is given by a barrier strategy. We were also able to determine the asymptotic behavior of the barrier function  $b(t)$  for  $t \rightarrow T$ .

In the paper, which we want to present, the investor again wants to maximize his expected discounted dividends, but in addition he is penalized, if he is ruined before the final time  $T$ . In this problem the value function  $V(t, x)$  is discontinuous, and we show that it is the unique viscosity solution of the corresponding Hamilton-Jacobi-Bellman equality. Moreover we characterize the optimal strategy as a barrier strategy and determine its asymptotic behavior for  $t \rightarrow T$ . Finally using the solution of the penalized problem we provide an algorithm for the solution of the following problem: For a given upper bound for the ruin probability, maximize the expected discounted dividends!

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# Estimation of a copula under censoring relying on nonparametric techniques with applications to model checking

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## Abstract

Copulas have become common tools for modeling the dependence between risks in insurance. Indeed, from Sklar's Theorem, the joint distribution function of a random vector can be linked to the marginal distributions of each of its component through a copula function. This property permits to separate the marginal modeling of each variable, and the modeling of the dependence structure. Nonparametric inference on the copula function allows to estimate the copula function without imposing too much restrictions on its shape. Moreover, even in the case where one wishes to fit a parametric model to the data, nonparametric estimators are a valuable tool to assess the validity of the model, see e.g. [1]. In this work, we discuss the extension of these nonparametric techniques when one or several variables are subject to censoring. This problem of censoring occurs in many situations in insurance, the most obvious being the modeling of lifetime data: in this case, some individuals may not be dead at the end of the observation period, or may leave the study, for example if they surrender their insurance contract. Many other situations (in non-life as in life insurance) present similar issues. Let us also mention the case where claim expenses are not exactly known when they exceed some fixed policy limit.

In the present paper, we propose new nonparametric estimators of the copula function adapted to a large number of multivariate censoring frameworks. We define a discrete copula estimators, and smoothed versions that can be seen as generalizations of the techniques proposed by [2] in absence of censoring. We derive asymptotic properties of these estimators, and show how they can be used for model checking. This work is supported by two real data applications in life and non-life insurance.

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## Optimal Timing For annuitization

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### Abstract

Fixed-payout life annuities are financial instruments paying a fixed amount periodically throughout the life of the recipient. Since they provide guaranteed income, life annuities help individuals to sustain a given level of consumption and are then an efficient protection against old age poverty. However, the payments are contingent on the recipient's survival and acquiring a lifetime payout annuity is an irreversible transaction. If the person were to die one instant after purchasing the life annuity, his beneficiaries would receive nothing in return. On another side, insurance companies or banks propose alternative financial investments such mutual funds or variable annuities that promise an attractive return on the long term. In this context, it is generally accepted that an individual should consider a two-step strategy. At younger ages, it is more profitable to invest in financial products to benefit from the high long term return of financial markets. While at older ages, the purchase of a life annuity offers a protection against a decrease of standards of living. Under the all-or-nothing arrangement where annuitization takes place at one distinct point in time, this work proposes a method to locate the optimal age at which to annuitize and to calculate the probability of conversion.

Gerrard et al (2012) analyze the optimal time of annuitization for a retiree having the possibility of choosing her own investment and consumption strategy, in a Brownian model and with constant force of mortality. Similarly, Brugiavini (1993), Kapur and Orszag (1999), Brown (2001), and Milevsky (1998), Milevsky and Young (2007) provide theoretical and empirical guidance on the optimal time to annuitize under various market structures. Hainaut and Devolder (2006) study the timing and optimal allocation between annuities and financial assets. Milevsky et al. (2006) derive the optimal investment and annuitization strategies for a retiree whose objective is to minimize the probability of lifetime ruin. Emms and Haberman (2008) discuss both the optimal annuitization timing and income drawdown scheme. Purcal and Piggott (2008) explain the low annuity demand by the relative importance of pre-existing annuitization. Stabile (2006) examines the optimal annuitization time for a retired individual subject to a constant force of mortality in an all-or-nothing framework. Di Giacinto and Vigna (2012) explore the sub-optimality cost of immediate annuitization in defined contribution pension funds.

Our paper considers the optimal timing of annuitization and differs from the previous contributions in several directions. First, the financial asset is modelled by a jump diffusion process instead of a geometric Brownian motion. Second, rather than using stochastic control, we rely on expected present value operators to infer a semi-closed form solution. Furthermore, our approach does not maximize the economic utility of cash-flows but rather their discounted values. Finally, the actuarial force of mortality is not constant over time. This assumption is more realistic than a constant one as done e.g. in Stabile (2006), Gerrard et al (2012), or Milevski and Young (2007). Finally, we also propose a method to approximate numerically the probability of annuitization.

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## Cash Balance Pensions: Valuation and Funding

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### Abstract

Cash Balance pensions appear quite similar to defined contribution plans. Both ostensibly involve the payment of employer/employee contributions into a fund for each member; the fund accumulates over time to create a lump sum benefit at the member's retirement. The difference between the CB plan and the DC plan is that the DC contributions are invested in assets, and the accumulated asset value is passed back to the employee. In the CB plan, the account is notional; the 'credited interest' earned on the employee's account is specified in terms of a market rate applying at each crediting date, and need not have any relation to the actual return on plan assets. So, for example, the employer may specify that all member accounts will earn 5% per year until retirement, or that the interest rate applied at each year end will be the published yield on 5-year government bonds, with an additional margin of, say, 0.25%.

According to the US Department of Labor, in 2007 there were over 10.5 million participants in Cash Balance pension plans, representing around 8% of the total number of participants in any US employer sponsored plan. Cash Balance plans are beginning to appear in other countries, including the UK and Japan. The attraction of Cash Balance plans for employers is very similar to the attraction of Defined Contribution plans (DC).

In the US, actuaries use traditional DB valuation methods and assumptions to determine the liability value for Cash Balance plans. This may create quite misleading values for the liability and normal contribution rates. In this work, we value the CB benefit as a financial liability, using the models and paradigms of financial economics, recognizing the interest rate guarantee as a financial option.

We use a two-factor Hull-White model to determine the liability. We derive analytic valuation formulae for crediting rates based on emerging long term spot rates, and use Monte-Carlo simulation to derive the liability valuation where crediting rates are based on par yields. Using the valuation methods, we investigate a range of funding and valuation schemes for CB pension plans, and highlight the differences between the financial valuation and the actuarial valuation.

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# Capital Allocation Problem for a New Ruin- Based Coherent Risk Measure

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## **Abstract**

This paper deals with one application of a ruin-based coherent risk measures to a basic problem of finance so called capital allocation problem. In this paper, we introduce a ruin-based coherent risk measure on the space of stochastic processes. As an application, we apply this risk measure to find the allocated capital for an insurer's surplus. In fact, we find that the capital allocation problem for this risk measure has a unique solution determined by Euler allocation method.

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# Optimal pension and insurance contract design: impact of mortality risk

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## Abstract

Some recent literature on optimal pension/life insurance studies whether contracts with guarantees can be preferred by a utility-maximizing policyholder. In the absence of mortality risk, the literature (e.g. [1]) shows that expected utility theory (EUT) fails to interpret demand for any forms of guarantees, while cumulative prospect theory (CPT) is able to support the existence of guarantees. In the present paper, we incorporate mortality risk in the life insurance contracts analyzed in [1] and investigate the impacts of *mortality risk* on optimal pension/life insurance design. Interestingly, our numerical results show that products with guarantees can be preferred by both EUT- and CPT-policyholders. Furthermore, a policyholder might have exactly the same preferences for the products, independent of whether he bases his decision on EUT or CPT. This argument holds particularly for long-term contracts.

To address mortality, we take endowment life insurance contracts as an example, in which the policyholder receives not only a survival but also a death benefit. Therefore we use a standard German pensioner life-table from the German actuarial society. Following [1], four types of benefits are considered: a benefit (with no guarantee) from Merton's investment strategy, an implicit put with a guarantee, a simple life contract with a guarantee and a possible terminal default, and a product with annual guarantees and possible defaults. After determining the fair parameter combinations, we compute the certainty equivalents for each contract, both under EUT and CPT. We show the implicit put and the simple life contract can yield higher certainty equivalents than Merton under both theories, while the contract with annual guarantees is never optimal.

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# Analysis of the latest Spanish survey of disability and dependence and its influence on the actuarial calculation of private Long-term care insurance

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## Abstract

In recent decades, industrialized societies have had a new demographic problem with strong social and economic consequences; we refer to the phenomenon of dependence of elderly people, "Long-term care". The convergence of several factors including increased life expectancy, declining fertility and family changes (childless couples or single, or the role of women within it with finding work), have led the traditional elderly care within the family, the so-called informal support, cannot be done in the same way as was done previously. Thus, Nations are making decisions to establish a framework for the risk coverage.

In the current crisis, countries like Spain cannot support the public cost needed for dependent care and that is why private insurance becomes more relevant and increasingly necessary. The paper presented in this meeting is part of a comprehensive study that for years this research group has done on long-term care in Spain. It aims to make an approach to the technical basis of actuarial private insurance.

Based on the Spanish survey of disability and dependence (EDAD 2008) this group make an own and specific statistical study that is appropriate to calculate among others, the different probabilities of risk, prevalence rates of dependency or building life tables for dependent individuals, and from there, applying appropriated actuarial techniques to achieve the objective proposed in this paper.

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# Optimal investment for Lundberg models: numerical methods and new examples

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## Abstract

We consider optimal investment in a Black-Scholes market for a Lundberg risk process in order to maximize survival probability. Extending work of H and Plum (2000, 2003), Azcue and Muler (2009) and Belkina et al. (2012) we solve the problem under weak assumptions via a recursive numerical procedure - similar to Euler's method - and a convergence argument which is based on viscosity solutions. New examples are presented for discrete claim sizes distributions as well as for constraints causing singularities in the value function. Part of the talk will appear in the Taksar memorial issue of *Stochastics* (2013), joint work with Alireza Edalati.

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# The impact of interest rates on lapse activity in life insurance: An international analysis

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## Abstract

In this presentation, we revisit the two major hypotheses for lapse behavior in life insurance: The emergency fund hypothesis (EFH) and the interest rate hypothesis (IRH). The EFH states that policyholders facing financial distress may be more likely to lapse their contracts to access the surrender cash value. The IRH argues that lapse rates are positively related to external rates of return such as market interest rates. A proper understanding of lapse behavior is of crucial importance as a misspecification not only negatively affects an insurer's liquidity, but also leads to adverse selection with respect to mortality. Based on a logistic regression model, we analyze lapse rates from 16 countries with maximum coverage over the years 1996-2011 with respect to several macroeconomic indicators such as gross domestic product growth, unemployment rate, and government bond yield. To our knowledge, this is the most diverse data set ever used in empiric lapse literature. As model specification tests yield ambiguous results, we present both fixed-effects and random-effects estimators. Our preliminary findings support the validity of the IRH. Particularly the government bond yield lagged by two years has a significant positive impact on lapse rates. For the EFH, however, our data shows only limited support. In addition, we investigate the impact of the recent financial crisis on lapse triggers and whether significant differences between advanced and emerging economies exist. To evaluate the predictive accuracy of the regression model, we use cross-validation methods and employ classification tables. Our findings are of relevance for insurance regulators and company managers as they contribute to the understanding of lapse dynamics in life insurance.

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## A Quantitative Comparison of Temperature Forecasts with Non-Gaussian Innovations

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### Abstract

The typical temperature models use the time series method together with the normal distribution error assumption. However, it is demonstrated that the distribution of temperature data series is not only asymmetry but also revealing the kurtosis characteristics. This study attempt to propose a temperature model with eight heavy-tailed distributions the subclass of the generalized hyperbolic distributions models, Students t-distribution , generalized hyperbolic skew Students t-distribution , variance gamma and normal inverse Gaussian, as well as the family of skew generalized error distribution, generalized error distribution, skew generalized t-distribution and Skewed t-distribution to capture the characteristic in the error terms of the time series model. We use the data in Paris and Lisbon which experienced serious heat wave disaster from 1995 to 2010. The out-of-sample analysis demonstrate that the model with heavy-tailed innovations are more appropriate to model temperature which provide another alternative to the foundation for pricing weather-related financial implements to managing the weather risk.

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# Robust Asset Price Bounds in Incomplete Markets

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## Abstract

One of the most important breakthroughs in modern asset pricing theory is that under certain conditions, such as dynamically complete markets, complex financial instruments can be perfectly replicated by sophisticated dynamic trading strategies involving simpler securities. Such a dynamic-hedging approach is the basis of the arbitrage-free valuation and the martingale pricing theory. However, in many realistic situations, perfect hedge or perfect replication is impracticable and impossible due to non-tradable underlying assets, institutional rigidities in financial markets, market frictions and discreteness in trading opportunities, which may further lead to the collapse of complete-market conditions. In addition, the fundamental assumption of the traditional dynamic-hedging approach in the complete market is the absence of any misspecification about the postulated models. Though the assumption is convenient and useful in some applications, it is rather restrictive.

In the paper, we derive robust and arbitrage-free, no-good-deal pricing bounds for derivatives in an incomplete market, along the lines of Cochrane and Saa-Requejo (2000), Hansen and Sargent (2001) and Anderson et al. (2003). Similar to the results of Cochrane and Saa-Requejo (2000), our derived pricing bounds rule out not only usual arbitrage opportunities, but also those investments, called good deals, with too high Sharpe ratios. Moreover, our derived pricing bounds are robust to a particular type of model misspecification, stemming from the parameter uncertainty of the underlying asset processes. More specifically, we consider a European option when the underlying asset is non-tradable, but is correlated with a traded asset that is used as an approximate hedge. We also assume that the non-tradable (traded) asset is driven by a geometric Brownian motion with model perturbation parameters. By controlling these perturbation parameters, we design a model set that consists of a broad range of alternative models whose relative entropy or Kullback-Leibler divergence from the benchmark nominal process (i.e., the geometric Brownian motion) is bounded by a specified value. This model set can then be used to characterize a particular form of model misspecification or model uncertainty for agents' decisions. Finally, the closed form solution of this problem is found via a dynamic game implementation: player one is the agent who makes a choices to maximize his/her utility under the no good deal assumption, while player two is a malevolent virtual agent who aims to minimize utility by controlling the model perturbation parameters. By solving this optimization problem, the agent insures against some endogenous worst-case scenario.

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# Projecting mortality: Identifiability with trend changes and cohort effects

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## Abstract

Projections of mortality rates are vital in the assessment of longevity risk. However, standard projection methods for the state variables in stochastic mortality models can often depend upon arbitrary choices made in order to identify the historic parameters of the model and give implausible predictions of future uncertainty in mortality rates. We discuss how to choose projection methods which are suitably identified, examine a selection of identifiable methods of projecting period and cohort effects, backtest these against observed data, and investigate their plausibility when projected decades into the future.

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## Fair Insurance Guaranty Premium in the Presence of Regulatory Forbearance

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### Abstract

Due to the global financial crisis in 2008 that resulted in systematic risks in the equity and credit market, it creates significant deprecation in the life insurers' balance sheet which affect insurers' solvency. In order to retain prudent supervision and market stability, the authority has announced capital temporal relief plan that may make insolvency insurer worse. Recent occurrences of financial distress to some insurers have raised questions about whether the current guaranty system that charge a flat levy rate in premium-based is adequate to protect policyholders. A risk-weighted levy rate in reserve-based has been proposed to establish reasonable contribution method which can avoid high risk insurers' moral hazard and protect the other insurers from further systematic risks.

A brief summary of the advantages of this paper is listed below: (1) By Monte Carol simulation method, detailed cash flow of insurer's asset allocation can be used to describe the risk preference of life insurer. (2) Our stock model incorporates jump diffusion and stochastic volatility in order to reflect that life insurers face increasing volatility in capital market. (3) Consider regulatory forbearance to represent government's attitude to life insurers.

We calculate fair guaranty premium through Monte Carol simulation method. We find that: (1) Fair premium increases as extending the period of regulatory forbearance. (2) As regulatory criterion raises fair premium decreases at first, but increases if regulatory criterion reaches certain level. (3) Increasing leverage ratio of the insurer results in increasing fair premium.

