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Abstract—This paper proposes a distributed optimization based dynamic tariff (DDT) method for congestion management in distribution networks with high penetration of electric vehicles (EVs) and heat pumps (HPs). The DDT method employs a decomposition based optimization method to have aggregators explicitly participate in congestion management, which gives more certainty and transparency compared to the normal DT method. With the DDT method, aggregators reveal their final aggregated plan and respect the plan during operation. By establishing an equivalent overall optimization, it is proven that the DDT method is able to minimize the overall energy consumption cost and line loss cost, which is different from previous decomposition-based methods such as multiagent system methods. In addition, a reconditioning method and an integral controller are introduced to improve convergence of the distributed optimization where challenges arise due to multiple congestion points, multiple types of flexible demands and network constraints. The case studies demonstrate the efficacy of the DDT method for congestion management in distribution networks.

Index Terms-- Congestion management, distributed optimization, distribution system operator (DSO), dynamic tariff, electric vehicle (EV), heat pump (HP).

I. NOMENCLATURE

Sets

N_B	set of aggregators
N_T	set of planning periods
N_L	set of lines
N_d	set of demand bus

Parameters

$A_{i,t,t\_}$	coefficient matrix, describe the relations between the power consumption and temperature change of the household ($°C$ /kW)
$B_{i,t}$	coefficient matrix of the price sensitivity (DKK/kWh/kWh)
D	power transfer distribution factor (PTDF)
$\tilde{E}_{i,t}, E_i$	customer to load bus mapping matrix

F_t	line loading limit of active power (kW)
$K_{i,t}^{a,\min}$	lower temperature limit ($°C$)
$K_{i,t}^{a,\max}$	upper temperature limit ($°C$)
R	resistance of each lines (ohm)
$\underline{V}$	voltage lower limit (p.u.)
V_0	voltage at node 0 (p.u.)
Y_{LL}	the matrix obtained by removing the first row and column of the nodal admittance matrix (S)
Z	the inverse matrix of Y_{LL} (ohm)
c_t	forecast baseline energy price (DKK/kWh)
$d_{i,t}$	discharging power of EVs due to driving (kW)
$e_i^{\min}$	lower limit of the state of charge (SOC) level (kWh)
$e_i^{\max}$	upper limit of the SOC level (kWh)
$e_{i,0}$	initial SOC level (kWh)
$p_{i,t}^{\min}, p_{i,t}^{\max}$	lower/upper charging power limit of EVs (kW)
$\hat{p}_i^{\min}, \hat{p}_i^{\max}$	lower/upper power limit of HPs (kW)
p_t^c	active conventional power at each load point (kW)
q_t^c	reactive conventional power at each load point (kVar)
$u_{i,t}$	initial temperature ($°C$)
α	coefficient, step size
$\beta_1, \beta_2, \beta_3$	coefficient for reconditioning and PI control

Vector Variables

$p_i, p_{i,t}$	charging power of EVs of one aggregator (kW)
$\hat{p}_i, \hat{p}_{i,t}$	power of HPs of one aggregator (kW)
r_t	regulation price, i.e., DDT rates (DKK/kWh)
s_t	total apparent power at each load point (kVA)
λ_t	Lagrange multiplier (LM) of line limit constraint
ρ_t	LM of load equation
ω_t	LM of voltage constraint

Other Symbols

n_*	cardinality of N_*, i.e. $n_* = \|N_*\|$
$\|*\|_1$	L-1 norm of vector *
$*^T$	transpose of matrix *
$\bar{*}$	element-wise conjugate of complex vector/matrix

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II. Introduction

ALONG with the rapid development of renewable energy resources (RESs), more and more distributed generators (DG) and flexible demands such as electric vehicles (EVs) and heat pumps (HPs) will be extensively deployed in future power systems for better balance of production and consumption. Potential congestion problems might occur in distribution networks due to simultaneous charging or discharging of flexible demands. In order to postpone or avoid the reinforcement of distribution networks, distribution system operators (DSOs) can use smart coordination methods to avoid or mitigate congestion. In recent years, a number of such coordination methods has been proposed such as direct control methods [1] and indirect control methods, i.e., market-based methods, including the dynamic tariff (DT) [2]–[5], distribution locational marginal price (DLMP) [6], [7], line shadow price (not nodal prices) method [8], subsidy-based methods [9]–[12], multiagent system methods [13], [14], and probabilistic congestion management methods [15]–[18].

Without considering probabilistic methods, which are able to enhance the underlying methods, the market based methods can be categorized into three fundamental types according to their business models: 1) DT methods [2]–[5], which are based on distribution network tariffs and function together with the existing energy market; 2) Distribution market methods, including DLMP [6], [7], which are new energy markets, and function parallel to or supplement the existing transmission-level energy market; 3) Subsidy-based methods [9]–[12], which give incentives to customers to change their planned/fixed schedules. Depending on the specific models, the multi-agent system methods may be one of the three types.

The motivation of proposing the distributed optimization based dynamic tariff (DDT) method is to provide more certainty and transparency than the normal DT method. In the DDT method, the aggregators are included in the optimization process, and they will reveal their final aggregated energy plans to the DSO and will keep the plans during the planning stage and operating stage. Thus, the DSO is more certain about the congestion management results, and the aggregators can manage the congestion cost more actively, implying more transparency of the DDT method. In the case study, a brief result of the uncertainty study of the normal DT method will be shown, while a more profound study can be seen in [18]. Another advantage of the DDT method is that the DSO does not need to know the cost functions of the aggregators. Therefore, it can protect the privacy of the aggregators, which is very important in a competitive market. As a contrast, the normal DT method needs to forecast the cost functions of the aggregators, as well as the constraints of the aggregators, which are sources of uncertainties as pointed out in [18].

