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Published in: IFAC-PapersOnLine

Link to article, DOI: 10.1016/j.ifacol.2017.08.356

Publication date: 2017

Document Version Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Adaptive Unscented Kalman Filter using Maximum Likelihood Estimation

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Abstract: The purpose of this study is to develop an adaptive unscented Kalman filter (UKF) by tuning the measurement noise covariance. We use the maximum likelihood estimation (MLE) and the covariance matching (CM) method to estimate the noise covariance. The multi-step prediction errors generated by the UKF are used for covariance estimation by MLE and CM. Then we apply the two covariance estimation methods on an example application. In the example, we identify the covariance of the measurement noise for a continuous glucose monitoring (CGM) sensor. The sensor measures the subcutaneous glucose concentration for a type 1 diabetes patient. The root-mean square (RMS) error and the computation time are used to compare the performance of the two covariance estimation methods. The results indicate that as the prediction horizon expands, the RMS error for the MLE declines, while the error remains relatively large for the CM method. For larger prediction horizons, the MLE provides an estimate of the noise covariance that is less biased than the estimate by the CM method. The CM method is computationally less expensive though.

Keywords: Unscented Kalman filter, Maximum likelihood estimation, Covariance matching technique, Adaptive filtering, Covariance estimation, Continuous glucose monitors.

1. INTRODUCTION

Identifying the uncertainties that affect a system is fundamental for monitoring and designing an optimal and adaptive estimator. In order to have a filter that is close to optimal, we need to know the covariance of the process and measurement noise. Methods for identifying noise covariances often include maximum likelihood estimation (MLE) (Zagrobelny and Rawlings, 2015b; Jørgensen and Jørgensen, 2007), covariance matching (CM) techniques (Maybeck, 1982; Weige et al., 2015; Partovibakhsh and Liu, 2015), and correlation-based approaches such as the autocovariance least-squares (ALS) method (Åkesson et al., 2008; Odelson et al., 2006a,b; Zagrobelny and Rawlings, 2015a). These methods often deal with only linear or linearized systems. The CM technique is commonly used for the nonlinear systems, because it is computationally inexpensive and it is flexible to accommodate the nonlinear models. It is a suboptimal covariance estimation though. The literature on the use of optimization-based covariance estimation approaches for nonlinear systems is sparse.

The purpose of this study is to use an optimization-based estimator, i.e., the MLE method, for identification of the noise covariance in a nonlinear system. Furthermore, we compare the MLE approach with a suboptimal estimation method, i.e., the CM algorithm, in the context of an adaptive unscented Kalman filter (UKF). We employ the estimation of the measurement noise covariance as the basis for deriving filter adaptation.

The paper is structured as follows. First, we present the unscented Kalman filter (UKF) for prediction, filtering, and generating the covariance matrix of the prediction errors. Then, we develop the MLE problem and the CM technique to estimate the covariance of the measurement noise. The multi-step prediction errors and their covariances generated by the UKF are used in the MLE and CM methods. We then apply the MLE and CM algorithms on an example. The example is a nonlinear metabolic model of a patient with type 1 diabetes. In the example, we estimate the noise covariance of a continuous glucose monitoring (CGM) sensor, and derive the adaptive UKF to filter the sensor measurements.

2. MATERIALS AND METHODS

2.1 The unscented Kalman filter

In the UKF, a set of sigma points are deterministically chosen to represent the mean and covariance of the states. The sigma points therefore approximate the probability distribution of the states as it goes through the nonlinear transformation (Särkkä, 2007; Julier and Uhlmann, 2004). Approximating the probability distribution of the states by the sigma points in the UKF has shown to produce less estimation bias compared to the linearization in the extended Kalman filter (EKF) (Simon, 2006).
Model The model of the state space in the stochastic differential equation (SDE) form and the measurement model are of the form
\[
\begin{align*}
\dot{x}(t) &= f(x(t), u(t), d(t)) \, dt + \sigma \cdot \, d\omega(t), \\
y_k &= g(x_k) + \xi_k, \\
\dot{\omega}(t) &= N_{iid}(0, I dt),
\end{align*}
\] in which \(x\) is the state, \(u\) is the input, \(d\) is the disturbance, and \(y\) is the measurement. We assume that \(\xi\) is a Gaussian zero-mean discrete-time measurement noise with covariance \(R\). The stochastic noise \(\omega\) is a standard Wiener process, and \(\sigma\) is the diffusion coefficient. \(I\) is an \(n \times n\) identity matrix, where \(n\) is the number of state variables in the model.

