Free Material Optimization of Composite Structures

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Wind Turbines section

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Structural optimization

- Find the lightest structure that is able to carry a given set of loads.

  minimize weight

  s.t. compliance ≤ given value

- Find the stiffest structure that is able to carry a given set of loads with limited amount of material.

  minimize compliance

  s.t. weight ≤ given value

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In Free Material Optimization (FMO)

The design variable is the full material tensor which can vary almost freely at each point of the design domain.

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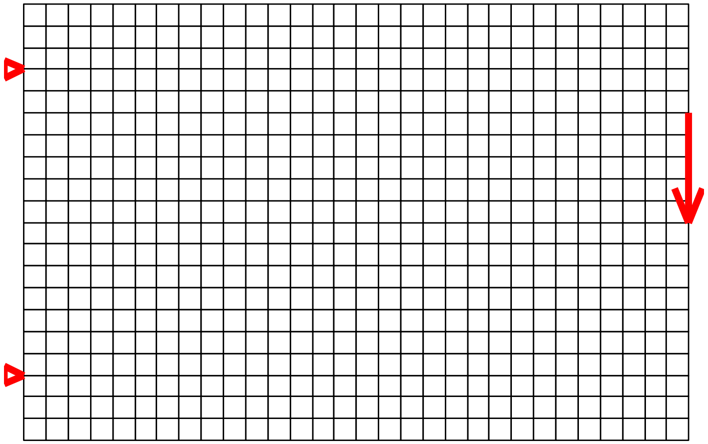
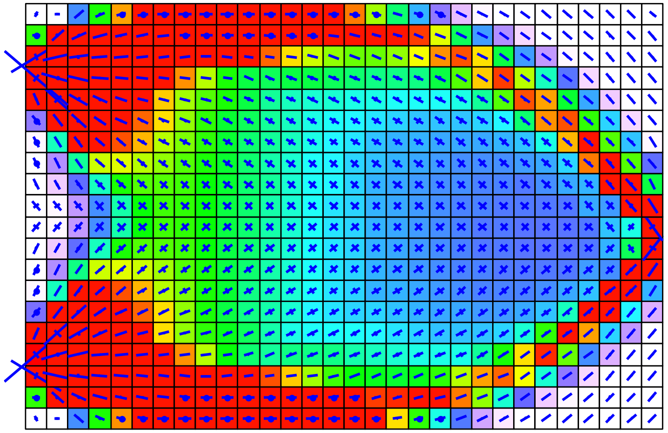
FMO yields optimal distribution of material as well as optimal local material properties.

A 2D example



Design Domain, bc and force



Optimal density distribution

The obtained design

- can be considered as an ultimately best structure,
- is difficult and expensive to manufacture,
- can be used to generate benchmarks and to propose novel ideas for new design situations.

FMO formulations

Mechanical assumptions

- static loads
- linear elasticity

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Basic FMO formulations for solid structures

Minimum compliance problem

$$\inf_{u \in V, E \in \tilde{\mathcal{E}}} \sum_{l \in L} w_l f_l^T u_l$$

$$\text{Subjec to} \quad A(E)u_l = f_l, \quad l \in L,$$

$$\sum_{i=1}^{m} Tr(E_i) \leq \overline{V}$$

Minimum weight problem

$$\inf_{u \in V, E \in \tilde{\mathcal{E}}} \sum_{i=1}^{m} Tr(E_i)$$

$$\text{Subjec to} \quad A(E)u_l = f_l, \quad l \in L,$$

$$\sum_{l \in L} w_l f_l^T u_l \leq \overline{\gamma}$$

$$\tilde{\mathcal{E}} = \{E \in (S) |, \underline{\rho \leq Tr(E_i) \leq \overline{\rho}, E \succeq 0}\}$$

Other different formulations can also be derived.