The DDT method is implemented through the dual decomposition method in [19], [20]. The dual decomposition method has been successfully applied for solving many large scale constrained optimization problems, especially when there are only a few constraints involving all decision variables while other constraints and the objective function are separable with respect to decision variables. The convergence and small optimality tolerance (how close to the optimal solution) are guaranteed if the step size is sufficiently small [21]. In the DDT method, the decision variables related to each aggregator are separable, and only the network constraints binding all decision variables. Therefore, the dual decomposition method is well suitable for the DDT method.

The most relevant work in the literature is the multi-agent system method [13], [14]. It is also based on the dual decomposition method, but the business models (cost allocation) and objectives are quite different from the DDT method. In the multiagent methods, the DSO and aggregators are agents, who optimize the energy planning separately; however, the objective is to minimize the distance to an initial schedule, which is actually an infeasible schedule when considering the network constraints. Unlike the DDT method, the multi-agent system methods are not able to ensure that the total energy consumption cost is minimized, as it is not in its objectives. In the DDT method, although the individual aggregators minimize their own costs separately, they are in line with the overall energy cost minimization as they are equivalent to the overall optimization (proved in Section IV).

There are other distributed control methods for demand coordination and/or congestion management in the literature. References [22], [23] have introduced distributed methods for coordinated EV charging; however, they are not based on the dual decomposition. A modified Benders decomposition method is employed in [24] for congestion management on transmission networks. However, this method is not suitable for the DDT method, because it requires the overall objective function. The same reason explains why the nonlinear Danzig-Wolfe decomposition method for security constrained economic dispatch (SCED) [25] is not suitable for the DDT method either. In addition, the abovementioned methods are not able to provide a price signal that can be easily understood by the aggregators as in the dual decomposition method, where the dual variables are also known as shadow prices.

The main contributions of this paper are summarized as follows: 1) Propose the DDT concept for congestion management in distribution networks; 2) Propose algorithms to calculate DDT rates considering line loading limits, voltage limits and line loss reduction; 3) Prove that the DDT method is able to minimize the overall energy cost (and line loss cost) respecting the network constraints.

The rest of the paper is organized as follows. The DDT concept and mathematical formulation are presented in Section III. The equivalence to the centralized optimization and discussion are presented in Section IV. The method to improve the convergence is described in Section V. In Section VI, case studies are presented and discussed, followed by conclusions.

III. Distributed-optimization Based Dynamic Tariff

In this section, the market mechanism of the DDT method for congestion management of distribution networks is presented. Afterwards, the mathematical formulation of the DDT

concept is described.

A. Congestion Management through the DDT Concept

The essence of the DDT concept is still the same as the DT concept [4] from the economic point of view, i.e., it is a network tariff which is location and time varying, and can influence the behavior of flexible demands according to the network conditions. Same as DT, DDT does not contain energy price. Therefore, the aggregators who represent the owners of flexible demands need to buy electricity by participating in an electricity market, such as the day-ahead market in Nordic (Nord pool) or the day-ahead market of the California ISO in the USA. DDT is published before the closure of the day-ahead market. It means the aggregators are able to optimize the energy purchasing portfolio (bids submitted to the market) considering the cost due to DDT. This is how the DDT can influence the behavior of flexible demands. It is assumed in this paper that aggregators have contracts with the owners of flexible demands, and can directly or indirectly control flexible demands. It is also assumed that aggregators as business units are economically rational and pursue the maximum profits by minimizing the energy consumption cost and network utilization cost. It will be shown in the next two sections that this network utilization cost is incurred by DDT, and can include both congestion cost and network-loss cost. DDT is determined by a DSO, who is responsible for the secure operation and maintenance of distribution networks.

The difference between DDT and DT is significant, especially in the implementation part. In the DT method, the tariff is determined by the DSO through a centralized optimization, namely a DCOPF, where the DSO needs to forecast the availability as well as the energy requirements of flexible demands, and then to formulate a cost minimization objective function and relevant network constraints. If the forecast has very high accuracy, the equivalence between the DSO optimization and aggregator optimization without network constraints can be established [4]. In the aggregator optimization, the network constraint is reflected in the DT, which is why the DT method can be effective in congestion management. However, in reality, the forecast cannot be perfect. To solve this issue, [18] proposed an uncertainty management algorithm for the DT method when forecast errors exist. In the present paper, the proposed DDT method can solve this issue more reliably. In the DDT method, the tariff is not determined by the DSO solely through a centralized optimization. Instead, it is determined by iterative interactions between the DSO and aggregators. The detailed procedure of employing the DDT method for congestion management is illustrated in Fig. 1. In the beginning, the aggregators need to obtain the forecasted energy prices and the DSO needs to forecast the network status, including the conventional (inflexible) demands. The DSO initiates the iterative process by sending out tentative DDTs. Then, the aggregators will separately make their own optimal purchasing plans by minimizing the energy cost and network cost. Then, the aggregators will send back the tentative demand responses (DRs) to the DSO. With the new information, the

DSO will modify the DDTs and the iterative process will continue till the network constraints are satisfied and the final DDTs are determined.