Prediction This section explains the multi-step prediction with the UKF. The prediction steps are \(j = 1, 2, \ldots, N_p\) and \(N_p\) is the prediction horizon. The scaling parameter \(\lambda\) determines how far the sigma points are scattered away from the mean. \(\lambda\) and \(c\) are defined as
\[
\lambda = \alpha^2(n + \kappa) - n, \quad c = \alpha^2(n + \kappa) - n.
\]
The associated weights, \(W\), for the \(2n + 1\) sigma points are given by
\[
\begin{align*}
W^{(i)}_0 &= \lambda/(n + \lambda), \\
W^{(i)}_c &= \lambda/(n + \lambda)(1 - \alpha^2 + \beta), \\
W^{(i)}_m &= 1/(2(n + \lambda)), \\
W^{(i)}_c &= 1/(2(n + \lambda)), \\
W_m &= [W^{(0)}_m \ldots W^{(2n)}_m]^T.
\end{align*}
\]
A deterministic approach, based on the Cholesky factorization of the covariance \(P\), samples the probability distribution to generate the sigma points \(\hat{X}\).
\[
\begin{align*}
\hat{X}_{k+j} &= \hat{x}_{k+j-1|k} \ldots \hat{x}_{k+j-1|k} \\
+ \sqrt{\lambda} \left[ \sqrt{P_{k+j-1|k}} - \sqrt{P_{k+j-1|k}} \right] \\
= [m^{(i)}_i \ldots m^{(2n)}_i], \\
& \text{for } i = 0, \ldots, 2n.
\end{align*}
\]
The nonlinear function \(f\) propagates each of the sigma points according to
\[
\frac{d\hat{X}_{k+j-1|k}}{dt} = f(\hat{X}_{k+j-1|k}, u(t), d(t)), \quad t \in [t_{k+j-1} t_{k+j}].
\]
The parameters \(\alpha\), \(\kappa\), and \(\beta\) are set to \(\alpha = 0.01\), \(\kappa = 0\), and \(\beta = 2\). The weighted average of the transformed sigma points gives the predicted mean
\[
\hat{x}_{k+j|k} = \hat{x}_{k+j|k} W_m.
\]
The covariance of the estimation error is computed by propagating \(dP_{k+j-1}\) according to
\[
\frac{dP_{k+j-1|k}}{dt} = \sum_{i=0}^{2n} W^{(i)}_c \left( m^{(i)}_m(t) - m_x(t) \right) \left( f(m^{(i)}_m(t), u(t)) - m_f(t) \right) T + \sigma \sqrt{T}, \quad t \in \left[ t_{k+j-1} t_{k+j} \right]
\]
where \(m_x\) and \(m_f\) are
\[
\begin{align*}
m_x(t) &= \sum_{i=0}^{2n} W^{(i)}_m m^{(i)}_m(t), \\
m_f(t) &= \sum_{i=0}^{2n} W^{(i)}_m f(m^{(i)}_m(t), u(t)).
\end{align*}
\]
The propagated error covariance is then
\[
P_{k+j|k} = P_{k+j-1|k} + 1.
\]
To increase accuracy, new sigma points are generated from the predicted state mean and covariance as indicated in
\[
\hat{X}_{k+j} = [\hat{x}_{k+j|k} \ldots \hat{x}_{k+j|k}] \\
+ \sqrt{\lambda} \left[ \sqrt{P_{k+j|k}} - \sqrt{P_{k+j|k}} \right] \\
= [\hat{m}^{(i)}_i \ldots \hat{m}^{(2n)}_i], \\
& \text{for } i = 0, \ldots, 2n.
\]
The measurement model transforms each of the new sigma points
\[
\hat{Y}_{k+j} = g(\hat{X}_k) = [\mu^{(i)} \ldots \mu^{(2n)}], \quad i = 0, \ldots, 2n.
\]
The weighted average of the measurement sigma points gives the predicted measurement
\[
\hat{y}_{k+j|k} = \hat{Y}_{k+j} W_m,
\]
and the \(j\)-step prediction error is given by
\[
\epsilon_{k+j|k} = y_{k+j} - \hat{y}_{k+j|k}.
\]
The measurement model is
\[
S^{W}_{k+j} = \text{covariance of } \hat{Y}_{k+j} \text{ and is computed as}
\]
\[
S^{W}_{k+j} = \sum_{i=0}^{2n} W^{(i)}_c \left( \mu^{(i)} - \hat{y}_{k+j|k} \right) \left( \mu^{(i)} - \hat{y}_{k+j|k} \right)^T.
\]
The updated state covariance is calculated by
\[
S_{k+j} = S_{k+j} + R_{k+j}.
\]
Filtering The equation set (5) describes filtering and measurement update with the UKF. \(S^{W}_{k+1|k}\) is the cross-covariance of \(\hat{X}\) and \(\hat{Y}_{k+j|k}\), and can be estimated as
\[
S^{W}_{k+1|k} = \sum_{i=0}^{2n} W^{(i)}_c \left( \hat{m}^{(i)} - \hat{x}_{k+1|k} \right) \left( \mu^{(i)} - \hat{y}_{k+1|k} \right)^T,
\]
and \(K_{k+1}\) is the filter gain as follows
\[
K_{k+1} = S^{W}_{k+1|k} S^{-1}_{k+1}.\]
The updated state mean is computed as
\[
\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} (y_{k+1} - \hat{y}_{k+1|k}).
\]
The updated error covariance is given by
\[
P_{k+1|k+1} = P_{k+1|k} - K_{k+1} S_{k+1} K_{k+1}^T.
\]
Multi-step prediction error and its covariance matrix Let \(\{y_k\}_{k=1}^{N}\) denote the measurements and \(N_p\) denote the prediction horizon. Let the time indices be \(k = 0, 1, \ldots, N - N_p\), and the prediction index be \(1 \leq j \leq N_p\). This implies that \(1 \leq k + j \leq N\). Let \(\epsilon_{k+N_p}\) denote the vector of the prediction errors in the \(N_p\)-sample prediction window as
\[
\epsilon_{k+N_p} = \begin{bmatrix} \epsilon_{k+1|k} & \epsilon_{k+2|k} & \ldots & \epsilon_{k+N_p|k} \end{bmatrix} = \begin{bmatrix} y_{k+1} - \hat{y}_{k+1|k} & y_{k+2} - \hat{y}_{k+2|k} & \ldots & y_{k+N_p} - \hat{y}_{k+N_p|k} \end{bmatrix}.\]
Analogously to the linear systems, the cross-covariances of the multi-step prediction errors may be computed by (Jørgensen and Jørgensen, 2007; Kailath et al., 2000)