Additional constraints

Depending on the problem formulation additional constraints can be included such as constraints

- on local stresses

$$\sum_{k=1}^{G} \left\| E_i B_{ik} u \right\|^2 \leq s_\sigma$$

- on local strains

$$\sum_{k=1}^{G} \left\| B_{ik} u \right\|^2 \leq s_\sigma$$

- on displacement

$$Cu \leq d$$

- etc

Optimization method

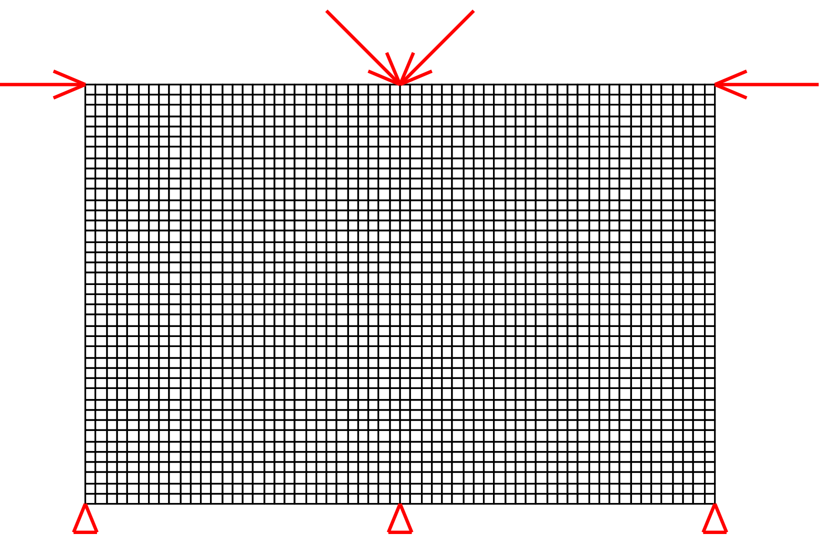
The resulting optimization problem is a nonlinear semidefinite program, a non-standard problem with many matrix inequalities.

Existing robust and efficient primal-dual interior point methods for nonlinear programming has in this project been extended to solve FMO problems.

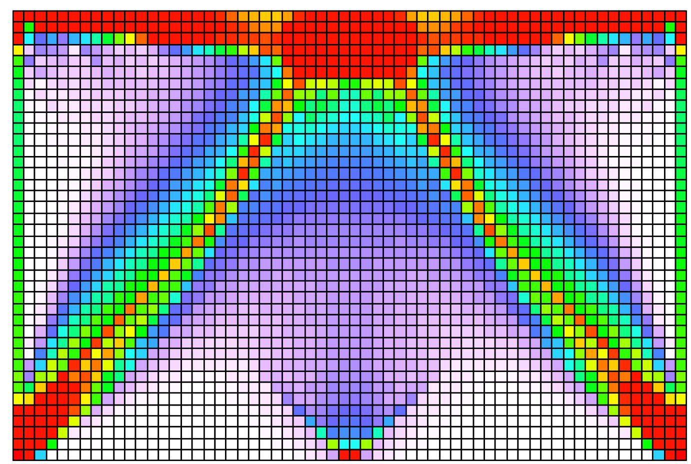
Numerical Results

Design domain, bc and loads

4-loads



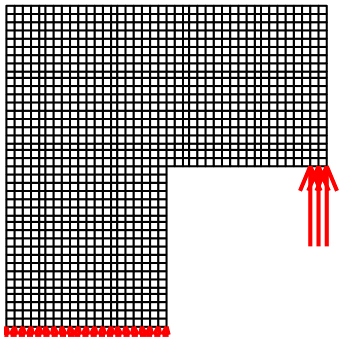
Optimal material density distribution



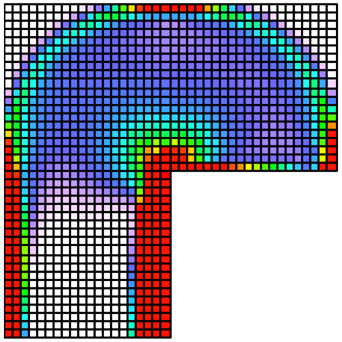
Numerical Results ….

