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## Operational modal analysis based prediction of actual stress in an offshore structural model

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### Abstract

In this paper the accuracy of predicting stresses directly from the operational responses is investigated. The basic approach to the stress prediction is to perform an operational modal analysis (OMA) and then applying a modal filtering to the operating response, so that the modal coordinates of all significant modes are known. Next, the experimental mode shapes are expanded using a finite element (FE) model together with the local correspondence principle to estimate the displacements in all degrees of freedom of the FE model, and strain is predicted using the strain mode shapes. The accuracy of the approach is assessed by comparing the predicted and measured strains.

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**Keywords:** Operational Modal Analysis; Modal Based Fatigue Estimation; Modal Expansion; Monitoring

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### 1. Introduction

Offshore structures accumulate damage during their operation because they are subjected to environmental and operational forces. The various forces vary and fluctuate continuously and this causes propagation of cracks that can lead to fatigue failure. The loading creates an irregular load history, which is difficult to predict. The practical design of offshore structures allows for these uncertainties by considering the load as a stochastic process. The calculation of the load history is based on norms and standards and it represents a simplified version of the reality.

Many offshore platforms in the North Sea are reaching the end of their design lifetime but the actual integrity of such structures is unknown. These oil platforms are expensive structures and the owners are faced with a loss of profit by either abandoning an oil reservoir or building a new platform. There is a significantly potential profit if the lifetime can be increased.

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A lot of effort has been put into the field of *Structural Health Monitoring* through the last three decades [1–4]. Unfortunately, there are several issues that complicate monitoring of offshore structures [2]. The hostile environments make the platforms abstruse and it hinders inspections. The corrosive saltwater damages underwater sensors and platform machinery introduces noise, which might cause measurements to become non-stationary. The system changes over time due to, for instance, ingress of water, marine growth, fluid storage levels and soil properties. A vibration based monitoring system is one possible solution to these problems. *Modal Based Stress Estimation* can extend the lifetime of offshore structures by reducing uncertainties on the stress history. There exist different techniques for full field strain estimation where the *Modal Expansion* [1,4–6] and the *Kalman filter* [6–8] are among the most used algorithms. Maes et al found that these methods are interchangeable [6].

This paper shows a way to reduce the uncertainty on the stress history by *Modal Expansion*. We will predict the strain history from the structural responses caused by ambient and operational excitation. The monitoring system is based on vibration measurements of the topside of a platform. This method uses a modal decomposition and a modal expansion technique so we are able to predict the strain history in any arbitrary point of a structure. In this paper we will demonstrate this method on a scale model of an offshore platform and we will validate the results by comparing the obtained results with strain gauge measurements.

## 2. Theory

### 2.1. Strain Estimation

We have a system response vector,  $\mathbf{y}(t)$ , a set of experimental mode shapes,  $\mathbf{A}$ , and a set of corresponding mode shapes from a highly correlated finite element model,  $\mathbf{B}$ . We can express the response of the linear system as a linear combination of the estimated mode shape vectors,  $\mathbf{A}$ , and the modal coordinates,  $\mathbf{q}(t)$ .

$$\mathbf{y}(t) = \mathbf{A}\mathbf{q}(t) \quad (1)$$

The modal coordinates are estimated by the pseudo inverse operator, as follows

$$\hat{\mathbf{q}}(t) = \mathbf{A}^+\mathbf{y}(t) \quad (2)$$

The experimental mode shape is smoothed by the use of the *Local Corresponding principle* [9]

$$\hat{\mathbf{A}} = \mathbf{B}_a\mathbf{P} \quad (3)$$

where  $\mathbf{B}_a$  is the finite element mode shape matrix, which contains the same active degrees of freedom as the experiment, and  $\mathbf{P}$  is the transformation matrix. We replace the reduced mode shape matrix with the full mode shape matrix for the entire finite element model,  $\mathbf{B}$ . This is equivalent to expand the experimental mode shapes in Eq. (3).

$$\hat{\mathbf{A}}_{full} = \mathbf{B}\mathbf{P} \quad (4)$$

where  $\hat{\mathbf{A}}_{full}$  is the experimental mode shape matrix, which is expanded to contain the same degrees of freedom as the finite element model. It has been shown that the modal coordinates for strain and displacement are the same [10]. Therefore, we can use the modal coordinates from Eq. (2) to estimate the strain response of the structure. We use the full strain mode shapes from the finite element model to estimate the strains in any point of the structure.

$$\hat{\boldsymbol{\varepsilon}}(t) = \mathbf{B}_\varepsilon\mathbf{P}\hat{\mathbf{q}}(t) \quad (5)$$

where  $\mathbf{B}_\varepsilon$  is the full strain mode shape matrix from the finite element model and  $\hat{\boldsymbol{\varepsilon}}(t)$  is the predicted strain response of the entire structure.