$$S_{k+i,k+j} = \langle e_{k+i|k}, e_{k+j|k} \rangle$$

$$= \begin{cases} \hat{Y}_{k+i} W \hat{X}_{k+j}^T & \text{if } i > j, \\ \hat{Y}_{k+i} W \hat{X}_{k+j}^T + R_{k+j} & \text{if } i = j, \\ \hat{Y}_{k+j} W \hat{X}_{k+i}^T & \text{if } i < j, \end{cases} \quad (7)$$

The covariance matrix of $e_{k+N_y}$ is calculated by

$$\mathcal{R}_{k+N_y} = \langle e_{k+N_y}, e_{k+N_y} \rangle = \begin{bmatrix} S_{k+1,k+1} & S_{k+1,k+2} & \cdots & S_{k+1,k+N_y} \\ S_{k+2,k+1} & S_{k+2,k+2} & \cdots & S_{k+2,k+N_y} \\ \vdots & \vdots & \ddots & \vdots \\ S_{k+N_y,k+1} & S_{k+N_y,k+2} & \cdots & S_{k+N_y,k+N_y} \end{bmatrix}.$$

The covariance $\mathcal{R}_{k+N_y}$ is an $n_y \times n_y$ matrix, in which $n_y$ is the size of the measurement vector $y$ and the distribution $N(0, \mathcal{R}_{k+N_y})$. By having $\mathcal{R}$, $\epsilon$, and $\epsilon$ from the UKF, we estimate and tune the covariance of the measurement noise $\mathcal{R}$ by MLE and the CM method.

