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# Electro-Thermal Model of Thermal Breakdown in Multilayered Dielectric Elastomers

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## Abstract

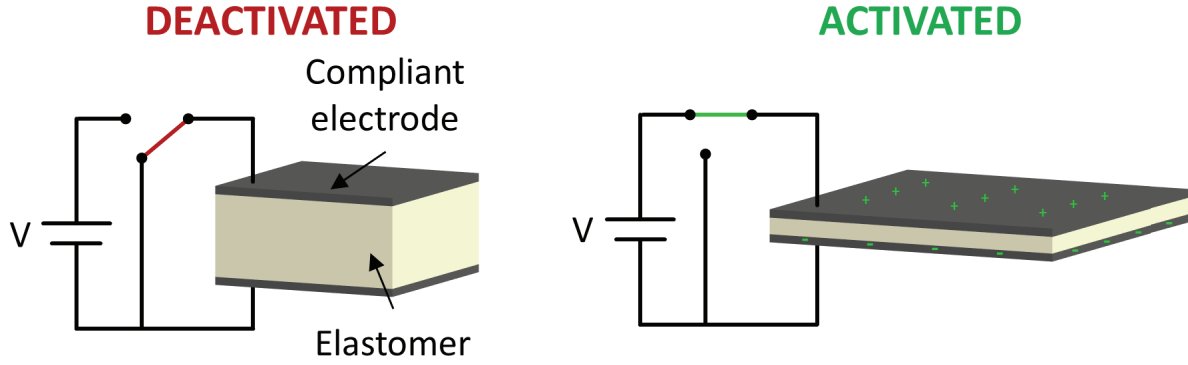
Energy transduction of dielectric elastomers involves minute electrical and mechanical losses, both of which potentially increase the temperature within the elastomer. Thermal breakdown of dielectric elastomers occur when heat generated therein cannot be balanced by heat loss on the surface, which is more likely to occur in stacked dielectric elastomers. In this paper an electro-thermal model of a multilayered dielectric elastomer able to predict the possible number of layers in a stack before thermal breakdown occurs is presented. Simulation results show that point of breakdown is greatly affected by an increase in surrounding temperature and applied electric field. Furthermore, if the stack diameter is large, thermal insulation of the cylindrical surface is a valid approximation. Two different expressions for the electrical conductivity are used, and it is concluded that the Frank-Kamenetskii expression is more conservative in prediction of point of breakdown than the Arrhenius expression, except at high surrounding temperature.

**Topical Heading:** Transport Phenomena and Fluid Dynamics

**Keyword:** Dielectric elastomer, Thermal breakdown, Electro-thermal model, Electrical conductivity, Multilayered

## Introduction

Dielectric elastomers (DEs) are promising materials for use in various electromechanical applications as actuators, sensors, and generators<sup>1,2</sup>. Applications have a wide spectrum of sizes, ranging from micro-fluidic pumps<sup>3,4</sup> over mm-sized Braille displays<sup>3,5</sup> through



**Figure 1:** Illustration of the working principle of a dielectric elastomer actuator.

26 envisioned wave energy converters consisting of 400 meter-long tubes<sup>6,7</sup>.

27 A DE consists of a thin elastomer film sandwiched between two compliant electrodes,  
 28 forming a capacitor capable of converting electrical energy to or from mechanical energy,  
 29 i.e. a transducer<sup>2</sup>. When an external voltage is applied to the electrodes, the gener-  
 30 ated electrostatic pressure causes the electrodes to attract one another, thus decreasing  
 31 thickness of the elastomer but increasing area, due to it being a nearly incompressible  
 32 material. In this way, electrical energy is converted into mechanical energy. When the ex-  
 33 ternal voltage is removed, the elastomer regains its original shape<sup>1,8</sup>. Figure 1 illustrates  
 34 the working principle of a DE actuator.

35 Common materials used for the elastomers include acrylic, polyurethane, natural  
 36 rubber, and silicone, with silicone elastomers being those most often used, due to their  
 37 high efficiency, reliability, and fast response times<sup>8</sup>. Two common types of electrodes for  
 38 DEs are carbon grease and thin metal films such as gold or silver<sup>9</sup>.

39 Since DEs are highly flexible, they may be configured in many different ways, de-  
 40 pending on the desired application, driving force, and operating strain<sup>10</sup>. Some common  
 41 examples in this regard include extender, unimorph, bimorph, diaphragm, and tube con-  
 42 figurations<sup>3,11</sup>. By stacking DEs on top of each other, such that elastomer and electrode  
 43 layers alternate, it is possible to increase the obtainable mechanical force in actuator  
 44 mode<sup>9,12</sup> or increase the amount of harvested energy when in generator mode<sup>13</sup>.

45 When applying an electric field to a DE, several types of ageing may occur, and  
 46 these can be divided into two main categories: Slow degradation mechanisms and im-  
 47 mediate breakdown mechanisms<sup>14</sup>. Slow degradation mechanisms lead to electrical trees

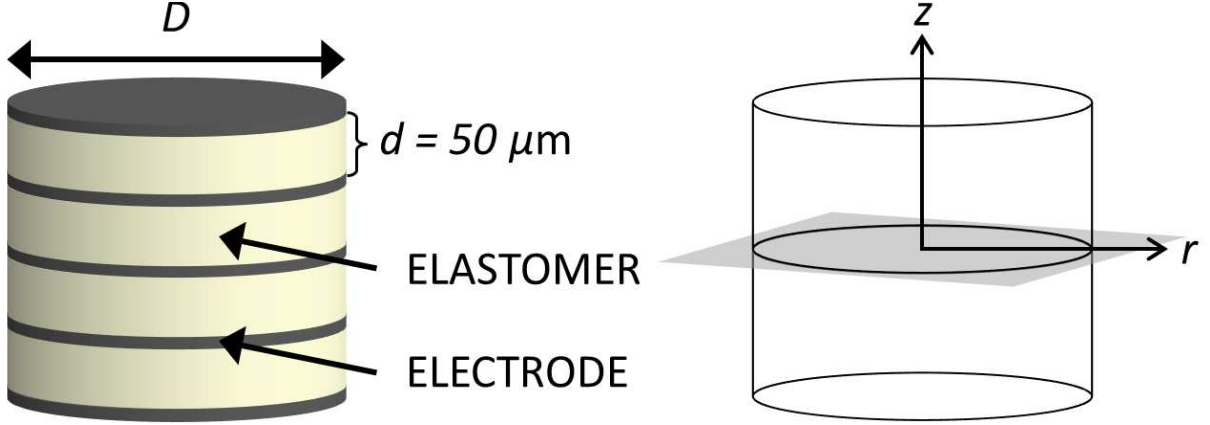
48 and water trees<sup>15,16</sup>, which may take more than one hour from initiation until material  
49 breakdown occurs, while breakdown mechanisms are somewhat instantaneous and en-  
50 tail partial discharge<sup>17</sup>, electromechanical breakdown<sup>18</sup>, electrical breakdown<sup>19,20</sup>, and  
51 thermal breakdown<sup>21-23</sup>.

52 Several studies, both experimental and model based, have been made in which the  
53 combination of electrical and mechanical forces has been investigated, in order to examine  
54 the electrical<sup>24-28</sup> and electromechanical<sup>26-32</sup> breakdown of dielectric elastomers. Elec-  
55 trical breakdown occurs when the amount of electrical carriers in the material increases  
56 exponentially, while electromechanical breakdown is the result of an uneven thinning of  
57 the material upon application of an electrical field. However, to the best of our knowl-  
58 edge, the combined effect of electrical and thermal energy on the breakdown of dielectric  
59 elastomers has received very limited attention. Thus, in this work we establish the basis  
60 for investigating thermal breakdown in multilayered dielectric elastomer, by combining  
61 electrical and thermal energies in a 2D model in the commercially available FEM soft-  
62 ware COMSOL Multiphysics<sup>®</sup>. After establishing the FEM model, a parameter study is  
63 conducted in which the effect of various parameters on the point of thermal breakdown  
64 is evaluated.

