Output Variability Caused by Random Seeds in a Multi-Agent Transport Simulation Model

7th International Workshop on Agent-based Mobility, Traffic and Transportation Models, Methodologies and Applications

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Random Seeds

- Normally random seeds do not receive much attention.

- But should they?

Motivation

- Transport model outputs are fundamental for policy support.
- Their results need to be trustworthy.
  - Do random numbers hinder this?

Outline

1. Literature Review
2. Methodology
3. Case Study & Results
4. Conclusions & Future Work

Outline

Selected Studies

Nagel, 1997*

- Two seeds.
- TRANSims.
- Solely observes difference.

Castiglione et al., 2003†

- 100 seeds.
- San Francisco model.
- Use partial means, and thus incomparable.

Cools et al., 2011‡

- 200 seeds.
- FEATHERS.
- Trip level: $C.V.'s \lesssim 1\,\%$.

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*Nagel, Kai (1997). "Experiences with iterated traffic microsimulations in Dallas". In: *Traffic and granular flow '97*. Ed. by Michael Schreckenberg and D. E Wolf. Heidelberg: Springer, pp. 199–214. arXiv: 9712001v1 [arXiv:adap-org]

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Additional Studies – Conclusions

- More or less consensus in the literature that the Monte Carlo variability is a non-issue for link loads.
  - Aditionally backed up by **Veldhuisen et al., 2000***, **Lawe et al., 2009**†and **Ziems et al., 2011**‡.

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‡**Veldhuisen, Jan, Timmermans, Harry, and Kapoen, Loek (2000).** **"Microsimulation Model of Activity-Travel Patterns and Traffic Flows: Specification, Validation Tests, and Monte Carlo Error".** **In: *Transportation Research Record: Journal of the Transportation Research Board* 1706, pp. 126–135.**

‡**Lawe, Stephen, Lobb, John, Sadek, Adel, Huang, Shan, and Xie, Chi (2009).** **"TRANSIMS Implementation in Chittenden County, Vermont".** **In: *Transportation Research Record: Journal of the Transportation Research Board* 2132, pp. 113–121.**

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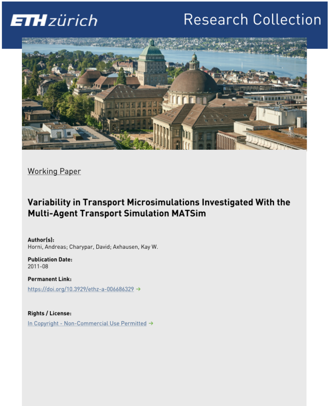
A Very Similar Study...

Horni et al., 2011 *

- Discovered after final submission deadline.
  - Thus not referenced in the proceedings paper.
- Actual study on MATSim.
- 30 random seeds.
- Link load average $C.V. \approx 4\,\%$.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Working Paper

Variability in Transport Microsimulations Investigated With the Multi-Agent Transport Simulation MATSim

Author(s):
Horni, Andreas; Charypar, David; Axhausen, Kay W.

Publication Date:
2011-08

Permanent Link:
https://doi.org/10.3929/ethz-a-006686329 →

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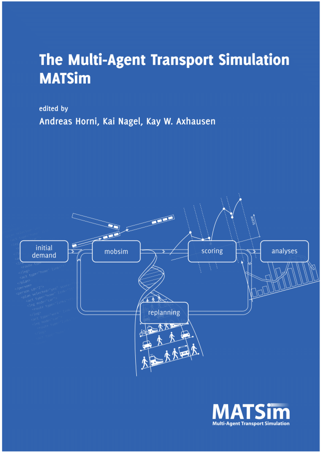
What's Missing?

Contributions

- Compare between-seed variation to within-seed variation.
- Analyse each seed separately.
- Larger dataset for link load analysis (network and number of seeds).

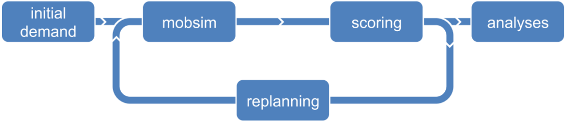
Outline

The Multi-Agent Transport Simulation MATSim



MATSim*

- Agent-based.
- Activity-based.
- Large-scale applicable.
- Scenarios around the world.
- Open source.



*Horni, Andreas, Nagel, Kai, and Axhausen, Kay W, eds. (2016). *The Multi-Agent Transport Simulation MATSim*. London: Ubiquity Press.

How Are Random Numbers Used in MATSim?

Used to Determine *which*...