Throughout the whole process shown in Fig. 1, the DSO doesn't know the energy cost function of the aggregators. This is a significant difference from the normal DT method, where centralized-optimization is employed and the cost function is known to the DSO. This feature also makes the DDT method slightly different from the normal optimization decomposition methods [20], [26], where the overall optimization problem is known. In the DDT method, the overall optimization problem is not known to the DSO, neither the aggregators. Each aggregator optimizes its own planning problem, which is smaller and easier to be solved because of no coupling network constraints. This is where the term 'distributed-optimization' in the DDT concept comes from.
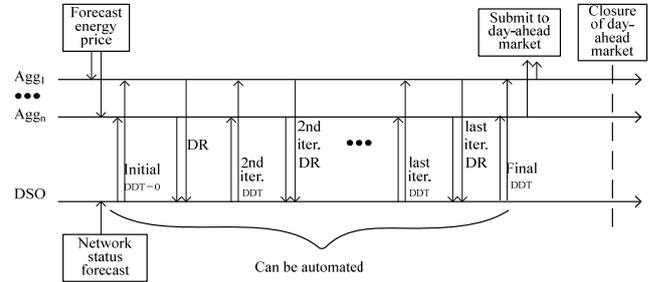

Fig. 1. Illustration of the DDT method for congestion management

Another difference between the DDT and DT methods is that the former requires aggregators keep the energy consumption level within the capacity that is revealed in the last iteration DR. This is why the DDT method can be more reliable than the DT method in congestion management. Further comparison of the two methods will be discussed in section IV.

B. Mathematical Formulation of DDT Method

1) Formulation at the aggregator side:

In the normal DT method [4], the optimizations at the aggregator side are not part of the calculation of DTs, but only for validation of the effect of the DT method for congestion management. However, in the DDT method, the optimizations at the aggregator side are part of the determination of the DTTs. The aggregators are purely economic units who do not consider any of the network constraints. They make energy schedules based on the requirements of flexible demands and prices, including the forecast energy prices, and fixed cost (such as grid tariffs and tax), and DDTs from the DSO. In order to facilitate the study, residential EVs and HPs are chosen to be the flexible demands. EVs are assumed to have very good battery storage systems; therefore the energy losses due to leaking and charging/discharging process can be neglected. HPs are assumed to have relatively poor thermal storage systems and the heat dissipates continuously due to the natural heat transfer processes (from high temperature objects to low temperature objects of the household structure and ambience). Therefore, the combination of EVs and HPs can represent many types of flexible demands in reality, e.g., refrigerators

and air conditioners (similar to HPs), and dish washer (shiftable loads, similar to EVs).

An aggregator can use a quadratic function to represent the energy consumption cost which is shown in (1). The quadratic term in (1) is due to the price sensitivity coefficient matrix $B_{i,t}$. The method to determine $B_{i,t}$ can be found in [4]. The reason of introducing the price sensitivity part in the cost function is also explained in [4]. Although one aggregator is small enough to be a price-taker in the electricity market, the incentives for an aggregator to make an optimal consumption pattern may be the same for many other aggregators. For instance, if the forecast energy price is particularly low for one hour, then every aggregator will tend to consume energy at that hour, which will make the aggregators as a group no longer the pure price-takers. Price sensitivity can capture this coaction effect of the energy planning. The network cost in (1) is represented by the term with the regulation price r_t, i.e., the DDTs. The optimal planning problem for aggregator i is formulated as (1)-(5).

$$\min_{p_{i,t}, \hat{p}_{i,t}} \sum_{t \in N_T} (\frac{1}{2} p_{i,t}^T B_{i,t} p_{i,t} + (c_t \mathbf{1} + E_i^T r_t)^T p_{i,t} + $$
$$\frac{1}{2} \hat{p}_{i,t}^T B_{i,t} \hat{p}_{i,t} + (c_t \mathbf{1} + E_i^T r_t)^T \hat{p}_{i,t}) \quad (1)$$

subject to,

$$e_i^{\min} \leq \sum_{t\_ \leq t} (p_{i,t\_} - d_{i,t\_}) + e_{i0} \leq e_i^{\max}, \forall t \in N_t, \quad (2)$$

$$p_{i,t}^{\min} \leq p_{i,t} \leq p_{i,t}^{\max} \quad \forall t \in N_T, \quad (3)$$

$$K_{i,t}^{a,\min} \leq \sum_{t\_ \leq t} A_{i,t,t\_} \hat{p}_{i,t\_} + u_{i,t} \leq K_{i,t}^{a,\max}, \forall t \in N_T, \quad (4)$$

$$\hat{p}_{i,t}^{\min} \leq \hat{p}_{i,t} \leq \hat{p}_{i,t}^{\max} \quad t \in N_T. \quad (5)$$

Constraints (2) - (3) are the limits of EVs. Constraint (4) represents the thermal limits of households and (5) gives the input power limits of HPs. Constraint (4) is derived from the thermal process analysis of the household and HP as shown in [4].

