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Logic without unique readability - a study of semantic and syntactic ambiguity

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▶ MARTIN MOSE BENTZEN, *Logic without unique readability - a study of semantic and syntactic ambiguity.*

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One of the main reasons for introducing a formal language is to remove ambiguity, the possibility of assigning several meanings to a linguistic expression. Typically, this is achieved through ensuring unique readability of formulas by using brackets (or another convention, such as Polish notation). Unique readability implies meaning uniqueness, exactly one valuation of a sentence given an interpretation of basic formulas and recursive truth conditions. Obviously, in natural language this one-to-one correspondence between syntax and semantics is absent, the unique readability assumption does not hold true universally. Whereas e.g. scope ambiguities in natural languages have been studied extensively, ambiguous formal languages have not been the focus of in depth research. Here, we lift the assumption of unique readability by omitting the brackets from propositional logic, making it possible to formally distinguish between syntactic and semantic ambiguity. A valuation then amounts to a semantic disambiguation, and rather than a unique valuation (truth value), there is a set of valuations corresponding to ways a formula could have been constructed. We show what happens to familiar concepts of logic such as definability, satisfiability and validity. Here follows two simple examples illustrating the relation between syntactic and semantic ambiguity. In some cases unique readability can be regained through careful construction of formulas. E.g., although an attempt to define $p \rightarrow q$ as $\neg p \vee q$ would be syntactically and semantically ambiguous, one may define it as $q \vee \neg p$, which can be read only one way (but obviously this construction is not stable under substitution). Syntactic ambiguity does not imply semantic ambiguity, although it is typically the case. For instance, although the formula $p \wedge \neg p \wedge p$ can be read in three ways, it has only one possible meaning (a contradiction).