Inow Measurements by Nacelle Mounted Lidars for Wind Turbine and Farm Control

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Inflow Measurements by Nacelle Mounted Lidars for Wind Turbine and Farm Control

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Inflow Measurements by Nacelle Mounted Lidars for Wind Turbine and Farm Control

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A major challenge in the operation and control of wind turbines is the adjustment to changes in the wind condition. Currently, a variety of sensors is being utilized to determine characteristics in the inflow to maximize the power production of the turbine while keeping its structural loads within the design limits. Among those, light detection and ranging (lidar) is a novel technology that allows to remotely sense the wind and has the possibility to overcome drawbacks of more traditional nacelle anemometry. Lidar systems mounted on the nacelle of wind turbines can be used in a forward-looking setup to probe the incoming wind field at several positions. By making suitable assumptions important inflow characteristics can be derived, such as wind speed, turbine yaw misalignment, vertical shear and gusts.

Several approaches to determine the dominant frequency in a Doppler spectrum exist. Here we investigate the performance of the maximum, centroid and median methods on the spatial averaging effect of the lidar and its accuracy compared to a sonic anemometer. Surprisingly, the maximum method, which uses the least information from the Doppler spectrum, performed best at reducing the spatial averaging. However, this benefit is gained at the expense of an increased root mean squared error (RMSE) compared to the sonic measurement. The overall best performance was achieved by the median method, which showed the lowest RMSE and had a slight mitigation of the spatial averaging effect compared to the centroid method.

By providing a preview of the rotor-effective wind speed (REWS), lidar systems can be used to assist pitch controllers to reduce structural loading on turbines. In this project, the coherence of the REWS estimated from turbine and lidar measurements is evaluated experimentally and compared to a model based on the Mann turbulence and a model for the spatial averaging of continuous-wave lidars. The comparison shows improved agreement with the field data compared to previously used models. It can be applied as a computationally efficient tool to optimize the focus point positions of a lidar system. In that way the coherence of the REWS estimated from turbine and lidar can be maximized.

Wakes can severely violate the flow assumptions applied when deriving inflow characteristics from lidar measurements. The effect on the yaw misalignment measurement is investigated in this work. Large biases occur in half-wake situations, where one of the beams of a lidar system is affected by a wake. It is thus necessary to detect and correct situations where the lidar is influenced by wakes. Here a wake detection algorithm is proposed that uses the broadening of the Doppler spectrum due to small-scale turbulence that is generated inside wake flows. As a detection parameter the line-of-sight equivalent turbulence intensity is used to quantify the amount of turbulence within the probe volume.
and the standard deviation is defined as the spectral width of the Doppler spectrum. The algorithm is tested on a test turbine and it was possible to detect all half- and full-wake situations. Also, an empirical correction method using with the undisturbed wind direction information from a meteorological mast is proposed and it was shown that the bias in the yaw misalignment measurement can be removed for the case investigated.
Dansk sammendrag


Hvis der er ”kølvand” fra andre møller i indstrømmingen til en mølle, kan det ødelægge lidarens evne til at bestemme vindretningen præcist, især hvis kølvandet kun påvirker en del af målepunkterne. Det er derfor vigtig at opdage, hvis et af lidar-målepunkterne er inde i et kølvand. Vi foreslår en metode, som bruger den effekt at Doppler spektret bliver bredere, hvis der er turbulens i målevolumenet. Vi bruger en line-of-sight turbulensintensitet til at kvantificere mængden af turbulens i målevolumenet. Metoden er eksperimentelt testet og kan detektere situationer hvor kølvandet dækker hele indstrømmingen eller kun en del. Ved at bruge denne information udvikles en metode, der kan korrigere vindretningen målt af lidaren.
This thesis was prepared at the department of Wind Energy at the Technical University of Denmark in fulfillment of the requirements for acquiring a Ph. D. degree. The research described in this thesis was funded by Danmarks Innovationsfond in form of an industrial Ph. D. project under grant 5016-00182.

This thesis summarizes the research work completed during the three year long Ph. D. project at the Meteorology and Remote Sensing section at DTU Wind Energy. The Ph. D. project lasted from 1st January 2016 to 31st December 2018 and was supervised by Prof. Jakob Mann and co-supervised by Dr. Qi Hu. It was accompanied by a research project called Lidar Detection of Wakes for Wind Turbine Optimization funded by Energiteknologiske Udviklings- og Demonstrationsprogram (EUDP) (project number: 64016-0020).

Risø campus, Roskilde, 21st December 2018

Dominique Philipp Held
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At Windar Photonics, I want thank the Wind Analytics and Turbine Integration team, namely Antoine Larvol, Guillermo González Rilova and Nikos Kouris. Also the time I spent with Sayathan Chattopadhyay during his Master thesis was really enjoyable. Further, I want to thank all the friends and colleagues that accompanied me through the three years.