### 2.2 Maximum likelihood estimation

By taking the negative log likelihood of the multivariate normal distribution of $e_{k+N_y}$, we write the MLE optimization problem as

$$\min_{\mathcal{R} \geq 0} V(\mathcal{R}) = \ln \det(\mathcal{R}_{k+N_y}) + \epsilon_{k+N_y}^T \mathcal{R}_{k+N_y}^{-1} \epsilon_{k+N_y}. \quad (9)$$

When $N_p$ is large, computing $\ln \det(\mathcal{R}_{k+N_y})$ is challenging in terms of computational time and numerical accuracy. Alternatively, Cholesky factorization offers a faster approach. The Cholesky factorization decomposes the positive definite $\mathcal{R}_{k+N_y}$ into $\mathcal{R}_{k+N_y} = LL^T$ with $L$ being a lower triangular matrix. Then

$$\ln \det(\mathcal{R}_{k+N_y}) = 2 \sum_{i=1}^{n_y N_y} \ln(L_{ii}), \quad (10)$$

in which $L_{ii}$ are the diagonal entries of $L$. Computing inverse of $\mathcal{R}_{k+N_y}$ for calculating $\epsilon_{k+N_y}^T \mathcal{R}_{k+N_y}^{-1} \epsilon_{k+N_y}$ is computationally heavy. To avoid matrix inversion, we compute $\epsilon_{k+N_y}^T \mathcal{R}_{k+N_y}^{-1} \epsilon_{k+N_y}$ by solving a system of linear equations $\mathcal{R}_{k+N_y} Z = \epsilon_{k+N_y}$ via back substitution and finding $Z$. Then

$$\epsilon_{k+N_y}^T \mathcal{R}_{k+N_y}^{-1} \epsilon_{k+N_y} = \epsilon_{k+N_y}^T \mathcal{R}_{k+N_y}^{-1} \epsilon_{k+N_y} Z. \quad (11)$$

### 2.3 Covariance matching technique

In the prediction window for the prediction steps $j = 1, 2, \ldots, N_p$, we compute $\hat{S}_{k+j}$ that is the theoretical covariance of $e_{k+j|k}$. The covariance matrix $\hat{S}_{k+j}$ is computed as

$$\hat{S}_{k+j} = \hat{S}_{k+j}^{yy} + R_{k+j}, \quad (12a)$$

in which the covariance matrix $\hat{S}_{k+j}^{yy}$ is calculated according to (4d). The sample covariance of $e_{k+j|k}$ is $\hat{S}_{k+j}$, which is given by

$$\hat{S}_{k+j} = \frac{1}{M - 1} \sum_{q=k+j-M+1}^{k+j} e_{q|q-j} e_{q|q-j}^T. \quad (12b)$$

where $M$ is the length of the data sequence used for estimating the sample covariance. We set $M$ to 15. The estimated measurement noise covariance is

$$\hat{R} = \frac{1}{N_p - 1} \sum_{j=1}^{N_p} (\hat{S}_{k+j} - \hat{S}_{k+j}^{yy}). \quad (12c)$$

In both estimation methods, the estimated $R$ is used in the UKF for the next $w$ samples. $w$ is the size of the moving step of the prediction window and is set to 50% of $N_p$. The size of the prediction window is the same as the prediction horizon $N_p$.

The root-mean square (RMS) error of the estimated noise covariances evaluates the performance of the two covariance estimation methods. The RMS error is computed by

$$V_{rms} = \sqrt{\frac{1}{N} \sum_{k=1}^{N} \| \hat{R}_{true,k} - \hat{R}_k \|_2^2}, \quad (13)$$

where $N$ is length of the data sequence.

### 3. EXAMPLE: IDENTIFYING THE NOISE OF CONTINUOUS GLUCOSE MONITORING SENSOR

The CGM sensor measures interstitial glucose from the subcutaneous (SC) tissue. The sensor measurements are corrupted by random noise and artifacts originated from several sources including sensor electronics, miscalibration of the sensor, and biofouling.