**Keywords:** fair premium; jump diffusion; stochastic volatility; regulatory forbearance.

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# A Simulation-based Stochastic Programming Model for Hedging Financial Derivatives in a Lévy Market

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## Abstract

In this paper, we propose an alternative approach for hedging problems in a Lévy market. A simulation-based stochastic programming model, which is composed of describing uncertainties using a Monte Carlo method, approximating hedging strategies using a hedging strategy function, and solving a mathematical programming problem, is applied to hedging derivatives. For hedging derivatives, we need to determine the hedging strategy function to take into account the complexity of derivatives. In numerical experiments on a call option, we consider the problem of minimizing the expected squared error between the payoff and the value of the hedging portfolio at maturity (quadratic risk), and propose a hedging strategy function using polynomials. Through comparison with an analytical approach, we verify that the proposed method is valid as an alternative approach for hedging problems. In addition, we demonstrate that this method has broad utility in hedging derivatives by applying it to shortfall risk and a structured bond. Furthermore, this paper also proposes that a quasi-Monte Carlo method should be used to describe uncertainties in financial markets. Numerical results indicate that the quasi-Monte Carlo method is more efficient in solving mathematical programming problems than the Monte Carlo method.

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# A Bivariate Shot Noise Self-Exciting Process for Insurance

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## Abstract

In this paper, we study a bivariate shot noise self-exciting process. This process includes both externally excited joint jumps, which are distributed according to a shot noise Cox process and two separate self-excited jumps, which are distributed according to the branching structure of a Hawkes process with an exponential fertility rate, respectively. A constant rate of exponential decay is included in this process as it can play a role as the time value of money in economics, finance and insurance applications. We analyse this process systematically for its theoretical distributional properties, based on the piecewise deterministic Markov process theory developed by [1], and the martingale methodology used by [2]. The analytic expressions of the Laplace transforms of this process and the moments are presented, which have the potential to be applicable to a variety of problems in economics, finance and insurance. In this paper, as an application of this process, we provide insurance premium calculations based on its moments. Numerical examples show that this point process can be used for the modelling of discounted aggregate losses from catastrophic events.

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# On the Theory of Life Insurance Decisions under Recursive Utility

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## Abstract

In a classical Black-Scholes market, we establish a connection between two seemingly different approaches to continuous-time utility optimization. We study the optimal consumption, investment, and life insurance decision of an investor with power utility and an uncertain lifetime. To separate risk aversion from elasticity of inter-temporal substitution, we introduce certainty equivalents. We propose a time-inconsistent global optimization problem, and we present a verification theorem for an equilibrium control.

In the special case without mortality risk, we discover that our optimization approach is equivalent to recursive utility optimization with Epstein-Zin preferences, studied in [1] and [2] for general preferences. We find this interesting since our optimization problem has a more natural interpretation than the recursive utility optimization problem and since recursive utility, in contrast to our approach, gives rise to severe differentiability problems.

Furthermore, our approach is apt for a set-up with mortality risk and utility from inheritance, and because of the established connection for a certain-lived investor, we propose to regard our approach as a generalization of recursive utility optimization with Epstein-Zin preferences to a set-up with mortality risk and life insurance. Finally, our approach is a generalization of the classical time-additive utility optimization in [3] to time-non-additive power utility.

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# Stochastic Model Efficiency Applications: Cluster-Distance Sampling to Tackle Sampling Errors and Bias

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## Abstract

Driven by the rising need for model efficient analyses that rely on principle-based approaches and coherent stochastic modeling, researchers have been seeking innovative alternatives to help increase model sophistication, dynamics, capacity, speed, and precision. These challenges have motivated our efforts to develop open-source research and implementation software applications for efficient stochastic modeling. One such software application is CSTEP, which stands for “Cluster Sampling for Tail Estimation of Probability”. CSTEP uses the method of representative scenarios to sample scenarios from a population of stochastic scenarios that are related to the financial outcome of interest, as described in [1]. CSTEP incorporates a mathematical continuity relationship between a high-dimensional scenario vector space and their one-dimensional stochastic financial model outcome functional. Use of CSTEP allows a researcher to construct a sample-run distribution of the financial outcome from a sample of representative scenarios that closely resembles the full-run distribution of the financial outcome, particularly at the tails of the distribution. This method allows for a model efficient analysis of data.

We have two goals in this paper. First, we strengthen the theory of high dimensional continuity which describes how the distance between two economic scenarios can be measured as was originally presented in [1]. We “contract” the uniform continuity bound coefficient via redefining the distance definition between scenarios, forcing scenarios to connect model outcomes by strengthening the weak continuity encountered in practice. We may “model-intelligently” pick the representative scenario sample that best replicates the outcome probability distribution, specifically at the tails, in the distance-redefined new scenario space. Second, we present a case study using CSTEP to reveal insights regarding the probability distribution of statutory ending surplus for a real block of annuity business projected from 10,000 stochastic interest rate scenarios. Specifically, we consider new variations of scenario sampling methodologies for model efficiency as an extension of [1]: the economic significance method and the economic present value method which each utilizes projected future cash flows that incorporate the asset runoff speed into the distance measure. We compare these methods to each other and to previous methods for measuring scenario distance, such as the relative present value method, by comparing central metrics (mean, median, variance), percentiles, and tail metrics (value at risk, conditional tail expectation) of the sample-run distribution of ending surplus, obtained using CSTEP, to the full-run distribution. We demonstrate that for our data, the economic significance method results in a sample-run distribution with central metrics that most closely match those of the full-run distribution, and that the economic present value method results in tail metrics of the sample-run distribution that most closely match those of the full-run distribution. We assert that the use of CSTEP allows a variety of scenario sampling ex-

periments to be efficiently conducted and analyzed, and allows for the investigation of small-sampling bias and random-sampling error incurred at the tails of the distribution.

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# Uncertainty on Survival Probabilities and Solvency Capital Requirement: Application to Long-Term Care Insurance

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## Abstract

In this paper, we focus on uncertainty issues on survival probabilities of LTC-claimants and its consequences on solvency capital requirement. Among the risks affecting long-term care portfolios, special attention is addressed to the table risk, i.e. the risk of unanticipated aggregate mortality, arising from the uncertainty in modeling LTC-claimants survival law. The table risk can be thought as the risk of systematic deviations referring not only to a parameter risk but, as well, to any other sources leading to a misinterpretation of the life table resulting for example from an evolution of medical techniques or a change in rules of acceptance. In fine, the idea is to introduce the risk of systematic deviations arising from the uncertainty on the conditional probability of death directly. We analyze the consequences of an error of appreciation on the LTC-claimants survival probabilities in terms of level of reserves and describe a framework in an Own Risk and Solvency Assessment perspective to measure the gap between the risk profile from the standard formula to the risk analysis specific to the organism.

**Keywords.** Own Risk and Solvency Assessment, Solvency Capital Requirement, Long-term care insurance, Risk of systematic deviations, Table risk, Semi-parametric model, Proportional hazard, Frailty.

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## Ruin time-constrained optimal dividend payments

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### **Abstract**

We consider the problem of optimal dividend payments in the Cramér-Lundberg model in the case where the payments are unrestricted (singular control type) with constraint in the ruin time. Specifically, we consider controls such that the probability of the ruin time less than a fixed time is small. We attempt to characterize the value function for this problem using convex duality and optimization techniques. Numerical examples conclude the presentation.

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## On premium estimation using the C&RT/Poisson model and its extensions

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### Abstract

Premium estimation is one of the main issues in insurance mathematics. Estimation of mean and variance of a total claim amount of a portfolio can be considered as necessary prerequisites for this. In turn, dividing the portfolio into homogeneous subportfolios can be considered as a first step towards finding those estimates. We consider the problem of estimating the claim intensity and propose a regression trees based approach for clustering the portfolio into homogeneous subportfolios in a situation where the durations of the policies differ and overdispersion is present. Several other generalizations are discussed. A case study involving Estonian casco insurance is included.

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## Measurement of Factor Risks in Annuities

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### Abstract

Calculation of risk contributions of sub-portfolios to total portfolio risk is essential for risk management in insurance companies. Thanks to risk capital allocation methods and linearity of the loss model, sub-portfolio (or position) contributions can be calculated efficiently. However, factor risk contribution theory in non-linear loss models has received little interest. Our concern is the determination of factor risk contributions to total portfolio risk where portfolio risk is a non-linear function of factor risks. We employ different approximations in order to convert the non-linear loss model into a linear one. We illustrate the theory on annuity portfolio where the main factor risks (or risk drivers) are the interest-rate risk and mortality risk.

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# Financial Fairness and Conditional Indexation

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## Abstract

We consider a pension scheme with conditional indexation of benefits in an overlapping generations model. In such a system pension benefits are adjusted with reference to an index, for example, an inflation index or an average salary index. However, this adjustment is subject to a constraint related to the solvency of the pension fund. We model this constraint by assuming that any indexation of benefits depends on the asset-liability ratio of the pension fund at indexation times. Since this ratio will depend on contributions as well as on asset returns, we find that contributions of future generations will have an impact on the value of pension promises made to a generation which is just joining the pension scheme. We will investigate the fairness of such a pension system and present an approximation formula for the calculation of the fair contribution that a new generation should make when it is joining the scheme. We find that the proposed approach for calculating fair contributions leads to a situation in which investment risks are shared to some extent between generations despite each generation just paying the fair value of promised benefits.

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## Improving pension product design

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### Abstract

Pension products characterized by linking an individual's savings directly to market returns represent the most popular, growing pension domain globally. These products are widely sold in contribution-defined pension schemes, labor market pensions, and individual schemes. For instance, in Denmark, there are almost 53 billion EUR invested in these products. However, available products are designed with a tendency to assume greater risk the longer it is until retirement, but are not adjusted to individual preferences and circumstances.

This paper develops an optimal asset allocation strategy, retirement benefits and insured sum for a defined contribution plan by adjusting to individual needs, such that the expected utility of total retirement benefits and the utility of leaving money upon death are maximized. The benefits consist of three sources of payments: state pension, labor market pension and private pension. We argue that the controlled processes should not only depend on the plan member's age (or time left to retirement), nor only on her risk preferences, but should capture personal characteristics. Among other factors, we include current wealth, expected lifetime salary progression (mandatory pension contributions), voluntary pension contributions, expected social benefits, choice of assets, type of retirement distribution schedules, marital status and bequest motive.

The problem is solved via a model that combines two optimization approaches: stochastic optimal control and multi-stage stochastic linear programming (SLP). The first method is common in financial and actuarial literature, but results in theoretical values. However, SLP, which is characteristic for operations research, has highly practical application. As an example of an optimal pension product design, we present the operations research methods, which have potential to stimulate new thinking and add to actuarial practice.

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# Insurance premiums under Cumulative Prospect Theory

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## Abstract

Determining mean-value principle zero utility principle is based on a belief that people use a subjective utility function while making decisions under risk and can properly evaluate the probabilities of gains and losses. However, in reality, these assumptions are usually incorrect. Numerous experiments carried out by Kahneman and Tversky confirm the fact that under risk and uncertainty, people make decisions using a function which assigns virtual value to monetary outcomes. Kahneman and Tversky noticed that people decide which outcomes they see as basically identical, set a reference point and consider lower outcomes as losses and larger as gains. They suggest replacing the utility function with a value function that depends on relative payoff. According to them, such a function should be convex for negative and concave for positive arguments. In contrast to expected utility theory, the value function measures losses and gains but not absolute wealth.

Kahneman and Tversky also discovered that people distort probabilities while making decisions under risk and uncertainty. The rank-dependent utility model accounts for the problem of overweighting very small probabilities. The formalization of this idea relies on distorting the cumulative distribution function instead of distorting the single probabilities. Kahneman and Tversky used the concept of the rank-dependent utility model and created Cumulative Prospect Theory in which they assume that probabilities of gains and losses are distorted in a different way.

The aim of the presentation is to define mean-value principle and zero utility principle under Cumulative Prospect Theory. It turns out that many well-known premium principles can be derived from these definitions. We also briefly discuss the properties of the introduced premium principles. The main mathematical tool used for the analysis is the generalized Choquet integral.

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# Expected shortfall in XVIII and XIX centuries: contributions of German and Danish actuaries

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## Abstract

Expected shortfall  $ES(t) = E[(X - t)_+]$  is one of the popular risk measures. Although its importance for risk management applications was recently recognised, the approach is not new. It was developed by actuaries in late XVIII and XIX centuries.

The first researcher who offers this approach in late XVIII century was J.N. Tetens (1736 - 1807). He used the value  $ES(E[X])$  under the title *Risico der Casse*. This approach was offered in the context of actuarial mathematics. An actuary actually was interested in the numerically equivalent value  $-E[(E[X] - X)_+]$  which is understood as the negative surplus of (life insurance) contract. This was a primitive ALM approach.

Further during XIX century, this approach was developed in using other value of  $t$ . The logics were based on a reimbursement of the negative surplus, which generates series of additional contracts based on the transitional rule  $t_k = ES(t_{k-1})$ ,  $k \in \mathbb{N}$ . This is linked to real option vision to insurance contract.

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# Valuing Employee Share Options Using the Exercise Multiple Approach and Binary Power Options

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## Abstract

This paper proposes an analytic formula for valuing Employee Share Options (ESOs) using the exercise multiple approach suggested by Hull [1] for modelling the early exercise behaviour of holders of ESOs. ESOs typically have a vesting date, vesting conditions or hurdles and a period (the exercise window) during which they can be exercised. The option valued is an American style call option during the exercise window from the vesting date to the maturity date. In Australia valuation of ESOs is often done by actuarial consulting firms, using Monte Carlo Simulation methods. This paper presents an analytic alternative method for doing the valuation.

The exercise multiple approach assumes that the option is exercised early as soon as the stock price reaches some multiple of the exercise price. Voluntary decisions to exercise early are driven by the extent to which the option is in the money. Under this approach, we show that the ESO can be thought of and valued as a combination of non standard barrier options. We also show this ESO can be expressed as a combination of European binary power options using a PDE technique called the method of images. The binary power option is a building block for the valuation of the ESO. Valuation formulae for these binary power options are presented and we show how these are used to obtain an analytic formula for the value of the ESO assuming exercise decisions are purely driven by the barrier level of the stock price being reached.

The ESOs may also be exercised early if the holder of the ESO dies, retires due to ill health, or experiences a financial / liquidity crisis which forces them to exercise early. We can model this behaviour by estimating the rates of exit from the population of ESO holders using a multiple decrement table approach as is used in superannuation / pension fund financial projections. We need exit rates for exit by death, ill health and liquidity crisis. We show how we can combine the analytic formula for the ESO with the population projection to obtain an ESO valuation allowing for early exercise decisions driven by both the breaching of the barrier level and by other causes such as death, ill health retirement or financial crisis. This approach can be extended to situations where the ESO has hurdles applying at the time of vesting defined in terms of the performance of the underlying stock and relative performance of the stock compared with the performance of other stocks or a stock market index.

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# Evaluation of discretely sampled Asian option Greeks by convolution

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## Abstract

Practitioners are interested in accurate and efficient ways to calculate the prices of derivative contracts, but also their sensitivities (also known as the “Greeks”) obtained by differentiating the contract price with respect to parameters of interest. Common Greeks are the so-called delta and gamma which represent the first and second derivatives of the contract price with respect to the value of the underlying,  $S_0$ , and the vega which is given by the first derivative of the contract price with respect to the volatility of the underlying. Based on the empirical work of [1], lack of use of the Greeks implies insufficient knowledge of the risk-return profiles of the options and can lead to poor performance on option investments. Apart from their use in risk management and hedging, the price sensitivities also contribute directly to the price quotes since the bid-ask spread is often proportional to some Greeks.

In this research, we investigate the problem of computing the price sensitivities of European-style arithmetic Asian options monitored at discrete time points. Asian options trade frequently in various markets, including the equity (e.g., see [2]), commodity (e.g., see [3]) and insurance (e.g., see [4]) markets. Their appeal stems mainly from the fact that the option payoff depends on the average value of the underlying, so that the risk of market manipulations as the option approaches maturity is reduced.