- ... in-going links of a node to be handled first.
- ... agents use *which*...
  - ... plan mutation strategy.
  - ... plan selection strategy.
- ... plan an agent choose (from choice set).

- For choosing between reasonable alternatives.
  - MATSim ought note to be particularly prone to high output variability.


Experiment Design

- 100 runs (100 different random seeds).
- 100 iterations in each run.
- A total of $100 \times 100 \times |L|$ link loads produced.
- $x_l^{s,i}$ is daily link load on link l in iteration i with seed s.

	It. 1	It. 2	$\cdots$	It. 98	It. 99	It. 100
Seed 1	$x_l^{1,1}$	$x_l^{1,2}$	$\cdots$	$x_l^{1,98}$	$x_l^{1,99}$	$x_l^{1,100}$
Seed 2	$x_l^{2,1}$	$x_l^{2,2}$	$\cdots$	$x_l^{2,98}$	$x_l^{2,99}$	$x_l^{2,100}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
Seed 99	$x_l^{99,1}$	$x_l^{99,2}$	$\cdots$	$x_l^{99,98}$	$x_l^{99,99}$	$x_l^{99,100}$
Seed 100	$x_l^{100,1}$	$x_l^{100,2}$	$\cdots$	$x_l^{100,98}$	$x_l^{100,99}$	$x_l^{100,100}$

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Seed 1	$x_l^{1,1}$	$x_l^{1,2}$	$\cdots$	$x_l^{1,98}$	$x_l^{1,99}$	$\mathbf{x_l^{1,100}}$
Seed 2	$x_l^{2,1}$	$x_l^{2,2}$	$\cdots$	$x_l^{2,98}$	$x_l^{2,99}$	$\mathbf{x_l^{2,100}}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
Seed 99	$x_l^{99,1}$	$x_l^{99,2}$	$\cdots$	$x_l^{99,98}$	$x_l^{99,99}$	$\mathbf{x_l^{99,100}}$
Seed 100	$x_l^{100,1}$	$x_l^{100,2}$	$\cdots$	$x_l^{100,98}$	$x_l^{100,99}$	$\mathbf{x_l^{100,100}}$

- **We (primarily) use those from the last iteration.**

Measures I – C.V.

- Our initial analysis measure is the coefficient of variation,

$$c_v^{x_l} = \frac{\text{Standard Deviation}}{\text{Mean}} = \frac{\sqrt{\frac{1}{|S|-1}\sum_{s\in S}\left(x_l^{s,100} - \bar{x}_l\right)^2}}{\bar{x}_l}, \qquad l \in L.$$

  - With $\bar{x}_l$ being the mean link load of link $l \in L$,

$$\bar{x}_l = \frac{\sum_{s\in S} x_l^{s,100}}{|S|}, \qquad l \in L.$$

Measures II – Proportions

- We introduce additional measures:
  - Proportion of links that deviate more than $q \cdot 100\%$ from their mean when using seed $s \in S$,

$$r_q^s = \frac{\sum_{l \in L} \left[\frac{\left| x_l^{s,100} - \bar{x}_l \right|}{\bar{x}_l} > q \right]}{|L|}, \qquad s \in S, \ q > 0.$$

  - Proportion of seeds in which the link load of link $l \in L$ deviates more than $q \cdot 100\%$ from the mean,

$$r_q^l = \frac{\sum_{s \in S} \left[\frac{\left| x_l^{s,100} - \bar{x}_l \right|}{\bar{x}_l} > q \right]}{|S|}, \qquad l \in L, \ q \geq 0.$$

Measures IIIa – Between-Seed vs Within-Seed Variation

- Finally, we introduce two auxiliary measures:
  - Between-seed variation:

$$B_l = \frac{1}{|S|-1} \sum_{s \in S} \left(x_l^{s,100} - \bar{x}_l \right)^2, \qquad l \in L.$$

  - Within-seed variation:

$$W_l = \frac{1}{|S|} \sum_{s \in S} \left(x_l^{s,99} - x_l^{s,100} \right)^2, \qquad l \in L.$$

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Measures IIIb – Between-Seed vs Within-Seed Variation

$B_l =$

	It. 1	⋯	It. 99	**It. 100**	Between-seed mean
Seed 1	$x_l^{1,1}$	⋯	$x_l^{1,99}$	$\left(\mathbf{x_l^{1,100}}\right.$	$\left. - \bar{\mathbf{x}}_\mathbf{l}\right)^2$
⋮	⋮	⋱	⋮	⋮	⋮
Seed 99	$x_l^{99,1}$	⋯	$x_l^{99,99}$	$\left(\mathbf{x_l^{99,100}}\right.$	$\left. - \bar{\mathbf{x}}_\mathbf{l}\right)^2$
Seed 100	$x_l^{100,1}$	⋯	$x_l^{100,99}$	$\left(\mathbf{x_l^{100,100}}\right.$	$\left. - \bar{\mathbf{x}}_\mathbf{l}\right)^2$