### 3.1 The state space model

We used the Medtronic Virtual Patient (MVP) model in the SDE form for the state space representation of the patient’s metabolism and also for simulating the CGM sensor (Kanderian et al., 2009). This model describes the pharmacokinetics (PK) of SC insulin and the insulin-glucose interaction. We also included the blood glucose -interstitial glucose dynamics in the model. The model also contains the two-compartments of carbohydrate (CHO) absorption (Willinska et al., 2010). The model is described as

$$dI_{sc}(t) = \frac{1}{\tau_1} \left( I(t) - I_{sc}(t) \right) dt + \sigma_{SC} \cdot d\omega_{SC}(t), \quad (14a)$$

$$dI_{p}(t) = \frac{1}{\tau_2} \left( I_{SC}(t) - I_{p}(t) \right) dt + \sigma_{p} \cdot d\omega_{p}(t), \quad (14b)$$

$$dI_{eff}(t) = \left( -P_2 \cdot I_{eff}(t) + P_2 \cdot S_l \cdot I_p(t) \right) dt + \sigma_{eff} \cdot d\omega_{eff}(t), \quad (14c)$$

$$dG_B(t) = \left( - (GEZI + I_{eff}(t)) \cdot G_B(t) + EGP + R_n(t) \right) dt + \sigma_G \cdot d\omega_G(t), \quad (14d)$$

$$dG_I(t) = - \frac{1}{\tau_3} \left( G_B(t) - G_I(t) \right) dt + \sigma_{GI} \cdot d\omega_GI(t), \quad (14e)$$

$$dD_1(t) = \left( q(t) - \frac{1}{\tau_m} D_1(t) \right) dt + \sigma_{D1} \cdot d\omega_{D1}(t), \quad (14f)$$

$$dD_2(t) = \frac{1}{\tau_m} \left( D_1(t) - D_2(t) \right) dt + \sigma_{D2} \cdot d\omega_{D2}(t), \quad (14g)$$

$$R_n(t) = \frac{1}{V_G \tau_m} D_2(t). \quad (14h)$$
$ID$ is the SC insulin input ($\mu U$/min), $I_{sc}$, $I_p$, and $I_{eff}$ are the SC insulin concentration (nU/L), the plasma insulin concentration (nU/L), and the effect of insulin (min$^{-1}$), respectively. $G_B$ is the blood glucose concentration and $G_I$ is the interstitial glucose concentration both in mg/dL. $q(t)$ is the CHO ingestion rate (g/min), $D_1$ and $D_2$ are the glucose masses (mg) in the accessible and inaccessible compartments, and $R_d$ is the glucose appearance rate (mg/dL/min). $\tau_1$ (49 min) is the time constant to the insulin movement from administration site to the SC tissue, $\tau_2$ (47 min) is the time constant related to the insulin movement from the SC tissue to plasma, $\tau_3$ (10 min) is the time constant related to the glucose movement from plasma to SC tissue, $C_I$ (2010 mL/min) is the insulin clearance, $P_3$ (1.06 $10^{-2}$ min$^{-1}$) is the delayed insulin action on the blood glucose, $S_I$ (8.11 $10^{-4}$ mL/µU/min) is the insulin sensitivity, $G_{EZI}$ (2.20 $10^{-3}$ min$^{-1}$) is the glucose effectiveness at zero insulin, $EGP$ (1.33 mg/dL/min) is the endogenous glucose production rate at zero insulin, $\tau_m$ (47 min) is the peak time of meal absorption, and $V_g$ (253 dL) is the volume of distribution for glucose.

ID contains basal and bolus insulin. The patient eats breakfast at 8:00 hrs, lunch at 13:15 hrs, dinner at 18:00 hrs, and snack at 22:00 hrs. The CHO content of the meals are 72 g for breakfast, 131 g for lunch, 51 g for dinner, and 70 g for snack, respectively. The insulin bolus to cover meal are 3.5 U for breakfast, 7 U for lunch, 2.5 U for dinner, and 3.5 U for snack, respectively.

### 3.2 The measurement model

The CGM sensor samples interstitial glucose. Therefore, the measurement model comprises $G_I$ measurements which are affected by the measurement noise $\phi$ as indicated by

$$y_k = G_{I,k} + \phi_k.$$  \hspace{1cm} (15a)

