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Imaging geochemical heterogeneities using inverse reactive transport modeling: An example relevant for characterizing arsenic mobilization and distribution

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Highlights

- Approach to estimate the spatial distribution of reactive minerals in the subsurface.
- Multi-scale applications for As-bearing pyrite distribution using oxygen measurements.
- Computationally efficient inversion coupling flow and multi-species reactive transport.
- As-bearing pyrite distributions characterized successfully with PCGA inverse modeling.
Abstract

The spatial distribution of reactive minerals in the subsurface is often a primary factor controlling the fate and transport of contaminants in groundwater systems. However, direct measurement and estimation of heterogeneously distributed minerals are often costly and difficult to obtain. While previous studies have shown the utility of using hydrologic measurements combined with inverse modeling techniques for tomography of physical properties including hydraulic conductivity, these methods have seldom been used to image reactive geochemical heterogeneities. In this study, we focus on As-bearing reactive minerals as aquifer contaminants. We use synthetic applications to demonstrate the ability of inverse modeling techniques combined with mechanistic reactive transport models to image reactive mineral lenses in the subsurface and quantify estimation error using indirect, commonly measured groundwater parameters. Specifically, we simulate the mobilization of arsenic via kinetic oxidative dissolution of As-bearing pyrite due to dissolved oxygen in the ambient groundwater, which is an important mechanism for arsenic release in groundwater both under natural conditions and engineering applications such as managed aquifer recharge and recovery operations. The modeling investigation is carried out at various scales and considers different flow-through domains including (i) a 1D lab-scale column (80 cm), (ii) a 2D lab-scale setup (60 cm × 30 cm) and (iii) a 2D field-scale domain (20 m × 4 m). In these setups, synthetic dissolved oxygen data and forward reactive transport simulations are used to image the spatial distribution of As-bearing pyrite using the Principal Component Geostatistical Approach (PCGA) for inverse modeling.
1. Introduction

Accurate identification of physical and geochemical aquifer properties controlling groundwater contaminant behavior is critical for reliable contaminant plume prediction, remediation design and operation, and effective management of groundwater resources. Spatially distributed physical and geochemical heterogeneities control solute transport in groundwater; thus, they are of pivotal importance for understanding the fate of contaminants in the subsurface. Over the last few decades, numerous studies have focused on identifying spatially distributed physical properties of aquifers including hydraulic conductivity, porosity, and specific storage. Examples of widely used characterization techniques include pressure-based methods, such as hydraulic tomography [1–4], and geophysical-based methods including electrical resistivity tomography [5,6] and ground penetrating radar [7,8]. The pressure-based methods use pressure measurements, which are directly sensitive and mechanistically linked to the physical parameters of interest through the groundwater flow equation, to infer the spatial parameters, i.e., hydraulic conductivity fields. The geophysical-based methods can be used to map lithologic heterogeneities in regions that are difficult to install bore wells and implemented jointly with pressure-based methods. These techniques have been applied to successfully estimate or "image" the physical heterogeneity of the subsurface in field cases (e.g., [1]). Here, and throughout this manuscript, the term “imaging” is used to describe the estimation of the spatial distribution of an aquifer property of interest.

The spatial distribution of biogeochemical properties exerts a key control on contaminant transport. For instance, the heterogeneous distribution of organic matter in aquifer systems is necessary for understanding the sorption of both organic compounds
and inorganic contaminants, including arsenic [13–19]. Similarly, the location of reactive mineral phases can control the fate of metals and metalloids [20–22], mineral precipitation/dissolution rates [23], and bioclogging [24] in porous media. Therefore, to quantitatively understand contaminant transport and to implement successful groundwater management strategies, it is important to characterize the spatial distribution of geochemical properties. However, only a limited number of studies have been conducted to infer spatially heterogeneous geochemical fields, likely due to (1) the limited amount of sparse measurements available at the field scale, (2) the challenge in formulating quantitative descriptions of controlling geochemical processes in the framework of groundwater flow and reactive transport models, and (3) the complexity and computational burden associated with multi-species reactive transport modeling-based inversions. Recent works overcome the former by applying geophysical-based methods to estimate the spatial distribution of reactive elements and mineral phases [25–31] as well as reactive facies [32,33]. While these methods have been successfully applied to field settings with promising results, these approaches still require collecting a large amount of geochemical data to develop a site-specific petrophysical relationship (from geophysical to hydro-geochemical properties) or correlation while the true relationship may not be unique or one-to-one [34]. In this study, we propose an effective geochemical imaging method using reactive transport based inverse modeling as well as the information content of sparse groundwater chemistry data perturbed by the controlling geochemical processes. We demonstrate the capability of the proposed method to image spatially distributed geochemical properties of several synthetic domains using sparse aqueous concentration data. We focus on geochemical
environments relevant to the mobilization of arsenic via mineral dissolution in aquifer systems. In the last few decades, numerous studies have focused on understanding geochemical controls of As mobilization. While these studies have greatly contributed to developing a geochemical framework for As reactivity in the environment, arsenic contamination of groundwater remains problematic in aquifers throughout the world. Research efforts can be enhanced by developing inexpensive and efficient methods for quantifying As distribution in aquifer sediments.