$W_l =$

	It. 1	⋯	It. 98	**It. 99**	**It. 100**
Seed 1	$x_l^{1,1}$	⋯	$x_l^{1,98}$	$\left(\mathbf{x_l^{1,99}}\right.$	$\left. - \mathbf{x_l^{1,100}}\right)^2$
⋮	⋮	⋱	⋮	⋮	⋮
Seed 99	$x_l^{99,1}$	⋯	$x_l^{99,98}$	$\left(\mathbf{x_l^{99,99}}\right.$	$\left. - \mathbf{x_l^{99,100}}\right)^2$
Seed 100	$x_l^{100,1}$	⋯	$x_l^{100,98}$	$\left(\mathbf{x_l^{100,99}}\right.$	$\left. - \mathbf{x_l^{100,100}}\right)^2$

Measures IIIc – Between-Seed vs Within-Seed Variation

- These allow calculating the importance of the between-seed variation,

$$\tilde{R}_l = \sqrt{\frac{B_l + W_l}{W_l}}, \qquad\qquad l \in L.$$

  - $\tilde{R}_l \to 1 \quad \Rightarrow \quad$ Not important.
  - $\tilde{R}_l = \sqrt{2} \quad \Rightarrow \quad$ Equally important.
  - $\tilde{R}_l \gg \sqrt{2} \quad \Rightarrow \quad$ Strongly dominates the within seed-variation.
- Thus, $\tilde{R}_l$ can indicate whether it is better to...
  - ... run more iterations (low $\tilde{R}_l$) or
  - ... run more seeds (high $\tilde{R}_l$).

Outline

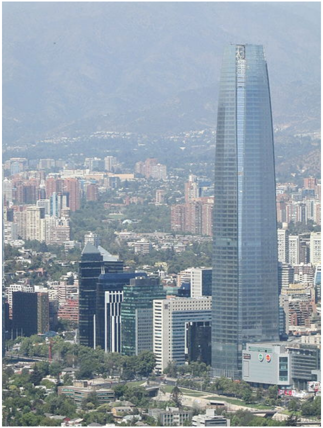
Case Study

Santiago de Chile Open Data Scenario v2b*

- 10% Population Sample: 665,201 agents.
- 22,981 link network.
- Schedule based public transport.

Default "Out-of-the-Box" Configurations

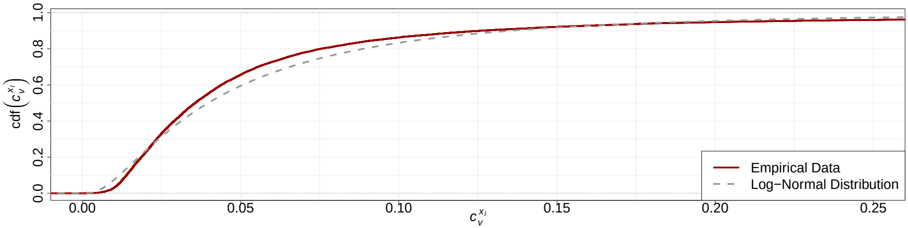
- Mode choice: Walk, public transport, car.
- 100 iterations.
- After 80 iterations:
  - Choice sets are locked.
  - Plan level MSA initialised.



*Kickhöfer, Benjamin, Hosse, Daniel, Turner, Kai, and Tirachini, Alejandro (2016). "Creating an open MATSim scenario from open data: The case of Santiago de Chile". In: *VSP Working Paper 16-02*. VSP Working Paper

Coefficients of Variation: All links.

