Integration of different CHP steam extraction modes in the stochastic unit commitment problem

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Appendix to Integration of different CHP steam extraction modes in the stochastic unit commitment problem

I. INTRODUCTION TO THE IMPROVED HYBRID DECOMPOSITION

In this document we explain in detail how the suggested improvements for the scenario partition and decomposition method, variant 1 (SPDA1) proposed in [1] are carried out. The improvements consist in applying heuristics to find a suitable number of scenario partitions or clusters for the specific problem and find a partly fixed first stage-decision to initialize the problem solution. These heuristics are based in the Progressive Hedging algorithm and rounding techniques. The Progressive Hedging algorithm was first introduced by [2] and has been applied to solve large-scale stochastic programming problems in different applications such as forest planning [3], resource allocation problems [4] and unit commitment problems [5]. The Progressive Hedging is an iterative process in which first the problem is solved for each scenario individually and the solutions obtained for the first-stage decisions are averaged for all scenarios. From these solutions a multiplier is created and afterwards, the problem is solved again for each scenario including this multiplier as a penalty in the objective function. Using a squared proximal term to calculate the distance between the first stage decision and the solutions obtained for the first-stage and second-stage decisions. It solves one per partition as in [1]. The master problem (MP) is formed by both first-stage and second-stage decisions. It solves one per partition. Therefore, the entire set of scenarios Ω is divided into different subsets named Ωp, which is comprised of all the scenarios ω ∈ Ω that belong to partition p ∈ P. The hybrid unit commitment writes as follows.

\[
\min_{X,T,Y,γ_p} \sum_{t ∈ T} \sum_{g ∈ G} \left( a_g x_{g,t} + C^SU_g y_{g,t} + C^SD z_{g,t} \right) + \sum_{p ∈ P} \rho_p γ_p \tag{6a}
\]

s.t. \( γ_p ≥ \sum_{t ∈ T} \sum_{g ∈ G} b_g P_{g,t,ω} + \sum_{t ∈ T} \sum_{m ∈ M} C^L L_{m,t,ω} \)

\( + \sum_{t ∈ T} \sum_{g ∈ G} \sum_{m ∈ M} C^CHP_{g,m} y_{g,m,t,ω} \)

\( + \sum_{t ∈ T} \sum_{g ∈ G} \sum_{m ∈ M} L_{g,t} (P_{g,m,t,ω} + \varphi_{g,m} q_{g,m,t,ω}) \)

(∀p ∈ P; ∀ω ∈ Ωp)

(1a) − (1n), (2a) − (2v)

where \( ρ_p \) represents the probability attached to each partition that is calculated as follows.

\[ ρ_p = \sum_{ω ∈ Ω_p} π_ω \ (∀p ∈ P) \]

The auxiliary variable \( γ_p \) equals the worst-case system cost for partition \( p \) and therefore the second term in the objective function (6a) represents the expected value of the worst-case scenarios at each partition \( p \) ∈ \( P \). To formulate the decomposition algorithm, we need to distinguish between the master problem and the subproblems. Both are formulated as in [1]. The master problem (MP) is formed by both first-stage and second-stage decisions. It solves one per partition \( p ∈ P \) and for iteration \( i \) it writes as follows.

\[
\min_{X^i,Y^i,γ_p} \sum_{t ∈ T} \sum_{g ∈ G} \left( a_g x_{g,t}^i + C^SU_g y_{g,t}^i + C^SD z_{g,t}^i \right) + \gamma_p \tag{7a}
\]

s.t. \( γ_p ≥ \sum_{t ∈ T} \sum_{g ∈ G} b_g P_{g,t,ω}^i + \sum_{t ∈ T} \sum_{m ∈ M} C^L L_{m,t,ω}^{shed} \)

\( + \sum_{t ∈ T} \sum_{g ∈ G} \sum_{m ∈ M} C^CHP_{g,m} y_{g,m,t,ω}^{shed} \)

\( + \sum_{t ∈ T} \sum_{g ∈ G} \sum_{m ∈ M} L_{g,t} (P_{g,m,t,ω}^{shed} + \varphi_{g,m} q_{g,m,t,ω}) \)

(∀ω ∈ Ωp) \[1− (1a), \ (2a) − (2v) \]

(7c)

Where \( X^i = \{x_{g,t}^i, y_{g,t}^i, z_{g,t}^i\} \) and \( Y^i = \{u_{g,m,t,ω}^i, y_{g,m,t,ω}^{shed}, \varphi_{g,m} q_{g,m,t,ω}^i, p_{g,t,ω}^{shed}, L_{m,t,ω}^{shed}, W_{p_{g,m,t,ω}}^i, P_{g,m,t,ω}^{shed}, q_{g,m,t,ω}^{shed}, \varphi_{g,m} q_{g,m,t,ω}^i, \varphi_{g,m} q_{g,m,t,ω}^{shed}, \varphi_{g,m} q_{g,m,t,ω}^i, \varphi_{g,m} q_{g,m,t,ω}^{shed}; \ ∀ω ∈ Ω_p^i \} \). One subproblem (SP) per scenario \( ω ∈ Ω_p \) is
solved determining the second-stage decision variables.