Arsenic is a toxic metalloid and a naturally occurring contaminant that poses a significant threat to groundwater quality. The release of native As from sediments into surrounding pore water can be attributed to shifts in water chemistry via four principal mechanisms: (1) ion-displacement (2) shifts in pH to alkaline values exceeding 8.5 (3) reduction of arsenate to the more labile arsenite and (4) dissolution of arsenic-bearing minerals [35–37]. The spatial distribution of As-bearing heterogeneities is a controlling factor in all the above-mentioned processes. Here, we focus on a mechanism (4) which is relevant in natural and managed aquifers with fluctuating redox environments [38–46]. Specifically, we are interested in imaging As-bearing sulfidic minerals such as arsenopyrite and arsenian pyrite which can be a source of dissolved As in groundwater, particularly in shifting redox environments including injection of oxidizing recharge water into previously anoxic aquifers as part of aquifer storage and recovery operations [47–50].

The outline of the paper is as follows: in Section 2, we present the proposed methodology for forward and inverse modeling. The forward reactive transport model describes the mobilization of As via kinetic oxidative dissolution of As-bearing pyrite
due to dissolved oxygen (DO) in the ambient groundwater. The inverse model is based on the computationally-efficient Principal Component Geostatistical Approach (PCGA) [3,51]. Considering increasing complexity and accounting for the scale-dependent spatial distribution of As-bearing minerals (Figure 1), the modeling investigation was carried out at multiple scales and considered different flow-through domains including (i) a 1D lab-scale column (80 cm), (ii) a 2D lab-scale setup (60 cm × 30 cm) and (iii) a 2D field-scale cross section (20 m × 4 m). In Section 3, we illustrate the results obtained using DO observations at selected locations, and forward and inverse modeling to produce the best estimate of the spatial distribution of As-bearing pyrite. We also analyzed the estimation uncertainty and performance of the PCGA-based inverse method. Concluding remarks are provided in Section 4.

Figure 1: Spatial distribution of arsenic-bearing sulfidic minerals (grey in color) at a 6 m depth (right) in sediments of the upper Mekong Delta, Cambodia. The distribution of sulfidic minerals has scale-dependencies as seen by the centimeter-variation within the sediments (left).
2. Methods

The Principal Component Geostatistical Approach (PCGA) is a fast Jacobian-free geostatistical inversion approach that uses the leading principal components of the prior information to save computational costs. In this study, we propose the application of this method with reactive transport simulations of arsenic release. We used a forward reactive transport model including equilibrium and kinetically-controlled reactions and applied the methodology to 1-D bench-, 2-D bench-, and 2-D field-scales to estimate underlying As-bearing pyrite distributions and the corresponding estimation uncertainties. The method was implemented in MATLAB using MODFLOW [52] and PHT3D [53] to develop the flow and reactive transport forward model. In the following subsections, we describe the forward model, the PCGA inversion method, and the model parameters and domain properties of the various domains analyzed.

2.1 Forward model

The forward reactive transport model includes a reactive network developed with the geochemical code PHREEQC [54] and coupled to groundwater flow and multicomponent transport using PHT3D [53]. PHT3D uses a finite difference approximation to solve numerically the governing mass conservation equations, which for reactive solute transport of mobile aqueous components and immobile entities (e.g., minerals) read as:

\[
\frac{\partial C_n}{\partial t} = \frac{\partial}{\partial x_\alpha} \left( D_{\alpha\beta} \frac{\partial C_n}{\partial x_\beta} \right) - \frac{\partial}{\partial x_\alpha}(v_\alpha C_n) + \frac{q_s C_n^s}{\theta} + r_{reac,n} \\
\frac{\partial C_n}{\partial t} = r_{reac,n}
\]
where \( C_n \) is the total aqueous component concentration of the \( n^{th} \) component [53]. \( v_\alpha \) is the seepage velocity in direction \( x_\alpha \), \( D_{\alpha\beta} \) is the hydrodynamic dispersion coefficient tensor, \( q_s \) is a volumetric flow rate representing sources and sinks, \( \theta \) is porosity, is the concentration of the source or sink, and \( r_{reac} \) is a term describing the source/sink chemical reaction rate.

