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# Stability of facetted translation shells

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**Summary**

This article is discussing the spatial stability i. e. rigidity of double curved shell surfaces under different support conditions. It is based upon a method developed by Henrik Almegaard, as part of the theory concerning the stringer system [ALM04a].

The investigations are limited to facetted translation surfaces (Figure 1).

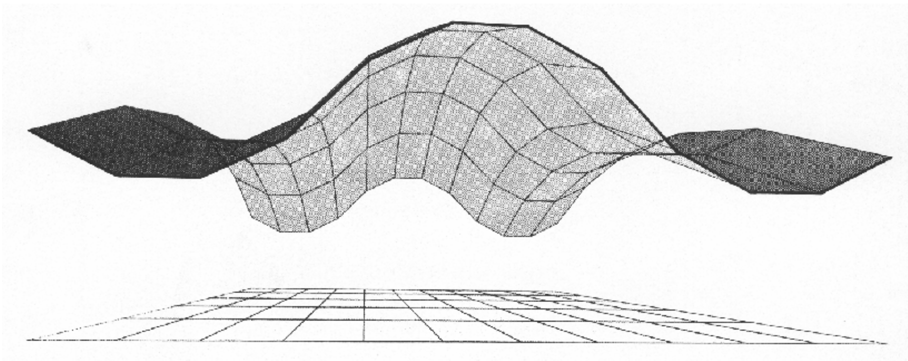


Figure 1. Facetted translation surface.

The translation surface is facetted by polygon lines in the two principal directions. The lines define a grid upon the surface, making a plane faceting of quadrangular elements. The advantages of this type of structures have been shown in i.e. [VAN95]. Professor Schlaich has used this type of structures for a number of daring glass structures i.e. [SCH97].

The question of boundary conditions for membrane shells has been dealt with earlier by professor Tarnai [TAR80, 81, 83], using the theory of partial differential equations. The results found in this article are generally in accordance with these theoretical results. The method presented here is a more practical oriented tool for the designing engineer, based upon simple structural criteria. It offers a possibility to evaluate the influence of different types of supports as well as different types of curvatures. The investigations in this article are limited to translation surfaces, but the results can be used to evaluate the support conditions for other related geometrical surfaces.

**Assumptions**

The translation facetted surfaces will be looked upon as single layer space trusses and the following definitions concerning rigidity and stiffness, referring to the definitions in [SZA92], will be used (Figure 2).

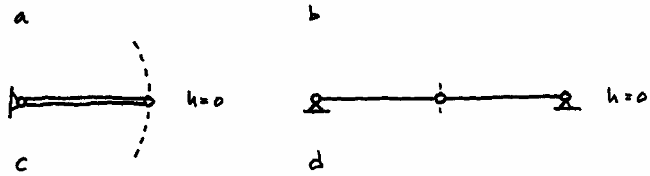



Figure 2. Plane systems illustrating the definitions bellow. a) and b) Statically, kinematically and geometrically determinate systems. c) Statically, kinematically and geometrically indeterminate system. d) Statically and kinematically indeterminate but geometrically determinate system.

*Rigid structure:* a structure that is either statically determinate or statically overdeterminate.

*Statically determinate structure:* a kinematically and geometrically determinate structure.

*Statically indeterminate structure:* either a finite mechanism, that is neither kinematically nor geometrically determinate, or an infinitesimal mechanism, that is a geometrically determinate structure but kinematically indeterminate structure.

*Statically overdeterminate structure:* a kinematically and geometrically overdeterminate structure.

Since the geometry of a statically determinate structure can be infinitesimally close to the geometry of a similar geometrically determinate but kinematically indeterminate structure, the designer's interest is not just rigidity but also stiffness.

S*tiff structure:* a rigid structure of which the geometry is so far from the geometry of a similar but kinematically indeterminate structure, that the internal forces arising from any expectable external load are limited.

If the structure is constructed of elastic material, the deformations will be limited as well. The stiffness of the structure is then related to the size of the internal forces in the way that the smaller deformations the stiffer structure, considering the same load.

**Successively built truss systems**

The principle of successively built truss systems is based on the fact that three bars determine the following joint, if the three bars are out of plane. Because of the particular geometrical conditions in the facetted translation surface this principle makes it simple to control the stability of the truss system, when built successively.

A *support* is defined as a support that secures a joint in exactly one direction.