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List of Publications

Journal Papers

1. Compareion of methods to derive radial wind speed from a continuous-wave coherent lidar Doppler spectrum
   D P Held and J Mann
2. Lidar Estimation of Rotor-Effective Wind Speed - An Experimental Comparison
   D P Held and J Mann
   submitted to Wind Energy Science
3. Detection of wakes in the inflow of turbines using nacelle lidars
   D P Held and J Mann
   submitted to Wind Energy Science

Conference Contributions

1. What is the best way to derive the wind velocity from a Doppler spectrum?
   D P Held, J Mann and N Angelou
   18th International Symposium for the Advancement of Boundary-Layer Remote Sensing (ISARS), (2016)
2. Detecting wind turbine wakes with nacelle lidars
   D P Held, A Larvol and J Mann
3. Rotor-effective wind speed estimated by a forward-looking lidar
   D P Held, A Larvol, E Dellwik and J Mann
   1st Wind Energy Science Conference (WESC), (2017)
4. Wind turbine rotor-effective wind speed estimated by nacelle-mounted Doppler wind lidars
5. **Wake detection in the turbine inflow using nacelle lidars**
   D P Held, N Kouris, A Larvol and J Mann

6. **Optimal yaw strategy for optimized power and load in various wake situations**
   A M Urbán, T J Larsen, G Chr Larsen, D P Held, E Dellwik and D Verelst

**Technical Reports**

1. **Cost-efficient lidar for pitch control**
   E Dellwik, M H Hansen, D P Held, Q Hu, J K Jensen, A Larvol, J Mann, M Mirzaei, P J Rodrigo, M Sjöholm, H Villanueva
   DTU Wind Energy Report (I), (2017)
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CHAPTER 1

Introduction

The transition from fossil fuels to sustainable energy sources is one of the biggest challenges in 21st century and many countries around the world have committed to ambitious climate goals. For example, under the lead of the United Nations Framework Convention on Climate Change a set of aims to mitigate anthropogenic climate change effects, the most notorious being a 1.5°C limit on global average temperatures, has been agreed upon and as of June 2018 195 member stated have signed the agreement\(^1\). From recent developments of global renewable energy sources it becomes obvious that wind energy will be one of the most important pillars to fulfil these goals. It contributes by the second biggest amount to the global renewable electricity production, just after hydroelectricity, and experiences strong growth rates [1]. To further increase the attractiveness of wind energy the levelised cost of energy needs to be decreased.

![Figure 1.1: Recent development of leading commercial wind turbine diameter sizes and generator ratings [2].](image)

Specifically, in order to lower the cost of energy production turbines have increased tremendously in size, see Figure 1.1. A trend likely to continue in the future, bigger rotor sizes allow to capture more energy of the wind making wind turbines more efficient. However, as wind turbine blades become longer their mass increases substantially and solutions to reduces blade weight are necessary. To realize lighter blades, innovative

sensors and control strategies were developed to minimize static and dynamic loads on turbines [3].

Out of these innovations, nacelle-mounted light detection and ranging (lidar) devices have been proposed to provide preview information to the wind turbine control system [4]. Though the application of lidar system in wind energy still lies mainly in site assessment of the wind resource and performance testing of turbines, lidar-assisted control possesses great potential in increased power production and load reduction [5]. The turbine control system can utilize the preview provided by lidars to better adjust its operational points to the changing nature of the wind and thus reduce fatigue loading and protect the turbine from extreme conditions.

1.1 A Brief History of Laser Anemometry

The measurement principle of laser anemometry involves transmitting narrow bandwidth laser light, of which a small portion is scattered by aerosols suspended in the atmosphere. According to the optical Doppler effect, the aerosols impart a frequency shift which is proportional to their velocity. Estimates of the frequency shift can be obtained by suitable signal processing. Most generally, the system architecture can be separated into two concepts: direct (or incoherent) and coherent detection method.

In the direct detection method optical frequency analyzers are used to determine the Doppler shift directly from the backscattered light. The output of the optical frequency estimators can then be measured by appropriate detectors.

In contrast, for the coherent detection method the received backscattered light is mixed with a portion of the emitted light, called the local oscillator, to create a beating signal, which can be detected by photo detectors. Spectral analysis then yields the induced Doppler shift. Coherent detection can also be divided into heterodyne and homodyne detection. The former, where the local oscillator is shifted by a certain frequency, allows to determine the sign of the frequency shift. Hence, aerosols moving towards or away from the system can be distinguished. This is not feasible for the homodyne detection, where the local oscillator is left unaltered.

Historically, laser anemometry has been developed since the 1960s (not long after the invention of the laser). The concept of a coherent Doppler lidar (CDL) has already been presented in Biernson and Lucy [6]. They gave design requirements for optical radar systems using pulsed laser sources for enhanced receiver sensitivity. Over the next decades systems with different laser types have been developed. But with the invention of the CO$_2$ laser, it became the favoured laser source, due to their energy efficiency and stable single frequency output (among other aspects) [7]. For example in Lawrence et al. [8] an atmospheric experiment with a CO$_2$ laser based lidar was presented and comparisons to a cup anemometer were analyzed.

By the mid-1980s high-powered diode lasers became available that allowed for long-range applications due to reduced absorption in the atmosphere, but they also had advantages in size and efficiency. This led to applications in boundary layer measurement and aviation. The current use of the 1.5$\mu$m wavelength was stimulated by advances
and cost reductions in parts developed by the telecommunication industry, improving compactness and efficiency [7]. Nowadays, all commercially available lidars developed for the wind energy industry operate around this frequency. Since the inception of lidars both pulsed and continuous-wave (cw) lasers have been used and today there seems to be no preference towards either as both sources have their respective advantages and disadvantages.