2.1.1 Physical properties

The influence of heterogeneity of physical properties for transport in porous media has been investigated in a number of contributions from pore to field scales (e.g., [55–59]). In order to focus on the estimation of chemical heterogeneities, the modeled domains were assumed to be physically homogenous with a uniform flow regime. The physical model properties are included in Table 1.

**Table 1: Geometry of the domain and parameters for the forward reactive transport model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1D bench</th>
<th>2D bench</th>
<th>2D field</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical properties</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Width (m)</td>
<td>0.8</td>
<td>0.6</td>
<td>20</td>
</tr>
<tr>
<td>Height (m)</td>
<td>0.01</td>
<td>0.3</td>
<td>4</td>
</tr>
<tr>
<td>Rows</td>
<td>1</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>Columns</td>
<td>80</td>
<td>60</td>
<td>100</td>
</tr>
<tr>
<td>Grid size in x and y (( \Delta x, \Delta y ))</td>
<td>0.01</td>
<td>0.01/0.01</td>
<td>0.4/0.08</td>
</tr>
<tr>
<td>Porosity</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Hydraulic Conductivity (m/day)</td>
<td>17.5</td>
<td>17.5</td>
<td>35.5</td>
</tr>
<tr>
<td>Molecular diffusion coefficient (m(^2)/day)</td>
<td>( 3.5 \times 10^{-5} )</td>
<td>( 3.5 \times 10^{-5} )</td>
<td>( 3.5 \times 10^{-5} )</td>
</tr>
<tr>
<td>Longitudinal dispersivity ( a ) (m)</td>
<td>( 7.7 \times 10^{-4} )</td>
<td>( 7.7 \times 10^{-4} )</td>
<td>( 2.0 \times 10^{-2} )</td>
</tr>
<tr>
<td>Transverse dispersivity ( b ) (m)</td>
<td>-</td>
<td>( 7.7 \times 10^{-5} )</td>
<td>( 2.0 \times 10^{-3} )</td>
</tr>
<tr>
<td>Seepage velocity (m/day)</td>
<td>0.5</td>
<td>0.5</td>
<td>3.7</td>
</tr>
</tbody>
</table>

**Solid-phase components**
Concentration pyrite \(^b\) (mg/kg soil) | 7498.8 | 7498.8 | 2249.6
Concentration As (mg/kg soil) | 70.3 | 70.3 | 21.1
As in pyrite (wt %) | 0.94% | 0.94% | 0.94%

<p>| Influent aqueous concentrations (mol L(^{-1})) |</p>
<table>
<thead>
<tr>
<th>pH</th>
<th>HCO(^3)-</th>
<th>Cl(^-)</th>
<th>Fe(_T)</th>
<th>Na(^+)</th>
<th>DO</th>
<th>SO(_4^{2-})</th>
<th>As(_T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>(1.0 \times 10^{-3})</td>
<td>(1.0 \times 10^{-3})</td>
<td>0</td>
<td>(1.82 \times 10^{-3})</td>
<td>(2.75 \times 10^{-3})</td>
<td>(1.0 \times 10^{-6})</td>
<td>0</td>
</tr>
</tbody>
</table>

\(^a\) values consistent with those reported in high-resolution laboratory [60] and field investigations [61,62]
\(^b\) values within range of concentrations listed in [63]

2.1.2 Chemical reaction framework

The forward simulations were based on a reaction network including 57 aqueous equilibrium speciation half reactions, 2 kinetic mineral reactions, 13 surface complexation reactions and reproduced scenarios in which an initially anoxic solution is displaced by an aerated solution of the same ionic composition containing \(2.75 \times 10^{-4}\) mol/L of dissolved oxygen. We focus on the redox reaction resulting in the oxidative dissolution of As-bearing pyrite due to the introduction of oxygenated water. The chemical composition of the influent solution (Table 1) is identical to the initial solution except the initial solution is anoxic and does not contain oxidized species. The reaction network is similar to networks used in previous studies of arsenic reactive transport (e.g. [64,65]) The introduction of oxygen into the previously anoxic domain results in the kinetic oxidative dissolution of As-bearing pyrite. Briefly, the rate of pyrite oxidation was defined by the following expression developed by [66] and previously used in field applications [48,67]:

\[
r_{pyr} = C_{O_2}^{0.5} C_{H^+}^{-0.11} \left(10^{-10.19} \frac{A_{pyr}}{V}\right) \left(\frac{C}{C_0}\right)_{pyr}^{0.67}
\]

where \(r_{pyr}\) is the rate of pyrite oxidation (mol/L/s), and \(C_{O_2}\) and \(C_{H^+}\) are the aqueous concentrations of oxygen and hydrogen ions, respectively (mol/L). \(A_{pyr}/V\) is the ratio of...
pyrite surface area to the volume of solution and is set to 115 [48]. $C/C_0$ is a factor accounting for surface area changes resulting from the dissolution of pyrite. While reaction (1) is slightly pH dependent, the bicarbonate in the groundwater buffers the solution pH. The release of As(III) was simulated as stoichiometrically linked to the oxidation of pyrite [50,64,65] which follows the following reaction:

$$FeS_2 + 3.75O_2 + 0.5H_2O \rightarrow Fe^{3+} + 2SO_4^{2-} + H^+ \quad (3)$$

The molar fraction of As to pyrite was set to 0.015 and was within previously reported ranges of sedimentary materials [63,68].

