Bin-packing problems with load balancing and stability constraints

Trivella, Alessio; Pisinger, David

Publication date:
2017

Document Version
Peer reviewed version

Link back to DTU Orbit

Citation (APA):
Trivella, A., & Pisinger, D. (2017). *Bin-packing problems with load balancing and stability constraints*. Paper presented at INFORMS Transportation and Logistics Society 2017, Chicago, United States.

Bin-packing problems with load balancing and stability constraints

Alessio Trivella*, David Pisinger
Department of Management Engineering
Technical University of Denmark, Kgs. Lyngby, Denmark
*Email: atri@dtu.dk

1 Introduction

The *Bin-Packing Problem* (BPP) is one of the most investigated and applicable combinatorial optimization problems. The problem consists of packing objects of different sizes into a finite number of similar bins, such that the number of used bins is minimized. Applications of the bin-packing problem apper in a wide range of disciplines, including transportation and logistics, computer science, engineering, economics and manufacturing. The problem is well-known to be $\mathcal{NP}$*-hard* and difficult to solve in practice, especially when dealing with the multi-dimensional cases.

Closely connected to the BPP is the *Container Loading Problem* (CLP), which addresses the optimization of a spacial arrangement of cargo inside a container or transportation vehicle, with the objective to maximize the value of the cargo loaded or the volume utilization. The CLP focuses on a single container, and has been extended in the literature to handle a variety of different constraints arising from real-world problems. Consider for example the problem of arranging items into an aircraft cargo area such that the barycenter of the loaded plane is as close as possible to an ideal point given by the aircraft's specifications. The position of the barycenter has an impact on the flight performance in terms of safety and efficiency, and even a minor displacement from the ideal barycenter can lead to a high increase of fuel consumption [1]. Similar considerations apply when loading trucks and container ships.

The aim of this work is to integrate realistic constraints related to e.g. load balancing, cargo stability and weight limits, in the multi-dimensional BPP. The BPP poses additional challenges compared to the CLP due to the supplementary objective of minimizing the number of bins. In particular, in section 2 we discuss how to integrate bin-packing and load balancing of items. The problem has only been considered in the literature in simplified versions, e.g. balancing a single bin or introducing a feasible region for the barycenter. In section 3 we generalize the problem to handle cargo stability and weight constraints.

2 The integrated packing and balancing problem

Packing and balancing the load of a set of items represent two conflicting objectives and it can be convenient to incorporate them in a single problem. In [2] we develop an integrated approach for solving the multi-dimensional bin-packing problem with load balancing. The goal is to arrange the items in the smallest number of bins, while ensuring the best overall load balancing of the used bins, i.e. ensuring that the average barycenter of the loaded bins falls as close as possible to an ideal point, for instance the center of the bin. We use:

- a Mixed-Integer Program (MIP) model. Despite having a quadratic number of variables and constraints, the model is considerably more complex than the BPP as several new sets of support binary variables and conditional constraints are necessary to optimize load balancing. The model is able to solve only instances involving up to 20 items.
- a multi-level local search heuristic able to deal with large instances. The algorithm takes advantage of the Fekete-Schepers representation of feasible packings in terms of particular classes of interval graphs [3], and iteratively improves the load balancing of a bin-packing solution using three different search levels:
    1. The first level explores the space of transitive orientations (TROs) of the complement graphs associated with the packing. This is made possible by exploiting nice theoretical properties of interval graphs and a relation TRO-packing.
    2. The second level modifies the structure of the interval graphs.
    3. The third level exchanges items between bins by repacking proper n-tuples of weakly balanced bins, coded inside a variable neighborhood search framework.

Extended computational experiments can be found in [2]. The results reveal that an effective load balancing can be obtained, with a running time rarely exceeding 3-5 minutes even for difficult instances with up to 200 items (C program on an Intel Core i5 with 8GB RAM). When the optimal barycenter is the geometric center of the bin, for instance, more than 95% of the initial imbalance can be removed by using the three search phases.
In Table 1 is a summary of the main features of the two approaches and the differences between them. It will be useful when considering additional constraints in the next section.

Table 1: MIP vs. local search.