The approach for the computation of the European Asian option price sensitivities we propose builds on backward price convolutions (see [5]). The method is exact and does not rely on finite difference approximations. In more details, our approach is based on direct differentiation of the risk neutral conditional expectation of the discounted payoff of the Asian option expressed in terms of the Carverhill-Clelland-Hodges factorization. We assume that the dynamics of the log-returns are driven by commonly used Lévy processes, such as the Brownian motion, the NIG [6] and CGMY [7] models, and prove that differentiation under the integral sign with respect to parameters of interest is permissible. In particular, we consider the option’s delta, gamma and, for first time in the literature, the nontrivial case of the option’s vega for a lognormal underlying. The suggested method also indicates capacity for extension to sensitivities with respect to other model-specific parameters.

We demonstrate the accuracy of our method by performing numerical tests under the different Lévy models and comparing against the outcome from Monte Carlo simulations implemented with control variates (see [8]). Finally, we analyze the impact of misspecifying the underlying asset price dynamics on the performance of delta-gamma hedging strategies by computing the overnight profit/loss for a representative contract.

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# Stochastic modelling of disability insurance in a multi-period framework

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## Abstract

We propose a stochastic semi-Markovian framework for disability modelling in a multi-period discrete-time setting. The logistic transforms of disability inception and recovery probabilities are modelled by means of stochastic risk factors and basis functions, using counting processes and generalized linear models. The model for disability inception also takes IBNR claims into consideration. We fit various versions of the models into Swedish disability claims data.

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# Minimization of a function of a quadratic functional with application to optimal portfolio selection

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## Abstract

We present an explicit closed form solution of the problem of minimizing the combination of a linear functional and a function of quadratic functional, subject to a system of affine constraints. This is of interest for solving significant problems of financial economics which are related to optimal portfolio selection.

The solution essentially generalizes the previous results of the authors concerning optimal portfolio selection with translation invariant and positive homogeneous risk measures (see [2] ). The important representatives of such risk measures are value-at-risk (VaR) and tail condition expectation (TCE).

The classical mean-variance model, and the recently introduced and investigated tail mean-variance model (see [1] ) , are special cases of the general setup considered here. The results are illustrated with the data of stocks from NASDAQ/Computers.

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# The effect on guaranteed cost for variable annuities when there are asset allocation options

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## Abstract

Variable annuities have been one of the most important insurance products of many countries over the last two decades. While there is investment risk associated with variable annuities, many of them offer guarantees of principal and/or downside protection. Premiums are allocated to some specific index fund at one time. However, many variable annuities provide asset allocation options today, which mean that policyholders can rebalance their portfolio under specific terms of contracts. As a result, it is important to realize the effect of asset allocation options on the guaranteed cost. Previous literatures assume a specific underlying asset return process when considering the guaranteed cost of variable annuities. To our limited knowledge, no literature consider the guaranteed cost incorporating the asset allocation option, this study tries to fill this gap. In this study, two assets, one with high risk and high return and one with low risk and low return, are available for a variable annuity with guaranteed minimum withdrawal benefits. The asset return processes are assumed to be lognormal. A policyholder, who has a power utility function, tries to maximum his utility by rebalancing his portfolio, with short-selling constraint, once a year. We apply a numerical approach to solve the asset allocation problem of the policyholder and recognize the potential effect, the raising guaranteed cost, when there is an asset allocation option.

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# A compound renewal model for the medical malpractice insurance

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## Abstract

A compound renewal model for the aggregate discounted payments and expenses assumed by an insurer is proposed for the medical malpractice insurance, where the real interest rates could be stochastic and the dependencies between the expenses, the payments and the delays of payments are examined through the theory of copulas.

As a first approach to this problem, we present formulas for the first moments of this aggregate risk process. Examples are given for Erlang claims interoccurrence times, Erlang delays of payments, Pareto payments and claims, and the influence of the dependency is illustrated by the Joe copula. Finally, the distribution, VaR and TVaR are also considered through simulations.

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# Valuation Of Currency Options Under A Regime-Switching Gaussian HJM Model

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## Abstract

Spot foreign exchange (FX) rates usually exhibit jumps and regime switching in a finite number of states due to change in macroeconomic factors or financial crises. A regime-switching jump diffusion (RSJD) model proposed in this paper is modeled by a Markovian system that is capable of capturing the feature of the dynamic spot-FX rate. We also examine term-structure data to show that the regime-switching of forward interest rates is distinct from that of the spot-FX rate, and thereby leading to different associated impacts (or regime-switching impacts). Hence, a regime-switching Gaussian Heath-Jarrow-Morton (HJM, (1992)) model is introduced via another Markov chain. The RSJD model is used along with the forward Esscher-transform technique for the arbitrage-free condition with which European currency call options are priced under the Markov-modulated Gaussian HJM model (MMHJM). Empirical study shows that the RSJD model combined with the MMHJM model is a more complete and appropriate model for pricing currency options, and the regime-switching impacts of the spot-FX rate and forward interest rates should be properly taken into account.

**Keywords:** Heath-Jarrow-Morton model; Markov modulated Poisson process; Esscher transform; Currency Options

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# Rationality-based Evaluation of Life Insurance Contracts with Surrender Guarantees via LSM Approach

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## Abstract

We study unit-linked life insurance contracts with surrender guarantees. The prices of such contracts are influenced by multidimensional risk factors including the financial market risk, the mortality risk and the surrender risk. The surrender risk is represented by a surrender intensity which switches between its lower and upper bounds depending on the contract situation and the policyholders' own financial needs. We extend the regression-based algorithm introduced by Bacinello, Biffis and Millosovich[1] by accounting for this bounded monetary rationality of the policyholders in surrendering their contracts. This enables us to investigate the effects of different risk factors on the surrender behavior of the policyholders.

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# ECOMOR and LCR Reinsurance with Gamma-like Claims

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## Abstract

Assuming that the claim sizes of an insurance company have a common distribution with gamma-like tail, we study the asymptotic tail behaviour of the reinsured amounts under ECOMOR and LCR reinsurance treaties, respectively. Our novel results include a precise asymptotic expansion for the tail probability of the reinsured amounts under the ECOMOR treaty, and tight asymptotic bounds for the LCR case. As a by-product we derive a precise asymptotic expansion for the tail of the product of independent regularly varying random variables.

*Keywords:* Asymptotics; Gamma-tail distributions; Reinsurance; LCR and ECOMOR treaties; Tail probabilities

*Mathematics Subject Classification:* Primary 62P05; Secondary 62E10, 91B30

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## Adverse Selection in China's Automobile Insurance Market

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### Abstract

This paper investigates adverse selection in China's insurance market. Utilizing the data set for the comprehensive automobile insurance coverage from 2008 to 2011, the paper examines the basic coverage-risk prediction of adverse selection theory that policyholders who purchase insurance coverage tend to be much riskier. The empirical results provide strong evidence to demonstrate the existence of adverse selection. Moreover, the robust test also consolidates the results even by controlling the attributes of the automobiles.

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## Gerber-Shiu analysis under a perturbed risk model with interclaim times depending on the claim sizes

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### Abstract

Under the classical compound Poisson risk model and the Sparre-Andersen risk model, one crucial assumption is that the interclaim times and the claim sizes are independent. However, this assumption might be inappropriate in practice. In this paper, we consider a continuous-time risk process where the interclaim-time distribution depends on the size of the previous claim. Also, the surplus process of the insurer is perturbed by a Brownian motion. Explicit solutions for the Gerber-Shiu discounted penalty function are derived for arbitrary claim sizes utilizing the roots of a generalized Lundberg's equation. Special cases of the Gerber-Shiu function when claim sizes come from the  $K_n$ -family are considered. A numerical example is provided for illustrative purposes.

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# Value at Risk for longevity under an AR-(G)ARCH mortality framework

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## Abstract

We propose an autoregression (AR) mortality model which makes the linear assumption between two mortality sequences with a time lag each other and further consider the conditional heteroskedasticity of error terms. In the empirical testing, we find that considering the conditional heteroskedasticity is substantial in mortality fitting of our model using the mortality data for Japan, the UK, and the USA; however, it does not produce a lot of differences between the predicted and real mortality rates but the confidence intervals with different width. Our model further performs quite satisfactory compared with the Lee-Carter model. We also provide the robustness for our model using other 7 countries. Finally, we use Japan data which show more differences between the structures of error terms as an example to demonstrate Value at Risks (VaRs) for longevity regarded as additional reserves for an annuity provider or pension funder, and compare the effects of conditional heteroskedasticity on VaRs.

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# Discrete-time Finite-state BSDEs with Infinite-time Horizon

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## Abstract

This work studies discrete-time finite-state Backward Stochastic Difference Equations (BSDE) (proposed by [1]) with infinite-time horizon. The existence and uniqueness theorem for the solution to the discrete infinite-time BSDE is established. We also apply this theory to the premium principles for contingencies with infinite-time horizon.

## References

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## Measuring herd behavior between stock prices

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### Abstract

Never put all your eggs in one basket. Investors are well aware of this advice and prefer to compose a blend of different stocks to invest in. This is a prudent strategy because heavy losses in one asset can be countered by gains of others. However, this diversification effect is fading away when there is an increased co-movement between the stocks. It is well documented that periods of increased co-movement are tied to periods of high market stress. As a result, the diversification benefit is evaporating when it is needed the most. Therefore, having a notion about the strength of the co-movement between stock prices gives market participants the opportunity to take the necessary cautionary actions.

We construct a new class of dependence measures, called *herd behavior measures*. Each measure estimates the strength of the co-movement between a set of dependent random variables by quantifying the standardized distance between the random vector and its comonotonic modification, using the distribution of the sum. We propose two general classes of herd behavior measures. A first class compares the expected utility of the random sum with its comonotonic modification. A second class compares the distorted expectation of the random sum with its comonotonic modification.

Each herd behavior measure can be used to define an index which measures the co-movement between stocks composing an index. Each measure will be model-free and based on option prices. These option prices contain the views of the market about future movements of the financial market. Therefore, the resulting estimate of the degree of herd behavior will take a forward looking view. We capture the expectation of the market about today's degree of herd behavior in a single, intuitive, number.

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## On a bivariate process with a dividend barrier strategy

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### Abstract

In this talk, we present a two-dimensional risk model under a barrier strategy. We use copulas to represent the dependence between two business lines when a "common shock" strikes. We define the time of ruin to be the first time that either of the two lines has its surplus level below zero. Under such a model, we derive a discrete approximation procedure to calculate the expected discounted dividends until ruin and apply a bivariate extension of the Dickson-Waters discretization (Dickson and Waters (1991)) with the use of a bivariate Panjer type recursion (Walhin and Paris (2000)). We have a thorough discussion of different dependencies via common shocks, copulas and proportional reinsurance with detailed numerical examples. We also illustrate that the joint optimal dividend barriers for the bivariate process are dependent on the initial surplus levels. Moreover, a modified type of dividend barrier strategy which can lead to increased dividends and longer survival time is proposed.

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# A regime-switching framework for the valuation of a guaranteed annuity option

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## Abstract

We introduce a Markov-modulated Gompertz model for the evolution of mortality rates. Our mortality modelling motivation was based on the empirical investigation of US data. A continuous-time finite-state Markov chain is employed to describe the evolution of mortality model parameters, which are estimated using a filtering-based method. The proposed model is deemed adequate in describing the mortality data given the goodness-of-fit metrics and likelihood-based selection criteria. We construct a valuation setting for guaranteed annuity option, where the interest and mortality risk factors are driven by two independent Markov chains. Thus, both factors for pricing are regime-switching. To get closed-form valuation formulae, the change of reference probability measure technique is employed. Numerical results under various different regime pair conditions are provided and they compared are with those obtained from Monte Carlo simulation to evaluate efficiency of the proposed approach.

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# Characterizations of counter-monotonicity and upper comonotonicity by (tail) convex order

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## Abstract

We characterize counter-monotonic and upper comonotonic random vectors by the optimality of the sum of their components in the senses of convex order and tail convex order respectively. In the first part, we extend the characterization of comonotonicity by [1] and show that the sum of two random variables is minimal with respect to convex order if and only if they are counter-monotonic. Two simple and illuminating proofs are provided. In the second part, we investigate upper comonotonicity using tail convex order as the main tool. By establishing some useful properties of this relatively new stochastic order, we prove that an upper comonotonic random vector must give rise to the maximal tail convex sum, thereby completing the gap in [2]. The relationship between tail convex order and risk measures along with conditions under which the additivity of risk measures is sufficient for upper comonotonicity are also explored.

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## Ruin theory with correlated lines of business

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### **Abstract**

In this talk, we present different results regarding ruin theory with correlated lines of business. We study impact of correlation on several risk and profit indicators. We tackle some issues related to assistance one line may provide to another one in the red.

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# Nonparametric estimation of the survival function for couples of lifetimes

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## Abstract

Pension contracts with reversion clauses depend on two random lifetimes: that of the policyholder, and that of his/her wife/husband. While studying the lifetime of a single individual is a classical problem for insurers, bivariate mortality analysis is much more delicate. Indeed, in lifetime data analysis, statistical inference faces the problem of censoring and/or truncation. In the univariate case, the Kaplan-Meier estimator is a common nonparametric method to estimate the survival function. However, in presence of bivariate censoring and bivariate truncation, nonparametric estimation of the joint survival function is more delicate, the most classical estimator being the one proposed by [1]. Nevertheless, this estimator is not a true survival function (it attributes negative masses to some points in the plane). As a consequence, this estimator can not be used for several applications including bootstrap.

In this work, we present a new method to perform a nonparametric estimation of the distribution of a couple of lifetimes under bivariate censoring and truncation, which is described in [2]. We extend the methodology described in this paper to incorporate a "generation effect", in order to understand the evolution of the distribution of the two lifetimes through time. We derive asymptotic convergence properties for this estimator, and show its practical behavior on a real dataset. This nonparametric estimator is then used for performing model-checking of parametric survival copula models, with critical values of the test procedure computed through a bootstrap methodology.

## References

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## Love and Death : A Freund Model with Frailty

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<sup>2</sup>*SCOR and CREST*

### Abstract

We introduce new models for analyzing the mortality dependence between individuals in a couple. The mortality risk dependence is usually taken into account in the actuarial literature by introducing an Archimedean copula. This practice implies symmetric effects on the remaining lifetime of the surviving spouse. The new model allows for both asymmetric reactions by means of a Freund model, and risk dependence by means of an unobservable common risk factor (or frailty). The model is applied to insurance products such as joint life policy, last survivor insurance, or contracts with reversionary annuities.

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# A Spatial Cross-Sectional Credibility Model with Dependence Among Risks

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## **Abstract**

A Bühlmann-Straub type credibility model with dependence structure among risk parameters and conditional spatial cross-sectional dependence is studied. Predictors of future losses for the model under both types of dependence are derived by minimizing the expected quadratic loss function. Non-parametric estimators of structural parameters are considered in the spatial statistics context. Predictions and estimations made for the proposed model are examined and compared to other models in an application with crop insurance data and a simulation study.

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# State-Dependent Fees and the Surrender Option in Variable Annuities

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## **Abstract**

Guaranteed minimum maturity benefits (GMMB) are similar to financial options on the fund value of a variable annuity. They are usually financed through a fee set as a fixed percentage of the fund, which is paid continuously throughout the term of the contract. From the policyholder's point of view, when the option is deep out-of-the-money, this fixed fee represents an important incentive to lapse. We first present an integral form for the value of a contract with a surrender option, and we analyse the optimal surrender region. We discuss the link between the continuous fee and the value of the surrender option, and present ideas for more advanced fee structures that aim to decrease the value of the surrender option, thus reducing the incentive to lapse. In particular, we introduce a new type of fee structure for variable annuities, where the fee rate depends on the moneyness of the guarantee. We present some numerical examples and sensitivity analysis.

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# Valuation and Risk Assessment of Disability Insurance Using a Discrete Time Trivariate Markov Renewal Reward Process

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## Abstract

During the last decades, the importance and demand for disability insurance has continuously risen, which allows the application of new methods of premium calculation due to an increased data basis. The previously used formulae, e.g. the Norwegian model or the Manchester Unity model, can now be replaced by multivariate Markov reward processes in order to provide a sound framework not only for premium calculation but also for risk analysis. This constitutes a major issue for life insurers in order to more adequately assess the value and risk associated with these contracts, which is especially relevant against the background of Solvency II. However, even though disability insurance is becoming increasingly important especially in the German market, there has been only very little research on this topic to date.