In order to determine a joint in space it has to be determined in three directions. This means that two conditions have to be fulfilled:

1. The joint shall be supported by three supports or three bars or a combination hereof.

2. The three directions belonging to these shall not lie in the same plane.

From these conditions a rigid truss system can be constructed by successively adding joints, bars and supports. It has to be done so that every added joint is secured in three directions either by bars from existing joints or by new supports or by a combination of hereof. The principle of successively built spatial truss systems is illustrated in (Figure 3).

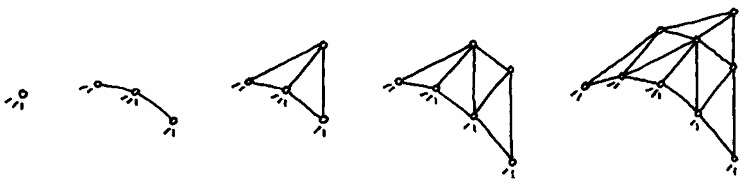


Figure 3. The principle of successively built spatial truss systems. The first joint is supported in three directions, the second joint is secured by one bar and supported in two directions. The rest of the system is built by subsequently adding joints, so that each joint is kinematically and statically determined by either one bar and two supports, two bars and one support or three bars. A support is illustrated by a short line.

This principle has been further developed and has resulted in the theory concerning stringer systems for membrane shells [ALM 04a].

**Facetted translation systems**

As mentioned above, the translation surface is facetted by polygon lines in the two principal directions. The lines define a grid upon the surface, making a plane faceting of quadrangular elements. The structural system in a facetted surface with quadrangular facets can be considered a truss system with four way joints that is braced with quadrangular plates (Figure 4a). The plates transmit only shear forces to the adjoining bars. This system is statical equivalent to a triangulated bar and joint system where every plate has been replaced by a diagonal bar (Figure 4bc).

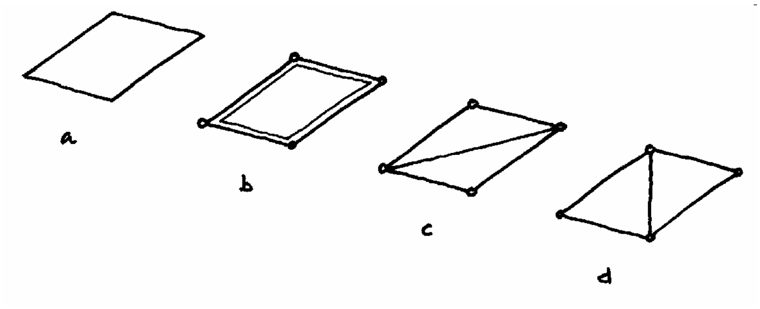
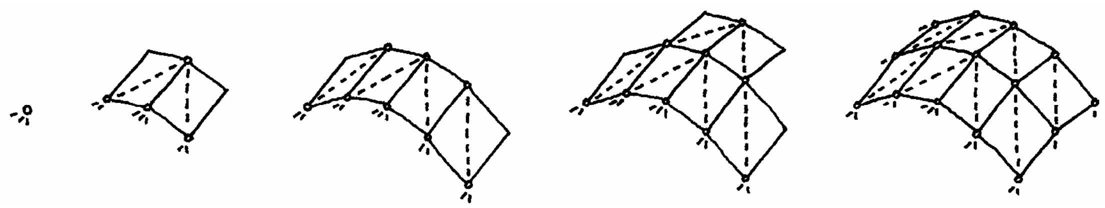



Figure 4. a) A quadrangular facet. b) Structural system consisting of bars and a plate. c) and d) Equivalent structural systems consisting of bars.

Considering the translation facetted surfaces as successively built truss systems it should be noticed that:

- The three bars that are securing a joint can not be placed in the same facet, as a facet is plane, why only two facets out of plane can secure a joint.
- The diagonal bar in a facet can be placed in any of the two diagonals of the quadrangle.

Then two successively built systems appear to be of interest (Figure 5).

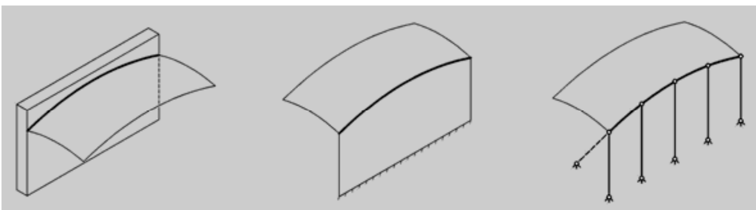


A B

Figure 5. Successively built translation truss systems. A: System that is supported in two directions at every joint along one edge. B: System that is supported in two directions at every joint along one edge and in one direction at every joint along the two edges on each side of this.