With out stress constraints

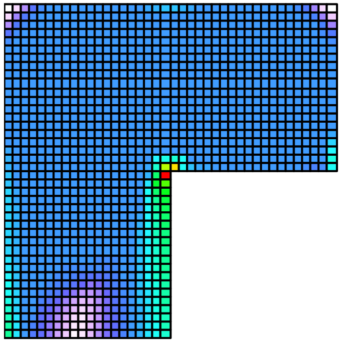
Design Damain, bc and forces



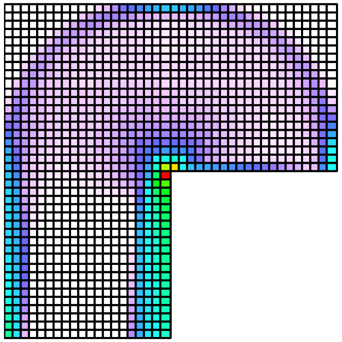
Density distribution of optimal design



Optimal strain norms

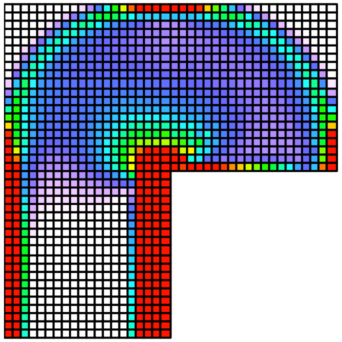


Optimal stress norms



Numerical Results ....

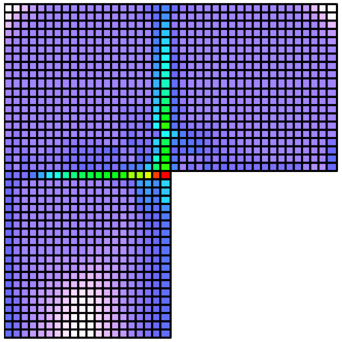
Density distribution of optimal design



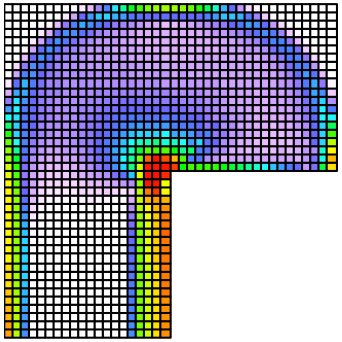
With stress constraints,

- max stress is decreased by 50%
- 11 more iterations

Optimal strain norms



Optimal stress norms



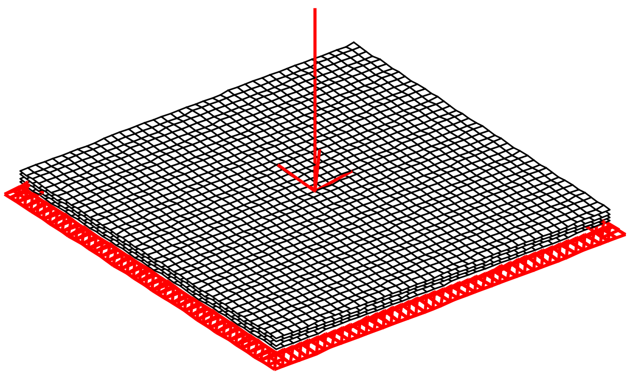
- Higher stresses are distributed to the neighbor regions,
- So is also material distribution