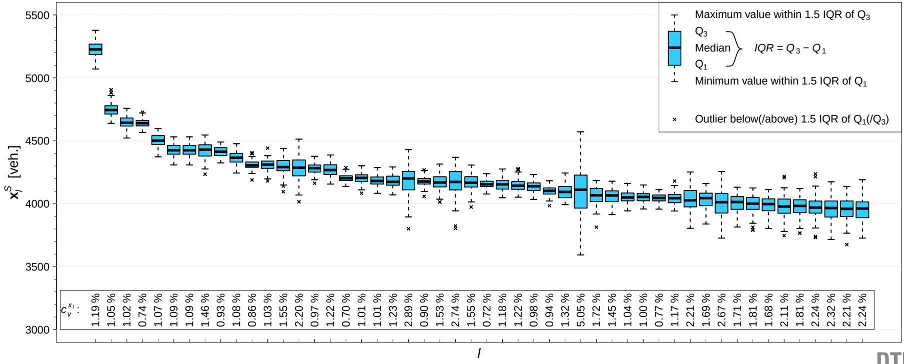
- Generally not high – somewhat in accordance with earlier studies.
  - 2/3 of links have a coefficient of variation $\leq 5\,\%$.
  - $\sim 15\,\%$ of links have a coefficient of variation $> 10\,\%$.



$$c_v^{x_l} = \frac{\sqrt{\frac{1}{|S|-1}\sum_{s\in S}\left(x_l^{s,100}-\bar{x}_l\right)^2}}{\bar{x}_l}, \quad l \in L.$$

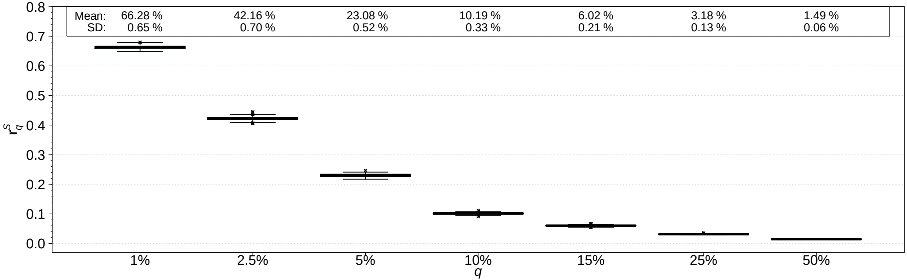
Coefficients of Variation: 50 busiest links.

- The 50 busiest links seem more stable.
  - Coefficient of variation ≈ 1 %.



Are Some Seeds Worse than Others?

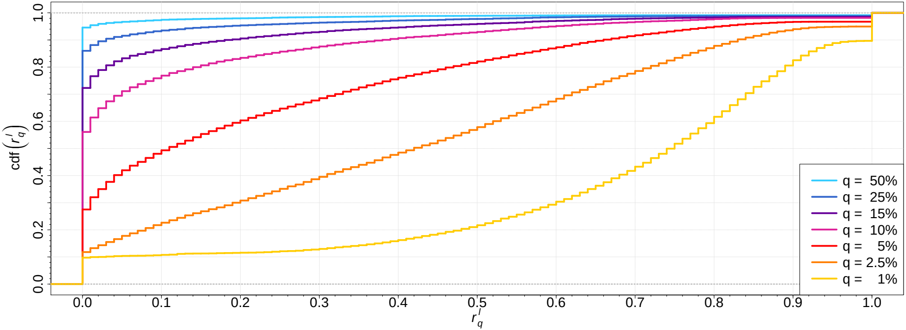
- All seeds perform more or less equally well.
  - ... i.e. 42 will do just fine.



$$r_q^s = \frac{\sum_{l \in L} \left[\frac{\left| x_l^{s,100} - \bar{x}_l \right|}{\bar{x}_l} > q \right]}{|L|}, \quad s \in S, \; q > 0.$$

How Often Does It Go Wrong? I

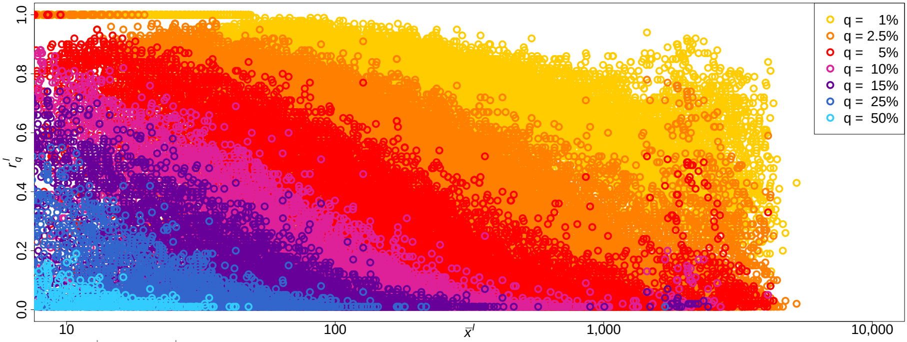
- ~20 % of links have a relative error of ≥ 5 % for ≥ 50 % of the seeds.
- ~10 % of links have a relative error of ≥ 15 % for ≥ 20 % of the seeds.