During the modeled dissolution of As-bearing pyrite, dissolved As(III) was oxidized to As(V) which was the dominant As specie at steady-state. Additionally, ferrous iron was released and subsequently oxidized to ferric iron. This resulted in the precipitation of ferrihydrite (HFO) which can sorb As and retard the transport of this contaminant. The rate of HFO formation is kinetic and was expressed as

$$r_{HFO} = k \left( 1 - \frac{IAP}{K_{sp}} \right) \quad (3)$$

where $r_{HFO}$ is the rate of formation of HFO, $k$ is an empirically derived constant set to $10^{-14}$ mol m$^{-2}$ s$^{-1}$ [65], and $IAP/K_{sp}$ is the saturation ratio of ferrihydrite. The surface complexation of As(III) and As(V) species on ferrihydrite was modeled as an equilibrium-controlled process with a sorption site density of 0.2 mol/mol ferrihydrite and a surface area of 600 m$^2$/g [69].

While other geochemical processes such as ligand exchange, competitive sorption can affect arsenic transport (e.g., [16,70–72]), we focused on the oxidative dissolution of reduced minerals as the main controlling mechanisms. We assumed DO concentrations
were controlled by the consumption of oxygen during the oxidation of sulfidic minerals. Accordingly, we used distributed observations of DO to image the heterogeneous distribution of As-bearing pyrite inclusions. In natural settings, DO consumption will be a result of numerous geochemical processes inclusive of the oxidation of organic matter and formation of oxide minerals such as manganese oxides which are both processes that can affect the cycling of As in sediments (e.g., [18,73–75]). However, field-scale studies have successfully simulated DO measurements by focusing on oxidative dissolution of sulfidic minerals as the controlling process [64]. Therefore, we focus only on the oxidative dissolution of As-bearing sulfides and precipitation of Fe (hydr)oxides as DO consuming processes. Other processes can be accounted for with the error added to simulated DO measurements as described below.

2.2 Inverse model

We used the PCGA to determine the spatial distribution of As-bearing pyrite from a limited number of observations of DO concentrations while other parameters in Equations (1) and (3) were assumed to be known and are provided in text. The PCGA is a fast large-scale and joint inversion approach based on the geostatistical inverse method [76,77], which is widely used for spatially distributed subsurface property calibration problems. Here, we include a brief description of the geostatistical method followed by a description of the PCGA.

2.2.1 Overview of the geostatistical approach

The objective of the geostatistical approach is to determine a vector of unknowns from a vector of observations in a Bayesian framework. The observation equation is given by:
\[ y = h(s) + \nu \]  

where \( y \) is the vector of observations, \( h \) is the forward model, \( s \) is the unknown variable, and \( \nu \) is a Gaussian term accounting for error in the observations and forward model. The prior probability of \( s \) is assumed to be Gaussian with an unknown mean and a prior covariance matrix \( Q \). Using this assumption, the posterior pdf of \( s \) is calculated using Bayes' theorem and the negative log likelihood of the posterior pdf forms the objective function. By minimizing the objective function, the maximum a posteriori (MAP) estimate or most likely value of \( s \) is obtained. This minimization of the objective function is a nonlinear optimization problem that is commonly solved using iterative Gauss-Newton method where the initial guess is repeatedly updated to obtain a new As-bearing pyrite distribution fitting the data. Specifically, in this application focused on imaging reactive As-bearing minerals, \( y \) is the oxygen concentration at monitoring well, \( h \) is reactive transport model, and \( s \) is spatially distributed As-bearing pyrite concentration.

While the geostatistical method is well suited for small- to moderate-scale inverse problems, it becomes computationally challenging and costly for large-scale inversions. Furthermore, the geostatistical method requires computation of the derivative of the forward problem (Jacobian matrix, i.e., sensitivity of the observations to the unknowns) which typically requires intrusive changes in the forward model code which is challenging for multi-physics problems that utilize coupled forward models.