After solving the optimal problem, the aggregator will have a tentative optimal plan $(p_{i,t}^*, \hat{p}_{i,t}^*)$. Then (6) can be used to formulate the aggregated DR, which will be sent to the DSO.

$$p_{i,t}^a = E_i(p_{i,t} + \hat{p}_{i,t}), \forall t \in N_t \quad (6)$$

2) Formulation at the DSO side:

At the DSO side, there is no optimization model involved, which is quite different to the normal DT method. After receiving the DR results, $p_{i,t}^a$, from the aggregators, the DSO will firstly determine the total apparent power load, s_t, for each bus using (9) and (10). Then, it will use a dc load flow method to determine the power flow of each feeder and the voltage level of each bus by (7) and (8), respectively. The method to estimate an approximate voltage level for each bus (the left side of (8)) is proposed by [27]. The inequality of (7) and (8) is to compare the power flow and voltage level with the network limits, respectively.

$$D \operatorname{Re}(s_t) \leq F_t, \forall t \in N_T, \quad (\lambda_t) \quad (7)$$

$$1 - \frac{1}{V_0^2} \operatorname{Re}(\overline{Zs_t}) \geq \frac{V}{V_0}, \forall t \in N_T, \quad (\omega_t) \quad (8)$$

$$\operatorname{Re}(s_t) = p_t^c + \sum_{i \in N_B} p_{i,t}^a, \forall t \in N_T, \quad (\rho_t) \quad (9)$$

$$\operatorname{Im}(s_t) = q_t^c, \forall t \in N_T \quad (10)$$

The Matrix Z is the inverse matrix of the partial nodal admittance matrix Y_{LL}, which is a submatrix of the full admittance matrix (Y_{00} represents the slack bus),

$$Y = \begin{bmatrix} Y_{00} & Y_{0L} \\ Y_{L0} & Y_{LL} \end{bmatrix}.$$

Vectors λ_t and ω_t in (7) and (8) can be considered as marginal prices of the network cost with respect to power flow limits and voltage limits, respectively. The marginal prices are positive if the corresponding network constraint has an effect on the DR of the aggregators, i.e., when the constraint is binding; otherwise, it will be zero. Even though the DSO does not need to model an optimization problem, it is interested in checking the network limits by establishing and evaluating the network constraints (7) and (8).

Vectors λ_t and ω_t play an important role in determining proper DDT rates. They are determined iteratively. Initially, they are zero, i.e., $\lambda_t^{(1)} = 0$, $\omega_t^{(1)} = 0$ and $r_t^{(1)} = 0$. Then after receiving DR results $p_{i,t}^{a\,(k)}$ from the aggregators (k refers to the k-th iteration, and $p_{i,t}^{a\,(k)}$ is the optimal solution based on k-th DDT, i.e., $r_t^{(k)}$), $s_t^{(k)}$ can be determined. Then, λ_t, ω_t and r_t can be updated by,

$$\lambda_t^{(k+1)} = \lambda_t^{(k)} + \alpha(D \operatorname{Re}(s_t^{(k)}) - F_t), \forall t \in N_T, \quad (11)$$

$$\omega_t^{(k+1)} = \omega_t^{(k)} + \alpha (-1 + \frac{1}{V_0^2} \operatorname{Re}(Z\overline{s_t^{(k)}}) + \frac{V}{V_0}), \forall t \in N_T \quad (12)$$

$$r_t^{(k+1)} = D^T \lambda_t^{(k+1)} + \frac{\operatorname{Re}(Z^T)}{V_0^2} \omega_t^{(k+1)}. \quad (13)$$

In (11) and (12), α represents a proper step size, and the term after α is the residual of constraints (7) and (8), respectively. The justification of (11)-(13) will be discussed in Section IV. There is an implicit requirement for the marginal prices λ_t and ω_t, i.e., they must be nonnegative; therefore, they are modified by,

$$\lambda_t = (\lambda_t)^+, \quad (14)$$

$$\omega_t = (\omega_t)^+, \quad (15)$$

i.e., if they are negative, they will be replaced with zero.

When the iteration converges, the residuals in (11) and (12) will be nonpositive, which means constraints (7) and (8) are satisfied. There are $\left| \lambda_t^{(k+1)} - \lambda_t^{(k)} \right| \leq \tau$, $\left| \omega_t^{(k+1)} - \omega_t^{(k)} \right| \leq \tau$ and $\left| r_t^{(k+1)} - r_t^{(k)} \right| \leq \tau$, where λ_t and ω_t are modified values using (14) and (15), and τ is a small tolerance. The final DDT rates are the same as the last iteration DDT rates (Fig. 1).

IV. EQUIVALENCE TO CENTRALIZED-OPTIMIZATION BASED DYNAMIC TARIFF AND DISCUSSION

A. Equivalence to Centralized-optimization Based Dynamic Tariff

In order to study the nature of the distributed optimization employed in the DDT method, an overall optimization problem is established in this section, where the distributed optimizations at the aggregator side are put together. The details of the cost function and constraints are kept unknown and represented by general symbols, f_i and $(p_i, \hat{p}_i) \in \Omega_i$, respectively.