$$r_q^l = \frac{\sum_{s \in S} \left[\frac{\left| x_l^{s,100} - \bar{x}_l \right|}{\bar{x}_l} > q \right]}{|S|}, \quad l \in L, \ q \geq 0.$$

How Often Does It Go Wrong? II

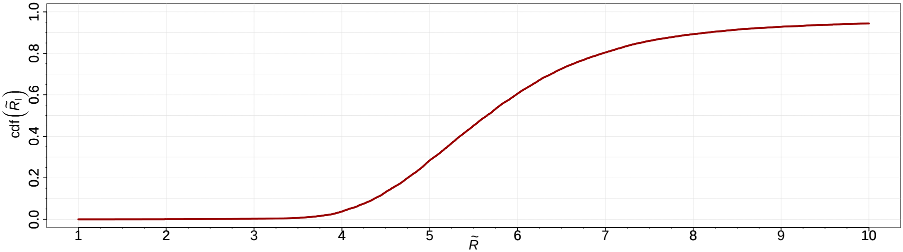
- Generally worst for the smallest links.
  - However, many busy links are quite volatile.



$$r_q^l = \frac{\sum_{s \in S} \left[\frac{\left| x_l^{s,100} - \bar{x}_l \right|}{\bar{x}_l} > q \right]}{|S|}, \quad l \in L, \; q \geq 0.$$

Between-Seed Variation vs Within-Seed Variation I

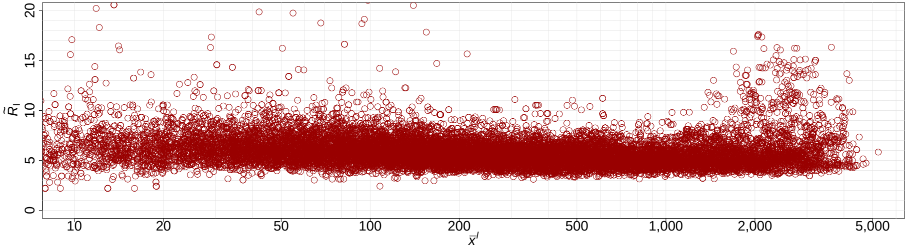
- Practically all links have $\tilde{R}_l \geq 4$.
  - Between seed-variation dominates within seed-variation.
  - Despite only using 100 iterations.



$\tilde{R}_l = \sqrt{\frac{B_l+W_l}{W_l}}, \quad l \in L.$

Between-Seed Variation vs Within-Seed Variation II

- Problem intensifies for busiest links.



$\tilde{R}_l = \sqrt{\frac{B_l + W_l}{W_l}}, \quad l \in L.$

Outline

1. Literature Review
2. Methodology
3. Case Study & Results
4. Conclusions & Future Work
   - Conclusions
   - Future Work

Conclusions I

Findings

- Coefficient of variation is generally low, and a decreasing function of link load.
- Large relative errors do happen – also for the busiest links.
- No bad seeds.
- Between-seed variation is much larger than within-seed variation.

Conclusions II

How to Deal with It?

- Use multiple seeds.
  - May be better to run 10×100 iterations than $1 \times 1,000$.
- Average out results across all seeds.
  - But point estimates may not correspond to an actual solution.
- Present results as a distribution.
  - As suggested in Chapter 48* of the MATSim book.
  - Contributes to deeper understanding of uncertainties.

*Flötteröd, Gunnar (2016). "MATSim as a Monte-Carlo Engine". In: *The Multi-Agent Transport Simulation MATSim*. Ed. by Andreas Horni, Kai Nagel, and Kay W. Axhausen. London: Ubiquity Press. Chap. 48, pp. 361–370.

Future Work

Spatial Analysis

- Current study ignores spatial interdependencies.
  - Visual overview.
  - Actual geostatistical analysis (e.g. Kriging).

Network Sensitivity

- Sensitivity to small changes in the network.
  - Does the output variability overshadow the effects of infrastructural changes?
  - Extremely relevant for scenario analysis.

Thank You for Your Attention

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Random Number Generation in MATSim?

Linear Congruential Generator (LCG)

Pseudo random numbers are constructed in MATSim using Java's default LCG with minor modifications,

$$X_{n+1} = aX_n + c \mod m.$$

- X_n = latest draw.
- X_{n+1} = next draw.
- $a = 25,214,903,917$.
- $c = 11$.
- $m = 2^{48}$.

In order to get a uniformly distributed number X_{n+1} is divided by m.