1.2 Wind Measurements by Lidar Systems

1.2.1 Principles of Lidar Systems

A lidar system allows to focus laser light on a position in space and then measure the backscatter by particles within the measurement volume. The concept of a coherent homodyne detection method is illustrated in Figure 1.2. Laser light is created and then focused onto a target point in the atmosphere. The aerosols within the target volume backscatter a small portion of the laser light, which can be captured by the receiving optics. It is then mixed with light sent out and analyzed at the photo detector. In a monostatic setup, laser, detector and the transmitting and receiving optics are combined in one device.

![Figure 1.2](image)

Figure 1.2: Illustration of the coherent detection method from [9]. $v$ is the velocity of a particle within the measurement volume and $v_{\text{LOS}}$ is the projected wind speed onto the laser beam.

According to the optical Doppler effect, the line-of-sight (LOS) velocity component $v_{\text{LOS}}$ of the backscattering particles leads to a frequency shift of the light:

$$\delta v = \frac{2v_{\text{LOS}}}{\lambda}, \quad (1.1)$$

where $\delta v$ is the Doppler shift and $\lambda$ is the speed of light. Here all particles within the measurement volume contribute to the shift in laser light frequency.

The signal processing chain is shown in Figure 1.3. First, the back-scattered light mixed with the local oscillator is collected by a photo detector. The frequency content of the signal can be analyzed through the usage of a Fast Fourier Transformation algorithm to yield the frequency components of the detected signal. This is called the Doppler spectrum. To enhance the carrier-noise-ratio several thousand of these Doppler spectra have to be averaged. Then peak finding methods can be applied to determine the
1.2 Wind Measurements by Lidar Systems

Figure 1.3: Illustration of the lidar signal processing chain from [4].

The dominant frequency in the Doppler spectrum and thus retrieve the LOS component wind speed.

For a cw lidar the measurement of $v_{LOS}$ can be represented mathematically as the convolution between the projected LOS component $\mathbf{n} \cdot \mathbf{u}$ and a weighting function $\varphi(s)$:

$$v_{LOS}(r) = \int_{-\infty}^{+\infty} \varphi(s) \mathbf{n} \cdot \mathbf{u}(sn + r) ds,$$

(1.2)

where $s$ is the distance from the focus point along the beam, $\mathbf{n}$ is the beam unit vector, $r$ is the focus position and $\mathbf{u} = (u_1, u_2, u_3)$ is the 3-dimensional wind field. The weighting function can be approximated by a Lorentzian distribution [10]:

$$\varphi(s) = \frac{1}{\pi} \frac{z_R}{z_R^2 + s^2},$$

(1.3)

where $z_R$ is the so-called Rayleigh length, which determines the shape of $\varphi(s)$ and increases quadratically with focus distance, i.e. $z_R \propto |r|^2$. The Rayleigh length can range from a few centimeters at very small focus distances to tens of meter at focus distances of 100 m or more.

Some limitations exist compared to more traditional cup and sonic anemometers. The convolution with the weighting function $\varphi$ introduces a low-pass filtering effect on the measured $v_{LOS}$ and turbulent fluctuations in $\mathbf{u}$ are attenuated. Thus, measured variances (and turbulence intensities) will be underestimated. Further, a lidar system focusing at one position can only measure one component of the wind vector, namely the LOS component. Thus, usually scans along trajectories or probing of different points are employed. Together with assumptions of the flow field, inflow characteristics can be derived. This process is usually termed wind field reconstruction.

Different methods to determine the dominant frequency in a Doppler spectrum can be defined. For pulsed lidar systems maximum likelihood estimators are commonly applied [11]. However, for cw lidars estimators of this kind have not been used because a model for the shape of the Doppler spectrum is necessary. Currently, no such model exists for cw lidars. Thus, in early development stages simply the maximum of the Doppler spectrum was used as a dominant frequency to derive the LOS speed. Later on the
1.2 Wind Measurements by Lidar Systems

The centroid method have been used in industrial applications (e.g. in Harris, Hand, and Wright [4]), while research instruments have been using the median methods (e.g. in [12]).

In a study conducted during this project, the influence of the three frequency estimators (maximum, centroid and median method) on the volume averaging effect was investigated, see Chapter 2 or Held and Mann [13]. It has been shown that the maximum method, though using the least amount of information from the Doppler spectrum, led to the largest mitigation of the probe volume effect. On the other hand, it also showed the largest root mean squared error (RMSE) against a sonic measurement. The median method had the overall lowest RMSE and showed a slight reduction in the probe volume effect compared to the centroid method. Thus, from this study is was suggested to use the median method if high-frequency time series are wanted. When comparing 10 minute averages the performance of all three methods was comparable.

1.2.2 Lidars applied to Wind Energy

In the wind energy industry a few different lidar architecture types dominate the market. Broadly speaking they can be divided into three classes, each using different scanning strategies to retrieve wind characteristics:

- Ground-based and floating profilers
  - Doppler beam swinging
  - Velocity azimuth display
- Nacelle- or spinner-mounted lidars
  - Discrete probing at several focus positions
- Ground-based scanning lidars
  - Plan position indicator
  - Range height indicator
  - Dual or Triple Doppler
  - User-defined trajectory

Ground-based and floating profilers are mainly used to replace meteorological masts to satisfy the increasing measurement height requirements for wind turbines. They are used for wind energy resource assessment in the planning phase of wind farms and to achieve wind turbine performance validation. In these applications, their cost benefit to masts, especially offshore, and flexibility are key reasons for their usage. Their impact on the industry is also reflected in the inclusion into the IEC 61400-12 standard.