The optimization models at the aggregator side can be abstracted as: $\forall i \in N_B$,

$$\min_{(p_i, \hat{p}_i) \in \Omega_i} f_i(p_i, \hat{p}_i) + \sum_{t \in N_T} r_t^T \tilde{E}_{i,t}(p_i + \hat{p}_i). \quad (16)$$

For brevity, vectors $p_{i,t}$ ($t = t_1, t_2, ..., t_n$) are stacked one on top of another to form one vector, written as p_i; same for $\hat{p}_i$. The network cost $r_t^T \tilde{E}_{i,t}(p_i + \hat{p}_i)$ in (16) is listed separately, because this part is supposed to be known to the DSO, noticing that $p_{i,t}^a = \tilde{E}_{i,t}(p_i + \hat{p}_i)$. Then, an overall optimization can be formed as (excluding the network cost),

$$\min_{(p_i, \hat{p}_i) \in \Omega_i} \sum_{i \in N_B} f_i(p_i, \hat{p}_i), \quad (17)$$

s.t. (7)-(10) and,

$$p_{i,t}^a = \tilde{E}_{i,t}(p_i + \hat{p}_i), \forall i \in N_B, \forall t \in N_T, (\pi_{i,t}). \quad (18)$$

It can be seen that the overall optimization problem is to minimize the total energy consumption cost with respect to the network constraints. The dual function of the overall optimization can be written as,

$$g(\lambda, \omega, \rho, \pi) = \inf_{(p_i, \hat{p}_i) \in \Omega_i} \sum_{i \in N_B} f_i(p_i, \hat{p}_i) + $$
$$\sum_{t \in N_T} (D^T \lambda_t - \rho_t + \text{Re}(Z^T)\omega_t / V_0^2)^T \text{Re}(s_t) + \quad (19)$$
$$\sum_{i \in N_B, t \in N_T} (\rho_t - \pi_{i,t})^T p_{i,t}^a + \sum_{i,t} \pi_{i,t}^T \tilde{E}_{i,t}(p_i + \hat{p}_i)$$

In order to be meaningful, g should not be $-\infty$; therefore, there is $D^T \lambda_t - \rho_t + \text{Re}(Z^T)\omega_t / V_0^2 = 0$ and $\rho_t - \pi_{i,t} = 0$. This means,

$$\pi_{i,t} = D^T \lambda_t + \text{Re}(Z^T)\omega_t / V_0^2 \quad (20)$$

The dual function can be simplified as,

$$g(\pi) = \inf_{(p_i, \hat{p}_i) \in \Omega_i} \sum_{i \in N_B} f_i(p_i, \hat{p}_i) + $$
$$\sum_{i \in N_B, t \in N_T} \pi_{i,t}^T \tilde{E}_{i,t}(p_i + \hat{p}_i). \quad (21)$$

From (13) and (20), it can be seen that, at each iteration, $r_t = \pi_{i,t}$. This means (21) is equivalent to (16), noticing that (21) is separable with respect to i. From here, one can see that at each iteration, the distributed optimization represented by (16) is in fact an evaluation of the dual function for a given r_t. When the iteration process converges, the value of the dual problem (maximize the dual function g) can't be improved;

therefore, an optimal solution of the dual problem is found. Due to the introduction of the price sensitivity part in the cost function of the aggregators, f_i is strictly convex; therefore, the dual gap is zero, which means the primary problem, i.e., the overall optimization, is also solved. Hence, the DDT method is able to find an optimal energy plan which minimizes the overall energy consumption cost with network constraints satisfied.

From (19), it can be seen that $D \text{Re}(s_t^{(k)}) - F_t$ and $-1 + \text{Re}(Z \overline{s_t^{(k)}}) / V_0^2 + \underline{V} / V_0$ are subgradients of g with respect to λ_t and ω_t, respectively. They are used in (11) and (12) for updating λ_t and ω_t.

B. Comparison with the Normal DT Method

In section IV.A, it is proven that the distributed optimization is equivalent to the overall optimization (when the iteration converges), which is a centralized optimization. Therefore, if the cost functions f_i happen to be the same as those in the normal DT method, which also employs a centralized optimization, the congestion management results will be the same. But due to different business models for the DDT method and normal DT method, cost functions are not the same. In the DT method, cost functions are results of the DSO forecast, while in the DDT method, the cost functions remain unknown to the DSO.

In [5], line loss reduction is included in the DT method. Assume that the line loss cost function is $\sum_{t \in N_T} h_t(\text{Re}(s_t))$. The cost function can be estimated with a linear method [28], a piece-wise linear method [29] or second order polynomials, such as $h(\text{Re}(s_t)) \approx \frac{c_t}{V_0^2} \text{Re}(s_t^T) D^T R D \text{Re}(s_t)$, where R is a diagonal matrix whose main elements are the line resistance.

Rewriting the dual function (19) or using KKT conditions, there is (22), which means the DDT updating formula (13) can be modified to include the gradient of the line loss cost function and the line loss reduction is incorporated with the DDT method.

$$\rho_t = D^T \lambda_t + \text{Re}(Z^T)\omega_t / V_0^2 + \nabla h_t, \forall t \in N_T \quad (22)$$

In [5], feeder reconfiguration is included in the DT method. However, it can't be incorporated with the DDT method. The optimal feeder reconfiguration needs to be determined through an optimization problem; however, the primal problem (cost function) in the DDT method is not known and can't be solved.

C. Comparison with the dual decomposition Method

One can see that the DDT method has adopted the idea from the dual decomposition method [20], [26]. However, they are not exactly the same. In the dual decomposition problem, the primary problem (the overall optimization) is known, but in the DDT method, it is not. For this reason, many algorithms of the dual decomposition method involving the manipulation of the cost function can't be employed here.