In this paper, we extend previous literature by proposing a discrete time trivariate Markov renewal reward process that, in comparison with semi-Markov reward processes, additionally captures the dependence between the duration in two successive states. This allows us to more precisely evaluate a disability insurance policy on account of the consideration of major drivers in disability insurance, which has not been done so far. Thus, our model incorporates all rewards paid such as premiums and benefits as well as the duration since the time of disablement, which has a considerable impact on disability probabilities, mortality rates, durations and reactivation [5]. We further provide a closed-form solution and a simulation approach for computing all higher moments of the proposed model that are needed for a thorough risk analysis of disability insurance contracts.

Based on assumptions relying on empirical results from [2], [3] and [5], higher moments for multiple risk groups are analyzed in combination with different premiums paid by the policyholder. Additionally, the influence of the waiting time distribution for each state transition on the risk inherent to the insurance contract is examined. In the numerical analysis, a special case of the proposed Markov reward process, i.e. the semi-Markov reward process, is analyzed. Finally, we compare the results of this special case with those obtained by the model of [4] and with those acquired by an actuarial approach as laid out in [1].

The results of the special case and the two benchmarks do not differ because they describe the same semi-Markov model. Nevertheless, the proposed model theoretically enables us to incorporate the dependence between the durations in two successive states and therefore, our new approach allows new prospective insight with respect to the assessment of an insurer's risk situation.

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## Aggregation of dependent risks with continuous marginals

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### Abstract

In the context of actuarial science and quantitative risk management, we investigate risk aggregation problems for a portfolio of dependent risks with continuous marginals. We first consider contexts for which we obtain explicit expressions for the cumulative distribution function of the sum of dependent risks. Among them, we examine specific multivariate distribution (e.g. Downton-Moran bivariate exponential). We also consider portfolio of dependent risks whose joint distribution is defined with a copula and with continuous marginals (e.g. Ali-Mikhail-Haq copula). In the contexts for which no explicit expressions for the cumulative distribution function of the sum of dependent risks are obtained, we propose numerical methods to compute the cumulative distribution function of the sum. The findings are illustrated with numerical examples.

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# How a Single-Factor CAPM Works in a Multicurrency World

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## Abstract

As shown in A.D. Wilkie's paper entitled "Why the capital-asset pricing model fails in a multi-currency world" there is no unique single-factor capital-asset pricing model (CAPM) in a multi-currency world. As he points out, the standard CAPM assumes that all investors measure risk and return in the same currency. He shows that, if two investors measure risk and return in different currencies, the standard CAPM cannot describe the pricing of capital assets for both investors.

The advantage in using a single-factor model is that it does not treat currency risks as carrying different weight from investment risks; regardless of its source, risk is measured as variance and weighted accordingly. The aim of this paper is primarily to give actuaries a way ahead in the use of the single-factor CAPM in a multi-currency world for the purposes of the stochastic modelling of the assets and liabilities of long-term financial institutions, and the application of the model has been designed with that intention. However, it is envisaged that the model will also be of interest to other practitioners. Unlike international CAPMs developed in the literature to date, it assumes that, for every currency in which investors measure risk, there is a unique CAPM across all the markets in which they invest, and that, regardless of the currency in which they measure risk, all investors have homogeneous expectations and all investors participate in the formation of equilibrium. It develops a theory for multi-currency CAPMs by developing a CAPM for each set of investors that measure their returns in a particular currency. In the development of this theory the meanings of 'homogeneous expectations' and of 'equilibrium' are reconsidered in the context of a multi-currency world.

In this paper a single-factor multi-currency CAPM (SFM-CAPM) is developed. It is shown that, for a single-factor CAPM to work in a multi-currency world, there is a necessary and sufficient condition. That condition applies to the ex-ante variances and covariances of returns. The estimation of the variance-covariance matrix of returns by constrained maximum-likelihood estimation is discussed. Some difficulties with that approach are explained and an alternative approach, using ordinary least squares, is developed. The theory is applied to two major currencies and two minor currencies, namely the USA dollar, the UK pound, the South African rand and the Turkish lira. The application is designed for use by actuaries in the modelling of the assets and liabilities of long-term financial institutions. To that end the longest possible range of time periods is used and quarterly intervals are used rather than the relatively short time intervals typically used in the literature. Indications are given of the way in which the findings of this paper will lead to further research for the purposes of such modelling.

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# Costs and Benefits of Crash Hedging

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## Abstract

The worst-case scenario portfolio problem which has been introduced by Korn and Wilmott [1] will be considered in this talk. In the setting of Korn and Wilmott, approximations for the optimal crash hedging strategy will be derived. Furthermore, the costs and benefits of using the optimal crash hedging strategy instead of the classical optimal portfolio strategy will be calculated. Additionally, we compute the efficiency of the optimal crash hedging strategy and derive the break even crash size, that is the crash size where the investor is indifferent in either using the optimal crash hedging strategy or the classical optimal portfolio strategy.

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# Tontines that Maximize Utility but Limit Longevity Risk: With Reference to the First English Tontine of 1693

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## Abstract

Historical tontine annuities promised enormous rewards to the last few survivors of the pool, but at the expense of those who (gambled, lost and) died early. And, while this winner-take-all design *appealed to the gambling instinct*, it is a suboptimal way to manage individual longevity risk. Indeed, conventional actuarially fair life annuities paying relatively more early on, and relatively less later on – provide greater lifetime utility, compared to a tontine annuity. And yet, tontines do not have to be designed using an increasing lottery approach. In this paper we derive a tontine structure that maximizes lifetime utility, but one that doesn't expose the administrator to institutional longevity risk. We prove that the optimal product: (i.) distributes a declining cash flow to the tontine pool, (ii.) proportional to the expected survival probabilities, and (iii.) is independent of the retirees longevity risk aversion. Coincidentally, this structure is similar to the first tontine ever issued by the British government in 1693, long-viewed as unattractive and never used since. We conclude the paper with some suggestions and recommendations for the design of retirement income products that do not impose any actuarial reserves or capital requirements, but are still appealing to the rational lifecycle investor.

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# A survival analysis of contract lifetime and policyholder behaviour: prediction of lapses on a Whole Life portfolio

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## Abstract

Insurers have been much concerned about policyholder's behaviour for a long time because it strongly impacts their reserves. The new european directive Solvency II, which promotes the development of internal risk models (among which a complete unit is dedicated to the lapse risk), strengthens the necessity to deeply study and understand this risk. In this paper we perform a survival analysis in order to model the contract lifetime, while taking nicely into account the main risk factors impacting policyholders' decisions. We find out that classical survival regression models fail to predict the actual timing of surrenders because of too much heterogeneity, whereas the competing risk framework seems to provide interesting insights and more accurate predictions. Numerical results follow from using [1] model on an insurance portfolio embedding Whole Life contracts: this model reveals to be quite efficient as it recovers the empirical lapse rate trajectory, which enables to set up an adapted asset and liabilities management.

## References

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# Forecasting Mortality in Related Populations Using Lee-Carter Type Models: A Comparison

Ivan Luciano Danesi<sup>1</sup>, Steven Haberman<sup>2</sup>, Pietro Millosovich<sup>2</sup>

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## Abstract

We test and compare several mortality forecasting models for related populations. More precisely, we consider extensions of the standard Lee-Carter approach, where either the central death rates or the corresponding improvement rates are modelled. The models include common and population specific time indices and/or combinations of them. We apply this range of models to the Italian regions and perform a thorough analysis. Finally, some actuarial application is presented.

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# On the Speed of Depletion and Drawdowns of a Lévy Insurance Risk Model

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## Abstract

The field of risk theory has traditionally focused on ruin-related quantities. Although it is true that there are still many challenging questions, ruin related quantities do not seem to capture path-dependent properties of the reserve. In this talk we aim at presenting the probabilistic properties of drawdowns and the speed at which an insurance reserve depletes as a consequence of the risk exposure of the company. These new quantities are not ruin related yet they capture important features of an insurance position and we believe it can lead to the design of a meaningful risk measures. Studying drawdowns and speed of depletion for Levy insurance risk processes represent a novel and challenging concept in insurance mathematics. Indeed, drawdowns and speed of depletion are quantities that do not depend on the level but rather on path properties of the model which explain how fast the process can drop. This type of quantities has never been proposed before as measures of riskiness in insurance. Drawdowns have been only studied for diffusion processes in a finance setting [1], yet in insurance we need expressions for processes exhibiting jumps. Definitions of all these concepts are given as well as some examples of L evy insurance risk processes for which they can be calculated. Future work and open questions are also discussed.

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# Performance evaluation of a financial asset with a non-additive measure under Knightian uncertainty

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## Abstract

In recent years, the increased availability of complete transaction and quote records for financial assets has provided researchers with an opportunity to further their knowledge about financial return volatility and other risk measures through robust computation methods. Nonetheless, such high frequency sampled observations of asset prices contain a non-negligible component of microstructure friction due to the imperfections of trading processes. The presence of microstructure noise implies the inadequacy of the use of classical methods for risk measures based on straightforward descriptive statistics. This paper contributes to the literature on risk measures, more precisely on performance evaluation, by adapting it to high frequency data. Because of the imprecision on the returns induced by the microstructure noise, we assume that the decision maker has incomplete, inaccurate knowledge about the state of nature. Since the decision maker does not know the objective probability measure on the space of state of nature, he faces a Knightian uncertainty. The paradox of Ellsberg (1961) explains that the existence of Knightian uncertainty has an effect on people's action. The behavior of decision makers contradicts with the Von Neumann-Morgenstern paradigm as well as the Savage (1954) paradigm, which assumes a unique well-defined additive probability distribution that represents the beliefs of an individual. There are alternative approaches to deal with Knightian uncertainty, of which, that of Gilboa (1987) and Schmeidler (1989), suggesting the use of a non-additive probability or capacity in the sense of Choquet (1954). This article adopts the latter approach by, i.e. uses the credibility measure of Liu and Liu (2002) which is a special capacity fuzzy measure and the Choquet integral, to describe the occurrence of the states. Under this uncertain environment, the performance evaluation using high frequency information is treated, mainly in the framework of one side uncertain by analogy to the one side risk (see Farinelli and Tibiletti, 2008, for more details). Numerical methods are proposed for the empirical computation of these performance ratios based on observed real returns. Finally, an empirical study is carried out in order to show the effectiveness of our modeling approach.

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# Multivariate Distribution defined with Farlie-Gumbel-Morgenstern Copula and mixed Erlang Marginals: Aggregation and Capital Allocation

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## Abstract

In this paper, we investigate risk aggregation and capital allocation problems for a portfolio of possibly dependent risks whose multivariate distribution is defined with the Farlie-Gumbel-Morgenstern copula and mixed Erlang distributions marginals. In such a context, we first show that the aggregate claim amount has a mixed Erlang distribution. Based on a top-down approach, closed-form expressions for the contribution of each risk are derived using the Tail-Value-at-Risk and covariance rules. These findings are illustrated with numerical examples.

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# Pricing and modeling of Insurance Linked Securities

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## **Abstract**

In this presentation we explore aspects of modeling and pricing Insurance Linked Securities (ILS), an asset class providing the Capital Markets the opportunity to participate directly in genuine (re)insurance risk. A highly attractive asset class as there is very little correlation to other financial market risks. Motivated by the objective to establish rigorous rules-of-thumb allowing for tractable analysis, we employ economic ideas (Utility Theory) as well as Extreme Value Theory, and support our findings by empirical tests based on available data. Specifically, we fit Generalized-Pareto-Distributions (GPD) to the catastrophe risk associated with a security by exploiting certain parameters (attachment probability, expected loss and exhaustion probability) supplied by third party modeling agencies (typically AIR, RMS, Eqecat or Miliman). We then discuss possible approaches to deal with events which were not anticipated either by the modeling agencies or by the corresponding scientific community (with reference to the Tohoku 2011 Earthquake in Japan). Secondly, we examine the utility indifference pricing approach for both Catastrophe Bonds and Collateralized Reinsurance Transactions. These being the most prevalent instruments within the ILS space. We conclude by applying the preceding method to the 2012 Catastrophe Bond market and compare it with other existing methods from ILS-related and reinsurance literature.

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# Forecasting longevity gains of a population with a short time series of mortality rates: an application in insurance and pension plans

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## Abstract

In this paper, a multivariate structural model is proposed to forecast longevity gains of a population with a short time series of observed mortality rates, where a traditional approach to measure the trends of the series may not be appropriate. The model considers that the time series of mortality rates of different ages are not directly related to each other, but are subject to similar influences, such as better eating habits, life quality and medical care, all of which can significantly reduce future mortality rates. These similar influences are captured in the model assuming that both observable mortality rates and unobservable components (trends) of different age groups are contemporaneously correlated. Thus, a seemingly unrelated time series equation (SUTSE) framework is adopted, and the model is represented in state space form to estimate the unobservable components and the parameters of interest through the Kalman filter. The model estimates the trends of mortality rates through the common trends approach (Harvey, 1989).

First of all, one needs to find a population with a long time series of mortality rates, which is called as related population, that has similar mortality characteristics to the population studied. The proposed model is used to obtain the longevity gain distributions of the male and female Brazilian populations using populations of Portugal and of the United States as related populations to compute the model. The results are then compared to those obtained by the Lee-Carter method (1992). It can be seen that forecasted Brazilian mortality rates by the proposed model are higher than those obtained by applying the Lee-Carter method.

Additionally, it is shown how the models can be used by insurers and pension funds to manage the risk of declining mortality rates considering the longevity gain model. The results are also compared with those obtained by the Lee-Carter method. These differences are concretely illustrated through measuring complete life expectancies. Temporal evolutions of these life expectancies are estimated.

The distribution of present values of the retirement cash flows is obtained using Markov chain Monte Carlo (MCMC), as example how the proposed model can be used to measure the actuarial risks in practice. For retired groups, the values of the solvency provision and of the regulatory capital based on underwriting risks are measured, this latter is based on the tail of the distribution.

The preset values of future cash flows are simulated for groups of different sizes to emphasize the values of the idiosyncratic risk. The estimated underwriting capitals are showed in relation to the values of provision. In function of the idiosyncratic risk, we can conclude that the larger the retired group the less will be the value of the capital in relation to the



value of provision. Therefore, to assure solvency of insurers and pension funds, one also has to consider the idiosyncratic risk in the process of valuation of the capital.

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## Forecasting Surrender Rates through Copulas and Macro Economic Variables

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### Abstract

In this paper, it is proposed a model to forecast the surrender rates from insurance and pension plans with annuity payments through multivariate elliptical copulas and macro economic variables. The proposed approach is quite relevant nowadays since adequate surrender rates play an essential role in the realistic valuation of insurance and pension plans liabilities or actuarial risks. Both objectives are now highly demanded by solvency and accounting principles. Besides this, companies must forecast surrender rates in order to manage risks that arise due to mismatches between assets and liabilities.

Our database consists of male and female surrender rates, separated by age, from 2006 to 2010 of a big Brazilian life insurer. First of all, cluster analyses is used to partition surrender rates in age groups, for both male and female. Kim (2005) used economic variables as exploratory variables to model surrender rates through generalized linear models (GLM) using logit link function and complementary log-log link function. At a first stage, the same process is applied in our approach, but we also tested the probit link function. Differently from Kim (2005), the real short interest rates are the only macro economic variable to predict surrender rates. The advantage of such variable is that we can take future market expectation trough current term structure of interest rates. Thus, in our approach, we do not need to forecast explanatory variables to predict expected surrender rates.

The residuals of each group of ages from the GLM model are then modelled by ARMA-GARCH processes. Statistical tests (BDS test) are performed in order to check independency. In order to model the dependence structure among these residuals, we applied elliptical copulas (Gaussian and the T-Student). During economic crisis, it is observed uncommon surrender rates in the insurance industry. Therefore, we also used returns of the Brazilian stock market index (Ibovespa) as one of the marginal distributions in the dependence modelling.

As initially expected, we found negative dependence among Ibovespa residuals and the surrender rates residuals. The proposed approach is evaluated trough an out-of-sample back test according to Christoffersen (1998). Our approach may be used to simulate future surrender rates given a specific financial or economic scenario, which can be chosen in a stress test procedure. To simulate this scenario, it is used elliptical copulas conditioned on the Ibovespa residuals.

