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Short-term Variability of Cross-Spectral Analysis for Ship Responses in Waves

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ABSTRACT

Short-term variability of ship responses is investigated by cross-spectrum analysis. In a steady state condition, it is well-known that a certain length of sampled data is required for stable results of the spectral analysis. However, the phase lag between responses, in terms of the phase angle obtained from the (coupled) cross-spectra, has not been discussed in detail. In a previous but similar study, the authors pointed out that the short-term variability of the relative phase angle of the cross-spectra is harmful to sea state estimation using the wave buoy analogy, i.e. ship as a wave buoy. In the present report, an investigation based on the multi-variate auto-regressive model method has been applied to the same data. In the results, it has been observed that the short-term variability of the relative phase angle can be reduced.

1. Introduction

The authors of the present paper are advocates of the wave buoy analogy. In this analogy, measured responses from an advancing ship are used together with corresponding transfer functions to obtain estimates of the sea state at the exact position of the ship [(3), (4), (6)]. In general, results of the wave buoy analogy compare reason-ably well with results of other means for wave estimation [(7), (8)] but observations with poor agreement are also found; not to mention which means are the most accurate. This brings to question how much variation, due to aleatory uncertainty [(9)], the sea state itself may exhibit on a short-term scale in a 2-5 minutes period.

A direct measure for the aleatory short-term variation of a sea state in time and position could be obtained based on results by the wave buoy analogy. However, an indirect measure can be given in terms of measurements of ship responses, since any change in sea state will be directly observable in the wave-induced responses; assuming other operational parameters (speed, heading, etc.) to be constant and neglecting the fact that a ship is a (linear) wave filter. The advantage by this indirect approach is that model-ling uncertainties, of the wave buoy analogy or other similar means for wave measurement, will not influence results. In case of the wave buoy analogy, notably un-certainty related to the transfer functions of the ship could influence results. ‘Modelling uncertainties’ may, in this sense, be viewed as a kind of epistemic uncertainty [(9)].

In the previous study [(5)], short-term variability of ship responses was investigated by cross-spectrum analysis using the discrete Fourier transform (in short, DFT) applied to stationary full-scale time series data. Thereby, details of the transition of the variance and relative phase angles of the cross-spectra could be investigated specifically. In the study, short-term variability of the relative phase angle was observed, and it was concluded that the short-term variability of the relative phase angle is harmful to sea state estimation using the wave buoy analogy.

In the present report, the exact same data as previously studied by Iseki and Nielsen [(5)] is investigated by application of multi-variate auto-regressive (in short, MAR) method. In summary, the tasks and associated objective of the study are: The ship responses are investigated by iterative cross-spectral analysis with a short time shifting (see Fig. 1). The detailed transition of amplitudes and relative phase angles of the cross-spectra is monitored, and it should be investigated if the short-term variability of the relative phase angle can be reduced by applying the MAR method instead of DFT [(5)]. The short-term variability of the relative phase angle is illustrated, and the problems encountered are discussed.

2. Analysis procedure

In this section, the procedure of spectral analysis with MAR model is described. Generally, the MAR model of a p-variate stationary time series $\mathbf{y}(n)=\{y_1(n), y_2(n), \cdots, y_p(n)\}, \quad (n=0,1,2,\cdots,N-1)$ can be expressed as

$$\mathbf{y}(n)=\sum_{l=1}^{m}\mathbf{A}_l\mathbf{y}(n-l)+\mathbf{w}(n) \tag{1}$$

where $\mathbf{A}_l$ is a $p\times p$ autoregressive coefficient matrix for l lag component, m is the autoregressive order, and $\mathbf{w}(n)$ denotes the p-variate Gaussian white noise sequence with mean zero and diagonal covariance matrix $\mathbf{V}$.

$$\begin{aligned}&\mathbf{w}(n)=\{w_1(n), w_2(n), \cdots, w_p(n)\},\\ &\mathbf{V}=diag\left(\sigma_1(n), \sigma_2(n), \cdots, \sigma_p(n)\right)\end{aligned} \tag{2}$$

where $w_i(n)$ is supposed to be independent on $w_j(n)$ for $i\neq j$.