## 2.2. Quality Measurements

Aiming to assess the errors in the strain prediction, the Time Response Assurance Criterion (*TRAC*) [4] and Frequency Response Assurance Criterion (*FRAC*) [5,11] are used as quality measurements of the strain prediction.

$$TRAC_i = \frac{(\varepsilon_i(t)^T \hat{\varepsilon}_i(t))^2}{(\varepsilon_i(t)^T \varepsilon_i(t))(\hat{\varepsilon}_i(t)^T \hat{\varepsilon}_i(t))} \quad (6)$$

$$FRAC_i = \frac{(\varepsilon_i(f)^H \hat{\varepsilon}_i(f))^2}{(\varepsilon_i(f)^H \varepsilon_i(f))(\hat{\varepsilon}_i(f)^H \hat{\varepsilon}_i(f))} \quad (7)$$

where  $\varepsilon_i(f)$  is the direct Fourier transformed of the signal  $\varepsilon_i(t)$ . Both *TRAC* and *FRAC* are used to compare general shapes and tendencies in the data. Since neither of them take amplitude differences into account then we need some additional quality measurements to account for these discrepancies. Therefore, we use the coefficient of determination,  $R_{t,i}^2$ , which is used in model validation [12].

$$R_{t,i}^2 = 1 - \frac{\sum_{k=1}^N (\varepsilon_i(t_k) - \hat{\varepsilon}_i(t_k))^2}{\sum_{k=1}^N (\varepsilon_i(t_k) - E[\varepsilon_i(t)])^2} \quad (8)$$

where  $E[X]$  is the expected value of  $X$ . We can use the coefficient of determination in the frequency domain by the use of the absolute value of the Fourier transformed signal.

$$R_{f,i}^2 = 1 - \frac{\sum_{k=1}^N (|\varepsilon_i(f_k)| - |\hat{\varepsilon}_i(f_k)|)^2}{\sum_{k=1}^N (|\varepsilon_i(f_k)| - |E[\varepsilon_i(f)]|)^2} \quad (9)$$

## 3. Case Study

We will show the prediction of strain on a model of a tripod oil platform, which is a 1:50 scale model and made of polymethyl methacrylate, see Fig. 1 A). The scale model and the related model laws are explained further in [1].

### 3.1. Test Setup

12 uniaxial accelerometers are placed on the topside and the upper part of the main column, see Fig. 1 B). An additional 14 strain gauges are placed on the lower part of the main column and tripod, see Fig. 1 C). The data are acquired with a sampling frequency of 1651 Hz. The model was excited by brushing the topside with a nylon brush and this loading had a high level of white noise characteristics.

In order to remove noise around DC and measurement noise, we used a bandpass filter to filter the translation and strain measurements with cut-off frequencies at 6 and 80 Hz. The lower cut-off frequency is introduced to reduce the integration error. For the higher cut-off frequency, it is assumed that modes located above 80 Hz have an insignificant contribution to the strain response. Furthermore, the strain gauges have a high level of noise and this low signal-to-noise-ratio makes the modal response from modes above 80 Hz negligible. The electric grid in Denmark creates harmonic components at 50 Hz and we removed these from both acceleration and strain measurements by interpolating in the frequency domain.

### 3.2. Modal Identification

The data is decimated in the modal identification process to ease the calculations [13]. As a result, a new sampling frequency of 206 Hz is obtained. The correlation function and spectral density matrices are calculated using a segment size of 1024 [14]. The singular values of the spectral density are plotted in Fig. 2.

We used the *Eigensystem Realization Algorithm* to identify the modes of interest [15]. The Hankel block matrices have 3 block rows and 5 singular values are extracted in the singular value decomposition. This resulted in 5 identified modes that are listed in Table 1 and marked on Fig. 2 A). There is high correlation between the experimental and finite element mode shapes as shown on Fig. 2 B).