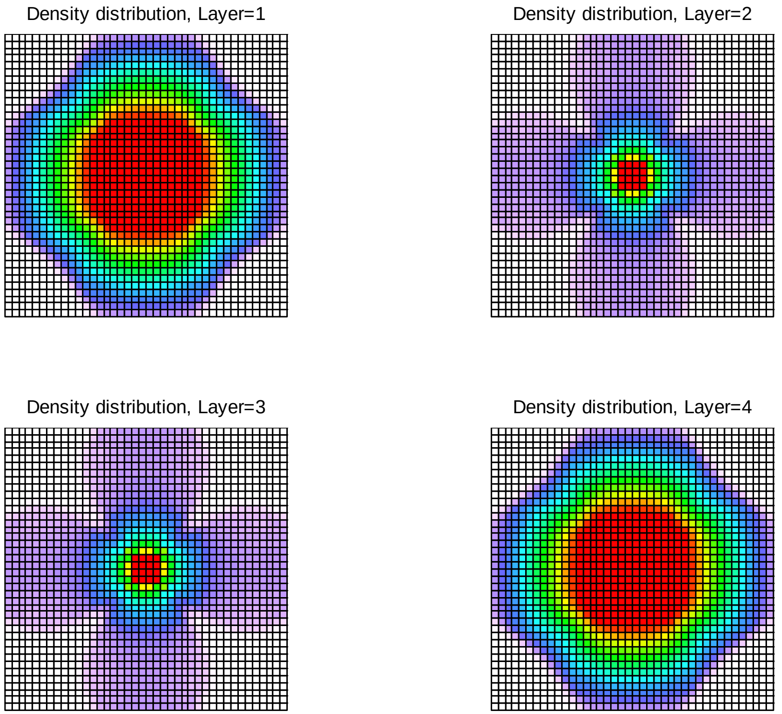
Numerical Results

Design domain, bc and loads

optimal material density distribution

4-layers





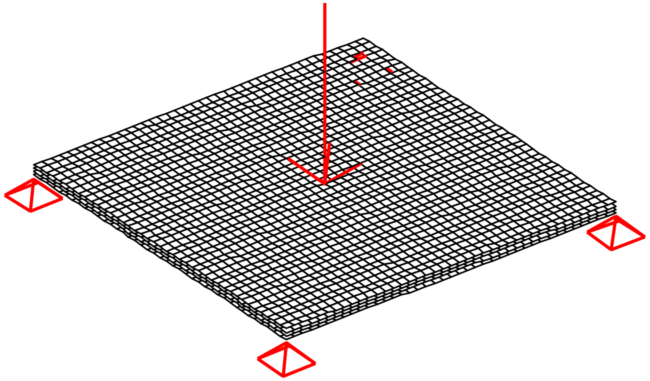
- Symmetric laminate
- High material distribution on top and bottom layers

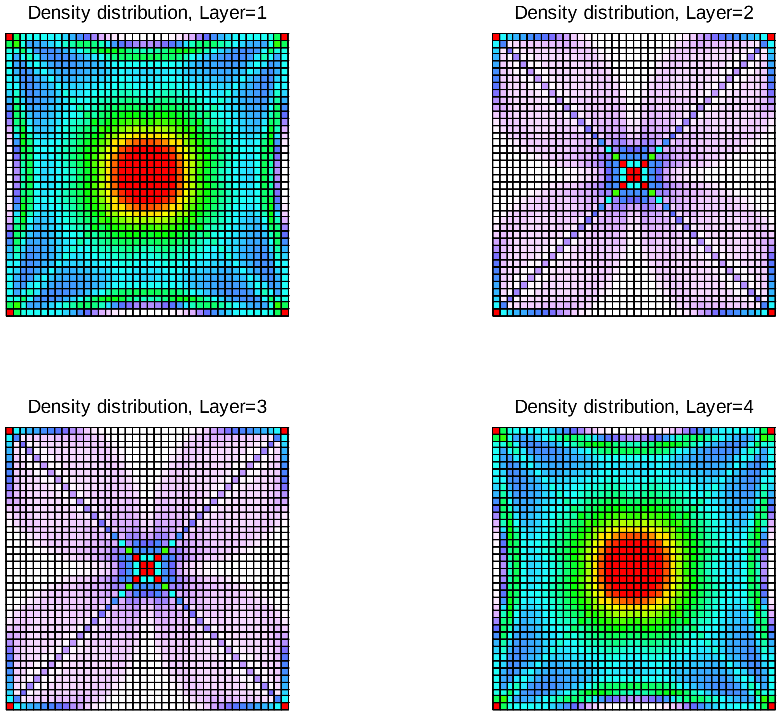
Numerical Results ....