The measurement noise $\phi$ has covariance $R_\phi$, and Facchinetti et al. identified it as the sum of two autoregressive processes given by (Facchinetti et al., 2014)

$$\dot{\phi}_k = c_k + \delta_{c,k},$$  \hspace{1cm} (15b)

$$c_k = 1.23c_{k-1} - 0.3995c_{k-1} + \delta_{c,k},$$  \hspace{1cm} (15c)

$$\dot{\delta}_k = 1.013\delta_{k-1} - 0.2135\delta_{k-1} + \delta_{\delta,k},$$  \hspace{1cm} (15d)

$$\delta_{c,k} \sim N(0, 11.3), \quad \delta_{\delta,k} \sim N(0, 14.45).$$

The model (1) corresponds to the model (14) with the state variables $x = [I_{sc} \ I_p \ I_{eff} \ G_B \ G_I \ D_1 \ D_2]^T$, the input $u = ID$, the disturbance $d = q$, the noise $\xi = \phi$, and the measurement $y$ being the CGM data. In this example, the measurement model $g$ is linear and $R_\phi$ is a scaler. For simulating the measurements $y$, we first simulated the model in (14) by using Euler Maruyama method (Higham, 2001). Then we added noise $\phi$ to the simulated $G_I$. We simulated one day of one-minute CGM data that consists of 1440 measurements. The aim is to estimate the unknown $R_\phi$ and adopt the UKF accordingly.

### 4. RESULTS AND DISCUSSION

We considered six different values for the prediction horizon $N_p$ (see Table 1). For each prediction horizon we performed 50 experiments, each experiment consists of simulating the model in (14) to generate one day (1440 min) CGM data. For simulation, we set $\sigma$ to 0.5% of $x_{ss}$, in which $x_{ss}$ is the steady state of the model. The experiments had different realizations of the measurement noise $\phi$ and process noise $d\omega$. Then we applied the MLE and the CM method on each experiment to estimate the covariance $R_\phi$. We initialized the UKF from the steady state of the model, and $P_0 = \sigma \sigma^T$. Table 1 and Table 2 summarize the results which are averaged over 50 experiments for each prediction horizon.

Fig. 1 compares the performance of the two covariance estimation methods in terms of the root-mean-square error. Fig. 2 illustrates the absolute deviation of the estimated covariance from the true covariance, for two estimation methods. Fig. 3 depicts the histogram of the estimated covariance over 50 experiments for the prediction horizon = 200 min. Fig. 4 shows the histogram of the CPU time for covariance estimation for 50 experiments and the prediction horizon = 200 min. Fig. 5 indicates the result of applying MLE and CM methods on an example experiment. Fig. 1 and Fig. 2 indicate that as the prediction horizon $N_p$ expands, the bias of the covariance estimate declines. This originates from the consistency property of the MLE. Because MLE is a consistent estimator, increasing the number of measurements by expanding the prediction horizon improves the estimation precision and reduces bias. This is well demonstrated in Fig. 1 and Fig. 2. The figures also indicate that the decrease in $V_{rms}$ and bias is approximately exponential for the MLE. Fig. 3 also shows that for sufficiently large $N_p$, the MLE has considerably less bias than CM method.
Proceedings of the 20th IFAC World Congress 2001). Then we added noise absorption, and compartments, and 3.5 U for snack, respectively. The insulin bolus to cover meal hrs, and snack at 22:00 hrs. The CHO content of the meals breakfast at 8:00 hrs, lunch at 13:15 hrs, dinner at 18:00 hrs, and snack at 22:00 hrs. The CHO content of the meals breakfast at 8:00 hrs, lunch at 13:15 hrs, dinner at 18:00 hrs.

\[ \text{insulin movement from the SC tissue to plasma, } \tau_p = 47 \text{ min} \] is the time constant related to the delayed insulin action on the blood glucose, \( \phi \). For simulating the measurements as indicated \( \phi \) has covariance \( R_p \), and \( \delta_p \), and \( \theta_p \). Also shows that for sufficiently large \( N \) bias is approximately exponential for the MLE. Fig. 3 indicates the result of applying MLE and CM methods on an example. Table 2. Estimating measurement noise covariance by covariance matching. The figures also indicate that the decrease in predicting the model in (14) to generate one day (1440 min) is the peak time of meal \( m \) which is a scaler.

\[ \text{prediction horizon} = 200 \text{ min.} \] Fig. 4 shows the histogram of the CPU time to perform 50 experiments, each experiment consists of per 50 measurements had different realizations of the measurement noise covariance averaged over 50 experiments. Table 1. Estimating measurement noise covariance by maximum likelihood estimation*. Table 2. Estimating measurement noise covariance by covariance matching*. The values are the mean for one-day data which is the average of the 50 one-day experiments. The values are the mean for one-day data which is the average of the 50 one-day experiments.