## 65 **Thermal Breakdown**

66 Thermal breakdown occurs when thermal energy generated within the stack can no longer  
67 be balanced by heat loss from the stack into the surroundings, and thus the temperature  
68 within the stack will increase towards infinity<sup>14</sup>. Thermal energy is mainly generated due  
69 to Joule heating of the material, i.e. heating due to electrical resistance in the material.  
70 The amount of thermal energy generated per unit volume,  $q$ , in a stacked DE with  $N$   
71 layers, each of thickness  $d$  and cross-section  $A$ , is given by Joule's law:

$$q = \frac{V^2}{R N d A} = E^2 \sigma \quad (1)$$



**Figure 2:** Left: A stacked DE with  $N = 4$  layers, each with a thickness of  $d = 50 \mu\text{m}$  and a diameter of  $D$ . Right: Geometric illustration of the stack, with the gray plate indicating the symmetry plane in the middle of the stack.

72 where  $V$  is the applied voltage,  $R$  the material resistance,  $E$  the applied electric field,  
 73 and  $\sigma$  the electrical conductivity of the elastomer. Thermal breakdown is especially  
 74 relevant when considering stacked DEs, since multiple layers result in a larger volume  
 75 and therefore more Joule heating, without an equal increase in surface area. Thus, heat  
 76 loss into local surroundings decreases when  $N$  increases.

## 77 Model Setup

78 As stated earlier, thermal breakdown is especially important when considering a multi-  
 79 layered DE. Thus, the configuration considered in this work is a stack of  $N$  circular discs  
 80 of DEs, as shown in Figure 2. The elastomer layers each have a thickness of  $d = 50 \mu\text{m}$   
 81 and a thermal conductivity of  $k = 0.15 \text{ W/mK}^{3,33}$ , which is assumed constant in terms of  
 82 both position and temperature. The electrode layers are approximately three orders of  
 83 magnitude thinner than the elastomer layers, and thermal conductivity of the conductive  
 84 electrodes is much higher than that of the elastomer. Therefore, it is assumed that the  
 85 electrodes will not be a limiting factor in heat transport within the stack, and so the  
 86 effect of the electrodes is therefore neglected in this work.

87 The steady-state energy balance for the system at hand is as follows:

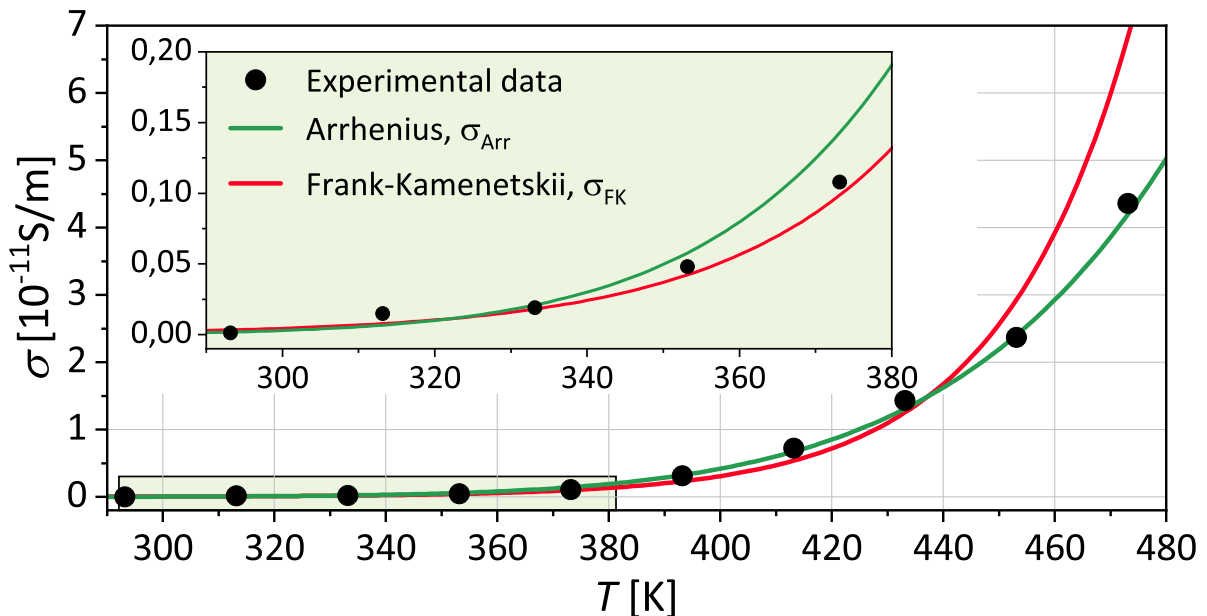
$$k \nabla^2 T + E^2 \sigma(T) = 0 \quad (2)$$

88 where  $E$  is the electric field,  $\sigma$  is the temperature dependant electrical conductivity, and  
 89  $T$  is the temperature.

90 First term on the left side of Eq. (2) states the thermal conduction within the stack,  
 91 while the second term on the left is the amount of thermal energy generated per unit  
 92 volume of the stack, as given in Eq. (1).

### 93 Electrical Conductivity

94 The elastomer material used for validating the model presented in this paper is Elastosil  
 95 RT625 from Wacker Chemie AG. The electrical conductivity of RT625 as a function of  
 96 temperature was measured through dielectric relaxation spectroscopy, using a Novocon-  
 97 trol Alpha-A high-performance frequency analyser at 1 Hz and 1 V/mm. The thickness of  
 98 the test sample was 1.316 mm and the diameter of the sample was 25 mm. The electrodes  
 99 used were 2 mm disposable gold-plated flat electrodes from Novocontrol Technologies with  
 100 a diameter of 20 mm for the top electrode and 40 mm for the bottom electrode. The  
 101 obtained experimental data are shown in Figure 3.



**Figure 3:** Experimental data (●) for the electrical conductivity of Elastosil RT625, fitted with Arrhenius expression (—), Eq. (5), and Frank-Kamenetskii expression (—), Eq. (6).

102 Typically the electrical conductivity of an elastomer varies with temperature in ac-

103 cordance to an Arrhenius-type relation:

$$\sigma_{\text{Arr}}(T) = \sigma_{0,\text{Arr}} \exp\left(-\frac{\beta_{\text{Arr}}}{T}\right) \quad (3)$$

104 where  $\sigma_{0,\text{Arr}}$  is the pre-exponential factor, and  $\beta_{\text{Arr}}$ , with a unit of temperature, is the  
105 ratio between the activation energy of conduction and Boltzmann's constant<sup>14</sup>. Due to  
106 mathematical difficulties involved in integrating  $\exp(-T^{-1})$ , it is common to use a Frank-  
107 Kamenetskii approximation of Eq. (3), which entails taking a Taylor-series expansion  
108 of the exponential around a reference temperature<sup>34</sup>. The resulting  $\sigma_{\text{FK}}$  function is as  
109 follows:

$$\sigma_{\text{FK}}(T) = \sigma_{0,\text{FK}} \exp(\beta_{\text{FK}} T) \quad (4)$$

110 where  $\beta_{\text{FK}}$  has units of inverse temperature.