Design domain, bc and loads

optimal material density distribution

4-layers





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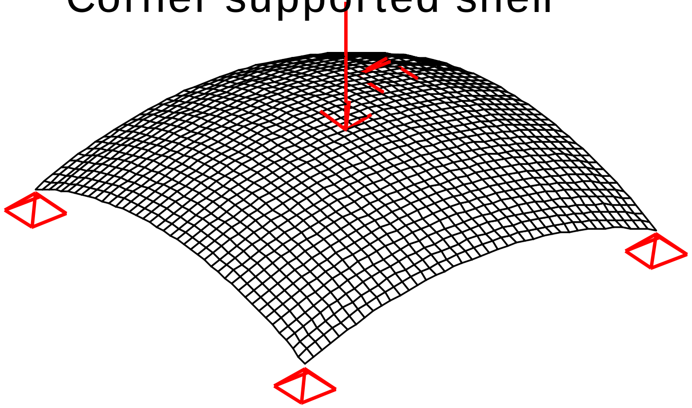
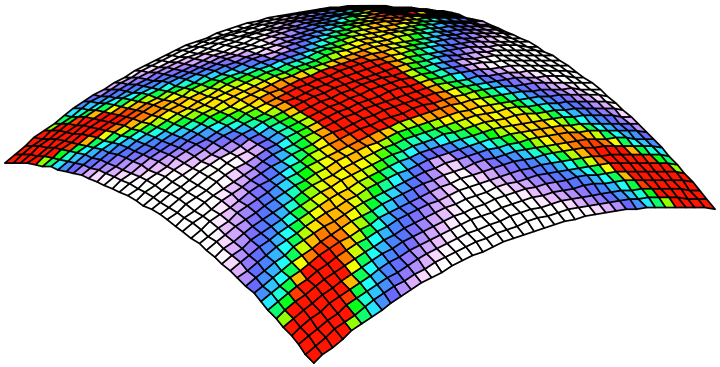
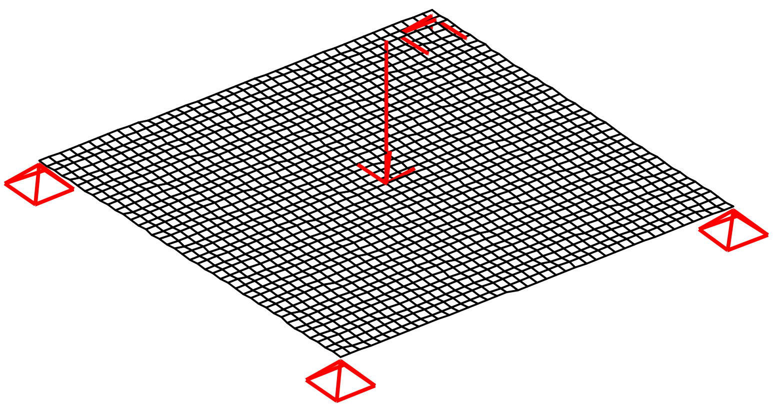
Numerical Results ....