Nacelle- or spinner-mounted lidars are able to provide measurements of the flow directly in front of a wind turbine. Their measurements can be used to assist the control system of individual wind turbines or wind farms. Further, power performance validations can also be realized if corrections for the influence of the turbine itself onto
the lidar measurements are applied. A typical installation of a nacelle lidar can be seen in Figure 1.4.

![Image of lidar installation on top of the nacelle of wind turbines]

**Figure 1.4:** Two picture showing the installation of lidars on top of the nacelle of wind turbines.

Scanning lidars provide the opportunity to measure large spatial scale of the flow as they are able to measure wind speeds up to several kilometres. Thus, it is possible to visualize flows within wind farms, scan areas behind the wind turbine to capture wakes or support short term wind speed predictions.

### 1.2.3 Measuring the Turbine Inflow with Nacelle Lidars

The previously mentioned limitation of measuring only one component of the usually three-dimensional wind vector leads to the necessity of assumptions if useful information about the incoming flow field is wanted from lidar measurements. For example, if the turbine yaw misalignment should be estimated, commonly the following assumptions are made:

1. No vertical wind speed component
2. Horizontally homogeneous flow

The first assumption is usually well satisfied if the terrain is not complex and if sufficiently long time averages are taken (usually one to 10 minutes). Further, if the lidar is probing the incoming wind at the hub height, the LOS component of the vertical wind speed is very small and is not problematic for e.g. a horizontally scanning 2-beam lidar.

Horizontal homogeneity means that the wind speed and direction are equal at the same height above the ground. This is illustrated in Figure 1.5 for a horizontally measuring 2-beam lidar. In the left panel flow aligned with the turbine is shown and the LOS measurements are indicated. Both beams only measure a fraction of the horizontal wind speed. The right panel shows a misaligned case. Here a reduction of the LOS component on beam 1 and an increase on beam 2 can be seen.

Based on the assumptions mentioned above the following set of equations can be
Figure 1.5: Horizontally homogeneous inflow which is aligned (left) and misaligned (right) with the turbine’s shaft axis. Red lines indicate the laser beams of a 2-beam nacelle lidar. The projection of the wind vector onto the laser beams is also illustrated.

derived:

\[
\begin{align*}
    v_{\text{LOS},1} &= U \cos \alpha - V \sin \alpha \\
    v_{\text{LOS},2} &= U \cos \alpha + V \sin \alpha \\
    \hat{U} &= \frac{v_{\text{LOS},2} + v_{\text{LOS},1}}{2 \cos \alpha} \\
    \hat{V} &= \frac{v_{\text{LOS},2} - v_{\text{LOS},1}}{2 \sin \alpha} \\
    \hat{U}_h &= \sqrt{\hat{U}^2 + \hat{V}^2} \\
    \hat{\phi} &= \text{atan} \left( \frac{\hat{V}}{\hat{U}} \right)
\end{align*}
\]

where \( \alpha \) is the half-cone opening angle between shaft axis and laser beam, \( U \) and \( V \) are the transversal and lateral wind speed components, \( U_h \) is the horizontal wind speed and \( \phi \) is the misalignment between the wind direction and the shaft axis. The hat notation (\( \hat{\cdot} \)) indicates estimates by the lidar. Yaw misalignment is defined as the angle the turbine needs to yaw in the clockwise direction (seen from above) to align itself with the flow, see Figure 1.5. Thus, it is possible to derive a turbine yaw misalignment by considering the LOS measurement at two positions in front of the turbine.

It is also possible to incline the beams upwards or downwards to the horizontal plane. An illustration of a possible setup is shown in Figure 1.6. One beam resides in each quadrant of the turbine rotor, which allows for the estimation of the horizontal wind speed at two different heights. Thus, shear can also be estimated.

Similar assumptions as mentioned earlier can be made to derive the horizontal wind speed components at two different heights. The flow is now assumed to be horizontally homogeneous within the probe volume, which also extends vertically. Again, the vertical
wind vector component is assumed to be zero. Compliance with the latter assumption is more critical for a 4-beam lidar than compared to the 2-beam system since the beams are inclined to the horizontal plane and vertical components will contribute to LOS measurements. The equations derived earlier can readily be used (eqs. 1.4-1.9). It has, however, to be noted that the half-cone opening angle $\alpha$ needs to be projected onto the horizontal plane and thus reduces to

$$\alpha_p = \tan^{-1}\left(\frac{d_f \sin \alpha \sin \beta}{d_f \cos \alpha}\right) = \tan^{-1}(\tan \alpha \cos \beta),$$

where $d_f$ is the focus distance of the lidar and $\beta$ is the azimuthal position on the cone, see Figure 1.6. If $\beta = 0$, the beam is pointing towards the top of the cone and in case of the lidar illustrated in Figure 1.6 $\beta$ is 45°, 135°, 225° and 315°. For details of the derivation see Larvol [14].