The total number of unknown coefficients is $p\times p\times m$ and the coefficients can be evaluated by the maximum likelihood method, the least square method, or the *Levinson-Durbin* algorithm [(2)] using N points of vector time series data $\mathbf{y}(n)$. The optimum value of the model order m can be obtained by minimizing *Akaike's* information criterion (AIC) [(1)].

After fitting the MAR model to the vector time series, the Fourier transform of Eq.(1) can be expressed as follows.

$$\mathbf{Y}(f)=\mathbf{A}(f)^{-1}\mathbf{W}(f) \tag{3}$$

where

$$\begin{cases}\mathbf{A}(f)=\sum_{l=0}^{m}\mathbf{A}_l e^{-2\pi i f l\Delta t}, \quad \mathbf{A}_0=-\mathbf{I}\\ \mathbf{Y}(f)=\sum_{n=0}^{N-1}\mathbf{y}(n)e^{-2\pi i f n\Delta t},\\ \mathbf{W}(f)=\sum_{n=0}^{N-1}\mathbf{w}(n)e^{-2\pi i f n\Delta t}\end{cases} \tag{4}$$

Actual value of $\mathbf{W}(f)$ can be obtained as the diagonal covariance matrix $\mathbf{V}$ expressed by Eq.(2) and each element can be evaluated by using prediction error.

The definitions of the power spectrum can be expressed as follows;

$$\mathbf{S}_{\mathbf{YY}}(f)=\Delta t\mathbf{Y}(f)\mathbf{Y}(f)^{*T}=\begin{pmatrix}S_{11}(f) & S_{12}(f) & \cdots & S_{1p}(f)\\ S_{21}(f) & S_{22}(f) & \cdots & S_{2p}(f)\\ \vdots & \vdots & \ddots & \vdots\\ S_{p1}(f) & S_{p2}(f) & \cdots & S_{pp}(f)\end{pmatrix} \tag{5}$$

where * and ^{T} are denoting the complex conjugate and the transpose matrix, Δt the sampling time. $S_{ii}(f)$ and $S_{ij}(f)$ $(i\neq j)$ denote the auto-power spectrum and the cross-power spectrum.

Considering that the cross-power spectrum is complex-valued, phase angle (i.e. the argument) of the cross-power spectrum can be defined as follows;

$$P_{ij}(f)=Arg\left(S_{ij}(f)\right) \tag{6}$$

The phase angle represents the phase difference between the two responses x and y. In studies related to the *wave buoy analogy*, the phase angle has indeed importance, since the angle contributes to the inference/estimation of (relative) wave direction. Therefore, the phase angle of the cross-spectrum is denoted the "relative phase angle" in this study.

Finally, the above mentioned spectral analysis is applied to the time series, which are assumed to be stationary. The concrete application is illustrated in Figure 1. The top blue colored bar denotes the measured steady state time series of ship responses. The spectral analysis is applied to the time series of constant time span and iterated many times with

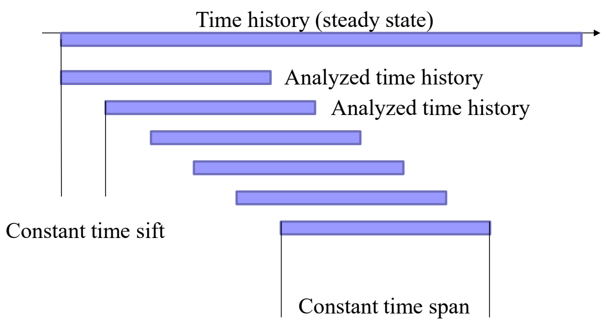

Figure 1 Concrete procedure of the Fourier analysis of stationary time series.

constant time shifting. Each shorter bar in the figure indicates a single spectral analysis on a batch of data. Based on the assumption of *stationary* time series, therefore, in theory, all the results of the spectral analysis must coincide with each other. An investigation of the aleatory short-term variation of the results is the core objective of this study.

3. Full Scale Ship Experiment

The full scale ship experiment was carried out on October 17th 2013 using the training ship Shioji-maru of Tokyo University of Marine Science and Technology. The principal particulars and a photo of the ship are shown in Table 1 and Figure 2, respectively. The location of the experimental area was off Sunosaki Cape in Chiba Prefecture, Japan. Ship motions and vessel position were measured using a fiber optic gyro and a GPS system, respectively. The data was sampled every 0.1s (10Hz) and recorded in the hard disk of a notebook PC through the RS-232C port.