The first two quadrangular elements are stabilizing each other as they are both supported along one edge. In system A this is the only supported edge, in system B also the two adjoining edges are supported.

Edges which are supported in at least two directions at every joint along the edge are called *double supported.* Edges which are supported in one direction at every joint along the edge are called *single supported.* A massive wall is an example of a structure that will do as a double support. A thin wall or a row of columns are examples of structures that will do as single supports (Figure 6).



a b c

Figure 6. Examples of supports. a) Double support. b) and c) Single supports.

The two types of supports are described more detailed in another paper at this symposium: Shell supports [ALM04b].

**Example 1. Shells with one free edge**

The system B can be used directly to build translational shells which are supported along three edges and have one free edge (Figure 7).

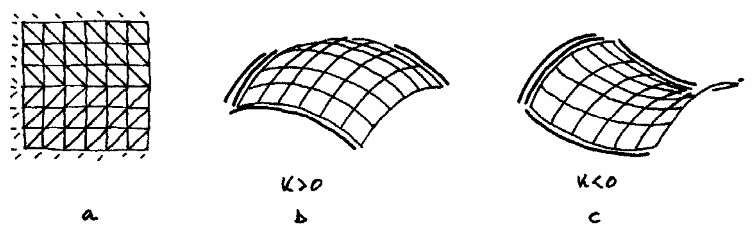



Figure 7. a) The successively built translation truss system. b) Facetted translation shell with positive Gaussian curvature. c) Shell with negative Gaussian curvature. Two lines along an edge indicate that it is double supported, one line along an edge indicate that it is single supported.

In general the shells of this configuration with positive Gaussian curvature (K) are less stiff than shells with negative Gaussian curvature.

**Example 2. Shells with two free edges**

The system A can obviously be used to build cantilevered shell structures. In this example four A systems are used to build a translational shell which is supported along two opposite edges and then has two free edges (Figure 8).

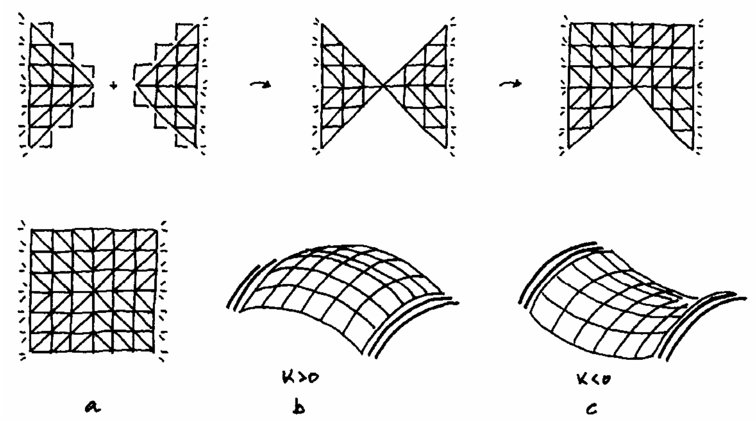



Figure 8. a) The construction of the translation truss system from 4 successively built systems. b) Facetted translation shell with positive Gaussian curvature. c) Shell with negative Gaussian curvature.

In general also for this configuration shells with positive Gaussian curvature are less stiff than shells with negative Gaussian curvature, but both these configurations are much stiffer than the previous.

**Example 3. Shell supported along all four edges**

In this example, a vertical supported translation facetted elliptic paraboloid, whose projection on the horizontal ground plan is a square, will be investigated (Figure 9)

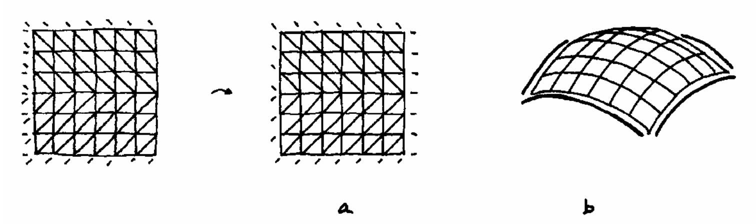



Figure 9. Vertical supported translation facetted elliptic paraboloid. a) The translation truss system. b) The facetted translation shell.

It is well known that such a structure considered as a membrane shell is both rigid and stiff.

Considering the truss system as a stringer system, one set of stringers is supported in the other end than in example 1. This means that the system can not be successively built – it has to be considered a closed surface.

As Cauchy has proved [CAU13] a closed convex triangular facetted surface is rigid. Hence a closed convex quadrangular facetted surface is also rigid. Indeed this is the stiffest configuration for shells of positive Gaussian curvature.