2.2.2 The principal component geostatistical approach

The PCGA expedites the geostatistical inversion by avoiding the direct evaluation of the Jacobian. As a result, the number of numerical simulations can be reduced by a factor of 10 or more with controlled accuracy in most cases. The PCGA avoids the direct
computation of the Jacobian by utilizing a low-rank approximation of the covariance \( \mathbf{Q} \) and a finite difference approximation of all matrix-vector products. \( \mathbf{Q} \) is approximated through:

\[
\mathbf{Q} \approx \mathbf{Q}_K \approx \mathbf{Z} \mathbf{Z}^T \approx \sum_{k=1}^{K} \zeta_i \zeta_i^T
\]  

(5)

where \( \mathbf{Q}_K \) is a rank-\( K \) approximation of \( \mathbf{Q} \), \( \mathbf{Z} \) is a \( m \) by \( K \) matrix and \( \zeta_i \) is the \( i \)-th column vector of \( \mathbf{Z} \). A fast method to obtain Equation (5) for large-scale covariance matrices is explained in [3]. Approximately "\( K \)" forward simulation evaluations are needed to obtain the inverse solution. By applying Equation (5), the PCGA can reduce the number of numerical forward simulations greatly, usually up to a few hundred simulation runs, which is highly beneficial for complex multi-species reactive transport problems, while inverse solutions are almost the same as those obtained from the conventional geostatistical approach as shown in [3].

To enforce positivity in the estimation of As-bearing pyrite concentrations (\( C_{\text{pyr}} \)) we used a square-root transformation:

\[
\mathbf{s} = \sqrt{\mathbf{C}_{\text{pyr}}}
\]  

(6)

This transformation can be seen as a simplified version of Box transformation [78], which has been used to contaminant source identification applications (e.g., [79,80]). The prior pdf of \( \mathbf{s} \) was assumed to be Gaussian with unknown mean and exponential covariance, \( \mathbf{Q} \). Numerical simulations and inversion were carried out on a Linux workstation equipped with Intel 16 core 3.1 GHz processors and 128 GB RAM.

### 2.3 Problem setup

#### 2.3.1 1-D bench
Beginning with a simplified domain, we simulated the oxidation of As-bearing pyrite from a 1-D laboratory column of approximately 80 cm in length. The column contained a 30 cm long inclusion of As-bearing pyrite in an otherwise homogenous domain as shown in Figure 2b. The spatial and temporal changes in chemistry during the forward reactive transport simulation are illustrated in Figure 2 for this 1-D domain. As a result of oxidative dissolution, dissolved As, Fe(III) and SO$_4^{2-}$ were released from the pyritic mineral lens (Figure 2a). Ferric iron was released at a lower concentration than SO$_4^{2-}$ due to the precipitation of ferrihydrite (Figure 2b) and the lower abundance of Fe than S in the pyritic minerals. The transport of As is also affected by the adsorption on ferrihydrite surfaces. The 1-D column simulation reached pseudo-steady state conditions after approximately 1.5-day simulation time (Figure 2c) in which the outlet concentrations of dissolved chemical species were stable due to the slow dissolution of As-bearing pyrite. After sufficient time, all the As-bearing pyrite will react and leave the system, but due to the slow kinetics of As-bearing pyrite dissolution the system reaches a pseudo-steady state in which the reaction byproduct concentrations are unchanged. Among the dissolved chemical species, we assumed that the steady-state oxygen concentrations were collected as the observation data for the inversion. This water chemistry parameter was selected due to its key importance for oxidative dissolution of reduced minerals and since currently available oxygen measurement sensors are relatively inexpensive, accurate and operable in real-time. Such sensors have been recently used to obtain high-resolution distributed measurements of oxygen in laboratory setups [81–84] and field applications [85]. Gaussian random noise with a standard deviation of 2% of the maximum oxygen consumption (10$^{-6}$ mol/L) was added to the
observations to represent measurement and conceptual modeling errors in a well-controlled column experiment. The number of observations, unknowns and magnitude of measurement error are listed in Table 2.