111 From the experimental data obtained, the following Arrhenius expression and Frank-  
112 Kamenetskii expression has been attained:

$$\sigma_{\text{Arr}}(T) = 1.261 \cdot 10^{-5} \frac{\text{S}}{\text{m}} \cdot \exp\left(-\frac{5968 \text{ K}}{T}\right) \quad (5)$$

$$\sigma_{\text{FK}}(T) = 1.327 \cdot 10^{-19} \frac{\text{S}}{\text{m}} \cdot \exp\left(0.0424 \text{ K}^{-1} \cdot T\right) \quad (6)$$

113 The two fitted expressions for the electrical conductivity is plotted along side the  
114 experimental data in Figure 3. From Figure 3 it can be seen that at low temperatures  
115 both the Arrhenius expression and the Frank-Kamenetskii expression are excellent at  
116 describing the temperature dependence of the electrical conductivity, but as temperature  
117 increases, the Frank-Kamenetskii fitting falls short. However, the mathematically simpler  
118 Frank-Kamenetskii expression is an overall good approximation of the Arrhenius function  
119 in the range from room temperature through 430 K.

## 120 Analytical Model

121 In order to obtain an analytical solution to the model put forth, it is assumed that the  
 122 cylindrical surface of the DE is thermally insulated, and that the temperature within the  
 123 stack only varies in height, i.e.  $z$ -direction (see Figure 2). Furthermore, the mathemat-  
 124 ically simpler Frank-Kamenetskii expression for the electrical conductivity obtained in  
 125 Eq. (6) is utilised. The energy balance put forth in Eq. (2) therefore simplifies to:

$$k \frac{d^2 T}{dz^2} + E^2 \sigma_{\text{FK}}(T) = 0 \quad (7)$$

126 By assuming that the temperature on the top and the bottom of the stack is constant  
 127 and equal to the temperature of the surroundings,  $T(z = \pm \frac{1}{2} N d) = T_0$ , a symmetry  
 128 plane in the middle of stack is employed, and an analytical solution to Eq. (7), can be  
 129 found. The analytical solution may be formulated as a relation between the maximum  
 130 temperature,  $T(z = 0) = T_{\text{max}}$ , and a non-dimensional parameter  $\lambda$  as follows:

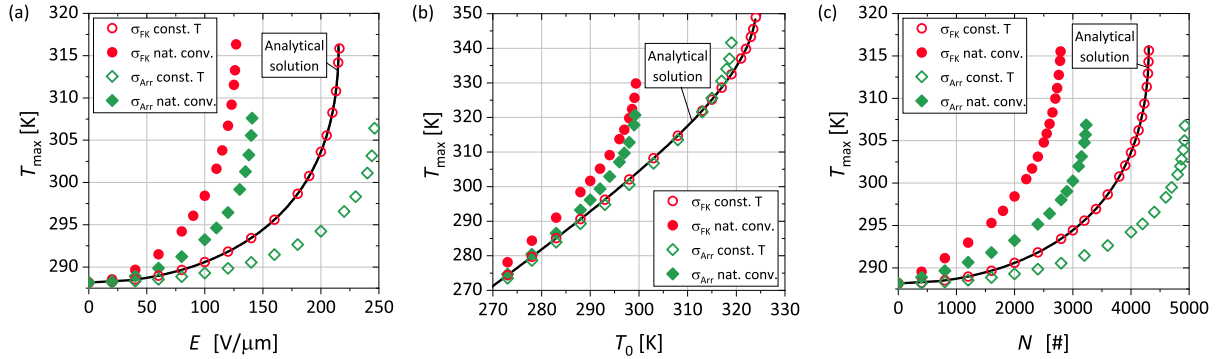
$$\lambda = 2 \exp(-\theta_{\text{max}}) \left( \operatorname{arctanh} \sqrt{1 - \exp(-\theta_{\text{max}})} \right)^2 \quad (8)$$

131 where  $\theta_{\text{max}} = \beta_{\text{FK}}(T_{\text{max}} - T_0)$  is a dimensionless maximum temperature, and:

$$\lambda = \frac{\sigma_{0,\text{FK}} \exp(\beta_{\text{FK}} T_0) \beta_{\text{FK}} (E N d)^2}{4k} \quad (9)$$

132 The parameter  $\lambda$  in Eq. (9) is in itself a function of several parameters, so in order to  
 133 illustrate how the analytical solution obtained in Eq. (8) varies in a physical setup, it is  
 134 plotted in Figure 4 as a function of: (a)  $E$ , (b)  $T_0$ , and (c)  $N$ . The base case values are  
 135  $E = 100 \text{ V}/\mu\text{m}$ ,  $T_0 = 288 \text{ K}$ , and  $N = 2000$ , from which one parameter is swept in each  
 136 figure while keeping the remaining two constant. As the value of the sweeping parameters  
 137 increase, the slope of the lines ( $\frac{dT_{\text{max}}}{dE}$ ,  $\frac{dT_{\text{max}}}{dT_0}$ , or  $\frac{dT_{\text{max}}}{dN}$ , respectively) approach infinity and  
 138 ends abruptly at some given parameter value, denoted as the points of breakdown. These  
 139 points, are the highest values of the sweeping parameters at which it is possible to obtain  
 140 steady-state solutions, since above these points the maximum temperature will be infinity.





**Figure 4:**  $T_{\max}$  within a stacked DE as a function of: (a)  $E$ , (b)  $T_0$ , and (c)  $N$ . Simulations using two  $\sigma$ -functions ( $\sigma_{\text{FK}}$  ( $\bullet, \circ$ ) and  $\sigma_{\text{Arr}}$  ( $\blacklozenge, \diamond$ )) and two boundary conditions on the top and bottom (constant temperature ( $\circ, \diamond$ ) and natural convection ( $\bullet, \blacklozenge$ )) are shown, as well as the analytical solution ( $-$ ). The base case values are  $E = 100 \text{ V}/\mu\text{m}$ ,  $T_0 = 288 \text{ K}$ , and  $N = 2000$ , from which one parameter is swept while keeping the remaining constant. Thermal insulation on the cylindrical surface is assumed.

141 For the analytical model the breakdown points are  $E_{\text{BD}} = 215 \text{ V}/\mu\text{m}$ ,  $T_{0,\text{B}} = 324 \text{ K}$ , and  
 142  $N_{\text{BD}} = 4305$ .

143 It should be emphasized that the breakdown values presented here are for a steady-  
 144 state breakdown, thus thermal breakdown will occur over time at parameter values higher  
 145 than the steady-state breakdown values.

## 146 Simulated Model

147 In order to obtain more complex and realistic results than the analytical solution, the  
 148 model stated in Eq. (2) is implemented into the commercial finite element simulation  
 149 software COMSOL Multiphysics<sup>®</sup>. In COMSOL Multiphysics<sup>®</sup> it is utilized that the  
 150 geometry of the multilayered stack of dielectric elastomer is 2D axisymmetric, and thus  
 151 a 2D axisymmetric model has been set up using the "Heat Transfer in Solids" module  
 152 applying a heat source with the quantity specified in Eq. (1). The energy balance solved  
 153 is:

$$k \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right] + E^2 \sigma(T) = 0 \quad (10)$$

154 where both the Arrhenius expression and the Frank-Kamenetskii expression for the elec-  
 155 trical conductivity has been used.

## 156 **Validation of Numerical Simulation**

157 The analytical solution from Eq. (8) is used to verify simulated results of the same setup  
158 in COMSOL Multiphysics<sup>®</sup>. It is therefore assumed that the cylindrical surface of the  
159 stack is thermally insulated, and the temperature on the top and bottom of the stack is  
160 constant and equal to the surroundings. Furthermore, the Frank-Kamenetskii expression  
161 for the electrical conductivity given in Eq. (6) is used.