V. RECONDITIONING AND ADDING INTEGRAL CONTROLLER FOR CONVERGENCE IMPROVEMENT

Although the convergence of the dual decomposition method is assured provided that the step size is sufficiently small [21], the speed of the convergence is quite slow in some cases. From (11)-(13), it can be seen that both λ_t and ω_t have contributions to the DDT rates. However, they have different weights: D^T and $\mathrm{Re}(Z^T)/V_0^2$ respectively. Because $D^T \gg \mathrm{Re}(Z^T)/V_0^2$, ω_t has much less importance in the DDT rates calculation. This is not true. In reality, the voltage constraints should have same importance as the line flow constraints in congestion management. According to [30], a scaling matrix K can be used to adjust the weights of λ_t and ω_t, and the negative gradient flow can be abstracted as (the step size is included) in Fig. 2, where $x = \begin{bmatrix} \lambda \\ \omega \end{bmatrix}$.
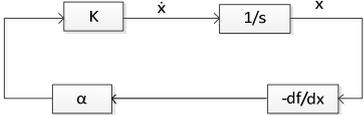

Fig. 2. Negative gradient flow with step size α (P controller)

In this paper, a scaling factor is used instead of the scaling matrix to have simplicity. Accordingly, (13) can be modified as (23) with an additional scaling factor β_1. Without β_1, when λ_t converges, ω_t may be far away from convergence, which slows down the overall convergence speed.

$$r_t^{(k+1)} = D^T \lambda_t^{(k+1)} + \beta_1 \frac{\mathrm{Re}(Z^T)}{V_0^2} \omega_t^{(k+1)} . \quad (23)$$

In addition, a diminishing integral controller (I controller) is added in parallel to the step size factor (P controller) for updating λ_t and ω_t in order to speed up the convergence. Accordingly, (11) and (12) can be modified as,

$$\lambda_t^{(k+1)} = \lambda_t^{(k)} + \alpha(D\,\mathrm{Re}(s_t^{(k)}) - F_t) +$$
$$\frac{\beta_2}{k}\sum_{j=1}^{k}(D\,\mathrm{Re}(s_t^{(j)}) - F_t), \forall t \in N_T \quad , \quad (24)$$

$$\omega_t^{(k+1)} = \omega_t^{(k)} + \alpha\,(-1 + \mathrm{Re}(Z\overline{s_t^{(k)}})/V_0^2 + \underline{V}/V_0) +$$
$$\frac{\beta_3}{k}\sum_{j=1}^{k}(-1 + \mathrm{Re}(Z\overline{s_t^{(j)}})/V_0^2 + \underline{V}/V_0), \forall t \in N_T \quad , \quad (25)$$

where α is the step size (P controller), and β_2/k and β_3/k are the gains of the I controller, which are diminishing as k grows. The negative gradient flow can be updated as in Fig. 3.
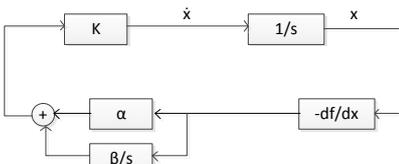

Fig. 3. Negative gradient flow with PI controller

VI. CASE STUDIES

A. Case study parameters

The single line diagram of the four-feeder distribution network of the Roy Billinton Test System (RBTS) [31] is shown in Fig. 4. Line segments of the feeder one are labeled in Fig. 4, among which L2, L4, L6, L8, L9, L11, and L12 refer to the transformers connecting the corresponding load points (LP1 to LP7). The study is focused on this feeder because it has the most diversity among all the feeders: 5 residential load points with different peak conventional demands and two commercial load points. The detailed data of these load points are listed in Table I. The peak conventional demands of residential customers are assumed to occur at 18:00 when people arrive home and start cooking. Assume that the EVs and HPs have unit power factor. The DSO has improved the power factor of the conventional consumption by reactive power compensations, and the remaining reactive power consumption is 10% of the conventional active power consumption. The line parameters are shown in Table II.

The key parameters of the simulation are listed in Table III. The lower voltage limit is set to be 0.948 p.u., in order to have a small margin (0.006~0.008 p.u.) compared to the assumed physical limit 0.94 p.u. The EV availability shown in Fig. 5 is from the driving pattern study in [32]. The household area is a random number between 100 and 200 (m^2).
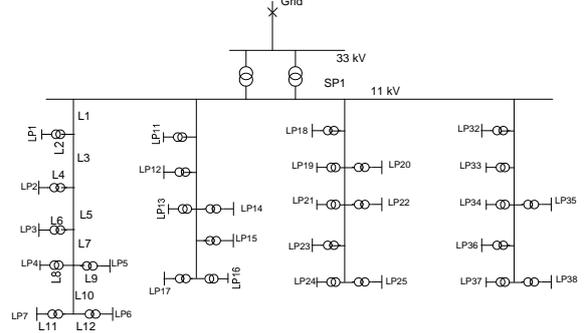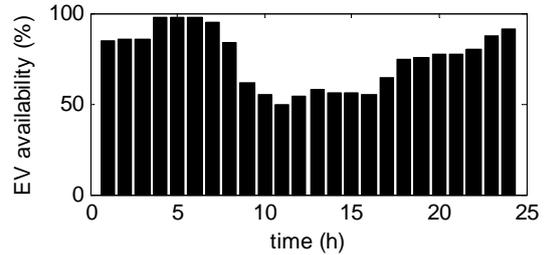

Fig. 4. Single line diagram of the distribution network

Fig. 5. EV availability

TABLE I
LOAD POINT DATA

load points	customer type	peak conv. act. power per point (kW)	peak conv. react. power per point (kVar)	number of customers per point
LP1-LP4	residential	886.9	88.69	200
LP5	residential	813.7	81.37	200
LP6,LP7	commercial	671.4	67.14	10