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# Extreme Value Theory and Risk Analysis: An Application for Emerging Financial Markets

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## Abstract

Financial crises occurred in recent years have highlighted the importance of risk measurement and studies on this area have been accelerated. Risk measurement enables us to evaluate potential risks and effects of these risks in financial markets. Value at risk (VaR) and expected shortfall (ES) rise as the most preferred risk measures to analyze financial risks. Furthermore, extreme value theory is one of the popular methods which appears in this context.

In this study, our aim is to analyze stock indexes of different emerging markets which includes Turkish IMKB-100. We model indexes by extreme value theory and calculate risk measures. Moreover, we evaluate performances of these indexes according to the obtained results and we analyze the position of Turkish stock market in detail.

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# Bonus-Malus Systems with a Mixture of Two Claim Severity Distributions

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## Abstract

One of the pricing strategies for Bonus-Malus (BM) systems relies on the decomposition of the claims randomness, namely, one part accounting for claims frequency and the other part for claims severity. Mixing of an Exponential with a Lévy distribution, we focused on modelling claim severity component as Weibull distributed. However, after comparing our results with [2] where Pareto severities were introduced, we propose as model a mixture of these two claim severity distributions accounting for the nice properties in the tail of a Pareto distribution. For a Negative Binomial number of claims, the BM premium rates are then derived. Results are illustrated by our numerical example.

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# Continuous Chain Ladder: Reformulating and generalising a classical insurance problem

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## Abstract

A novel approach to claims reserving is set out. Considering the data in continuous time, the paper presents the natural continuous version of the classical chain ladder method. One purpose of the paper is to establish that the classical chain ladder estimator can be understood as a structured histogram. We argue that this interpretation of the classical chain ladder method is extremely relevant for its practical implementation. Our histogram interpretation of the classical chain ladder estimator has a number of immediate implications: first, the bandwidth selection problem of the histogram (whether chain ladder is based on monthly, quarterly or yearly data) is following the laws of bandwidth selection of non-parametric smoothing problems well known in the mathematical statistical literature. Secondly, it is well known from mathematical statistics that histograms are not efficient when estimating smooth curves. The most common and most immediate improvement to histogram methods is simply the kernel smoothing method. Thirdly, classical chain ladder is indeed a granular method, because the histogram works on continuous data. Then one could stay in the “good old chain ladder histogram world” and first derive two under-smoothed histograms. One in the underwriting year direction and other in the development year direction. And then smooth these one-dimensional histograms. Unfortunately our study cannot recommend this. This is unfortunate, because this simple method of sieves would be intuitively appealing to many applied actuaries.

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## Forward transition rates in multi-state life insurance

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### Abstract

In the interest rate world it is common to consider forward rates which describe the market interest rate for future transactions. During the last decade the idea of forward interest rates has been transferred to the mortality framework, and meanwhile forward rates have become a helpful tool for the securitization of mortality and longevity risks. While there exists a lot of literature on forward mortality rates, multi-state insurance products such as disability insurance or joint life insurance are hardly discussed. A first attempt to define general forward transition rates was made in [1], where different approaches how to define forward transition rates are discussed, but the author concludes that in general there is no clear answer. In particular he points out to the fact that forward rates depend on the type of derivatives that are studied.

Our paper has two major objectives: a sound definition of forward rates that features some desirable properties and an investigation of the dependency structure of the underlying processes that are necessary for the existence of forward rates.

First, we theoretically discuss how forward transition rates can and should be defined. In particular, we follow the substitution concept and stress the notion that forward rates should be invariant with respect to some set of derivatives. These sets should include all common benefits (i.e., for staying in one state and for the transition into another state) and some standardized products (i.e., a forward representation of each single discounted hazard rate).

Second, we discuss the existence of forward rates in multi-state models. We consider the active-dead-model with and without cancellation, a simple disability model, and a model for a joint life insurance. In all models, we deduce sufficient and necessary conditions for the dependency of the underlying processes such that the invariance property holds. For the proof of necessary conditions we consider a broad class of processes, which includes most of the models used in literature.

We show that the independence assumption in the active-dead-model between mortality and interest rate, which is assumed in many papers, is always necessary. However, the assumption of independence between the transition intensities in a simple disability model causes at least one process to be deterministic. For the existence of forward rates the transition processes from active to dead and from disabled to dead need to have a specific common structure.

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# An Agent-Based Model of the Motor Insurance Market

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## Abstract

We construct an agent-based model of the motor (automotive) insurance market. This is a behavioral simulation model where insurers are heterogeneous agents with bounded rationality. The basic framework is that of an economic location model: insurers move over a two-dimensional business strategy space in terms of customers' age and price of their cars. Insurers calculate a premium based on exponential utility but in addition apply two behavioral rules: first, they compare themselves against local competitors in the business strategy space and move across this space to attract new customers; second, they learn about their customers' risks and apply a larger profit loading when faced with lower local competition. Agents therefore interact locally, learn from the competitive environment, and are adaptive.

Our research may be seen as an extension of [1] and [2]. In the former, insurer's pricing decisions are dependent on both the risk of its potential customers and the prices of their competitors with similar sizes. In the latter, a regime-switching Markov model is used to capture the asymmetrical features of the downward versus the upward parts of the underwriting cycle.

Our simulations show that the speed of movement across the business strategy space and the size of the local neighborhood against which insurers compare themselves are key parameters which affect the dynamics of the market. In particular, there are herding effects which, when combined with entry and exit from the market, can tip the market and generate cycles in the aggregate loss ratio.

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# Nash Equilibrium Pricing of General Insurance in a Competitive Non-Cooperative Market

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## Abstract

In the insurance industry, the number of products from different insurance companies has been significantly increased, and the strong market competition has also boosted the demand of a competitive premium. However, there is still little literature available in the actuarial science on modelling how the competition actually affects the determination of the company's premiums. Considering the exponential demand function, a model is constructed to investigate the optimal pricing strategy in a competitive non-cooperative market by calculating the Nash Equilibrium of an N-player game. The market average premium is specified as a geometric Brownian motion in this model and each insurer is assumed to maximize its utility of terminal wealth over the planning horizon. We illustrate this general framework by studying a two-player insurance game wherein a company's payoff depends upon the aggregate premium of other's strategies.

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# Asymptotic Consistency and Inconsistency of the Chain Ladder

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## Abstract

The distribution-free chain ladder reserving method belongs to the most frequently used approaches in the general insurance. It is well known, see [1], that the estimators  $\hat{f}_j$  of the development factors are unbiased and mutually uncorrelated under some mild conditions on the mean structure and under the assumption of independence of the claims in different accident years. In [2], we deal with some asymptotic properties of  $\hat{f}_j$ . Necessary and sufficient conditions for asymptotic consistency of the estimators of true development factors  $f_j$  are provided. A rate of convergence for the consistency is derived. Possible violation of these conditions and its consequences are discussed, and some practical recommendations are given. Numerical simulations and a real data example are provided as well.

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# Credible Risk Measures

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## Abstract

In the insurance and financial industry there are cases where different firms have similar but not identical risks. In this paper we introduce new risk measures called credible risk measures in order to recapture the risk of an individual insurer's portfolio (or financial sector) as well as the industry risk. Some of these measures are: the credible value at risk (CrVaR), the credible conditional tail expectation (CrCTE), the credible tail conditional median (CrTCM) and the credible quantile tail expectation (CrQTE). The new regulatory system, Solvency II requires the calculation of VaR and other risk measures to determine the insurance risk. Solvency II is focused on estimating the risk of individual insurance portfolio, separately. Credible risk measures provide more complete tools than the usual risk measure (eg. VaR, CTE) in capturing the individual insurer's risk and industry's risk. The advantages of these new credible measures in comparison with the existing risk measures are also presented.

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# Ruin problems for mixed Erlang claims in the presence of an upper barrier

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## Abstract

For the classical risk model, we present a new method to obtain closed-form expressions for various ruin-related quantities in the presence of an upper barrier, under the assumption that the claim-size distribution is a mixture of Erlangs. The method is based on Wald's fundamental identity and is easily carried out using computer algebra software.

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# Variance Swap with Mean Reversion, Multifactor Stochastic Volatility And Jumps

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## Abstract

Variance swap is a popular derivative security for managing volatility risk. This paper investigates variance swap pricing with a model which integrates three major features of financial assets, namely mean reversion in asset price, multi-factor stochastic volatility (SV) and simultaneous jumps in prices and volatility factors. Closed-form solutions are derived for vanilla variance swap and gamma swap while corridor variance swap and conditional variance swap are expressed in a one-dimensional Fourier integral. Our numerical test confirms that the derived solution is accurate and efficient compared to Monte Carlo simulation. Furthermore, as there exists empirical studies showing that multi-factor SV model better captures the implied volatility surface from option data, we also investigate the effect of multi-factor SV model on the pricing of variance swap. Our empirical result shows that the additional volatility factor contributes significantly to the price of variance swaps. Hence, it is in favor of multi-factor SV models for pricing variance swaps as it is more consistent with the implied volatility surface.

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# Heavy Metals: Pricing Health Insurance Under the Patient Protection Affordable Care Act (PPACA)

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## Abstract

The Patient Protection Affordable Care Act (PPACA), which is commonly called Obamacare, requires insurance companies to cover all applicants (regardless of age, sex, health status, the only exception being smoking status) and charge the same premium to applicants in the same age band. All qualified health insurance plans must offer, at a minimum, a set of health care benefits called the essential health benefits package. In addition, qualified plans must fall into four coverage levels or “metal tiers:” bronze, silver, gold, and platinum, with actuarial values of 60%, 70%, 80%, and 90%, respectively, with higher percentages denoting greater coverage. PPACA also requires that actuarial values be calculated using a continuance table based on a so-called “standard population” to mitigate the effects that a large group of very sick or very healthy insureds may have in distorting the plan’s calculations. An important consumer protection built into PPACA is the requirement that health insurers’ medical loss ratio be at least 80% for small groups and 85% for large groups.

Given a standard population, the factors affecting the amount of coverage provided by a plan are the plan’s annual deductible ( $d$ ), its coinsurance percentage (100%), and its maximum out-of-pocket expense per individual ( $M$ ). In this paper we define a qualified health plan,  $\mathcal{P}$ , as a vector  $\mathcal{P} = (G, d, c, M, \mathcal{B})$  where  $G$  is the plan’s monthly premium and  $\mathcal{B}$  is the plan’s health care benefits package offered to consumers. An important problem faced by health care actuaries is to design health care plans that simultaneously satisfy the PPACA requirements that qualified health plans must belong to one of the metal tiers and satisfy a specified medical loss ratio. Unfortunately, there are no published actuarial formulas that connect the elements of the plan vector  $(G, d, \alpha, M)$  to the plan’s desired actuarial value ( $\alpha$ ) and its minimum medical loss ratio ( $\rho$ ). The objective of this paper is to develop a model of a consumer’s sickness process based on a Poisson process, a consumer’s health care consumption. This model is then used to find the optimal qualified health plan,  $\mathcal{P}^*$ , that maximizes the insurer’s profits subject to the constraints that the plan’s actuarial value must belong to a specified metal tier, i.e.,  $(\alpha \in \{60\%, 70\%, 80\%, 90\%\})$ , and have a minimum medical loss ratio, i.e.,  $\rho \geq 0.8$ . The contributions of this paper are new to the economics and the actuarial literature.

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# Multivariate Risk Models and Queues, Part I: The two-dimensional model with simultaneous arrivals

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## Abstract

We focus on a particular connection between queueing and risk models in a two-dimensional setting. We first consider the joint workload process in a queueing model with two parallel queues and simultaneous arrivals at the queues. For the case that the service times are ordered (i.e., service times in the first queue are always bigger than service times in the second queue) we obtain the Laplace-Stieltjes transform of the joint stationary workload distribution. Using a multivariate duality argument between queueing and risk models, this also gives the Laplace transform of the survival probability of both books in a two-dimensional risk model with simultaneous claim arrivals and the same ordering between claim sizes. We also discuss a stochastic decomposition result for the joint workload vector, and give a brief outline how the two-dimensional risk model with a general two-dimensional claim size distribution (hence without ordering of claim sizes) is related to a known Riemann boundary value problem.

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## Distortion risk measures, ambiguity aversion and optimal effort

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### **Abstract**

We consider the class of concave distortion risk measures to study how choice is influenced by the decision-maker's attitude to risk and provide comparative static results. We also assume ambiguity about the probability distribution of the risk and consider a framework à la Klibanoff, Marinacci, and Mukerji (2005) to study the value of information that resolves ambiguity. We show that this value increases with greater ambiguity, with greater ambiguity aversion, and in some cases with greater risk aversion. Finally we examine whether a more risk-averse and a more ambiguity-averse individual will invest in more effort to shift his initial risk distribution to a better target distribution.

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# Generalized Panjer's Recursion For Dependent Claim Numbers

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## Abstract

We discuss a generalization of the collective risk model and Panjer's recursion. The model we consider contains a total claim number which is a deterministic sum of claim numbers of the individual lines of business. Thus the model is the sum of collective risk models. The distributions of the claim numbers are assumed to be Poisson mixture distributions. The mixture distribution is a gamma distribution and reflects a risk factor as in an extended CreditRisk<sup>+</sup> model which is mathematically also a sum of collective risk models. In our contribution we let the risk factors have certain dependence structures and prove that a modification Panjer's recursion is still applicable by finding an appropriate equivalent representation. Due to the dependence between the risk factors we obtain dependent claim numbers. To be more precise, these dependence structures between the risk factors may be of linear and stochastically linear nature. Such stochastically linear dependence scenarios are capable to produce also negative correlation between the risk factors. Using these results we compute risk contributions of the risks due to a risk factor with such dependence structures. This is joint work with Uwe Schmock.

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# Distortions of multivariate distribution functions and associated level curves: applications in multivariate risk theory

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## Abstract

We propose a parametric model for multivariate distributions. The model is based on distortion functions, i.e. some transformations of a multivariate distribution which permit to generate new families of multivariate distribution functions. We derive some properties of considered distortions. A suitable proximity indicator between level curves is introduced in order to evaluate the quality of candidate distortion parameters. Using this proximity indicator and properties of distorted level curves, we give a specific estimation procedure. The estimation algorithm is mainly relying on straightforward univariate optimizations, and we finally get parametric representations of both multivariate distribution functions and associated level curves. Our results are motivated by applications in multivariate risk theory. The methodology is illustrated on both real and simulated examples.

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## Determining the Optimal Level of Private Sector Investment in the Development of Agricultural Insurance

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### Abstract

Nowadays the important role of agricultural insurance in economical, social and political development has proven and counted as one of the human society developing indexes. Such an obstacle which is continuously introduced as a problem in developing the way in this economical part is a risk in the production, price (supply) and credit dimensions. This problem because of the occurrence of unpredictable event whether they are natural for obligatory, is organized in the investing way and establishing the agricultural part. Clearly, reaching to the developing goals in the agricultural sectors and as a result in the economical development of the country requires a change in this situation.

In order to develop this section, much attention is very important for developing the agricultural production insurance component especially providing the active participation background of private sector and removing the obstacle of the way which we shouldn't ignore of the cooperation of this sector with government in the agricultural insurance management and Lack of clarity over the respective roles of the public and private sector is the biggest problem that we have. It should be noted that agricultural insurance can't be addressed in the same way as other insurance lines purely private and purely state based approaches have failed.

Recent experience in developed countries has shown that Public Private Partnerships are required in order to increase penetration and be cost efficient.

The effective reduction and financing of catastrophic risks requires a combined response by both private and public sector players.

The development of agricultural insurance is complex and costly; thus access to technical and financial assistance for development is desirable. A critical minimum mass of potential insured and economies of scale are needed for the private sector to make the necessary investments. In addition, the adaptation of any agricultural insurance scheme is, in most cases, subject to costly financial losses that can jeopardize the continuity of such programs. The insurance sector alone does not have sufficient resources to make all the investments needed for a sustainable agricultural insurance scheme. The establishment of agricultural insurance pools is often justified in such circumstances. Agricultural insurance pools, jointly with government assistance, allow the industry to share the start-up and adaptation costs and to reach the economies of scale needed to implement sustainable agricultural insurance schemes.

An insurance scheme in the public sector has an advantage that it could have access to government budget and cooperation of other public institutions and banks.