162 The simulated results using  $\sigma_{\text{FK}}$  and  $T(z = \pm \frac{1}{2}Nd) = T_0$  are shown in Figure 4, and  
163 it can be seen that these are in full agreement with the analytical solution obtained for  
164 all three varied parameters.

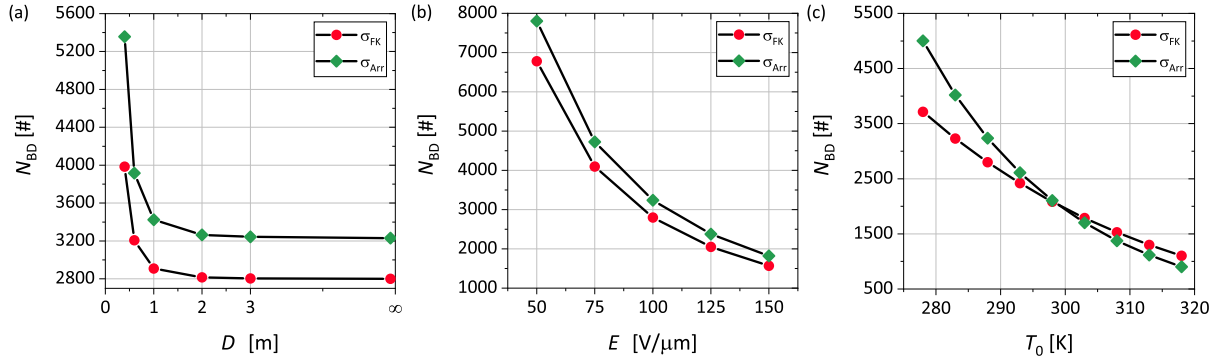
## 165 **Including Natural Convection as Means of Heat Transfer**

166 Instead of using the crude approximation that the temperature at the top and bottom  
167 of the DE stack is equal to the temperature of the surroundings, a more realistic model  
168 may be obtained by including natural convection on the top and bottom of the stack.  
169 The heat transfer functions used are as follows<sup>35</sup>:

$$h_t = 2.44 \text{ W/m}^2\text{K} (T_t - T_0)^{0.25} \quad (11)$$

$$h_b = 1.31 \text{ W/m}^2\text{K} (T_b - T_0)^{0.25} \quad (12)$$

170 where  $T_t$  and  $T_b$  are temperatures at the top and bottom plates, respectively. Simulated  
171 results using natural convection as boundary conditions are shown in Figure 4. In all three  
172 cases, when varying  $E$ ,  $T_0$ , or  $N$ , it is notable that results from simulations with natural  
173 convection have a lower breakdown point than simulations where  $T(z = \pm \frac{1}{2}Nd) = T_0$  is  
174 assumed. When assuming a constant temperature at the top and bottom, the assumption  
175 effectively used is that all excess thermal energy is removed from the surfaces, i.e. perfect  
176 heat transfer. However, with natural convection as the boundary condition, the heat  
177 transfer is no longer perfect, and so the temperature at the top and bottom will be higher  
178 than the surroundings, thereby leading to a higher  $T_{\text{max}}$  and thus lower breakdown point.



**Figure 5:**  $N_{BD}$  as a function of: (a)  $D$ , (b)  $E$ , and (c)  $T_0$  for a stacked DE with natural convection on the top and bottom, according to Eq. (11) and (12). Two fittings of the electrical conductivity has been used in the simulations; Frank-Kamenetskii ( $\bullet$ ) and Arrhenius ( $\blacklozenge$ ). The base case values are  $E = 100 \text{ V}/\mu\text{m}$ ,  $T_0 = 288 \text{ K}$ , and  $D = \infty$  (thermal insulation on the cylindrical surface), from which one parameter is swept while keeping the remaining constant.

## 179 Significance of Expression for Electrical Conductivity

180 Up to this point, the Frank-Kamenetskii equation for electrical conductivity,  $\sigma_{FK}$ , has  
 181 been used. However, as stated earlier, this expression is an approximation of the more  
 182 realistic Arrhenius equation,  $\sigma_{Arr}$ . Therefore, simulations using  $\sigma_{Arr}$  as given in Eq. (5)  
 183 were performed in COMSOL Multiphysics<sup>®</sup>, and the results obtained are shown in Figure  
 184 4.

185 By comparing results from simulations where  $\sigma_{FK}$  is used to ones where  $\sigma_{Arr}$  is used,  
 186 when varying  $E$  and  $N$  (Figure 4 (a) and (c)), it is evident that results using  $\sigma_{FK}$  always  
 187 underestimate the breakdown value of  $E$  or  $N$ , for both cases of boundary conditions.  
 188 When varying the surrounding temperature (Figure 4 (b)), results using  $\sigma_{FK}$  slightly  
 189 overestimate the maximum temperature at low  $T_0$ , but at high  $T_0$  the maximum temper-  
 190 ature is underestimated. Consequently,  $T_{0,BD}$  is overestimated with Frank-Kamenetskii  
 191 approximated results compared to Arrhenius fitted results. This crossover of the Arrhe-  
 192 nius and Frank-Kamenetskii fitted results is assigned to the crossover in the fittings of  
 193  $\sigma_{Arr}$  and  $\sigma_{FK}$  in Figure 3. However,  $\sigma_{FK}$  is an acceptable approximation of  $\sigma_{Arr}$  as long  
 194 as  $T_0$  is not too high.

## 195 Parameter Study

196 As mentioned earlier, thermal breakdown is more prone to occur when DEs are stacked, so  
197 a parameter study is performed in order to investigate how different design and operating  
198 parameters affect the possible amount of layers in a stacked DE before thermal breakdown  
199 occurs, hence  $N_{\text{BD}}$ .

### 200 Diameter

201 Instead of assuming thermal insulation on the cylindrical surface of the stack, natural  
202 convection is also assumed to occur on this surface, with the following heat transfer  
203 function<sup>35</sup>:

$$h_c = 1.97 \text{ W/m}^2\text{K} (T_c - T_0)^{0.25} \quad (13)$$

204 where  $T_c$  is the temperature on the cylindrical surface of the stacked DE. Figure 5 (a)  
205 shows results from simulations in COMSOL Multiphysics<sup>®</sup> when varying the diameter of  
206 the stack,  $D$ .  $N_{\text{BD}}$  decreases significantly when increasing  $D$  for both electrical conduc-  
207 tivity fittings. When the stack diameter approaches zero, a stack with an infinite amount  
208 of layers could theoretically be constructed, because the thermal energy generated within  
209 the stack never exceeds the amount of energy transferred away at the surface, due to the  
210 fact that the ratio between surface area and volume approaches infinity. On the contrary,  
211 when the stack diameter approaches infinity, the amount of layers possible asymptotically  
212 approaches the  $N_{\text{BD}}$  value obtained when assuming thermal insulation on the cylindrical  
213 surface of the stack. This value is 2800 layers when using  $\sigma_{\text{FK}}$  and 3230 layers when using  
214  $\sigma_{\text{Arr}}$ . For the remaining simulations, thermal insulation on the cylindrical surface is used,  
215 since this minimizes simulation time.