On the other hand, if different assumptions are made, identical LOS measurements can be interpreted to derive different wind characteristics. For example, if instead of horizontal homogeneity the assumption of a perfectly aligned wind turbine is made, then the lidar measurements can be used to derive horizontal wind shear instead. This can become problematic if the lidar should be used for yaw alignment and cyclic pitch control, where both yaw misalignment and horizontal shear information is necessary [5]. A different reconstruction approach based on fitting wind characteristics to observed LOS speed has been proposed in Borraccino et al. [15].

For the remainder of this section, estimations of important inflow characteristics are compared to sonic anemometers at DTU’s test site at Risø. For this the following experiments have been conducted:

1. A 2-beam Windar Photonics lidar was mounted on a 10m high mast and was aligned horizontally. Three-dimensional sonic anemometers have been mounted approximately 1 m below each focus point. In such a setup the performance of the lidar can be analyzed very precisely, because the LOS component of the wind can be measured by sonic anemometers. Details of the setup and results can be found in Held and Mann [13].
2. The same lidar was also mounted on the nacelle of a turbine in a realistic, commercial installation. Comparisons were performed against a meteorological mast and the turbine itself. Two experimental campaign were realized; the first installation lasted from 5th December 2015 to 12th January 2016 and a second system was mounted between 29th March 2016 and 4th May 2016. Details and results can be found in Held and Mann [16] and Held and Mann [17].

3. A development unit with 4 distinct focus points was also tested at the Risø test site to evaluate the possibilities to measure the horizontal wind speed at two different heights and estimate vertical shear. The experiment lasted between 21st Oct and 15th Dec 2016 and more information can be found in Dellwik et al. [18] and Held and Mann [16].

An overview of the differences between the two lidar systems can be found in Table 1.1. Note the higher Rayleigh length due to the increased focus distance for the 4-beam lidar. Both lidars perform a scan at every focus point within 1 s. The distance between the lidar measurement and the rotor plane allow for a preview of wind fluctuations, but also require an estimation of the time delay between the measurements.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>2-beam</th>
<th>4-beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focus distance $d_f$ [m]</td>
<td>37</td>
<td>62</td>
</tr>
<tr>
<td>Rayleigh length $z_R$ [m]</td>
<td>2.1</td>
<td>6.0</td>
</tr>
<tr>
<td>Half-cone opening angle $\alpha$ [°]</td>
<td>30</td>
<td>18</td>
</tr>
<tr>
<td>Azimuth angle $\beta$ [°]</td>
<td>90 and 270</td>
<td>45, 135, 225 and 315</td>
</tr>
<tr>
<td>Distance focus points - rotor $\Delta x$ [m]</td>
<td>32</td>
<td>59</td>
</tr>
<tr>
<td>Distance lidar - rotor $d_{Nac}$ [m]</td>
<td>$\approx 1$</td>
<td>$\approx 1$</td>
</tr>
<tr>
<td>Scan time per beam [s]</td>
<td>0.5</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**Table 1.1:** Information of lidar setup parameters for the 2- and 4-beam lidar. The azimuth angle refers to the position on the scanning cone surface with 0° being the top of the cone.

**Mean Wind Speeds**

First, the 10 minute mean LOS components of the wind speed measured very close to the focus points will be compared for the setup on a 10 m tower. The magnitude of the three-dimensional sonic measurements, which have been projected onto the LOS, are compared to the lidar measurements. The comparison can be seen in Figure 1.7. A filter has been applied to instances, where the LOS component speed was above 2 m/s since lidars with homodyne detection cannot measure Doppler shifts close to zero. For both beams it can be seen that the lidar measurements agree very well with the sonic anemometers. There are no significant biases and the regression fits show slopes of close to unity.

Next, the lidar measurements mounted on the nacelle of the turbine with a hub height of 44 m and diameter of 52 m are compared to a meteorological mast. A sonic
1.2 Wind Measurements by Lidar Systems

Figure 1.7: 10 minute average wind speed comparison between lidar and sonic anemometer LOS component.

An anemometer mounted at the hub height is used to measure the horizontal wind speed. By using the assumptions described earlier the lidar measurement can also be used to estimate the horizontal wind speed. Both the 2-beam and 4-beam lidar are compared and only measurements, where the turbine was facing the meteorological mast and operating at normal power production were used. Since both lidars measure within the induction zone, a correction for the slow-down of the wind speed in front of the turbine has been applied. Details of the correction can be found in Chapter 3 and Held and Mann [16].

Both lidar systems show an adequate estimation of horizontal wind speed. Similar to the previous result, no biases or systematic errors can be observed. This gives confidence in the validity of the flow assumptions. If no induction correction is applied, the lidar measurements would systematically underestimate the horizontal wind speed.

Figure 1.8: 10 minute average horizontal wind speed comparison between lidar and sonic anemometer mounted on the meteorological mast. *Left:* 2-beam lidar and *right:* 4-beam lidar.
1.2 Wind Measurements by Lidar Systems

Turbulence

Besides measuring mean wind speeds it is also important to know the amount of fluctuations within the averaging period, because they contribute to fatigue damage on the wind turbine structural components. In the wind energy, commonly turbulence intensities are used to characterize turbulence: \( TI = \frac{\sigma}{\bar{u}} \), where \( \sigma \) is the standard deviation of the wind speed fluctuations and \( \bar{u} \) is the mean wind speed. As shown previously, mean wind speeds can be measured very accurately with lidars. Here the standard deviation is investigated.