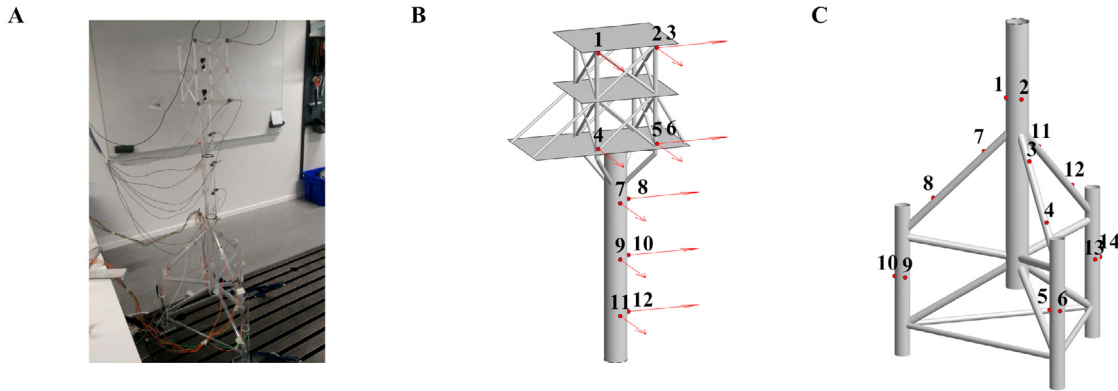


Fig. 1. A) Photo of the scaled offshore platform in the lab, B) Position of accelerometers, C) Position of strain gauges, which measure along the longitudinal direction of the elements

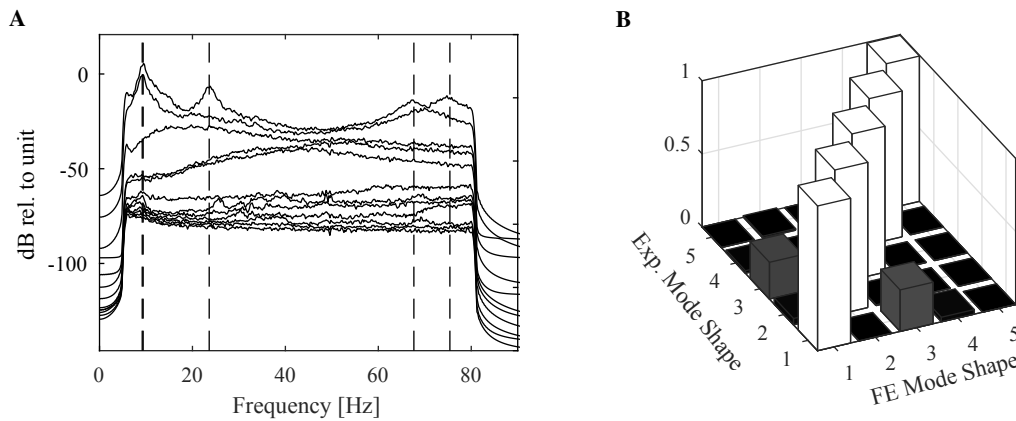


Fig. 2. A) Singular values of the spectral density matrix of the accelerometers and marked modes, B) MAC value between experimental and finite element mode shapes

Table 1. Modal Parameters

Mode Number	1	2	3	4	5
Frequency, [Hz]	9.19	9.39	23.6	67.7	75.4
Damping, [%]	5.63	5.94	3.99	4.18	3.61

### 3.3. Prediction of Strain

The acceleration was integrated twice to obtain the displacement vector,  $y(t)$ , using the differentiation properties of the Fourier transform [14]. The strain was predicted in all points where strain gauges were positioned, see Fig. 1 C. The predicted and measured strains were compared by the quality measurements and they are shown on Fig. 3. The FRAC values were calculated for the frequency range of 0-80 Hz.

The predicted and measured strains are plotted in Fig. 4 for two different channels of strain gauges. Channel 1 has the best correlation between predicted and measured strain and channel 12 has the worst correlation. The auto spectral density of the two channels are plotted in Fig. 5.

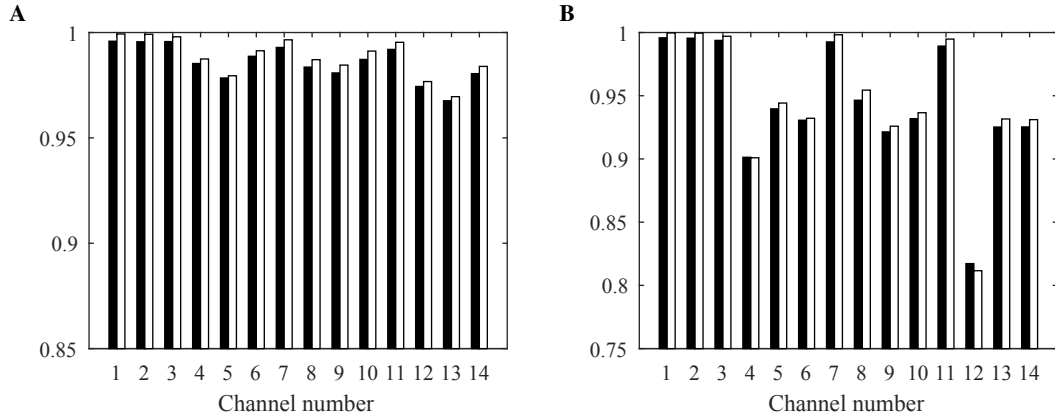


Fig. 3. A) TRAC (black) and FRAC (white), B)  $R^2_t$ , time domain, (black) and  $R^2_f$ , frequency domain, (white)