$$\min \sum_{i \in T} \sum_{g \in \mathcal{G}^t} b_g^i p_{g,t,\omega}^i + \sum_{i \in T} \sum_{g,m,t,\omega} C_L^P \varphi_{g,m,t,\omega}$$ (8a)

$$+ \sum_{i \in T} \sum_{g \in \mathcal{G}^m} a_g^i p_{g,m,t,\omega}^i$$

$$+ \sum_{i \in T} \sum_{g \in \mathcal{G}^m} \sum_{m \in \mathcal{M}} b_{g,m}^i (p_{g,m,t,\omega}^i + \varphi_{g,m,t,\omega}^i)$$

s.t. (1f) – (2a) – (2v)

$$\Omega_p = \{p \in \mathcal{P} : \varphi_{g,m,t,\omega}^i \leq 0, \forall g,m,t,\omega \}$$

(8b)

Where \( \Omega_p^0 = \{p \in \mathcal{P} : \varphi_{g,m,t,\omega}^i \leq 0, \forall g,m,t,\omega \} \).

III. SOLUTION APPROACH

The solution algorithm is described in the following. Note that the master problems (7a)-(7c) and subproblems (8a)-(8b) for each partition \( p \in \mathcal{P} \) are solved in parallel and that they are called instances of the SPDA1 algorithm.

1) Initialize iteration \( j = 0 \). Select the initial number of partitions \( k^0 \) applying hierarchical clustering to the set of scenarios \( \Omega \).
2) Create \( k^0 \) parallel instances of the SPDA1 algorithm.
3) Initialize iteration \( i \) to 0 and set \( \Omega_p^0 = \emptyset \).
4) Solve the master problem and return the optimal solution found for the vector of first stage decisions \( \chi_{p}^0 \). Obtain the Lower Bound (LB) as \( \sum_{g \in \mathcal{G}} \sum_{t} (a_g^i x_{g,t}^i + C_{SU} y_{g,t}^i + C_{SD} y_{g,t}^i + \gamma_p) \).
5) Solve the subproblems (SP) with the first-stage decision variables fixed at \( \chi_{p}^0 \). Once all the subproblems are solved, obtain the scenario \( \omega' \) that yields the highest system cost. Include this scenario in the reduce set of worst-case scenarios (\( \Omega_{p}^i \)) such that \( \Omega_{p}^{i+1} = \Omega_{p}^i \cup \{\omega'\} \) and obtain the Upper Bound (UB) as \( \sum_{g \in \mathcal{G}} \sum_{t} (a_g^i x_{g,t}^i + C_{SU} y_{g,t}^i + C_{SD} y_{g,t}^i + \gamma_p) \).
6) Check convergence. If \( |UB - LB| \leq \xi \), where \( \xi \) is the tolerance value, the iterative process stops. If \( |UB - LB| > \xi \) then \( i := i + 1 \) and go to step 4.
7) Once all partitions have converged, we obtain the first-stage decision vector for each partition \( \chi_{p}^i \).
8) Increase iteration number \( j := j + 1 \). Calculate the average value for the first-stage commitment decisions over all partitions \( \overline{\chi}_{p}^j = \frac{1}{|\mathcal{P}|} \sum_{p \in \mathcal{P}} p_p \chi_{p}^{j-1} \). Obtain squared distance \( \sigma^j = \| \overline{\chi}_{p}^j - \overline{\chi}_{p}^j - 1 \| \) (where \( \overline{\chi}_{p}^0 = 0 \)). If \( \sigma^j \leq \varepsilon \) we stop the iteration process for \( j \) and move a step forward. If \( \sigma^j > \varepsilon \), we increase the number of partitions \( k^j := k^{j-1} + 1 \) and go step 2.
9) Obtain the partly fixed commitment decisions using the rounding technique:

$$\overline{\chi}_{p}^\text{round} = \begin{cases} 1 & \text{if } \overline{\chi}_{p}^j \geq 1 - \alpha \\ 0 & \text{if } \overline{\chi}_{p}^j \leq \beta \\ \in [0, 1] & \text{if } \beta < \overline{\chi}_{p}^j < 1 - \alpha \end{cases}$$

10) Solve (6a)-(6c) for the scenarios finally retained in the set of worst-cases scenarios \( \Omega_{p}^i \) using \( \overline{\chi}_{p}^\text{round} \) as partly fixed commitment decisions.

REFERENCES