The public and private sectors each bring complementary strengths and each has an important role to play for any program to be successful. The roles of each engaged organization need to be clear, complementary and supported by the appropriate authority to carry out these roles.

Finally, in this article the optimal level of investment for private sector will determine and with investment of this sector and sharing the financial risk of losses, the development of agricultural insurance will close.

Keywords: Agricultural Insurance, Development, Optimal Investment, Public Sector, Private Sector.

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# Optimal Multisource Sequential Change-Detection

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## Abstract

We consider the problem of detecting a simultaneous abrupt change at some unobservable time both on the rate of a homogeneous Poisson process and the drift of a Weiner process. This problem arises, for example, from the surveillance of the incidence of some insurance risks simultaneously with an exogenous relevant factor. In this paper, the problem is to detect the change-point as quickly as possible after it happens while controlling the frequency of false alarms.

The optimality of the cumulative sums (CUSUM) test with respect to Lorden [2] framework is developed for such a problem when the two processes are independent. Similarly to El Karoui *et al.* [1] in the case of exclusive information of the Poisson process, a nice connection with the ruin problem is explored and the detection time is showed to be the ruin time of a perturbed surplus process in presence of a dividend barrier. Additionally, using similar reasoning as in the risk theory, we derive closed formulas for the performance functions of the detection procedure. Finally, a numerical analysis is provided based real-world datasets.

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# Decomposition of life insurance liabilities into risk factors - theory and application to annuity conversion options

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## Abstract

Life insurance liabilities are in general a function of various risk sources such as equity, interest, and mortality. It is common to measure the total risk by advanced stochastic models. However, the question of how to allocate the randomness of life insurance liabilities to different risk sources, and how to quantify and compare the individual risk contributions, is rarely discussed. Nevertheless, in order to be able to take adequate risk management strategies including product modifications, insurance companies need to assess the relative importance of each risk source in detail.

Simple approaches such as sensitivity analyses with respect to model parameters usually lack any comparability between the different risk drivers. Moreover, such approaches only quantify the individual risk contributions, but do not provide the full probability distribution of each risk factor. In literature, the most common method to decompose life insurance liabilities is the variance decomposition approach which consecutively decomposes the risk by means of conditional expectations and then uses the variance as risk measure. Neglecting the variance, the decomposition by means of conditional expectations as for example used in [1] yields the distribution of each risk factor. However, the results of this decomposition method depend on the sequence of conditioning, or in other words, this method is not symmetric in its arguments. Another approach based on Taylor expansion as proposed by [2] neglects the natural requirement that the sum of the single risk components should be equal to the total insurance liabilities. Furthermore, most methods proposed in literature are only applicable at a certain point in time or state of the considered contract, although the risk decomposition clearly changes over time.

In this paper, we first propose a general decomposition method mainly based on the martingale representation theorem and show how this method provides a dynamic allocation of the total risk to the different risk sources over time. A comparison of the proposed method to those mentioned above demonstrates how this dynamic decomposition overcomes all discussed drawbacks. In particular, it provides the full distribution of each risk factor which, for example, allows the application of any risk measure and the analysis of the dependency structure. Second, we apply the proposed decomposition approach to several types of annuity conversion options. In the context of unit-linked deferred annuity contracts, annuity conversion options guarantee the policyholder certain minimum conditions for converting the accumulated money into an annuity. Based on the framework introduced in [3] which allows a joint analysis of financial and longevity guarantees, we first quantify the total risk implied by the considered annuity conversion options and then determine the respective equity, interest and mortality risks by means of the proposed decomposition method. We show that different product designs imply significantly different risk decompositions and derive valuable insights for risk managers.

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# Asymmetric Bivariate Copulas: Transformations, Order and Measures of Concordance

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## **Abstract**

We study a group of transformations on the collection of all real functions on the unit square. These transformations map the collection of all bivariate copulas into itself. For every copula, they generate a variety of new copulas; this is of particular interest with regard to asymmetric copulas and may also be useful for proving that certain real functions on the unit square are indeed copulas. Some of these transformations turn out to preserve symmetry and order as well as the value of any measure of concordance, while the others reverse order and change the sign of a measure of concordance.

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## Intra-group Risk Sharing under Financial Fairness

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### Abstract

In 1962, Karl Borch [2] presented a parametrization of all Pareto efficient solutions of the risk sharing problem for a pool of reinsurance companies. The notion of intra-group risk sharing can be used in many other situations in which agents, either by their own choice or for legal reasons, form economic collectives in which they trade at least part of their risks internally rather than with the market at large. One may think for instance of partners in a startup company, generations in a pension fund, or business units within a corporation.

In many of such situations, the group members will find it desirable to let the exchange of risks take place on the basis of a pricing rule that is conceived as fair. Such a pricing rule may be derived from the external market (i.e. internal trades take place at external market prices, so that there are no implied cross-subsidies), or it may just be the result of an agreement among the members of the group.

The question thus arises whether there exist Pareto efficient allocations that are “financially fair”, i.e. which are such that all trades take place at zero value according to a given pricing rule. This question was answered positively by Yves Balasko in 1979 [1]. Balasko showed uniqueness of Pareto efficient financially fair (PEFF) allocations only under some conditions on the problem data; for the general case he showed that the number of PEFF allocations is locally constant and generically odd.

In this paper we revisit the PEFF allocation problem, concentrating in particular on the case in which preferences are given by expected utility. We prove that in this case the PEFF allocation is unique. Our methods are based on nonlinear Perron-Frobenius theory, rather than on differential topology as in the work of Balasko.

We obtain the PEFF allocation as the solution of a fixed-point problem on the unit simplex. The solution can be found by an iteration method. We show the convergence of this method, which provides an effective way of finding the PEFF allocation. Our proof technique also provides a demonstration of the existence of a Pareto efficient and financially fair allocation, independent of Balasko’s result.

In a few cases (exponential utility; power utility with equally risk averse agents) explicit solutions for the unique PEFF allocation are available. We show a numerical example in which a lognormal risk is tranché between power utility agents who have unequal levels of risk aversion.

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# Comparison of Approximations for Compound Poisson Processes

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## Abstract

The aim of this paper is to provide a comparison of the error in several approximation methods for the cumulative aggregate claim distribution customarily used in the collective model of insurance theory. In this theory it is usually supposed that a portfolio is at risk for a time period of length  $t$ . The claims take place according to a Poisson process of intensity  $\mu$ , so that the number of claims in  $[0, t]$  is a Poisson random variable with parameter  $\lambda = \mu t$ . Each single claim is an independent replication of the random variable  $X$ , representing the claim severity. The object of study is the cumulative distribution function of the random sum of  $N$  independent replications of  $X$ , i.e. a compound Poisson process representing the aggregate claim or total claim amount process in  $[0, t]$ . Due to the complexity of its computation, several approximation methods for this cdf have been proposed in the literature. In this paper, we only consider approximations that use information on the lower order moments of the involved distributions. This requirement rules out Esscher series approximation, Panjer recursion and the FFT. Therefore, we consider the Normal, Edgeworth, NP2, NP2a, Adjusted NP2, NP3, Wilson-Hilferty, Haldane A and B, Lognormal, Gamma, Translated Gamma, Bowers Gamma, Inverse Gaussian and Gamma-IG approximations. For these fifteen approximations put forward in the literature, we consider the difference between the true distribution and the approximating one and we propose to use expansions of this difference related to Edgeworth series to measure their accuracy as the Poisson intensity diverges to infinity. Using these expansions, several statements concerning the quality of approximations for the distribution of the aggregate claim process can find theoretical support. Other statements can be disproved on the same grounds. At last we investigate numerically the accuracy of the proposed formulas.

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# The Error in Panjer-type Approximation of Compound Poisson Processes

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## Abstract

A widely used technique to compute the cumulative aggregate claim distribution of interest in the collective model of insurance theory is to use Panjer recursion. In order to set the notation, we suppose that claims take place according to a Poisson process of intensity  $\mu$ , so that the number of claims in  $[0, t]$  is a Poisson random variable  $N$  with parameter  $\lambda = \mu t$ . Each single claim is an independent replication of the random variable  $X$ , representing the claim severity. The object of study is the cumulative distribution function of  $S_N = \sum_{i=1}^N X_i$ , the random sum of  $N$  independent replications of  $X$ , i.e. a compound Poisson process representing the aggregate claim or total claim amount process in  $[0, t]$ . The original paper by Panjer spawned a large literature and spurred several extensions of the method. However, when the claim distribution is continuous, the Panjer recursion requires the replacement of the claim severity  $X$  with a lattice random variable  $X^{(\Delta)}$  with lattice width  $\Delta$ . The error implied by this preliminary discretization is not completely understood. In this paper we develop an Esséen series expansion for the cumulative distribution function of the compound Poisson process  $S_N^{(\Delta)} = \sum_{i=1}^N X_i^{(\Delta)}$  (we recall that the Esséen series is the equivalent, for lattice random variables, of the Edgeworth series for continuous ones). The convergence of the Esséen series approximation to the true cumulative distribution function of  $S_N^{(\Delta)}$  is uniform when  $\lambda \rightarrow \infty$ . Moreover, when  $\Delta \downarrow 0$ , using the uniform bounds on the Bernoulli polynomials, it is possible to show that convergence of the Esséen series to the classical Edgeworth series is uniform. We then consider several techniques for the discretization of  $X$  and compare them according to the error in the resulting cumulative distribution function.

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# Adjusting the Analytic Hierarchy Process for Fuzziness

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## Abstract

The Analytic Hierarchy Process (AHP) [Saaty (1980, 1999, 2008)] is a theory of measurement through pair-wise comparisons that relies on judgment to derive priority scales. It provides the objective mathematics to process the subjective and personal preferences in making a decision. With the AHP, one constructs hierarchies, then makes judgments or performs measurements on pairs of elements with respect to a criterion to derive preference scales that are then synthesized throughout the structure to select the best alternative.

The AHP finds application in many areas of decision making; risk assessment, for example, being one of the current hot topics. Given the decision-making context, however, problems can arise because the decision-making process often is hindered by data limitations and ambiguities, such as incomplete or unreliable data, and vague and subjective information owing to a reliance on human experts and their communication of linguistic variables. Since fuzzy logic (FL) has been shown to be an effective tool in such circumstances [Shapiro (2004)], it seems natural to inquire into the merging of the AHP and FL.

This presentation reports the result of such an inquiry. It begins with a brief overview of AHP and its limitations when confronted with a fuzzy environment. This is followed with a discussion of various ways that AHP models can be adjusted to accommodate fuzziness. The presentation ends with a commentary on the findings.

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# Optimal investment-consumption-insurance with random parameters

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## Abstract

This paper discusses an optimal investment, consumption and life insurance purchase problem for an economic agent in a complete market with Brownian information. Specifically, we assume that the parameters describing the market model and the agent, such as interest rate, appreciation rate, volatility, force of mortality, income and discount rate, are all random processes. Our modelling framework is very general, which allows these random parameters to be non-Markovian functionals of the underlying Brownian motion. The agent's problem is to choose an optimal investment-consumption-insurance strategy so as to maximize the expected, discounted utilities from intertemporal consumption, legacy and terminal wealth over a finite time horizon. We develop a novel approach, which combines the stochastic Hamilton-Jacobi-Bellman equation and backward stochastic differential equation to solve this problem. Using the Feynman-Kac formula and the Malliavin calculus, we derive explicit expressions for the optimal investment-consumption-insurance strategy and the value function. To illustrate our results, we provide closed-form solutions to the problem with stochastic appreciation rate, stochastic mortality and stochastic volatility, respectively.

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# Edgeworth type expansion for renewal-type equations and application to risk theory

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## **Abstract**

An asymptotic expansion of a solution to a defective renewal equation is presented. The expansion is derived from a compound geometric decomposition of the solution and the Edgeworth type expansion of the corresponding compound geometric distribution under the asymptotics that the equation tends to the proper renewal equation. In the context of risk theory, this asymptotics corresponds to the “small loading asymptotics”. As an example, we shall give a higher-order approximation to Gerber-Shiu’s discounted penalty functions with numerical illustrations.

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# Option Pricing Without Tears: Valuing Equity-Linked Death Benefits

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## **Abstract**

Nowadays, many products sold by insurance companies are investment funds wrapped around with (exotic) options and guarantees. These financial options and guarantees should be priced, hedged, and reserved using modern option-pricing theory, which involves sophisticated mathematical tools such as martingales, Brownian motion, stochastic differential equations, and so on. This talk will show that, if the options or guarantees are exercisable only at the moment of death of the policyholder, the mathematics simplifies to an elementary calculus exercise.

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**Valuing equity-linked death benefits:  
An investigation with a regime-switching jump-diffusion process**

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**Abstract**

Motivated by the work of Gerber et al. (2012)[1], we revisit the problem of calculating the expected discounted value of a payment at the time of death under the regime-switching economy. More specifically, we model the asset dynamics underlying each life-contingent contract as the regime-switching jump-diffusion process. Contrary to the discounted density approach in Gerber et al. (2012) [1], we shall price each life-contingent contract under the Laplace transform framework. The key feature of the model is that analytical solution to the first-passage-time distribution exists under the case when the time-until-death random variable is exponentially distributed, independent to the asset dynamics. The direct consequence is that series of life-contingent products, such as call/put options, lookback options, barrier options, dynamic fund protection, and the dynamic withdrawal benefits all admit closed forms under the Laplace transform. The price of each life-contingent claim can then be recovered efficiently by the numerical Laplace inversion algorithm. The strength of working under Laplace transform framework becomes particularly apparent when the methodology discussed here can be carried out effortlessly to the cases when the contracts having finite expiry and when the time-until-death variable is uniformly distributed, thanks to the De Moivre's law. In summary, pricing scheme discussed in this paper enables us to enrich the original model in Gerber et al. (2012)[1] significantly without losing computational tractability. Numerical examples are provided to illustrate the versatility of the technique.

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# Stochastic comparisons of conditional distributions under positive dependence

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## Abstract

Let  $\mathbf{X} = (X_1, \dots, X_n)$  and  $\mathbf{Y} = (Y_1, \dots, Y_n)$  be two portfolios with respective aggregate risks  $S = X_1 + X_2 + \dots + X_n$  and  $S' = Y_1 + \dots + Y_n$ . We study, in the case where  $\mathbf{X}$  and  $\mathbf{Y}$  share a particular dependence structure, stochastic comparisons of conditional distributions of the individual risks  $X_i$  and  $Y_i$ , for  $i = 1, \dots, n$ , given that (a) the respective aggregate risks  $S$  and  $S'$  exceed their VaR and (b) other individual risks exceed their VaR. Our results are given in terms of the usual stochastic order, the stop-loss order, the dispersive order and the excess wealth order.

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# A Markov chain model of mortality projection calibrated to data

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## Abstract

In this paper, we present a mortality projection model where future stochastic changes in mortality are driven by a finite state Markov chain, which incorporates two separate Markov processes. The first process reflects the structural changes in mortality over time. The second one allows for the shorter term deviations from the overall mortality trend, like catastrophes. A basic parametric model like Makeham is chosen for the initial mortality, although generalizations can be accommodated. This model is in some way similar to the one discussed in the worked example by [1]. However, rather than involving specific causes of death which may diminish over time, we will look at mortality in aggregate terms only.

The focus in this paper is on fitting the model - which in general contains a relatively small number of parameters - to real mortality data from several countries and estimating the transition intensities of the Markov chain. Distributions of key mortality indices like complete expectation of life will be shown as well.

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## On the (in-)dependence between financial and actuarial risks

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### Abstract

Probability statements about future evolutions of financial and actuarial risks are expressed in terms of the ‘real-world’ probability measure  $\mathbb{P}$ , whereas in an arbitrage-free environment, the prices of these traded risks can be expressed in terms of an equivalent martingale measure  $\mathbb{Q}$ . The assumption of independence between financial and actuarial risks in the real world may be quite reasonable in many situations. Making such an independence assumption in the pricing world however, may be convenient but hard to understand from an intuitive point of view. In this pedagogical paper, we investigate the conditions under which it is possible (or not) to transfer the independence assumption from  $\mathbb{P}$  to  $\mathbb{Q}$ . In particular, we show that an independence relation that is observed in the  $\mathbb{P}$ -world can often not be maintained in the  $\mathbb{Q}$ -world.