Figure 3 shows the trajectory of the T.S. Shioji-maru during the experiment. To measure changes in ship motions with respect to the encounter angle of waves, the propeller pitch angle was set to 15 degrees. Measurement was carried out for 60 minutes involving three straight sections and changes in course. The sections A and B have 10 minutes duration and the section C has 20 minutes duration. The wave direction was SE as reported by Japan Meteorological Agency.

Table 2 shows the ship courses and the mean speeds-through-water, measurement duration, and true wind directions and wind speeds are also summarized. During the experiment, the observed waves were: height 1.0-1.5m, directions (relative to N) 150-160 and 335-350 degrees (mixed sea condition).

Table 1 Principal particulars of T.S. Shioji-maru

Length (P.P.)	46.00(m)
Breadth (M_{LD})	10.00(m)
Depth (M_{LD})	6.10(m)
Draught (M_{LD})	2.65(m)
Displacement	659.4(t)
Main engine	4 cycle diesel 1,030 kw × 700 rpm

Table 2 Ship course and the sea conditions

Run	Ship course (deg)	Ship speed (knot)	Dura-tion (min)	Wind dir.	Wind speed (m/s)
A	200	11.0	10	NNE	3.0
B	180	11.0	10	NNE	3.0
C	0	10.5	20	NNE	5.0

Figure 2 The training ship Shioji-maru.

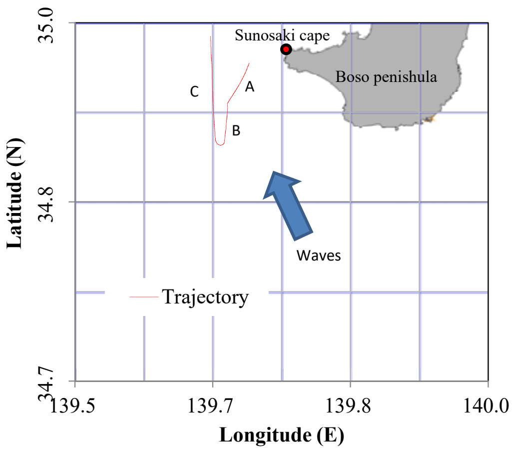

Figure 3 The experimental area at off Sunosaki cape and the ship trajectory.

4. Results of the analyzes

A specific software to continuously estimate auto- and cross-spectra with constant time (data points) shifting, has been developed in the previous report (5). In the present study, the algorithm and software have been modified to reflect the updated analysis procedure (see Section 2). A screenshot of graphical results produced by the software is shown in Figure 4. The three components of the power spectra and the time series are shown in the left part of the dialog box. The upper left graphs can be switched to the relative phase spectra. In the calculation of this figure, each analyzed time series has 600 data points with tapered cosine bell type data window and 1s (10 data points) time shifting. The most recent spectra are plotted by white color and the past spectra are drawn by blue color in the upper left part. In the right part of the screen, the ship course, speed and the diagonal parts of MAR coefficients are indicated (the model order *m*=6).

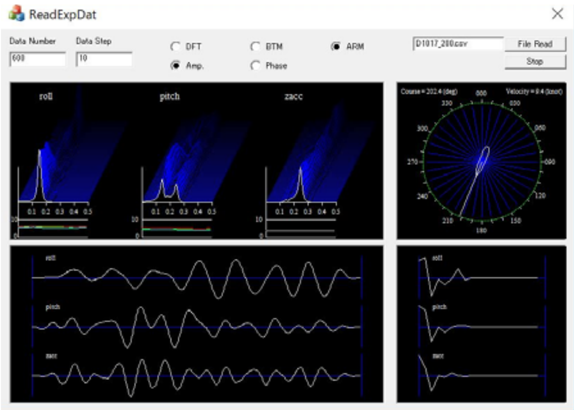

Figure 4 Screen shot of the graphical user interface of the software. (run A)

4.1 Auto-spectral analysis

Figure 5 shows transition of auto-spectra of roll, pitch and vertical acceleration (in short, zacc). The graphs are close view of the left upper graph of Figure 4. The difference between the upper and the lower graphs is only the time span. The upper graphs are showing the spectra calculated by 3-minute and the lower by 1-minute time span, respectively. In comparison, the longer time span (upper plots) seems to be straight along the graph's depth direction. On the other hand, the shorter time span (lower plots) shows temporal fluctuations along the depth directions.