Fig. 3. Histogram of the estimated covariance compared to the true covariance based on 50 experiments and the prediction horizon = 200 min. Fig. 4. Histogram of CPU time for covariance estimation based on 50 experiments and the prediction horizon = 200 min. Because the true noise covariance is not known in the prediction window and is to be estimated. Furthermore, the assumption of optimality for the nonlinear filters, i.e. the UKF and EKF, is not valid in general. This is due to the fact that the UKF approximates the state probability distribution and the EKF linearizes the state-space model, both procedures make the filter deviate from optimality. As Fig. 5(c) illustrates, the CGM data filtered with the maximum likelihood estimated covariance is closer to the ideally (known covariance) filtered CGM, compared to the CGM data filtered with the CM estimated covariance. The improvement is modest though. This is because the process noise covariance moderates the effect of variation of the measurement noise covariance on filtering. When the process noise is relatively small (small \( \sigma^2 \) in (14)), the filtered measurements are close to the one step model-predicted measurements, due to the small filter gain, without being.

| Prediction horizon (min) | \( R_{\phi,\text{true}} \) (mg\(^2\)/dL\(^2\)) | \( \hat{R}_\phi \) | \( \frac{|R_{\phi,\text{true}} - \hat{R}_\phi|}{R_{\phi,\text{true}}} \) % | \( V_{rms} \) | CPU time (s) |
|-------------------------|---------------------------------|----------------|---------------------------------|---------|-------------|
| 5                       | 103.8                           | 42.3           | 59.3                            | 118.2   | 378.0       |
| 20                      | 114.0                           | 29.6           | 74.0                            | 102.9   | 284.3       |
| 40                      | 105.9                           | 41.9           | 60.4                            | 78.4    | 277.1       |
| 80                      | 104.9                           | 65.3           | 37.7                            | 66.6    | 289.9       |
| 100                     | 102.6                           | 70.8           | 31.0                            | 59.9    | 301.2       |
| 200                     | 106.9                           | 98.8           | 7.5                             | 52.8    | 378.5       |

* The values are the mean for one-day data which is the average of the 50 one-day experiments.

| Prediction horizon (min) | \( R_{\phi,\text{true}} \) (mg\(^2\)/dL\(^2\)) | \( \hat{R}_\phi \) | \( \frac{|R_{\phi,\text{true}} - \hat{R}_\phi|}{R_{\phi,\text{true}}} \) % | \( V_{rms} \) | CPU time (s) |
|-------------------------|---------------------------------|----------------|---------------------------------|---------|-------------|
| 5                       | 103.8                           | 69.9           | 32.6                            | 123.3   | 220.5       |
| 20                      | 114.0                           | 105.5          | 7.4                             | 120.9   | 179.2       |
| 40                      | 105.9                           | 143.3          | 35.4                            | 150.2   | 172.7       |
| 80                      | 104.9                           | 159.6          | 52.1                            | 143.6   | 161.0       |
| 100                     | 102.6                           | 156.7          | 52.6                            | 128.0   | 157.2       |
| 200                     | 106.9                           | 178.2          | 66.7                            | 117.1   | 142.6       |

* The values are the mean for one-day data which is the average of the 50 one-day experiments.
method is somewhat larger than the computational cost associated to the MLE.

show that the MLE method outperforms the CM method.

compared the method with the CM algorithm which is 

their covariance matrix generated by the UKF. We also 
estimates the noise covariance. The inputs of the ML 
multi-step prediction errors and 

objective function are the multi-step prediction errors and 
estimates the noise covariance. A method based on the MLE 

we presented an adaptive UKF by tuning the measure-

noise covariance.

The absolute deviation of the filtered CGM data from 
the ideally filtered CGM. The ideal filter is the UKF 
experiment and the prediction horizon = 200 min. a) 
estimated covariance. b) The filtered CGM data. c) 
the ideally filtered CGM. The ideal filter is the UKF 
with the actual measurement noise covariance. 

profoundly affected by the variations of the measurement noise covariance.

5. CONCLUSIONS

We presented an adaptive UKF by tuning the measurement noise covariance. A method based on the MLE estimates the noise covariance. The inputs of the ML objective function are the multi-step prediction errors and their covariance matrix generated by the UKF. We also compared the method with the CM algorithm which is a suboptimal estimation technique. The results generally show that the MLE method outperforms the CM method. However, the computational cost associated to the MLE method is somewhat larger than the computational cost of the CM method.

REFERENCES