Line	r (ohm)	x (ohm)	x/r ratio
1	0.1210	0.0370	0.3058
2	0.3000	3.0000	10.0000
3	0.4233	0.0943	0.2228
4	0.3000	3.0000	10.0000
5	0.3722	0.0829	0.2227
6	0.3000	3.0000	10.0000
7	0.4403	0.0981	0.2228
8	0.3000	3.0000	10.0000
9	0.3000	3.0000	10.0000
10	0.3091	0.0689	0.2229
11	0.3000	3.0000	10.0000
12	0.3000	3.0000	10.0000

TABLE III
KEY PARAMETERS OF THE SIMULATION MODEL ([32], [33])

parameter	value
EV battery size	25 kWh
Peak charging power	11 kW (3 phase)
Energy consumption per km	150 Wh/km
Minimum SOC	20%
Maximum SOC	85%
Average driving distance	40 km
Coefficient of performance (COP) of HP	2.3
Min Temp. of the House	20 ℃
Max Temp. of the House	24 ℃
Voltage rating = V_0	11 kV
Lower voltage limit	0.948 p.u.
Transformer rating	1~3 MVA
L2 limit (kW)	1100
L3 limit (kW)	7000
L4 limit (kW)	2700
Price sensitivity (DKK/KWH/ KWH)	0.01
$\beta_1, \beta_2, \beta_3$	10, 10, 12

B. Case study results

The simulation was carried out using the GAMS optimization software [34] for the distributed optimization part at the aggregator side, and a Matlab script for the iteration control, i.e., the DDT calculation and convergence check at the DSO side.

1) Congestion Management Results:

Firstly, the distributed optimization was performed with initial zero DDT. The line loading results are shown in Fig. 6, where there is congestion at hour 17, 18, 19 and 24. Then the iteration starts. When the iteration converges, the final DDT can be determined. The line loadings of L2, L3 and L4 after using the DDT method are shown in Fig. 7, and they are all lower than the limits. As the voltage constraints are also included in the DDT calculation, the voltage profile of the critical bus, LP4, is above the lower voltage limit, which is shown in Fig. 8. It can be seen that the error between the approximated voltage and accurate one (by the load flow method) is less than 0.5%.

The base energy prices and final energy prices with the DDT for the customers at LP1-5 are shown in Fig. 9. Because there is congestion at hour 17-19 due to conventional peaks and the HP consumption, the DDT rates have to be very high at hour 19, 18 and 17 (rates: 19>18>17) such that the energy

price at hour 16 will become attractive enough to the HP consumption. Also, because the voltage at LP4 (critical bus) is more sensitive to loads at the bottom of the feeder than the top, DDT rates have relations: LP4>LP5>LP3>LP2. DDT rates at LP1 are results from both line loading limit of L2 and voltage limits of LP4; therefore, they are relatively high.
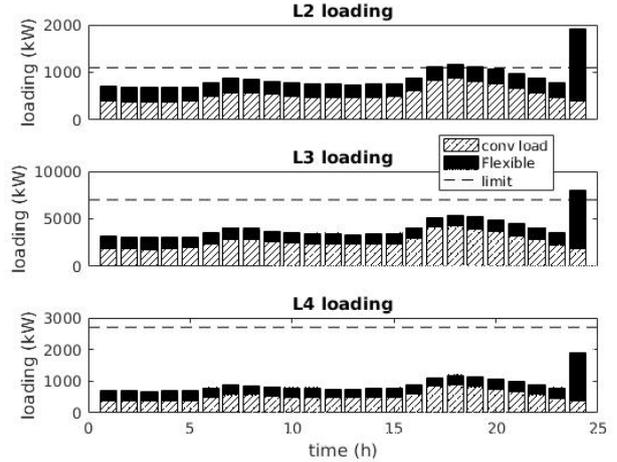

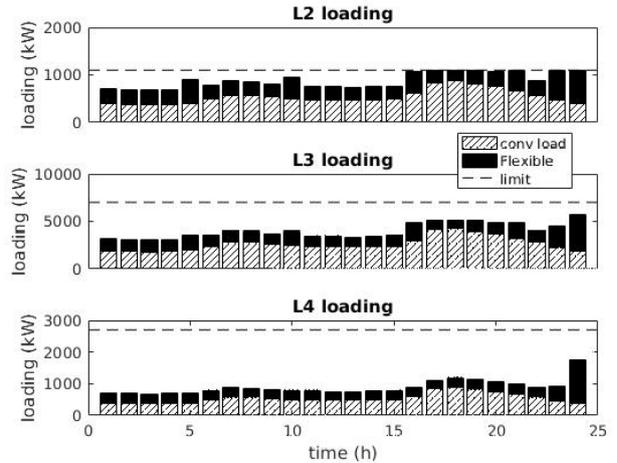Fig. 6. Line loading results with the DDT=0