In Section 1.2.1 the large probe volume as a consequence of the focusing of a cw laser beam has been discussed. The effect on the measurements is an attenuation of turbulent fluctuations. Thus, the time series measurement of the LOS component are low-pass filtered and variances are underestimated. To overcome this shortcoming, the use of average spectra has been suggested [19]. If a sufficiently long time average of the Doppler spectrum is taken, then the spectrum will approach the probability distribution function (PDF) of a point measurement [20, 21].

For the 2-beam lidar this has been tested at the setup on the ground. The estimation of the LOS component standard deviation by the lidar has been compared to the equivalent sonic measurement. The transducer distance of the sonic is 17.5 cm long, while the lidar’s Rayleigh length was 14.5 m. Similar to the mean wind speed calculations, time averages have been taken over 10 minutes. The comparison can be found in Figure 1.9.

![Figure 1.9](image)

Figure 1.9: 10 minute standard deviation comparison between the sonic anemometer LOS component and the lidar measurement. The estimation from the time series (Lidar TS) shows a systematic underestimation, while using the average Doppler spectrum (Lidar PDF) a high degree of similarity could be achieved.

It can be seen that the standard deviation calculated from the time series measurement is consistently underestimated by the lidar. On the other hand, the LOS standard deviation estimated from the averaged Doppler spectrum shows a substantially improved agreement and it can be seen that the data is clustered around the 45° line.

Turbulence intensities of interest are usually those along the longitudinal, lateral or vertical directions. Thus, estimations of the LOS component fluctuations need to be
converted to the principle directions. Two methods for nacelle lidars have been proposed in Peña, Mann, and Dimitrov [22] that utilize turbulent fluctuation measurement from several beams. The first method uses a model of the spectral tensor and the spatial averaging of the lidar to derive low-pass filtered turbulence statistics, while the second uses average spectra to derive unfiltered statistics.

**Yaw Misalignment**

Avoiding systematic turbine yaw errors is important to ensure maximum power production. Here, the estimates of from a mast-mounted sonic anemometer are compared to the estimates of the 2- and 4-beam lidar. Yaw misalignments can be calculated from lidar measurements are discussed above, while the misalignment from the sonic measurements is calculated as the difference between measured wind direction and turbine yaw angle

$$\hat{\phi}_s = \delta - \psi,$$

where $\delta$ is the wind direction measured by the sonic and $\psi$ is the turbine yaw angle. Both the sonic anemometer and the yaw direction sensor have been aligned to north.

Wakes in the inflow can have a significant effect on the lidar measurements and inflow direction from the wake sector have been removed. A method to detect if a lidar measurement is affected by a wake has been proposed in this work and more details can be found in Chapter 4 and in Held and Mann [17].

![Comparison of turbine yaw misalignment measured by a sonic anemometer at a meteorological mast and the 2-beam lidar (left) and the 4-beam lidar (right). The data has been averaged into bins of 1 m/s width and the error bars indicate ± one standard deviation.](image)

The result can be found in Figure 1.10 and it can be seen that all sensors follow a similar trend. At low wind speeds the misalignment shows values of approximately $5^\circ$, which are decreasing with increasing wind speed. This trend can be observed for both
the 2- and 4-beam lidar. The 2-beam lidar shows some deviations at low wind speed, but agrees very well at high wind speeds. Further, the measured standard deviation of the misalignment shows similar magnitudes for the lidar and sonic anemometer estimates. A slight reduction of the standard deviation with wind speed increases can be seen.

**Vertical Shear**

The advantage of inclining the laser beam from the horizontal plane is the sensing of the inflow at different heights. Thus, dependencies of the horizontal wind speed with height can be measured. In case of the 4-beam lidar, two measurement heights are available: ± 13.5 m from the hub height (or roughly 50% of the rotor radius). At the meteorological mast sonic anemometers have been mounted at the same height, making a direct comparison of the horizontal wind speeds possible, see Figure 1.11.

Different correlations can be found for the two heights; a detailed regression analysis can be found in Table 1.2. The lower measurement points show a high agreement with the sonic measurements. There is no significant over- or underestimation. On the other hand, the lidar measurement at the upper position register an underestimation of roughly 2-3% compared to the sonic anemometer. Currently no explanation of this underestimation can be given, but a higher induction effect due to higher wind speeds at the upper half of the rotor might explain the underestimation of the lidar. Here, further analysis is necessary.

To evaluate the shear a power law dependence of the wind speed with height is assumed IEC [23]

\[
U_h(z) = U_h(z_r) \left( \frac{z}{z_r} \right)^{\alpha_s},
\]

where \(z\) is the height above the ground, \(z_r\) is a reference height and \(\alpha_s\) is the wind shear exponent. The measurements from sonic anemometer and lidar at the two heights are

![Figure 1.11: 10 minute average horizontal wind speed comparison between 4-beam lidar and sonic anemometer mounted on the meteorological mast at two heights.](image-url)
Table 1.2: Resulting line fits of the comparison between horizontal wind speeds measured at two height by the sonic anemometers and the 4-beam lidar.