**Keywords:** Independence, real-world probability measure  $\mathbb{P}$ , risk-neutral probability measure  $\mathbb{Q}$ , financial risks, actuarial risks, insurance securitization.

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# Modelling of Diagnosis Inception Rates in Critical Illness Insurance

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## Abstract

We develop methodology for estimating diagnosis inception rates for critical illness insurance (CII), based on UK data supplied by the Continuous Mortality Investigation. Long delays between the diagnosis of a critical illness and the corresponding settlement are typical in CII and may lead to problems such as IBNR or IBNS. This, in turn, may affect the liabilities of an insurance company. Here we build on work in [1] and propose incorporating model uncertainty by exploring various delay distributions in a generalised linear-type model setting, including the log-normal, generalised gamma and generalised beta families of distributions. The models are fitted under a hierarchical Bayesian framework using Markov chain Monte Carlo methodology. Claim rates are then estimated by date of diagnosis using a parametric Poisson model which accounts for claims that have not been settled by the end of the observation period and thus are not present in the data [2, 3]. The sensitivity of the estimated rates and related future cash flow of policies to different delay distributions is also explored.

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## On a form of multivariate Pareto distribution: Properties and pricing

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### Abstract

The objectives of this talk are two-fold. First, I shall introduce a new class of multivariate dependent probability models with Pareto (univariate) margins, discuss its construction, properties of interest and connections to similar known models. Namely, with the help of copula functions, Laplace transforms and closure under minima, I shall elucidate various ways to formulate the new multivariate Pareto distribution. Then I shall demonstrate that the model proposed herein generalizes e.g., the classic Pareto considered in [1], as well as the ‘flexible’ Pareto developed in [2]. Last but not least, to explore the dependence structure of the new class of multivariate Pareto distributions, I shall derive the covariance, Kendal’s tau and Spearman’s rho in explicit form. In addition, some other notions of dependence, such as, tail monotonicity, association, orthant dependence and centered regression are to be verified in the framework of the model of interest.

Second, I shall consider the possibilities of (actuarial) pricing for risks following the new multivariate Pareto distribution by employing the ideas in [3], [4] and [5]. As the univariate Pareto is considered a classic example of a heavy-tailed probability distribution, I believe that the results in this part of the talk are of both theoretical and practical significance.

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# Georgian Social Insurance Fund Management Problems and Probabilistic and Statistical Analysis of Fund Capital Management

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## Abstract

Social protection and social insurance of the populations is the main activity of the government. Georgia as ex SSR country had social problems during market economy transition period. Absence of experience in management of social problems made Georgia experimental polygon. Until 1995 social protection system was implemented by identifies health priority programs. In 1996 was made the first step towards social insurance financing mechanisms setting into motion. However, this mechanism proved to be weak because of rapidly increasing unemployment, made powerless social insurance and financial base of the state budget. Since 2003, health insurance companies have stopped functioning and new structure was created - State United Social Insurance Fund, the source of the formation - social insurance taxes, was included in the treasury similar as net taxes of other subjects, i.e. This tax still was an intermediate category, despite its targeting approach. During 2003-2004 social insurance taxes couldn't be sat in it's the framework and failed to fulfill purposeful function, so in 2005 was abolished social insurance tax and established social tax, which was accumulated in the budget. In the existing mechanisms conditions to meet a minimum living standards was impossible during any fiscal year.

One of the reasons, along with various other reasons, was incorrect management of the Fund, in particular the formation of an incorrect assessment of the sources and the inappropriate use of resources. Existing social tax till 2008 by its purpose should have been spend on social sphere funding, but however, despite this principle high level protection it was obscure for employees what kind of social protection can they receive from state social fund, at the same time it helped to increase the salary fund shadow scale.

The only way for implementation social function by government is the use of insurance mechanism. On the one hand, the structure of Social Protection is like the insurance business, but has distinctive peculiarities, which is reflected in its non-commercial nature as well in obligation to finance the insurance social protection programs and pay insurance compensation for occurring insurance case, even though the capital fund is not enough. This task implementation requires a rational use of capital funds. Exactly for the solution mentioned problem in the thesis is presented and researched social insurance fund capital management mathematical models with exponential insurance reimburse and financing determined social programs; capital's probability density function, fund's functioning probabilistic characteristics are obtained, that makes it possible to determine the sufficiency of fund capital at all levels of its management. With the help of conclusion it is possible for particular period of time determine in insurance fund change of cash flow movement speed, on what basis in determined from state tax revenues assigns acceptance necessity and capacity.

# Delayed Heston Model: Improvement of Volatility Surface and Hedging of Volatility Swaps

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## Abstract

We present a new variance drift adjusted version [2, 4] of the Heston model [1] which leads to significant improvement of the market volatility surface fitting (compared to Heston). The numerical example we performed with recent market data shows a significant (44%) reduction of the average absolute calibration error (i.e., average of the absolute differences between market and model implied Black-Scholes volatilities) (calibration on Sep. 30th 2011 for underlying EURUSD). The main idea behind our model is to take into account some past-dependent history (a.k.a delay) of the variance process in its (risk-neutral) diffusion. We also focus on volatility swap pricing and hedging [3, 4].

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# Dividend Maximization and Ruin Probabilities under Incomplete Information

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## Abstract

In this presentation we discuss the valuation problem of an (insurance) company. Therefore we maximize its expected discounted future dividend payments.

We model the firm value process as a diffusion, but in contrast to classical contributions on this problem we assume the drift of the firm value to be unobservable under the considered observation filtration. We apply filtering theory to overcome uncertainty at the cost of an extra dimension of the underlying process. Nevertheless, we prove that the optimal value function is the unique viscosity solution of the associated Hamilton-Jacobi-Bellman equation.

This characterization allows a numerical treatment, which suggests that threshold strategies are at least  $\varepsilon$ -optimal.

Additionally, we characterize the finite time ruin probabilities in our model as the solution of a PDE.

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# Asymptotic Analysis for the Haezendonck–Goovaerts Risk Measure with a General Young Function

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## Abstract

Due to its definition, the Haezendonck–Goovaerts (HG) risk measure is intricate and fascinating. In this talk, for a risk variable following an extreme value distribution and for a general Young function, we derive a unified asymptotic formula for the HG risk measure for the confidence level close to 1.

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## Portfolio Optimization on Tails

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### Abstract

Given that financial crises do not arise due to frequent risks but rare events . The optimization of a portfolio of financial assets based on the entire distribution, usually assumed to be normal, ignores extreme risk. Several authors, such as [2],[1] have attempted to optimize portfolio extreme risk using the extreme value theory and copula function. In this paper we focus on the distribution tails that containing only losses and optimize the portfolio risk. To filter dily returns, we use DCC-MGARCH, Multivariate Gumbel copula, and Block Maxima approach. Our results show that this new approach of portfolio optimization based on tails gives a set of information useful for investor's decision making. These informations are: dynamic correlation and volatility analysis, extreme dependence structure and extreme risk measures. The minimum risk portfolio is more diversified than Markowitz's approach. The efficient frontier moves left providing more return and less risk. Our new approach can be considered as portfolio insurance in crisis periods.

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# Optimal Consumption Strategies under Deterministic Income

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## Abstract

We consider an economic individual endowed with an initial wealth, having an income and consuming goods and services. The wealth development rate is assumed to be a deterministic continuous function of time. The objective is to maximize the accumulated discounted consumption over a finite time horizon. In an insurance context this can be interpreted as a peculiar dividend maximization problem, since the underlying surplus process involves no stochastic ingredients. The development rate function can take positive as well as negative values and does not need to obey any monotonic behaviour. This feature is in contrast to existing results in the literature, which generally model the drift rate as a positive function of the state variable.

The optimization problem can be examined in a purely analytical way via the Hamilton–Jacobi–Bellman (HJB) approach. The characterization of the value function as the unique discontinuous viscosity solution to the associated HJB–equation then allows for a numerical treatment. But, one can do better, it is possible to derive an algorithm for explicit calculation of the value function and optimal strategy by balancing future prospects and immediate consumption. It turns out that the value function is in general not continuous. Finally the method is illustrated by two examples.

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# Strategies of mortality duration/convexity matching for hedging longevity/mortality risks

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## Abstract

In this paper, closed form formulas for the mortality durations and convexities with respect to an instantaneously proportional change and an instantaneously parallel shift in mortality rates are defined and derived. We propose duration/convexity matching strategies for determining the weights of two-product and three-product insurance portfolios. With the stochastic Lee-Carter model, we calculate and compare corresponding Values-at-Risks (VaRs) and (semi-)standard deviations of the surpluses for the underlying portfolio among these matching strategies. Illustrated numerical examples demonstrate that the duration/convexity matching strategies can hedge significantly the mortality/longevity risks.

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## Sensitivity analysis of internal risk models

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### **Abstract**

An internal risk model can be seen as consisting of (a) the *statistical model*, a random vector of risk factors; and (b) the *business model*, a real valued function defined on the risk factors, giving an output of interest (e.g. net asset value) for each realisation. Typically, the statistical model is high dimensional and the business model is non-linear. An empirical distribution for the output is obtained via Monte-Carlo simulation and a risk measure is calculated on that distribution, representing e.g. a capital requirement. While it is possible to evaluate the business model, the way that the risk factors interact and impact on the risk of the output may be opaque to the user, such that the model is a *black box*. In that context it is important to work out what the sensitivity of the output and its risk measure is to the risk factors. For business models that are linear in the risk factors, standard capital allocation methods deliver sensitivity analyses. We extend such approaches for distortion risk measures, by using smoothing methods to derive a reduced version of the business model from a Monte-Carlo sample of the risk factors and the quantity of interest. This allows decomposing a risk measure applied to the quantity of interest to risk contributions due to different risk factors. The impact of non-linear effects, such as inflation or parameter uncertainty can thus be quantified.

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# Portability vs Asset Price and Contribution Risk: A Continuous-Time Expected Utility Comparison of DB and DC Pension Plans

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## Abstract

This paper compares two different types of private retirement plans from the perspective of a representative beneficiary: Defined Benefit (DB) and Defined Contribution (DC) plan. One of the key differences is that DB plans carry portability risks, whereas DC plans bear asset and contribution risk. We model these differences explicitly in this paper and compare these two plans in a utility based framework. Our numerical analysis focuses on determining the critical job switching intensity which makes the beneficiary indifferent between the DB and DC plan.

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## Structural changes in mortality rates

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### Abstract

We consider the modelling and projection of mortality rates. Many mortality models exist, where death counts are generally assumed Poisson distributed, and for several countries these models have been estimated and back tested ([1, 2, 3, 4, 5]). Constructing mortality rate projections consists of two parts, namely (i) estimating a mortality model on historical data, and (ii) forecasting time-dependent parameters from the estimated model. The current mortality modelling literature has mainly focused on the first part by creating more extensive mortality models allowing for specific factors such as a cohort effect or mortality improvements of specific age groups.

However, recently focus has shifted to the projection of the time-dependent parameters. For example, [7] considers a regime switching model to project the period effect, while [8] considers a structural change model. We believe structural changes to be more appropriate when modelling period effects, because changes in life style or medical advances are likely to last for longer periods of time. However, [8] allows for only one structural change and considers specifically the Lee-Carter model. It is possible that multiple events have occurred in the past that effected mortality, which implies that multiple structural changes should be allowed for. Further, it remains unclear what the impact is when the method is applied to mortality models with multiple period effects.

We introduce a modelling strategy for the period effects that allows for objective, statistical detection of one or multiple structural changes, and this strategy can be easily applied to different mortality models. We show that for Dutch males it is better to allow for multiple structural changes. We further show that structural changes are less likely to be found for mortality models with multiple period effects. The period effects from those models are more volatile, which makes it difficult to find statistical evidence for the presence of structural changes.

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# The evaluation of the sum of dependent risks: a geometric-combinatorial approach

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## Abstract

The evaluation of the sum of dependent risks is a main issue for many applications in finance and insurance. One of the reason for such a growing interest is the approach of the regulatory standard on bank capital adequacy introduced by the so called Basel II [2], which is based on a quantile (Value-at-Risk) of the distribution function of the potential losses of a bank, collecting several risky positions. There would be no need to recall that in insurance applications the evaluation of aggregate losses of many risks is a very common task, as is the case, for example, with stop-loss reinsurance treaties. Such kind of subject is addressed by a wide range of literature contributions exploiting the concept of copula, which permits to separate the analysis of the dependence structure from that of the marginal distributions. Whereas the theory based on copula functions allows to measure the effects of dependence among different risks, especially when stress-testing is required, another relevant problem is to assess bounds for the probability of the sum of dependent risks exceeding some threshold. To this purpose [1] have introduced a new numerical procedure. In fact they proposed an algorithm whose implementation required a bounded density of the multivariate distribution and whose convergence was proven for dimension less or equal than 5. However, in a quite recent breakthrough, [3] has proven the convergence of the algorithm for any dimension. Building on that, we propose an improvement of the algorithm, which, exploiting a combinatorial approach, allows to consider a summation of a smaller number of terms. This way we aim at a considerably faster numerical procedure, which, avoiding some memory constraints, can make the algorithm manageable and competitive with Monte Carlo methods also for higher (i.e. larger than 5) dimensions.

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# Optimal investment and consumption when allowing for terminal debts

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## Abstract

The present paper analyzes an optimal consumption and investment problem for a representative agent who has different preferences for consumption and for terminal wealth. The utility for consumption is characterized by constant relative risk aversion but in contrast the risk aversion with regard to final wealth does not exclude the occurrence of negative wealth levels. We define the risk aversion for both positive and negative wealth levels and require them to be positive but not monotone. In fact, there is a point of maximal risk aversion at zero wealth and the agent may continue to consume when wealth becomes negative.

By the use of dual methods we can derive explicit solutions for this problem in a multi-asset economy which takes survival probabilities of the agent into account. This allows us to study the optimal patterns for consumption and investment and compare them to the case where terminal debt is not allowed.

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# Evolution of coupled lives' dependency across generations and pricing impact

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## Abstract

This paper studies the dependence between coupled lives - both within and across generations - and its effects on prices of reversionary annuities in the presence of longevity risk. Longevity risk is represented via a stochastic mortality intensity. Dependence is modelled through copula functions. We consider Archimedean single and multi-parameter copulas. We find that the dependence between man and woman decreases when passing from older generations to younger generations. Not only the level of dependence but also its features - as measured by the copula - change across generations: the best-fit Archimedean copula is not the same across generations. Moreover, for all the generations under exam the single-parameter copula is dominated by the two-parameter one. The independence assumption produces quantifiable mispricing of reversionary annuities. The misspecification of the copula produces different mispricing effects on different generations. The research is conducted using a well-known Canadian dataset of double life contracts.

Keywords: copula, goodness-of-fit, significance test, stochastic mortality, generation effect, reversionary annuity.

JEL classification: C12, C18, G22, J12.

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## Mean-variance optimization in DC plan with stochastic interest rate

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### Abstract

We solve a mean-variance optimization problem of a defined contribution pension scheme in the accumulation phase. The financial market consists of: (i) the risk-free asset, (ii) a risky asset following a GBM, and (iii) a bond driven by a stochastic interest rate whose dynamics follows a mean-reverting process. We solve the associated Hamilton Jacobi Bellman PDE and find a closed-form solution for the optimal investment strategy. The efficient frontier is provided in analytical form and its dependence on the model's parameters is analyzed. Numerical applications illustrate the behaviour of the optimal investment strategy when retirement approaches.

Keywords: mean-variance portfolio selection, efficient frontier, defined contribution pension scheme, Hamilton-Jacobi-Bellman equation, dynamic programming, stochastic interest rate.

JEL classification: C61, D81, G11, G23.