### 216 Electric Field

217 A second important parameter is the applied electric field and its effect on  $N_{\text{BD}}$ , as shown  
218 in Figure 5 (b). It is evident that  $N_{\text{BD}}$  decreases as  $E$  increases. This is explained by  
219 Eq. (1), which states that the amount of generated energy increases with electric field

220 squared. Thus, when more energy is generated,  $T_{\max}$  increases, and fewer layers can  
221 therefore be stacked before thermal breakdown occurs.

## 222 Temperature of Surroundings

223 The third examined parameter is the temperature of the surroundings, with the results  
224 as shown in Figure 5 (c). It is notable that  $N_{\text{BD}}$  decreases when  $T_0$  increases. The  
225 driving force for natural convection is the difference in temperature between surroundings  
226 and stack surface, as seen in Eq. (11) and (12). Thus, when the temperature of the  
227 surroundings is increased, the driving force is decreased. Consequently, the temperature  
228 within the stack is increased and thermal breakdown occurs. It should be noted that  
229 once again, the crossover of results with  $\sigma_{\text{FK}}$  and  $\sigma_{\text{Arr}}$  is seen, as explained earlier.

## 230 Discussion

231 The lowest value of  $N_{\text{BD}}$  obtained in the results shown in Figure 5 is  $N_{\text{BD}} = 904$  at  $T_0 =$   
232  $318 \text{ K}$ ,  $E = 100 \text{ V}/\mu\text{m}$ , and with thermal insulation on the cylindrical surface. However,  
233 this amount of layers is far beyond the amount currently seen in any applications; for  
234 example, SBM Offshore has built a wave energy harvester with a maximum number of  
235 300 layers<sup>7</sup>.

236 The overestimation of the possible number of layers predicted by simulations compared  
237 to experimental results may be explained by the fact that no mechanical deformation of  
238 the stack is taken into consideration in the model presented herein. Modelling the electro-  
239 mechanical coupling of dielectric elastomers is addressed multiple times in the literature,  
240 e.g by Hoffstadt and Maas<sup>36</sup>, Dorfmann and Ogden<sup>37,38</sup>, Zhao and Suo<sup>39</sup>, and Qu and  
241 Suo<sup>29</sup>. If the electro-mechanical coupling was included in the electro-thermal model pre-  
242 sented in this work, the electric field would increase due to compression of the elastomer  
243 layer. Along side an increase in the electric field, the amount of energy generated by  
244 Joule heating, i.e. Eq. (1), would also increase leading to higher temperatures within  
245 the stack and thus less possible numbers of stacked layers. Consequently, the point of

246 thermal breakdown,  $N_{\text{BD}}$ , would decrease considerably, if the electro-mechanical coupling  
247 had been included as well.

248 Furthermore, the elastomer material is assumed not to have any impurities or inho-  
249 mogeneous regions, which are inevitable in real-life products. Imperfections would lead  
250 to a lower  $N_{\text{BD}}$ , since these areas of impurities or inhomogeneous regions will inherently  
251 have a higher thermal conductivity and thus serve as hotspots in the material. Last but  
252 not least, the model presented herein does not take into account any thermal degradation  
253 processes that may take place in the elastomer when the temperature of the material is  
254 elevated. Thus, the importance of the results presented lies not in the specific values of  
255  $N_{\text{BD}}$  obtained but rather in the trend obtained when varying a given parameter.

## 256 Conclusion

257 In this paper, an electro-thermal model of a stacked DE with  $N$  layers has been presented,  
258 which is able to predict the point of thermal breakdown. Two types of fitting functions  
259 for electrical conductivity have been used, and it can be concluded that the use of the  
260 mathematically simpler Frank-Kamenetskii approximation is acceptable, albeit conserva-  
261 tive, except at high surrounding temperatures. A parameter study was conducted from  
262 which it can be concluded that if the diameter of the stack is large, it is suitable to assume  
263 thermal insulation of the cylindrical surface of the stack. Furthermore, it has been found  
264 that increasing the applied electric field or the temperature of the surroundings greatly  
265 decreases the possible number of layers in a stacked DE.

## 266 Acknowledgements

267 The authors gratefully acknowledge financial support from Aage and Johanne Louis-  
268 Hansens Fond, and they would also like to thank senior researcher Liyun Yu at DTU for  
269 performing the electrical conductivity measurements.

## 270 **Notation**

$\beta$	Factor in $\sigma_{\text{FK}}(T)$ [ $\text{K}^{-1}$ ] or $\sigma_{\text{Arr}}(T)$ [K]
$\lambda$	Dimensionless parameter in analytical solution [-]
$\sigma$	Electrical conductivity [S/m]
$\sigma_0$	Factor in $\sigma_{\text{FK}}(T)$ or $\sigma_{\text{Arr}}(T)$ [S/m]
$\theta$	Dimensionless temperature [-]
$A$	Cross-section area of DE stack [ $\text{m}^2$ ]
$d$	Thickness of DE [m]
$D$	Diameter of stack [m]
271 $E$	Electric field [V/m]
$h$	Heat transfer function [ $\text{W}/\text{m}^2\text{K}$ ]
$k$	Thermal conductivity [ $\text{W}/\text{mK}$ ]
$N$	Amount of layers [#]
$q$	Generated thermal energy pr. volume [ $\text{W}/\text{m}^3$ ]
$R$	Resistance [ $\Omega$ ]
$T$	Temperature [K]
$V$	Voltage [V]

## 272 **Subscripts**

Arr	Arrhenius equation
b	Bottom surface of DE stack
BD	Breakdown
c	Cylindrical surface of DE stack
273 FK	Frank-Kamenetskii approximation
max	Maximum
t	Top surface of DE stack
0	Surroundings

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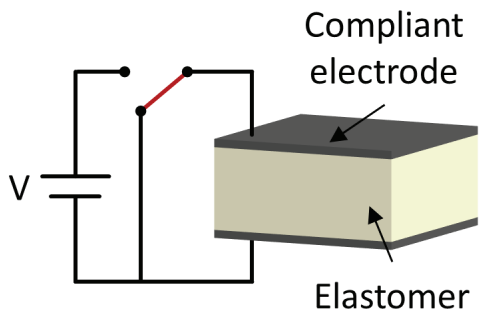
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## 354 List of Figures