To investigate the relationship between the time span and the fluctuations, the variances of each time span were calculated and compared. Figure 6 shows transition of the variance of the pitch motion with respect to the "constant time shifting". The data was measured on "run C" and can be considered as a stationary time series. Theoretically speaking, therefore, the variance must take constant value regardless of the time shifting. The total number of data points is 15000 (i.e., duration is 25 minutes) and the variances were calculated as the area of the pitching auto-spectra. It should be noted that the spectra are treated as one-sided spectra in this report, therefore, the vertical axes become twice compared to the previous report [(5)]. The horizontal axis denotes the shifted time in second. The six colored lines indicate results based on the time span of the analysis; for instance, the blue line "1 min." was evaluated by 1-minute time span (600 data points). On the contrary, the green line "6 min" was evaluated by 6 minutes time span (3600 data points). The MAR model orders were, somewhat arbitrarily, set to m = to concentrate attention to actual time histories of the variance. There are some large fluctuations around 500s and the longer than 3-minnute time span provides the smoother line. Based on physical expectation, the smoother line is quite reasonable and successfully estimated under stationary conditions. This tendency was almost the same in the previous report [(5)] based on the DFT.

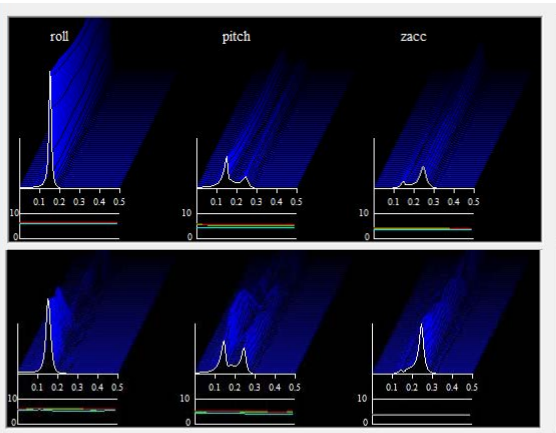

Figure 5 Transition of the auto-spectra of roll, pitch and vertical acceleration (zacc) (upper: min, lower: 1min).

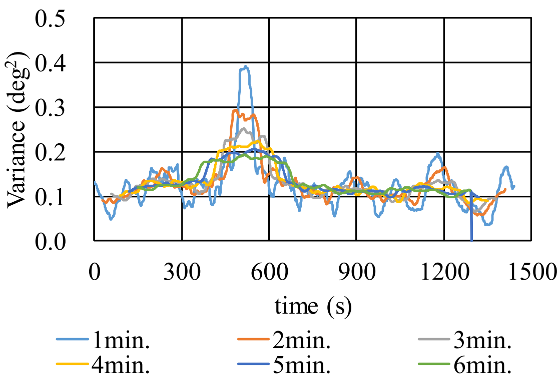

Figure 6 Transition of the pitching variance for each time spans (run C).

4.2 Cross-spectral analysis

The relative phase angle of a cross-spectrum defined by Eq.(6) was also investigated by the "constant time shifting" approach. Figure 7 shows transition of relative phase angles of cross-spectra of roll-pitch, pitch-zacc and zacc-roll. The graphs are the close-up views of the other mode of the left upper plot of Figure 4. It should be noted that the vertical axes indicate the relative phase angles from 0 to 2π. The upper plots are showing the phase spectra calculated by MAR model with 3-minute time span. On the other hand, the lower graph is the same phase spectra calculated by DFT [(5)]. Completely different tendency can be observed between MAR and DFT. The relative phase angle of MAR are indicated by continuous curves, except the gap between 0, while the results of DFT are indicated by scattered points. This different tendency may come from the fitting procedure of MAR modeling because the process of fitting can be recognized as a kind of averaging of the time histories.