Table 2: Parameters of the inverse model carried out using the PCGA approach

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1D bench</th>
<th>2D bench</th>
<th>2D field</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of observations</td>
<td>5</td>
<td>20, 50, 75, 150</td>
<td>250</td>
</tr>
<tr>
<td>Number of unknowns</td>
<td>80</td>
<td>1800</td>
<td>5000</td>
</tr>
<tr>
<td>Number of principal components, $K$</td>
<td>80</td>
<td>32, 64, 96, 192</td>
<td>96</td>
</tr>
<tr>
<td>Covariance kernel, $q$</td>
<td>exponential</td>
<td>exponential</td>
<td>exponential</td>
</tr>
<tr>
<td>Correlation length $\lambda$ (m)</td>
<td>0.6</td>
<td>$\lambda_x = 0.6, \lambda_z = 0.3$</td>
<td>$\lambda_x = 20, \lambda_z = 4$</td>
</tr>
<tr>
<td>Variance</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Measurement error (mol/L)</td>
<td>$10^{-6}$</td>
<td>$10^{-6}$</td>
<td>$2.5 \times 10^{-5}, 5 \times 10^{-5}$</td>
</tr>
</tbody>
</table>
Figure 2: (a) Spatial profiles of dissolved As, $\text{SO}_4^{2-}$ and Fe(III) concentrations at pseudo-steady state (2 days). (b) Solid-phase concentrations of pyrite at pseudo-steady state which diminished from initial concentration of 0.3 mol/L and Fe (hydr)oxides which form as a result of pyrite oxidative dissolution and subsequent oxidation of Fe(II). (c) Breakthrough curves of As and $\text{O}_2$ concentrations (mol/L) at the outlet of the domain.
2.3.2 2-D bench scale

Next, we imaged As-bearing pyrite inclusions in a 2-D bench-scale system. The 60 by 30 cm domain contained two 11 by 7 cm box-shaped inclusions in an otherwise homogenous system as shown in Figure 4a. We assumed that five vertical arrays of oxygen sensors were installed every 15 cm (x = 0, 0.15, 0.3, 0.45 and 0.6 m) (Figure 4a). A measurement error of 4% of the maximum oxygen consumption was added to the observed concentrations simulated from "true" As-bearing inclusions. Using this domain, we examined (1) the number of forward simulation model runs, i.e., the number of principal components (K) needed to obtain a reliable estimate of the unknown field and (2) the effect of the number of measurements on the accuracy of the best estimate.

2.3.3 2-D field scale

To demonstrate the potential of the proposed approach to map complex patterns of field-scale geochemical heterogeneities, we expanded our analysis to a 20 m × 4 m cross-sectional domain. For this scenario, the spatial distribution pattern of geochemical heterogeneity was generated stochastically: two realizations of Gaussian random fields from an anisotropic exponential covariance model [86] were thresholded to obtain two random binary As-bearing pyrite distributions as shown in Figure 7a.

3. Results and Discussion

3.1 1-D bench scale

The "true" field shown in Figure 3a was used to generate five steady-state synthetic concentration measurements with observation locations displayed as red circles in Figure 3c. Because this 1-D forward problem can be solved efficiently in 5 seconds, the PCGA with all K = 80 principal components (i.e., equal to the number of grid cells)
was used to estimate the underlying As-bearing pyrite concentrations to illustrate how the inversion approach performs for geochemical heterogeneity identification. In this case, low-rank approximation was not used and the approximation error only comes from the finite difference approximation in the inverse method. Inversion with smaller number of principal components is presented in the 2-D simulations. While the initial guess was set to a homogeneous As-bearing pyrite distribution with a concentration of 5000 mg/kg across the entire domain, several initial guesses were tested and the inversion converged to the same estimate for any reasonable choice of an initial guess.

**Figure 3:** (a) True concentration (mg/kg) of As-bearing pyrite, (b) estimated concentration (mg/kg), (c) steady-state oxygen concentration resulting from the
oxidative dissolution of true As-bearing pyrite and selected measurements (red circles) with noise for the inversion and (d) simulated steady-state oxygen concentrations resulting from the estimated As-bearing pyrite and reproduced oxygen concentrations (red circles) at the monitoring locations

A total of 329 forward model runs (converged with 4 iterations) were used to obtain the result on 16 processors in 5 minutes. Structural parameters such as noise level, prior variance and correlation length were chosen using Q2/Cr criteria [87] which provides a framework for selecting the model parameters to be used for the inversion. The resulting parameter values are reported in Table 2. The inversion successfully captures the location and magnitude of the true As-bearing pyrite concentration (Figure 3b) while the estimated distribution is smooth due to the Gaussian distribution assumption of the prior. The true As-bearing pyrite distribution is located within the 95% confidence interval (Figure 3b) of the inversion. Confidence interval was relatively wide at the constant concentration boundary condition (x = 0 cm) and the most downstream (x = 80 cm). Because of the fixed boundary condition at x = 0 cm and advective information propagation, the information of measurements at both locations is almost zero, resulting in the high uncertainty around the both ends. The estimated pyrite reproduces the DO measurements accurately within measurement error as shown in Figure 3d. This simple experiment demonstrates that a reasonable estimate can be achieved with a few hundred simulation runs even with only a few sparse oxygen measurements.

3.2 2-D bench scale
The true field for a 2-D bench-scale domain (Figure 4a) was used to generate the DO observation measurements in Figure 4a. These measurements were successively used for the PCGA-based inversion to image the location and the concentration of the As-bearing reactive mineral inclusions in the 2-D domain.