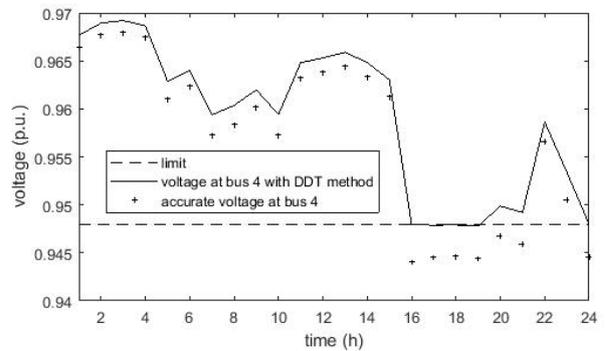Fig. 7. Line loading results after using the DDT method
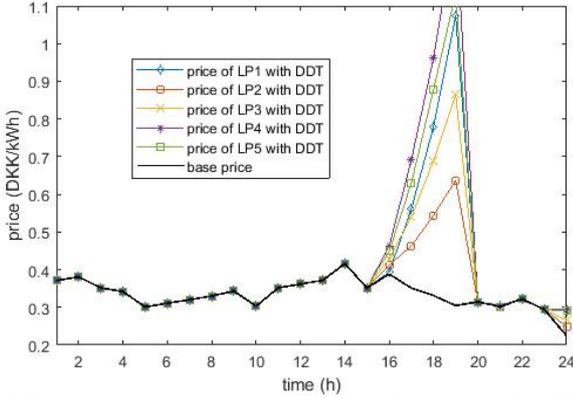

Fig. 8. Voltage profile of LP4

Fig. 9. Forecast system prices (base price) and final prices (including DDT)

2) Convergence Observation:

The key variables, including the line flow and voltage, the corresponding marginal prices λ_t and ω_t, and the DDT rates are observed in the iteration process. The results are shown in Fig. 10. It can be seen that the congestion at hour 24 is solved very quickly, as the cyan curves (hour 24) are settled very quickly (less than 20 iterations).
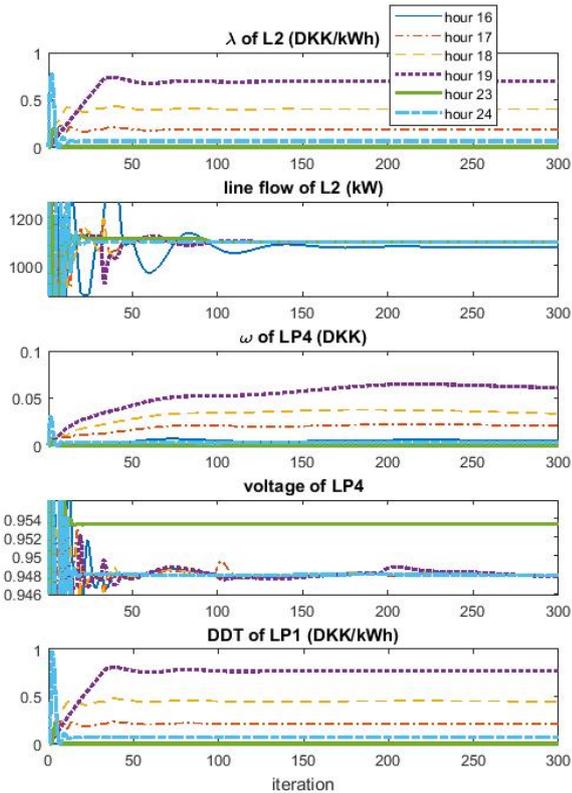

Fig. 10 Iteration observation of the key variables

However, the congestion at hour 17-19 is solved relatively slow. It takes about 200 iterations for the blue, magenta, yellow and purple (hour 16-19) curves, especially in the second, third and fourth sub graphs, to settle down. Two reasons are accountable for this. One is that the congestion at these hours

is actually caused by HPs, and HPs have poor heat storage efficiency compared to the battery storage of EVs. The other one is that the conventional peaks occur at those hours, and the congestion occurs at three consecutive hours and all the HP loads should be shifted to hour 16 in order to solve congestion at hour 17-19. This is particularly challenging for a distributed optimization method. Without reconditioning of the constraints, e.g., let $\beta_1 = 0$, there is no sign of convergence of variables ω_t and voltage after 300 iterations. In practice, the iteration process including the communication between the DSO and the aggregators should be automated, and the communication time for each iteration should be kept within 1 or 2 seconds.

3) Comparison with DT Method:

When using the DT method instead of the DDT for congestion management, uncertainty will occur if a forecast error exists. For instance, assume that the DSO forecast 6 kWh for the average EV demand; however, the real EV demands (the aggregators collected from their customers) are different under different scenarios shown in Table IV. The congestion management by the DT method may fail as shown in Table IV. However, the DDT method can always succeed, implying that the DDT method has more certainty about the congestion management results.

TABLE IV
COMPARISON OF DT AND DDT

Scenario	Relative Error: (Real Demand - Forecast)	Results by DT	Results by DDT
1	30%	Fail (over loading 9.9%)	Success
2	20%	Fail (over loading 6.8%)	Success
3	10%	Fail (over loading 3.4%)	Success
4	-10%	Success	Success

VII. CONCLUSIONS

This paper proposes the DDT method for congestion management in distribution networks with high penetration of EVs and HPs. The DDT method employs a decomposition-based optimization method to have the aggregators participate in the congestion management explicitly, which gives more certainty and transparency compared to the normal DT method. Simulation results show that employing the DT method instead of the DDT method for congestion management may lead to failure if there is a forecast error, and the overloading level depends on how big the error is. The reconditioning method and the PI controller can be used to improve the convergence especially when the optimization structure is complicate due to multiple congestion points and multiple types of flexible demands and network constraints. The case studies have demonstrated and validated the efficacy of the DDT method for congestion management. In the future work, the ACOPF, which is more accurate than the DCOPF, will be employed to determine the DDT.

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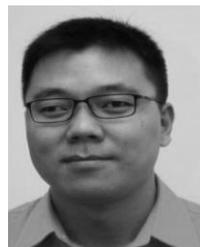
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