<table>
<thead>
<tr>
<th>Beams</th>
<th>Line fit</th>
<th>$R^2$ [%]</th>
<th>Line fit (unity slope)</th>
<th>$R^2$ [%]</th>
<th>Line fit (zero intercept)</th>
<th>$R^2$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper</td>
<td>$0.983x - 0.134$</td>
<td>98.37</td>
<td>$x - 0.291$</td>
<td>97.83</td>
<td>$0.972x$</td>
<td>97.61</td>
</tr>
<tr>
<td>Lower</td>
<td>$1.008x - 0.008$</td>
<td>99.68</td>
<td>$x - 0.010$</td>
<td>99.26</td>
<td>$0.999x$</td>
<td>99.32</td>
</tr>
</tbody>
</table>

used to calculate $\alpha_s$

$$\alpha_s = \frac{\log \left( \frac{U_h(h_1)}{U_h(h_2)} \right)}{\log \left( \frac{h_1}{h_2} \right)},$$  

where $h_1 = 30.45\,\text{m}$ and $h_2 = 57.55\,\text{m}$. The resulting comparison can be seen in Figure 1.12. As expected from the comparison of the horizontal wind speeds at the two heights, the shear shows an underestimation because the higher horizontal wind speed is underestimated by the lidar compared to the sonic anemometer.

Figure 1.12: Comparison of the wind shear exponent derived from 10 minute averages of the horizontal wind speed at two heights by the sonic anemometer and the 4-beam lidar.

**Gusts**

In this section the performance of measuring gust in the inflow of turbines by lidar systems is evaluated. In order to estimate the effect of the changing wind on the turbine the rotor-effective wind speed (REWS) has been calculated. The torque-balance method has been used to calculate REWS from measurements at the turbine and a precalculated power coefficient surface. The REWS estimation has also been calculated from the lidar measurements. Both processes are described in Chapter 3 and in Held and Mann [16].

Gust can be defined in several ways and here we follow the definition of the velocity increment procedure, because the time scale of the gust can be chosen. The time $\tau$ of a
1.2 Wind Measurements by Lidar Systems

window is chosen and only the first and last point of the window are considered. For each 10 minute period, the window is propagated through a 10 minute time series and the maximum difference between window end and start is defined as the gust for the 10 minute period:

$$\delta u_{\text{max}} = \max[u(t + \tau) - u(t)].$$

(1.14)

To identify the gusts the REWS estimated from turbine measurements has been used. The identified gust for each 10 minute period have then been ranked by the highest $\delta u_{\text{max}}$ and the three largest gusts are presented here. For a gust time-scale of 10s the results for the 2-beam lidar can be found in Figure 1.13 and for the 4-beam lidar in Figure 1.14. Also long-term gusts of length 60s have been considered. The plot for the 2-beam lidar can be found in Figure 1.15 and for the 4-beam in Figure 1.16.

![Figure 1.13: Examples of 10s gust in the turbine inflow measured by the 2-beam lidar and the turbine.](image1)

![Figure 1.14: Examples of 10s gust in the turbine inflow measured by the 4-beam lidar and the turbine.](image2)
Figure 1.15: Examples of 60 s gust in the turbine inflow measured by the 2-beam lidar and the turbine.

Figure 1.16: Examples of 60 s gust in the turbine inflow measured by the 4-beam lidar and the turbine.

The graphs show that the REWS estimation from the lidar is leading the REWS from the turbine because the lidar is measuring in front of the turbine. Thus, a preview of incoming gust is possible. The time delay between lidar and turbine measurement is calculated by an information theoretical delay estimator presented in Moddemeijer [24]. For more details see Chapter 3. The preview can be seen very clearly for the 10 s gusts, especially for the 4-beam lidar, which has a larger focus distance. In all three cases the lidar is capable of sensing the gust before it affects the turbine.

The 2-beam lidar shows a similar performance, but the preview times are shorter due to the reduced focus distance. In the center graph of Figure 1.13 an extreme increase of the wind speed from 11 to 17 m/s within only 10 s could be detected. It is also possible
to detect gust of longer time scale with both systems. For both lidars the estimation of
the wind speed increases is as accurate as for short-term gusts.

1.3 Lidar-Assisted Control of Wind Turbines and Farms

1.3.1 Individual Wind Turbine Control

The control functions of an individual wind turbine can be divided into two regimes. A
controller handling the actuation of blade pitch angles and generator power or torque (and
other possible actuators) aims at maximizing the current power production while keeping
the turbine within its design limits. It operates on very fast time scales (usually at 35-
50 Hz). It is assisted by a supervisory control system that handles the transition between
operational states of wind turbines (e.g. the start-up of a turbine), yaw alignment, noise
control, etc. These adjustments are executed on the order of several minutes.

Lidar systems have the potential to improve the controller performance by providing
a preview of the incoming wind field. Research on application of lidar sensors for wind
turbine control purposes began more than 10 years ago (see e.g. Harris, Hand, and
Wright [4]). A huge amount of simulation studies have been conducted in the meantime
to assess the improved performance of lidar-assisted control aiming at different control
objectives. These include:

- Yaw Control, e.g. in [25, 26, 27, 28, 29]
- Rotor Speed Control, e.g. in [25, 30, 31]
- Pitch Control
  - Collective Pitch Control, e.g. in [32, 33, 34, 35, 30]
  - Cyclic Pitch Control, e.g. in [36]
  - Individual Pitch Control, e.g. in [37, 38, 39]
- Model Predictive Control, e.g. in [40, 41, 42, 43]

Experimental field test, on the other hand, are still scarce. Tests on research turbines
for lidar-assisted pitch control [44, 45, 46] and lidar-assisted yaw alignment [47] have
been conducted. The results showed promising benefits in term of load reduction and
power production increase. However, integration of lidar system into the control system
still faces some major issues (availability, fall-back strategies, etc).