**Figure 4:** The effect of the number of principal components (K) on the pyrite distribution estimate. (a) shows the true pyrite distribution (first column) and
corresponding oxygen concentration (second column) along with the DO measurements which are highlighted with black markers. The best estimates of pyrite distribution, corresponding simulated oxygen concentration and variance of estimate are shown for K = (b) 512, (c) 96, (d) 64, and (e) 32.

3.2.1 Effect of number of principal components (K)

Initially, it was assumed that each sensor array collected a DO depth profile with measurements every 1 cm in depth, which amounts to a total of 150 measurements used in the inversion. To investigate an optimal number of principal components in this application, K = 512, 96, 64 and 32 principal components were used to estimate the underlying pyrite concentration. Sixteen multiples of K values were assigned to take advantage of parallel executions on the 16-core workstation. For K = 512, a total of 2576 forward simulations were executed to find a best estimate in around 10 hours; while for K = 32, total 176 simulations were run in 40 minutes.

The best estimates are presented in Figure 4b-4e. Even with only 32 principal components, the inversion was able to capture the location and magnitude of the underlying pyrite lenses. Also the best estimate of the oxygen consumption, obtained with the different numbers of principal components, closely reproduced the pattern observed in the true field. We used the root mean square error (RMSE) as a quantitative metric to assess the performance of the inversion. Calculating the RMSE of the best estimate as compared to the true distribution, we observed that it does not change significantly as a function of K (Figure 6a). This is due to the simple geometry of the true
concentration field, which only requires a relatively small number of simulations to obtain a reasonable estimate.

While the estimated location of the pyrite lenses can be accurately determined with a small number of principal components, the estimation uncertainty was generally underestimated because of the approximation used in Equation (4). Increasing K resulted in decreased RMSE of the uncertainty as shown in Figure 6c. Thus, it is recommended that more principal components be added to accurately quantify estimation uncertainty. While this requires more simulation runs, starting with a moderate number of K values and increase K when convergence is achieved, the correct estimation uncertainty can be gained.

3.2.2 Effect of number of measurement points

To evaluate the efficiency of the inversion approach, the effect of the number of data measurements used in the inversion was examined. Figures 5b-5e show the best estimates for the location and concentration of the As-bearing pyrite inclusions, the corresponding oxygen consumption and the computed estimation uncertainty with 150, 75, 50, 20 measurements which corresponds to 1, 2, 3 and 7.5 cm vertical spacing of measurement locations, respectively. Good estimates of As-bearing pyrite distribution were obtained for the cases considering at least 50 data points. The quality of the image deteriorates significantly for the case using only 20 measurement points. Inversion with 20 measurements (7.5-cm spacing which is larger than the 7-cm height of the pyrite lens) resulted in poor identification of the pyrite distribution. In this case, the method did not capture the bottom-right inclusion and also the concentration range of the image becomes rather dissimilar from the true field. This results from the impossibility of the very sparse...
observation network to capture the oxygen concentration gradients due to oxidative
dissolution in the lower part of the domain. Again, we used RMSE to quantitatively
evaluate the performance of the inversion as a function of the number of observation
points. We observed that the RMSE decreases significantly with increasing number of
observations (Figure 6b). However, it should be noted that such RMSE decrease tends to
be less pronounced with larger number of measurements for the distribution of
measurements examined in this domain. The results emphasize that even with a relatively
small number of observations (~50 measurement points), the proposed approach was able
to identify the location and magnitude of the As-bearing pyrite distribution while smaller
number of data measurements could result in errors in the estimation of the sulfidic
mineral location.
Figure 5: Effect of the number of measurements on pyrite estimates. The true pyrite and oxygen distributions are shown in (a). The best estimates of pyrite distribution, corresponding simulated oxygen concentration and uncertainty are shown for (b) 150, (c) 75, (d) 50, and (e) 20 total measurements. Black circles highlight the location of the dissolved oxygen measurements.
Figure 6: RMSE as a function of (a) the number of principal components, (b) number of measurement points used in the inversion and (c) RMSE relative to uncertainty of the best estimate for 512 principal components.