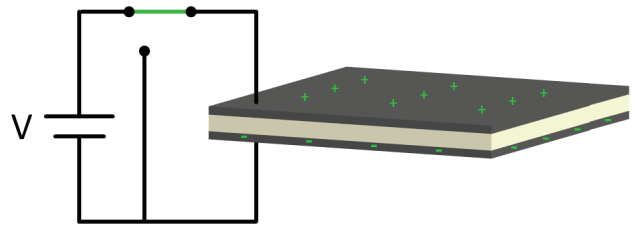
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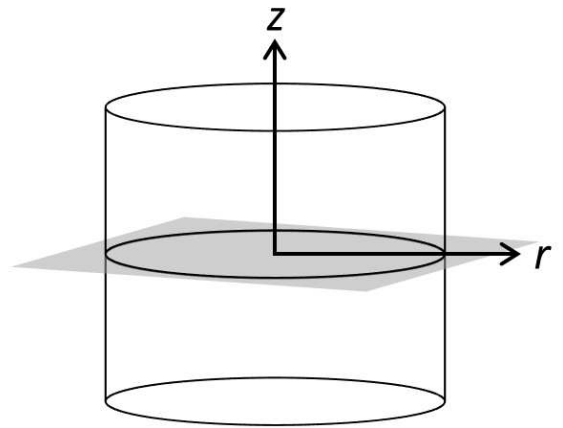
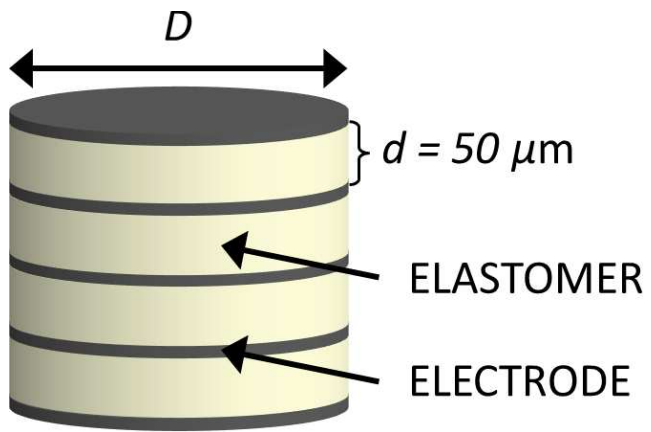
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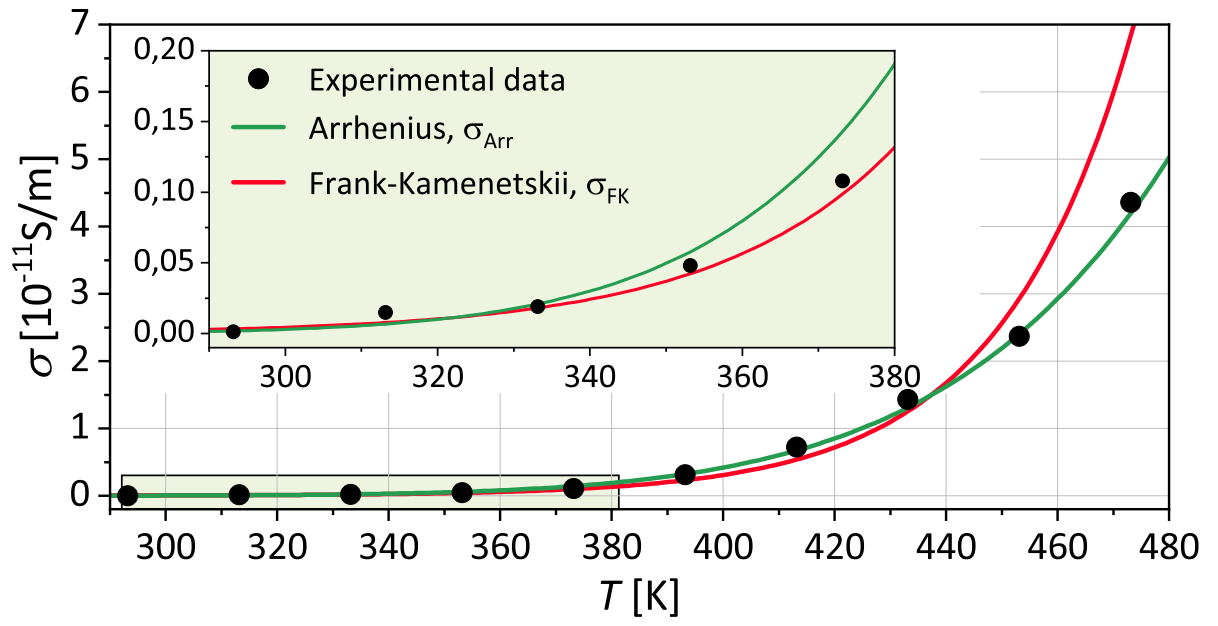
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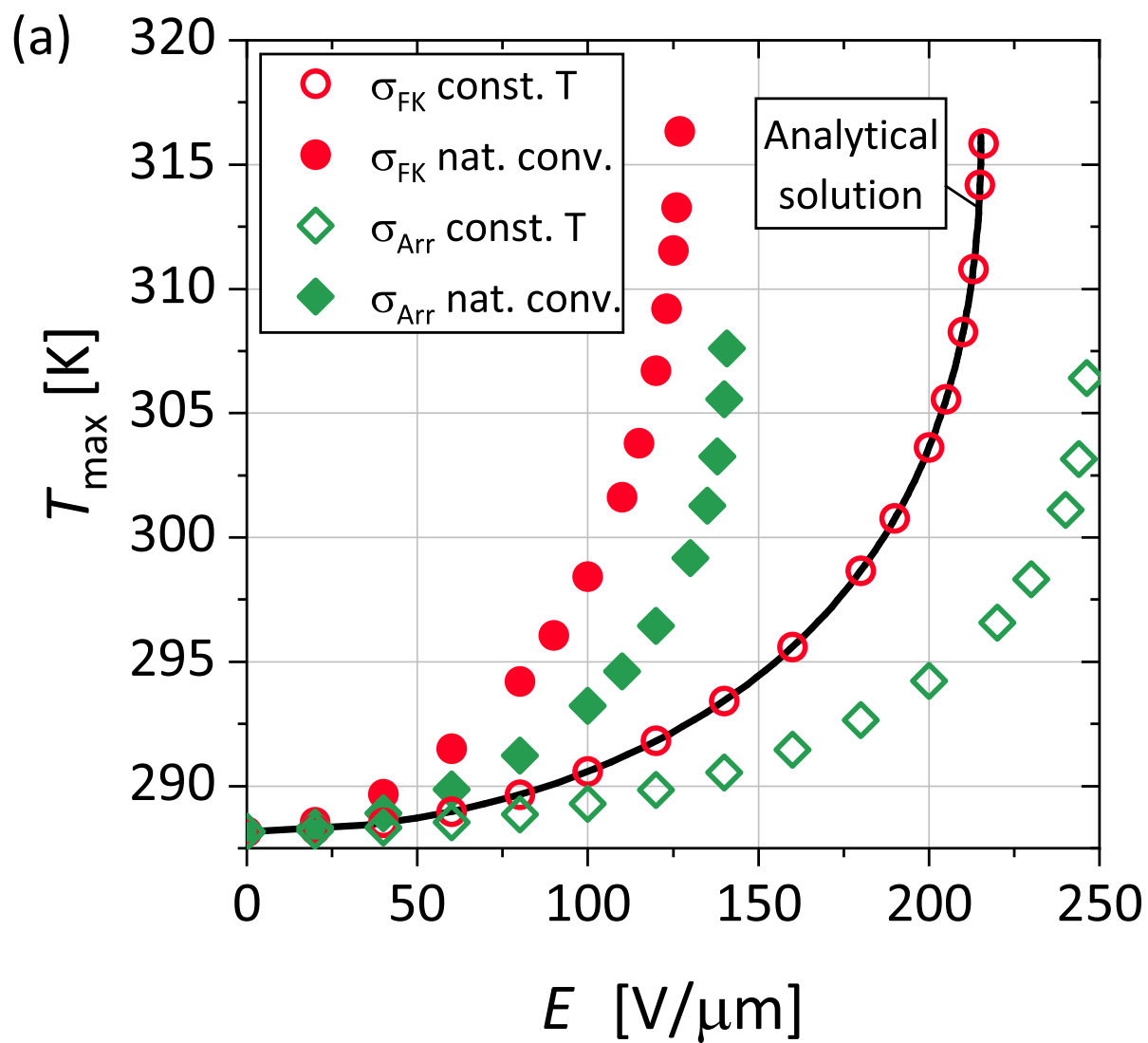