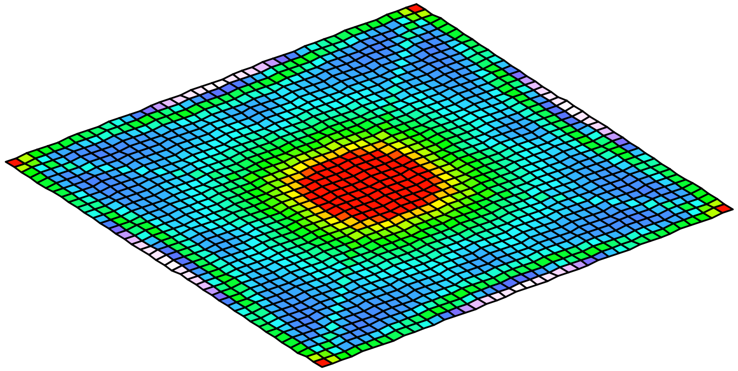
Corner supported shell





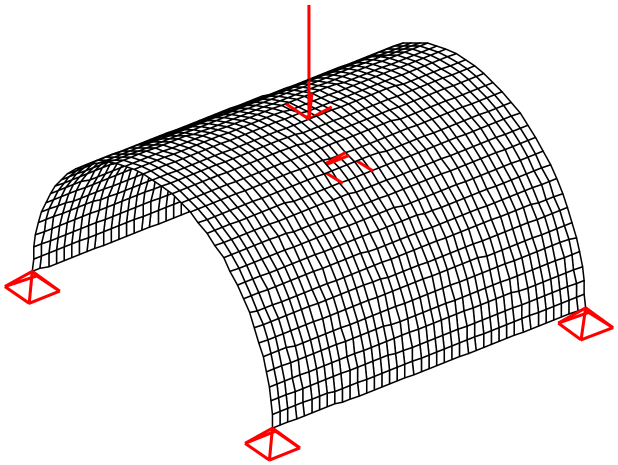
Corner supported plate

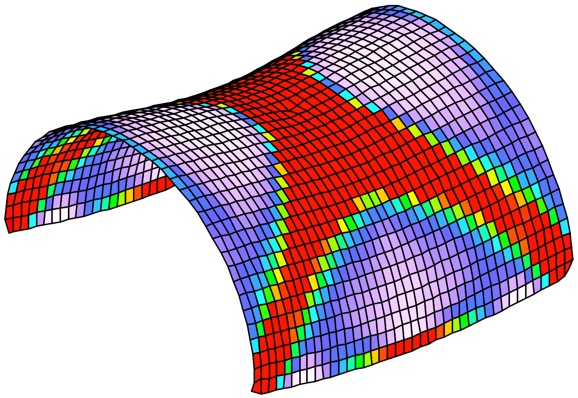




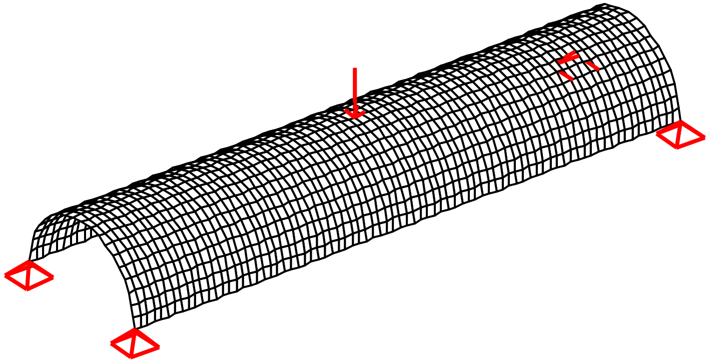
Numerical Results ....

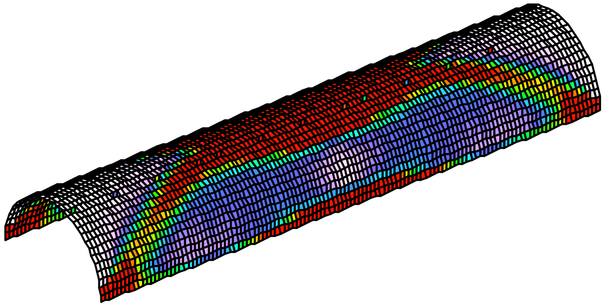
Corner supported half cylinder, length to width ratio 1:1





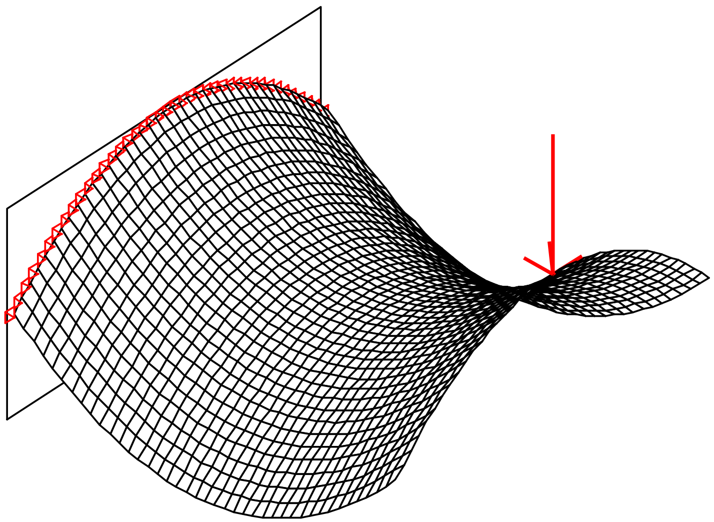
Corner supported half cylinder, length to width ratio 4:1

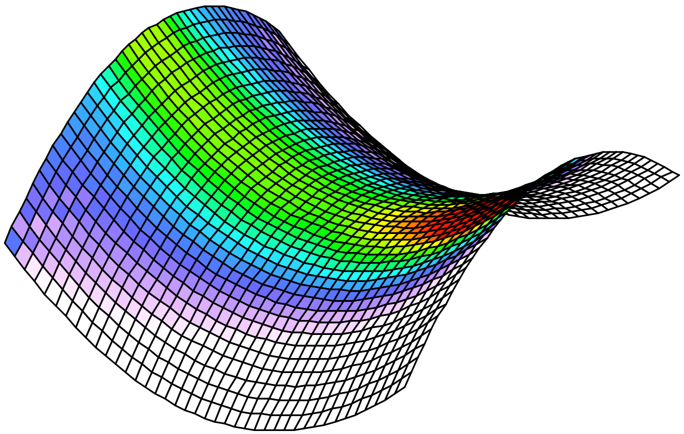




Numerical Results ….

Edge supported saddle surface shell





Future work

- Provide physical interpretation of the optimal solution
- Propose models and methods for FMO for beams with advanced cross

sectional analysis (with José Blasques and Mathias Stolpe at DTU Wind Energy)

- Include large deformations (geometric nonlinearity)
- Write articles, and PhD thesis.