In order to evaluate steady states of the relative phase spectrum, an index based on the concept of standard deviation is defined to evaluate the total movement of phase movement (in short, P.M.) at the n shifted time. The definition of the index is;

$$I_{PM}(n)=\sqrt{\frac{1}{N_f}\sum_{k=0}^{N_f}\left\{P_{ij}(f_k;n)-P_{ij}(f_k;n-1)\right\}^2} \tag{7}$$

where N_f denotes the total number of discrete frequencies and $P_{ij}(f_k;\mathrm{n})$ represents the relative phase spectrum at the n shifted time. Any fluctuation of this index can be interpreted as the short-term variability [5].

Figure 8 shows the transition of the index $I_{PM}(n)$ of rolling-pitching. The upper and lower graphs show the results of MAR and DFT with six different lengths of time spans (see also Fig.6). The vertical axes are the index $I_{PM}(n)$ and the horizontal axes denote the shifted time. Analyzed data is the same with Figure 6, run C and the time span is 1800s. Rough fluctuation of the index $I_{PM}(n)$ and no periodic transition can be seen in both plots. However, it can be clearly observed that the short-term variability of the relative phase angle is reduced. This seem to conclude that the MAR method can provide better results in applications using the wave buoy analogy.

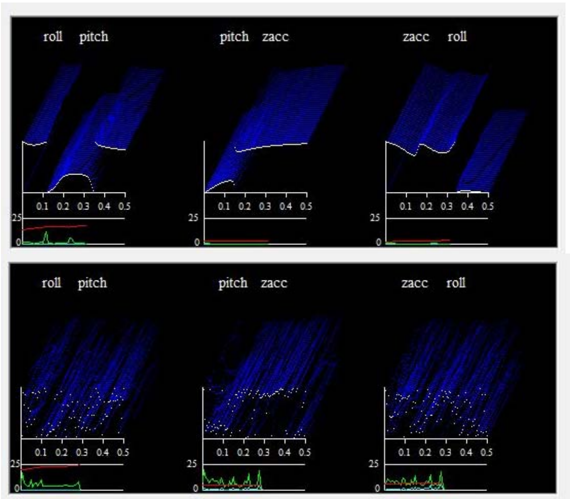

Figure 7 Transition of the relative phase angle between roll-pitch, pitch-zacc and zacc-roll. (upper: MAR, lower: DFT).

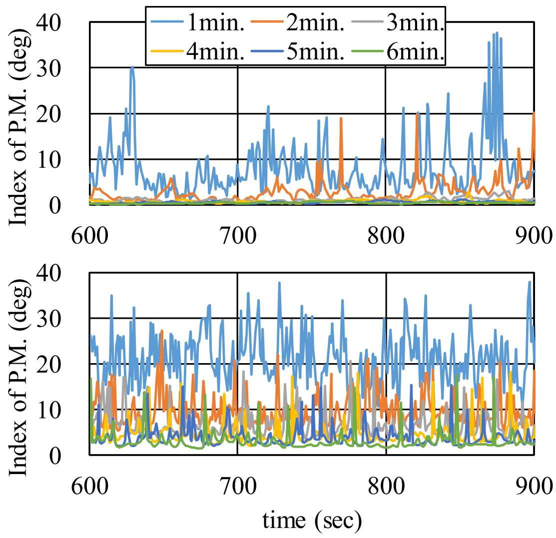

Figure 8 Transition of $I_{PM}(n)$ of roll- pitch cross-spectra for each time span (runC).

5. Conclusions

Short-term variability of ship responses was investigated by cross-spectrum analysis. Using stationary time series data, details of the transition of the variance and relative phase angles of the cross-spectra have been investigated with time shifting. The results obtained in this report are summarized below:

- In analyzes of (statistical) variance, the tendency is almost the same with the direct method, the discrete Fourier transform [5]. That is, the MAR-based results and those obtained by DFT are similar.
- Short-term variability of the relative phase angle is reduced by introducing the MAR modeling method.

In a larger application-oriented context, short-term variability of the relative phase angle is harmful to sea state estimation by the wave buoy analogy. Hence, in future work, the short-term variability should be investigated in more details.

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