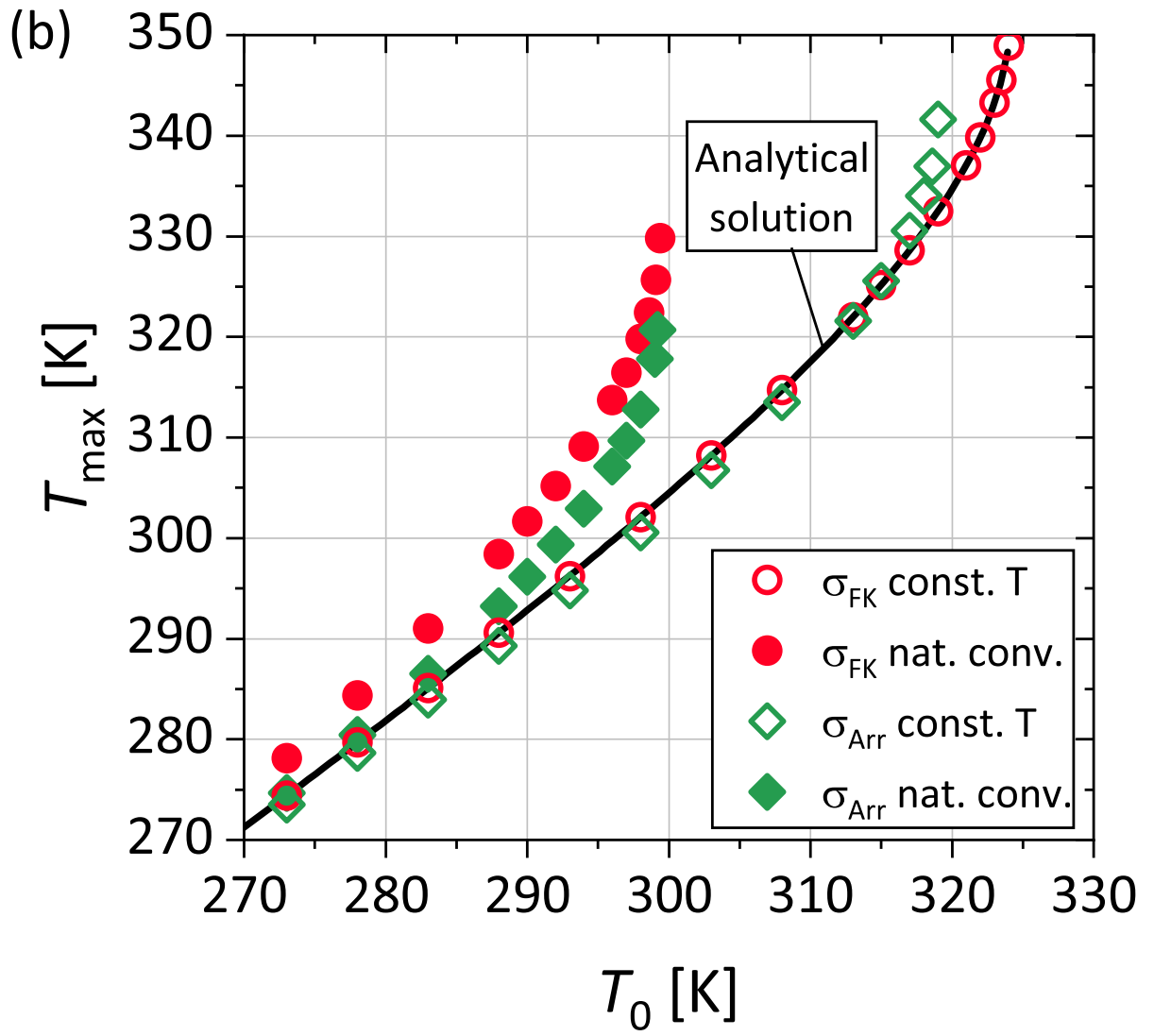
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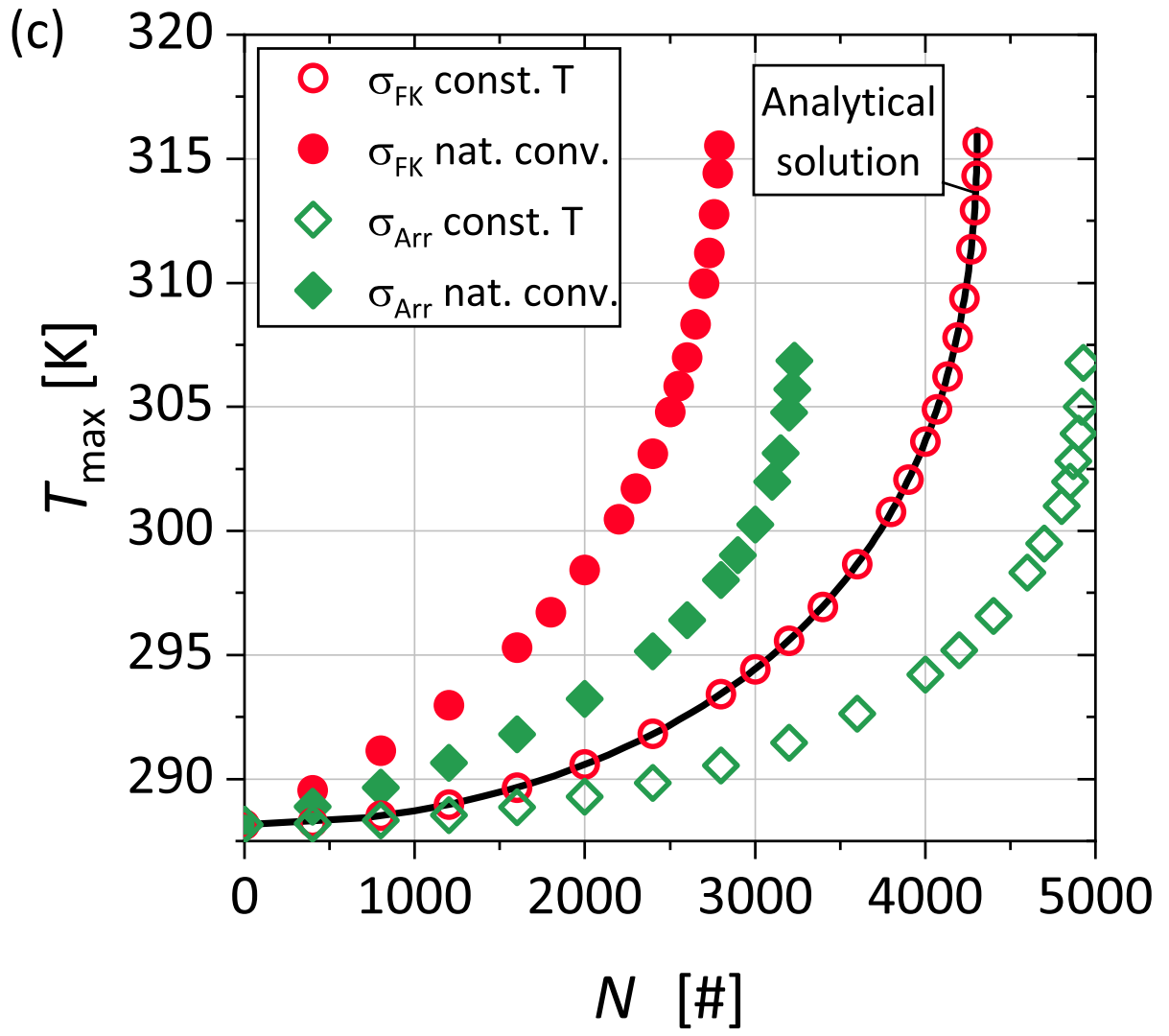
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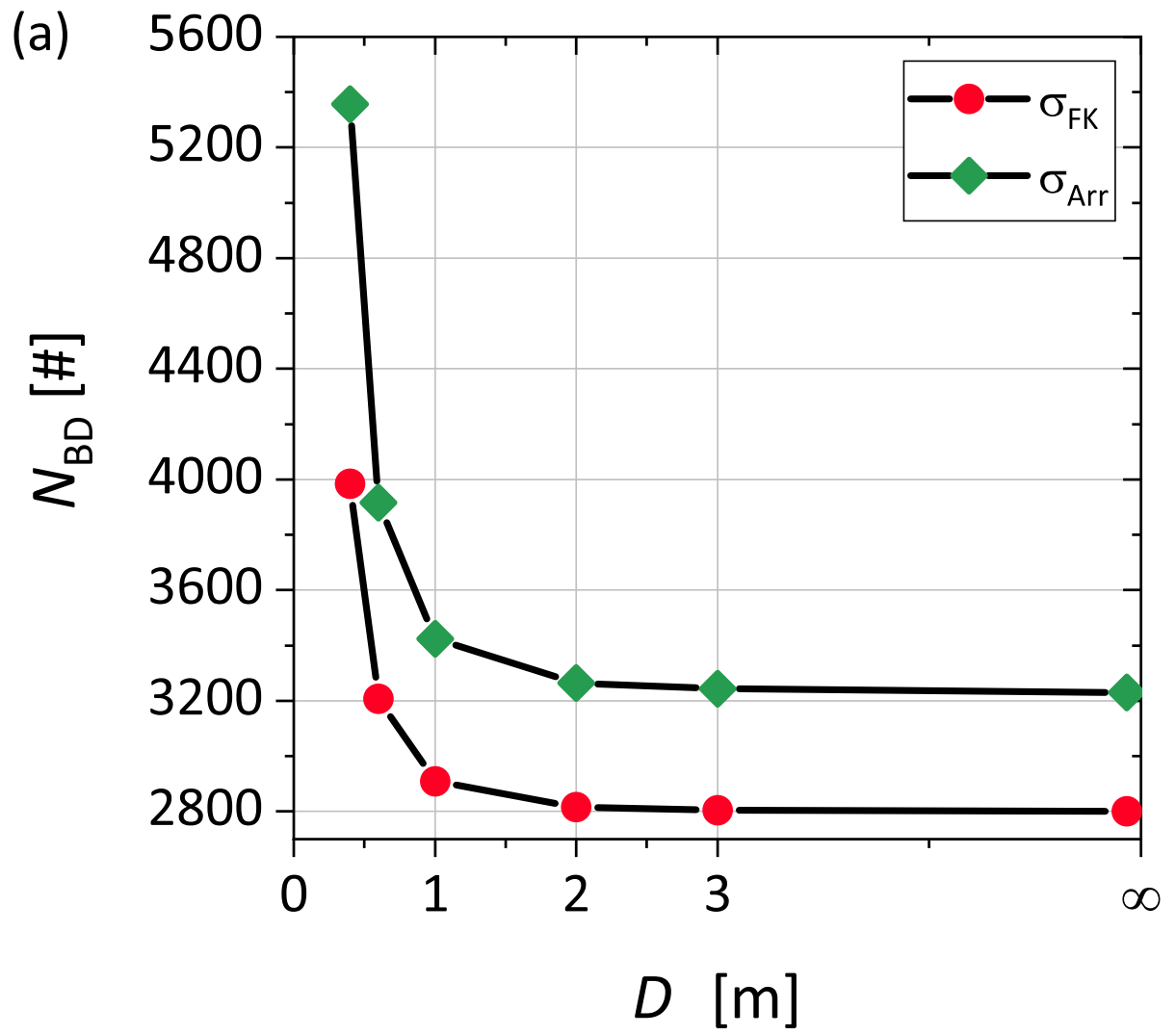