But for shells of negative Gaussian curvature this configuration has often proved to be statically and kinematically indeterminate. Among others professor Nielsen has shown that a hyperbolic paraboloidial truss shell with a rectangular ground plan, that is vertical supported i.e. single supported all around, is an infinitesimal mechanism, which means that it is statically as well as kinematically indeterminate [NIE64].

**Differences in stiffness**

The differences in stiffness for the previous examples can be illustrated by plane truss analogies (Figure 10).

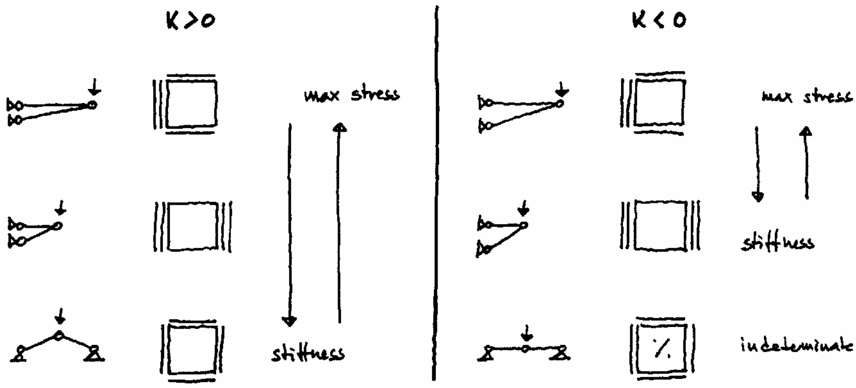



Figure 10. The stiffness of the previous examples illustrated by plane truss analogies.

It must be concluded, that the principle of successively built translation truss systems in general generate rigid structures independent of the curvature of the facetted surface, as long as the two facets securing a joint are out of plane. But for non-successively built systems the curvature is decisive.

**Shell configurations**

As a consequence of this, the available shell configurations can be indicated (Figure 11).

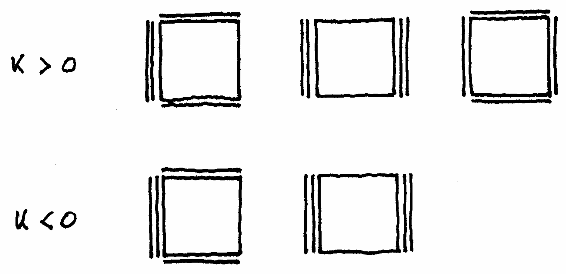



Figure 11. The available shell configurations depending on the Gaussian curvature K

The conditions for supporting a shell on another shell can be derived from simple subdivisions of the configuration used in example 1 showing that if the geometry is appropriate (Figure 12):

- A free edge of one shell can be a double support for another shell.
- Two edges that have to be single supported can support each other.

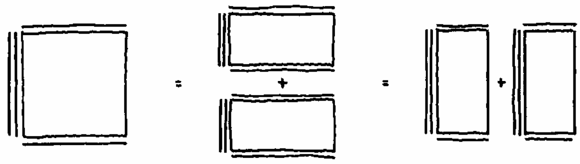


Figure 12. The configuration in example 1 can be subdivided into two configurations of the same kind in two ways.

With these rules the configurations can be used as building units to construct and analyse different possible structural systems for shell structures.

**Example 4. Shells with surfaces composed of areas with different Gaussian curvatures**

A translation shell designed from a sinus curve by the co-author as a proposal for a competition entry at a colloquium in September 1990 at the IL in Stuttgart led to a discussion raised by Mr. Isler concerning the rigidity of the structure (Figure 13).

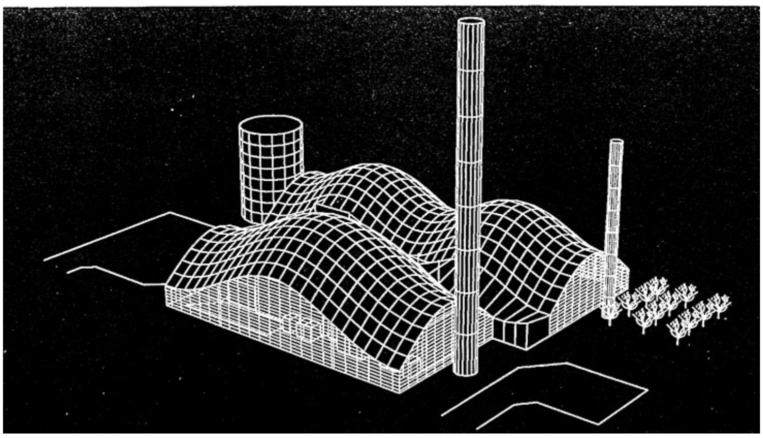


Figure 13. Proposal for a competition entry. This example concerns the translation shell at the back.