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# Modelling mortality by cause of death and socio-economic stratification: an analysis of mortality differentials in England

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## Abstract

It is well known that mortality rates and life expectancy vary across socio-economic sub-populations of a country. Higher socio-economic groups - whether defined by educational attainment, occupation, income or area deprivation - have lower mortality rates and longer lives than lower socio-economic groups. In many cases, high socio-economic subpopulations also experience faster rates of improvement in mortality. These socio-economic differences pose important challenges when designing public policies for tackling social inequalities, as well as when managing the longevity risk in pension funds and annuity portfolios. The successful addressing of these social and financial challenges requires the best possible understanding of what has happened historically and what is likely to occur in the future. A key step in this direction is to investigate how mortality trends from individual causes of death differ between socio-economic groups. In [1] it is demonstrated that multiple population extensions of the Lee-Carter model offer a compelling framework for the modelling of socio-economic differentials in all-cause mortality. Here, we explore the applicability of this approach in the modelling of mortality by cause of death and socio-economic stratification. Using England mortality data for socio-economic subpopulations defined using a deprivation index, we show that multiple population extensions of the Lee-Carter model can be satisfactorily employed both in the assessment of the magnitude of historical mortality differentials for the main causes of death and in the projection of the possible future evolution of these differentials.

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# Option Pricing, Esscher Transform and Risk Measures under Regime Switch Generalized Hyperbolic Model

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## Abstract

To simultaneously capture the stylized facts such as asset prices jump, volatility clustering and term structures of volatility, skewness and kurtosis, the main goal of this paper is to combine the generalized hyperbolic (GH) processes with the regime switching processes. The first contribution of this paper is to develop a discrete-time regime-switching GH model and to derive the characteristic function in closed form. To our best knowledge, there is no research to derive the conditional Esscher transform for option valuation in a discrete-time regime-switching GH model. Consequently, the second contribution of this paper is to derive the Esscher transform for fair valuation of long-term equity-linked guarantee products under the regime-switching GH model. For risk management, the third contribution of this paper is to derive the closed form solution of value at risk (VaR) and conditional tail expectation (CTE) for long-term equity-linked guarantee products. From the empirical results, we find that the regime-switching GH model successfully fits the model to the daily data from the Standard and Poors 500 (S&P 500) and the Toronto Stock Exchange 300 (TSE 300) indices. In addition, we demonstrate that the model risk have a slight effect on the fair values of European options and equity-linked guarantee products but significantly impact on the CTE.

*Keywords:* Conditional Esscher transform; Conditional Tail Expectation; Equity-Linked Guarantee Products; Generalized Hyperbolic; Regime Switching.

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## Convex Order on Risk Aggregation

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### **Abstract**

We introduce the admissible risk class as the set of possible aggregate risks when the marginal distributions of individual risks are given but the dependence structure among them is unspecified. The convex ordering upper bound on this class is known to be attained by the comonotonic scenario, but a sharp lower bound is a mystery for  $n \geq 3$ . In this talk we give a general convex ordering lower bound over this class. In the case of identical marginal distributions, we give a sufficient condition for this lower bound to be sharp. The results are used to identify extreme scenarios and calculate bounds on convex risk measures and other quantities of interest, such as expected utilities, stop-loss premiums, prices of European options and TVaR. Numerical illustrations are provided for different settings and commonly-used distributions of risks.

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# Forecasting multivariate realized stock market volatility: PCA or MFA?

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## Abstract

Forecasting multivariate realized volatility matrix typically involves the dimensionality reduction of the realized covariance matrices. Two methods are considered: the principal component analysis (PCA) and the matrix factor model (MFA) recently proposed by [1]. Unlike PCA, MFA is based on 2D data so that the volatility matrix does not need to be vectorized before dimension reduction. In this talk, we compare these two methods theoretically and empirically. We show that MFA can be viewed as PCA under certain condition on the covariance matrix. Our experiments on simulated and real-world stock market data sets reveal that MFA can only outperform PCA when the sample size is small, says less than 40.

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# Consumption-Investment Strategies with Non-Exponential Discounting: CRRA Utility

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## **Abstract**

This paper considers the consumption-investment problem with non-exponential discounting. It is well known that this problem is a time-inconsistent optimal control problem in the sense that it does not satisfy Bellmans optimality principle. We consider a non-Markovian model in which the parameters, i.e., the interest rate, mean rate of return and volatility of the stock, are stochastic. By studying a multi-person differential game, we characterise the equilibrium strategy by a BSDE.

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# Pricing and Hedging Variable Annuities under Stochastic Interest Rates and Stochastic Volatility

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## Abstract

While providing the possibility to share high investment return of the financial market, Variable Annuities (VAs) expose to high risk from the financial market, especially during the financial crisis, improper pricing and inefficient hedging led to the inadequate solvency for many insurers, as reported in [3], [4], [5]. Underestimated volatility risk and interest rate risk are the important reasons that lead to improper pricing and inefficient hedging. To reflect the volatility and interest rate risk, a hybrid model combining a CIR interest rate model with a Heston equity model is proposed. Under this model, a semi-closed formula for the price of VAs is obtained by using the Fourier-Cosine method proposed in [1] and [2], then the Greeks of VAs are derived as well. By the efficient semi-closed formula of the price and Greeks of VAs, we test the performance of the dynamic Greeks hedging and examine the impact of the stochastic volatility and stochastic interest rate to the pricing and hedging strategy of VAs.

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## Tail Asymptotic of Weibull-Type Risks

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### Abstract

With motivation from [1], in this paper we derive the tail asymptotics of the product of two dependent Weibull-type risks, which is of interest in various actuarial science and financial applications. Our results extend some recent findings of [2] and [3].

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# On optimal periodic dividend strategies in the dual model with diffusion

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## Abstract

The dual model with diffusion is appropriate for companies with continuous expenses that are offset by stochastic and irregular gains. Examples include research-based or commission-based companies. In this context, [1] show that a dividend barrier strategy is optimal when dividend decisions are made continuously. In practice however, companies that are capable of issuing dividends make dividend decisions on a periodic (rather than continuous) basis.

In this paper, we consider a periodic dividend strategy with Erlang( $n$ ) inter-dividend-decision times and continuous monitoring of solvency (as in [2], also in the dual model but without diffusion). A recursive, closed form representation of the expected present value of dividends until ruin is obtained when gains are hyperexponential.

Furthermore, we consider the global optimality of a barrier strategy when jumps are hyperexponential. Results are illustrated.

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# On the Time Value of Parisian Ruin Time in Sparre Andersen Model

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## Abstract

In this talk, we consider the time value of Parisian ruin [1] in the context of the Sparre Andersen (or renewal) risk process in which the inter-arrival times and the claim severities are mutually independent. It is demonstrated that the study of the Parisian ruin time is often linked to some related quantities in the corresponding dual risk model. Under the mild distributional assumption that either the inter-arrival time or the claim size is exponential (while keeping the other arbitrary), the introduction of appropriate excursions is crucial to our analysis. Expressions for the Laplace transform of the Parisian ruin time will be derived. If time permits, the time value of Parisian ruin in the dual risk model will also be discussed.

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# A time-consistent hedging strategy of mortality and longevity risk under mortality cointegration

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## Abstract

This paper investigate the time-consistent dynamic mean-variance hedging problem of an insurer using zero coupon bonds and longevity bonds. Insurance liabilities are modelled using a doubly stochastic Poisson process with an intensity rate being correlated and cointegrated to the underlying mortality rate of the longevity bond. The dynamic hedging problem is solved using the dynamic programming approach. Explicit solutions for the optimal hedging strategy are derived for cointegrated stochastic mortality models with both constant and state-dependent volatilities. Our solutions shows that the feature of cointegration is indispensable for mortality hedging.

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# Some distributional properties of a class of counting distributions with claims analysis applications

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## Abstract

We discuss a class of counting distributions motivated by a problem in discrete surplus analysis, and special cases of which have applications in stop-loss, discrete TVaR and claim count modelling. Explicit formulas are developed, and the mixed Poisson case is considered in some detail. Simplifications occur for some underlying negative binomial and related models, where in some cases compound geometric distributions arise naturally. Applications to claim count and aggregate claims models are then given.

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# The impact of a bank's asset allocation, early closure and capital forbearance on the premiums of deposit insurance

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## Abstract

This paper aims to value deposit insurance when the asset allocations of the bank's deposit invest the loans and the stocks under a stochastic interest rate setting. We derive a closed-form formula on the premium of the deposit insurance while considering early closure, capital forbearance, financial leverage and moral hazard. Moreover, we investigate the impacts of the bank's asset allocation, early closure and capital forbearance on premiums of deposit insurance. Finally, several numerical experiments are conducted to analyze the source of premiums, to compare to Merton's deposit insurance put (1997), and to explore the way that the bank's maintain ratio, debt-to-asset ratio, portfolio shares of risky assets and moral hazard from banks and depositors affect premiums.

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# Profit Maximizing-Risk Minimizing Reinsurance Policy Provision Via Adverse Selection

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## Abstract

In light of the richness of their structures in connection with practical implementation, in this talk, I shall base on the seminal works in economics (e.g. Landsberger and Meilijson (1999), Mirrlees (1971), Rochet and Chonee (1998), Rothschild and Stiglitz (1976) and Stiglitz (1977)) to introduce the Principal-Agent (multidimensional screening) models (also see Laffont and Martimort (2001)) to study a monopolistic reinsurance market with adverse selection; instead of adopting the classical expected utility paradigm, the novelty of our present work is to model the risk preference of each insurer (agent) by his Value-at-Risk at his own chosen risk tolerance level consistent with Solvency II. Under information asymmetry, the reinsurer (principal) aims to maximize his average profit by designing an “optimal” policy provision (menu) of “shirt-fit” reinsurance contracts for every insurer with hidden characteristics from a “huge” (continuum) family. Because of the non-convex nature of Value-at-Risk as a risk functional, a non-standard approach originated from Optimal Transport Theory has been developed. We then introduce the establishment of the optimality, over all feasible reinsurances, of the menu of all first-best strategies by utilizing a special feature of insurance layers. From a practical consideration, by using the indirect utility functions, the same Principal-Agent problem can be completely solved for the following commonly encountered subclasses of reinsurances: (i) stop-loss; (ii) quota-share; and (iii) reinsurance contracts with convex indemnity; in general, we conclude that the reinsurer will normally discriminate the insurers into two groups (of high or low profile) according to their risk tolerance levels, which agrees with the key conclusion found in the seminal works aforementioned in the classical expected utility setting.

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# The ruin time under the Sparre-Andersen dual model

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## Abstract

We study the Sparre-Andersen dual risk model in which the times between positive gains are independently and identically distributed and have a generalized Erlang- $n$  distribution. An important difference between this model and some other models such as the Erlang- $n$  dual risk model is that the roots to the generalized Lundberg's equation are not necessarily distinct. Hence, we derive an explicit expression for the Laplace transform of the ruin time, which involves multiple roots. Also, we apply our approach for obtaining the expected discounted dividends when the threshold-dividend strategy discussed by [1] is implemented under the Sparre-Andersen model with Erlang- $n$  distribution of the inter-event times. In particular, we derive an explicit form of the expected discounted dividends when jump sizes are exponential.

## References

- [1] NG, A.C.Y. (2009) On a dual model with a dividend threshold. *Insurance: Mathematics and Economics* 44, 315-324.
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# Optimal consumption and investment problem incorporate life insurance decision: The continuous time case

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## Abstract

This study considers the optimal consumption-investment-insurance problem for an individual when interest rates and labor income are stochastic. Due to the insurance industry has an important role in the world economy (the premium penetration is roughly 7%). This study investigates the individual who has life insurance demand for his/her posterity with future lifetime uncertainty. We extended [1] to allow the individual's insurance purchase, examining the impact of insurance demand to the individual's consumption and investment strategy. With constant relative risk aversion preferences, the household should own more life insurance with lower bonds position and the risk aversion parameter has little effect until the household nears retirement.

**Keyword:** Portfolio management, Life insurance, Labor income risk, Interest rate risk.

## References

- [1] MUNK, C., AND C. SORENSEN. (2010) Dynamic asset allocation with stochastic income and interest rates. *Journal of Financial Economics* 96, 433-462.
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# Quantifying the Impact of Partial Information on Sharpe Ratio Optimization

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## Abstract

Motivated by the fact that many investors have limited ability to update the expectation regarding future stock returns with the arrival of new information instantly, this paper provides a continuous-time model to study the performance of passive trading strategies. We derive the true Sharp ratio of the passive strategies in terms of the mean and variance of an explicit stochastic process. Based on this expression, we quantify the impact of partial information by performing a thorough comparative static analysis. Such an analysis provides a rationale for why investors with inaccurate information about stock return behave better in the mean-reverting environment than in the i.i.d. environment and why pessimistic investors can achieve better performance than optimistic ones. As a by-product, we propose an analytical approach to compute the “implied” parameters in stock return predictor for both i.i.d. and mean-reverting dynamics, which seems interesting for future research.

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# The Choice of Optimal Reinsurance of Flood Insurance

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## Abstract

The purpose of this paper is to do an empirical analysis about the optimal reinsurance portfolio of flood insurance in China, with the internal loss-sharing and the practical feasibility of the portfolio, in order to provide technical supports for the smooth development of China's flood insurance. The paper uses the Mean-variance principle in optimal reinsurance to research the mixed optimal reinsurance of proportional and non-proportional reinsurance, and then derives the expression of the mixed reinsurance portfolios optimal retention level. In the empirical study, the authors use stochastic simulation to get the loss distribution which accords with realistic disaster characteristics, rather than the Poisson distribution. The actual distribution is used to derive the optimal reinsurance portfolio, and the trend about the share part of the portfolio is analyzed.

**Key words:** Flood insurance, Reinsurance, Mean-variance principle, Mixed optimal reinsurance.

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## On the Dividend Strategies with Non-Exponential Discounting

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### Abstract

In this paper, we study the dividend strategies for a shareholder with non-constant discount rate in a diffusion risk model. We assume that the dividends can only be paid at a bounded rate and restrict ourselves to the Markov strategies. This is a time inconsistent control problem. The extended HJB equation is given and the verification theorem is proved for a general discount function. Considering the pseudo-exponential discount functions (Type I and Type II), we get the equilibrium dividend strategies and the equilibrium value functions by solving the extended HJB equations.

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# Management of Solvency Capital – A Research Based on 30 Life Insurance Companies in China

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## Abstract

Ever since April 2012, China Insurance Regulatory Commission sets about constructing China Risk Oriented Solvency System(C-ROSS). The two pillars of C-ROSS, namely Capital Adequacy Requirement and Risk Management Requirement, have proposed higher demands in respect of solvency capital management for insurance companies. This paper discusses whether or not the life insurance companies of China during the pre C-ROSS period are risk-orientated. To do this, we use the panel data of 30 life insurance companies of China from 2010 to 2011 to study the factors influencing their actual solvency margin. It comes to the conclusion that: (1) The actual solvency margin of life insurance companies significantly correlates with their solvency adequacy ratio, capital utilization ratio, gearing ratio and long-investment ratio; however, it insignificantly correlates with company size, profitability and underwriting risks. It reveals that the structural risks, such as asset allocation risk and financial risk, have bigger influence on life insurance companies than business risks do. Generally, the life insurance companies, with higher solvency adequacy ratio, lower capital utilization ratio, lower gearing ratio and heavier investment on term deposit and held-to-maturity investment, have higher actual solvency margin. (2) The actual solvency margin for life insurance companies insignificantly correlates with minimum solvency margin, which proves that these 30 life insurance companies, when managing their actual solvency margin, have not fully taken the minimum solvency margin into account. (3) At present, the life insurance companies of China, in control of their actual solvency margin, have considered their risk factors; in other words, the risk-orientated awareness has already implied in their behaviors of managing solvency capital and that is to say, C-ROSS has its realistic foundation in China.

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# A Cautious Note on Natural Hedging of Longevity Risk

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## Abstract

In this note, we examine the so-called natural hedging approach for life insurers to internally manage their longevity risk exposure by adjusting their insurance portfolio. Specifically, to analyze the impact of the forecasting model on the effectiveness of natural hedging, we compare results under several assumptions for the future evolution of mortality in the context of a stylized life insurer. In particular, aside from considering deterministic mortality rates and a simple factor model as in previous studies, we also use a non-parametric forecasting model that averts the assumption that all mortality rates are driven by the same factor(s).

Our primary finding is that higher order variations in mortality rates may considerably affect the performance of natural hedging. More precisely, while results based on a parametric single factor model—in line with the existing literature—imply that almost all longevity risk can be hedged, results are far less encouraging for the non-parametric mortality model. The key point is that “simple” mortality models—which may serve very well for certain purposes—strip away certain aspects in the data that are important for the analysis of natural hedging. At a broader level, we believe our results call for more caution toward model-based results in the actuarial literature in general.

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