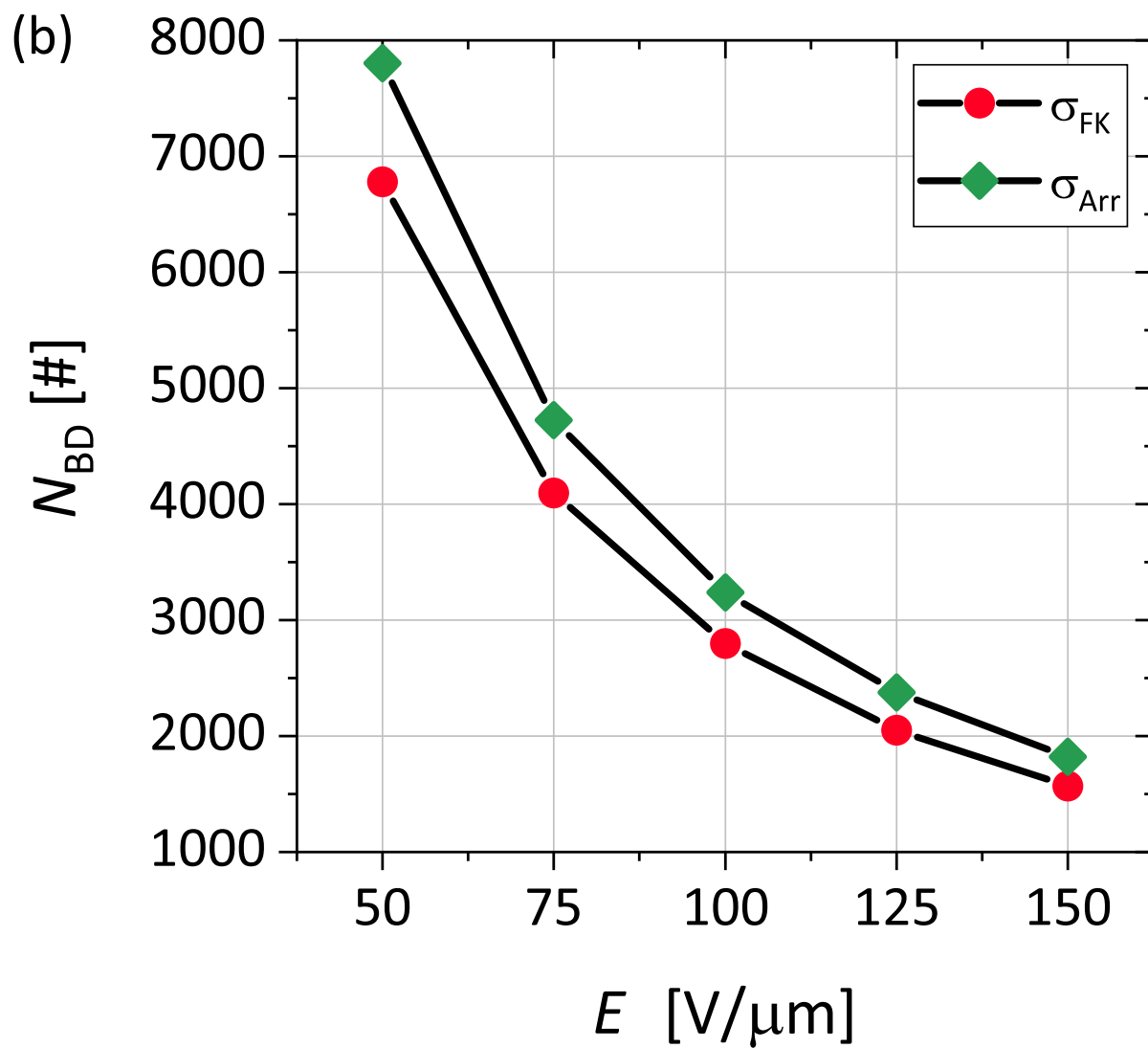
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