### 3.3 2-D field scale

Lastly, field-scale characterizations were performed to study potential applications of the inversion method. Two random heterogeneous fields were generated and tested as shown in Figures 7a and 7b. Oxygen sensor arrays are assumed to be located at x = 0, 4, 8, 12, 16 and 20 m and at each array, sensors were installed every 0.25 m corresponding to a total of 96 measurement points for this analysis. Larger error of 10% and 20% of the maximum oxygen consumption was added to the simulated concentrations to reflect higher uncertainty at the field-scale. The best estimates are shown in Figure 7. With measurement error of 10% and irregularly distributed heterogeneities, the inversion method can accurately identify the location of the As-bearing pyrite lenses. At 20% measurement error, difficulties were encountered in capturing the location and magnitude of all the heterogeneities, particularly small pyrite
lenses. We also performed forward simulations using the best estimate of the spatial distribution of pyrite to analyze the capability of the inversion results to reproduce the arsenic plumes from the true pyrite distribution. The estimated As-bearing pyrite locations and magnitude reproduced the main features and the multiple plumes of dissolved As within the simulated aquifer (Figure 8). With 10% measurement error, the location of all high As concentration plumes were accurately captured although the magnitude was slightly underestimated. With 20% measurement error the main arsenic plumes are still correctly located but not all chemical heterogeneities causing As release were captured; thus, not all As plumes were reproduced and the magnitude of As concentration was underestimated, in particular in the upper plumes.
Figure 7: Best estimate maps for randomly distributed heterogeneous fields of As-bearing pyrite for (a) Case 1 with (c) 10% error, (e) 20% error, and for (b) Case 2 with (d) 10% error and (f) 20% error.

Figure 8: Dissolved arsenic concentrations at pseudo steady state for (a) the true As-bearing pyrite distribution and (b) the estimated pyrite distribution obtained in Figure 7c with 10% measurement error and (c) the estimated pyrite distribution obtained in Figure 7e with 20% measurement error.

Field-scale imaging of chemically reactive heterogeneities can be useful for a variety of applications. For example, one potential application includes managed aquifer
recharge (MAR) projects where shifts in native aquifer chemistry can mobilize contaminants including As [48,50,63,88,89]. MAR sites in which common water quality parameters are regularly monitored are potentially well-suited sites for inverse reactive transport modeling methods to image reactive geochemical heterogeneities due to installation of water quality monitoring well networks. Additionally, MAR sites that employ aquifer storage and recovery (ASR) wells provide a unique reversal in flow regime which can further enhance inversion results. Generally, upstream locations have more information available through the downstream measurements. Thus, we tested a scenario with reversed flow conditions, by switching downstream and upstream locations. This improves the imaging results of previously down-gradient heterogeneities that contained higher uncertainty. Specifically, Figure 9 shows a simulated inversion for the heterogeneous field-case in which reverse flow data were used to image pyrite lenses (Figure 9b) and the forward and reverse flow measurements were used in conjunction (Figure 9c) to further enhance the imaging of the reactive mineral inclusions. Furthermore, in the reverse flow case, previously unidentified pyrite lenses (Figure 7f) are now characterized relatively well, and the combination of two different data sets with higher measurement error provides a comparable result to Figure 7d, which uses unidirectional flow but with smaller measurement error. RMSEs for forward flow (Figure 7f), reverse flow (Figure 9b) and joint (Figure 9c) cases with respect to the true field (Figure 7b) are computed as 623, 585 and 557 mg/kg respectively, while RMSE of the case with unidirectional forward flow and 10% measurement error (Figure 7d) is 514 mg/kg.
Figure 9: (a) Random, true As-bearing pyrite distribution, (b) the best estimate using only information for a reversed flow regime with 10% noise (flow right-to-left), and (c) the best estimate using data measurements from both left-to-right flow regime (Figure 7f) and the right-to-left flow regime.

4. Conclusion

Our results show that the proposed PCGA-based inverse reactive transport modeling approach can be used to image irregularly distributed As-bearing pyrite inclusions at various scales and has the potential to map complex distributions of reactive
subsurface properties based on the measurement of reactive dissolved constituents commonly measured in groundwater. While we develop this method using a simplified modeling approach, the method can be adapted to accommodate more complex reaction networks. This application and similar approaches will be facilitated by rapid advances in computational methods and sensor technology.

We demonstrated the capability and potential of inverse methods for geochemical tomography by focusing on As-bearing minerals, and there are numerous additional applications in which these methods can greatly help characterize subsurface geochemical heterogeneities. Possible directions for exploring include: testing the proposed approach with experimental observations, considering the value of information of transient observations taken in different sampling events, using observations of multiple parameters (e.g., dissolved oxygen with pH or other dissolved species) perturbed by specific geochemical reactions.

Of key importance is the deployment of these methods for the simultaneous characterization of both physical and chemical subsurface properties. In this study, we focused on characterizing reactive chemical properties as a proof-of-concept; however, coupling of physical and chemical subsurface imaging is essential for ensuring proper management of groundwater aquifers and enhancing the reliability of process-based reactive transport models. Additionally, the accurate determination of spatially distributed physical and geochemical properties in field systems provides valuable understanding of the influence of aquifer characteristics on contaminant behavior.
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