	MIP	Local search
algorithm type:	exact	heuristic
objective function:	only L_1	any, e.g. L_p
ideal barycenter:	any	any
instances handled:	small	medium/large

3 Including stability and weight constraints

In practical contexts, additional properties are relevant when packing containers to ensure the stability of the cargo (see e.g. [4, 5]). Mechanical stability can be enforced at a *static* or *dynamic* level, related to obtaining a stable motionless or moving cargo, respectively. Moreover, a number of weight constraints may apply depending on the specific transport application. In the following, we limit the discussion to static stability, which is by itself challenging, and provide an overview of the different stability and weight requirements considered.

Table 2: Static stability and weight constraints.

Property	Description
Lower gapless	Each box touches another box (or the bin) below it
Full base support	The entire base of each box is in contact to other boxes (or the bin)
Partial base support	A fix percentage of the box base is in contact to other boxes (or bin)
Barycenter support	The box barycenter is located above the surface of a supporting box
Mechanical equilibrium	The sum of external forces and torque acting on each box is zero
Load bearing	Maximum pressure that can be applied over the top face of a box
Weight limit	Items packed in each bin cannot exceed a given maximum weight
Weight distribution	The weight is distributed within the bin according to certain criteria

In the previous section, we provided two tools to solve different instances of the load-balanced bin-packing problem: an MIP model and a heuristic based on local search. The goal now is to understand whether it is possible (and straightforward) to embed the additional constraints of Table 2 into the two algorithms. In the remainder of the section, we briefly discuss two of the properties; the others will be examined more in detail in the paper following this abstract.

3.1 Gapless property

The local-search algorithm can be easily adapted to lower gapless requirements. Indeed, each TRO of the z-interval graph complement can be associated to a lower gapless packing. In other words, we can simply collapse the bunch of packings associated to a TRO to a single lower gapless solution, narrowing in this way the search space.

The gapless property is, in contrast, very hard to model in the MIP framework. However, if the ideal barycenter location is the center of the bin's base, than enforcing the lower gapless property is actually not needed, as an optimal solution necessarily satisfies it. If not, it would be possible to move downwards at least one box in one bin, keeping the others fixed, decreasing the total cost.

3.2 Weight limit

The MIP model contains binary variables $c_{ij} = 1$ if and only if item i is in bin j. Using these variables, it is easy to identify the total load of a bin and impose a limit by adding for each bin j the constraints $\sum_i c_{ij} m_i \leq \mathcal{W}^{max}$, where m_i is the mass of item i. The weight limit can also be formulated here as bin-dependent $\mathcal{W}^{max} = \mathcal{W}_j^{max}$.
Integrating weight limits in the heuristic algorithm affects both the construction phase and external search (recombination of bins). The two phases are in a sense similar, as the external search consists of applying a constructive heuristic to a certain number of bins. Thus, both phases can be modified by closing earlier bins where no more boxes can be added without violating the weight limit. If n bins cannot be recombined in the same number n of bins, each fulfilling the weight limit, we discard the solution and go to the next local search move.

3.3 Further work

Future work includes quantifying the impact of additional stability constraints on the volume utilization of containers and on the objective function of the integrated load-balanced bin-packing problem.

References

[1] Mongeau M. and Bes C., Optimization of aircraft container loading, *IEEE Transactions on Aerospace and Electronic Systems*, 39, 140-150, 2003.

[2] Trivella A. and Pisinger D., The load-balanced multi-dimensional bin-packing problem, *Computers & Operations Research*, 74, 152-164, 2016.

[3] Fekete S.P. and Schepers J., A Combinatorial Characterization of Higher-Dimensional Orthogonal Packing, *Mathematics of Operations Research* 29(2), 353-368, 2004.

[4] Ramos A.G, Oliveira J. F., Gonçalves J.F. and Lopes M.P., A container loading algorithm with static mechanical equilibrium stability constraints. *Transportation Research Part B*, 91, 565-581, 2016.

[5] Junqueira L., Morabito R. and Yamashita D.S., Three-dimensional container loading models with cargo stability and load bearing constraints, *Computers & Operations Research*, 39, 74-85, 2012.