In order to evaluate the stability of shell surfaces composed of areas with different Gaussian curvatures, the shell has to be divided into sections having either positive, zero or negative Gaussian curvature. The structure then can be analysed using the available configurations - depending on the Gaussian curvature - and the conditions for supporting shells on shells (Figure 14).

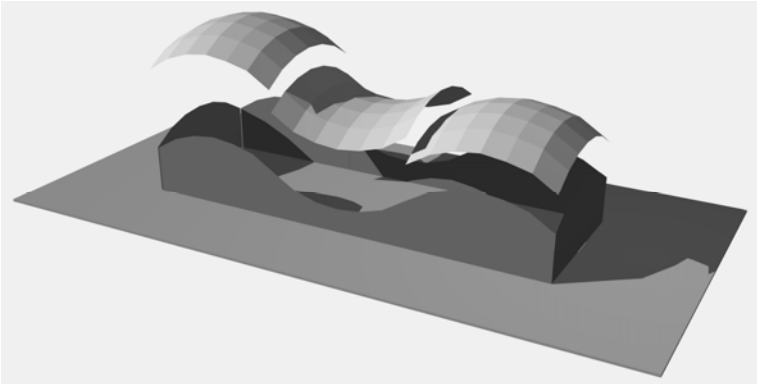


Figure 14. The considered structure is covered with one translation shell composed of three sections having respectively positive, negative and positive Gaussian curvature.

In this structure the section in the middle having negative Gaussian curvature must be considered single supported on the vertical walls as well as on the two adjoining shell sections of positive Gaussian curvature surface (Figure 15).

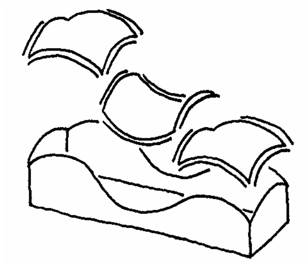


Figure 15. Configuration illustrating the support conditions for the three sections. One line along the edge of a shell indicate that the edge is single supported.

Hence it must be expected that either this configuration will not be rigid, or it will generate very large forces and deformations in the shell

Shell surfaces of negative Gaussian curvature though are always spatial stable if the corresponding stringer system is successively built. This means that this section of negative Gaussian curvature either has to be double supported along one side and single supported along the two adjoining sides, or double supported along two opposite sides as indicated by the configurations in (Figure 11). On the background of these configurations and the conditions for supporting a shell on another shell, the different possible combinations can be analysed, and it turns up that there are basically two configurations (Figure 16).

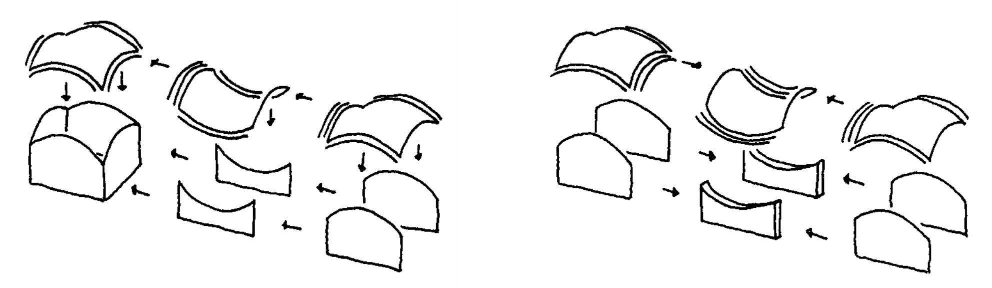


Figure 16. The two available configurations for the combined shells/systems

The combinations of supports in (Figure 16) are necessary and sufficient to ensure statically and kinematically determinacy of the shell. But as mentioned above relatively large forces and deformations will appear near the free edges of the sections of positive curvature, i.e. at the open gables. In practise these edges will have to be provided with additional supports – that could be extra supports taking up horizontal forces at the end of the edge or vertical single supports along the edge.

**Conclusion**

The principle of successive build truss systems on at translation surface opens a simple method to understand and ensure rigidity of shell structures of different curvature, and under different support conditions.

The shell configurations, indicating double and single supported as well as free edges, can be used to understand and identify possible structural systems for shell structures in the conceptual stage.

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