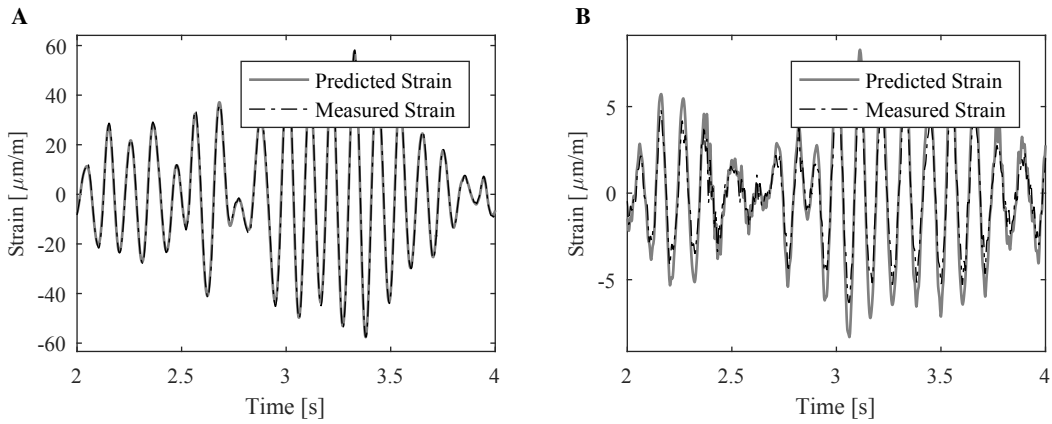


Fig. 4. A) Predicted and measured strain of channel 1, B) Predicted and measured strain of channel 12

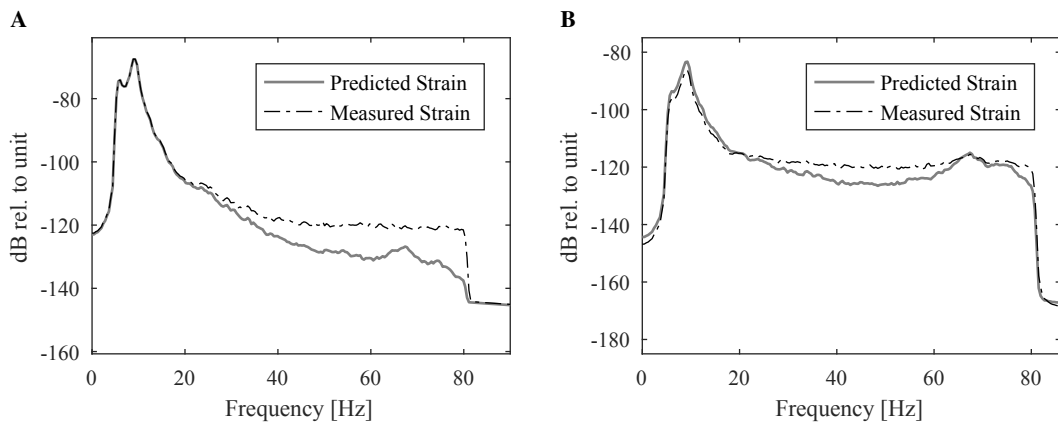


Fig. 5. A) Auto spectral density of the strain of channel 1, B) Auto spectral density of the strain of channel 12

#### 4. Conclusion

This paper presents a method of strain prediction based on vibration measurements above water and a finite element model. The method can be used to predict strain in points that would normally be inaccessible. The theory is validated

by experimental test on a scale model of a tripod platform. The test shows promising results and the strain history is predicted with high precision.

However, some issues need to be addressed in the experimental tests such as: (1) the high amount of measurement noise on the strain gauges makes them less than ideal as a reference point for the prediction of strain; (2) the noise floor of the sensors is located around -120 dB, see Fig 5, and spots with low strain, like channel 12, is close to this floor, which results in a low signal-to-noise ratio; and (3) the scale model does not take the hydrodynamic damping and effective mass of the waves into account. Furthermore, the quasi-static responses caused by waves should be examined in further work.

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