Within this project a model of the coherence between REWS estimated from turbine
and lidar data has been developed. In contrast to previous models which use the
Kaimal turbulence model, the Mann turbulence model is used. It has been validated
experimentally for the 2- and 4-beam installations and an improved agreement was found
compared to the models based on the Kaimal turbulence model. Details can be found in
Chapter 3.
1.3.2 Wind Farm Control

Wind farm control strategies deal with the optimization of parameters on wind farm level to achieve beneficial operation of the entire farm. As wind turbines are usually clustered in a confined area, the interaction between turbines is an important aspect during planning and operation of a wind farm. In contrast, the previously mentioned individual wind turbine control tries to optimize a single turbine irrespective of any surrounding turbines. This is commonly referred to as a greedy wind farm control approach.

Research on wind farm control can be divided into approaches. Strategies involving the turbine's power electronics deal with the control of frequency, voltage, active and reactive power delivered to the grid, battery storage and ancillary service to the grid. Controlling the mechanical and/or aerodynamic aspects on wind farm level include the optimization of the power extraction from the wind while minimizing structural loading. The majority of the published studies fall into the first category [48].

As lidars provide wind measurements, this section will highlight the potentials of mechanical/aerodynamic optimization. However, applications to electrical control are also imaginable (e.g. by providing wind speed forecasts [49]). The flow field within a wind farm can be manipulated by controlling the wakes emitted by the individual turbines with regard to wake deficit and direction. Two approaches are prevalent:

**Axial-Induction Control** The axial induction factor is the relative reduction in wind speed from undisturbed flow to the rotor plane. By either changing the collective blade pitch angle or the generator torque, the power extraction from the wind and consequently the axial induction factor can be manipulated. A reduction leads to a less severe wake deficit and the harmful effect of the wake on downstream turbines can be mitigated. Often static wake models are used to find optimal power set points for a certain inflow sector. The benefits of this strategy has been investigated in numerical simulations (e.g. in Gebraad and Wingerden [50]) and in scaled wind tunnel tests (e.g. in Medici [51]). Full-scale experiments are scarce; in Boorsma [52] a pitch angle offset lead to increased power production of a row of 2.5MW turbines.

**Wake steering** Wake steering achieves a mitigation of the wake influence on downstream turbines by misaligning the wake-emitting turbine and thus directing the wake away from the downstream turbines. The misaligned upstream turbine will experience a power production loss, but the wake deficit can be steered away from surrounding turbines and improved power production can be experienced. Numerical studies and wind tunnel experiments have shown a positive effect on the power production while the turbine loads were not substantially increased (see Gebraad et al. [53], Fleming et al. [54] and Campagnolo et al. [55]). However, the methods are very sensitive to wind direction changes and if uncertainties in the estimation of the wind direction are not included into the control system, power losses compared to greedy control strategy are possible [56].

The potential of both strategies though simulations in the control optimization
environment FLORIS developed at NREL and TU Delft is briefly investigated. The code uses engineering wake models for the wake deficit behind the turbine, the deflection of the wake and power loss due to yaw misalignment. Details can be found in Annoni et al. [57]. Here a fictional 4x4 wind farm consisting of NREL 5MW turbines with an interspacing of 7 rotor diameters is used. The used Gaussian wake model is derived analytically from the simplified Navier-Stokes (NS) equations and presented in Abkar and Porté-Agel [58]. It has been validated against CFD simulations. The wake deflection model uses a momentum budget on the Reynolds averaged NS equations [59] and has been validated with scaled wind tunnel measurements.

The flow field within the wind farm resulting from the greedy control approach can be seen in the left panel of Figure 1.17. A wind direction where the biggest wake effect can be expected has been chosen. The mean wind speed was 8 m/s and the ambient turbulence intensity was set to 6%.

![Flow field resulting from from the greedy wind farm control approach (left), axial-induction control (center) and wake redirection (right).](image)

The effect of the first column of turbines onto subsequent turbines leads to a reduced inflow and lower power production. The induction control and wake steering strategies found by maximizing the overall wind farm power production can be found in the center and right panel of Figure 1.17. For the induction control strategy the less severe wakes behind the first three turbine columns can be identified. The deflection of the wake away from downstream turbines avoids the detrimental interaction between turbine and wake in case of wake steering.

The performance of the control strategies can be found in Figure 1.18. The optimal induction control strategy showed that the induction factor should be reduced by almost 30% at the first column, around 10% at the second, 4% at the third and by 0% at the last column. Compared to the greedy strategy production losses can be observed at the first turbine column, while small increases are seen on the subsequent columns. The highest gain occurs at the second column. The overall power production increase for this control strategy is 1.5% compared to the maximizing the power at each turbine individually.

For the wake steering an offset of around $23^\circ$ is observed for the first three columns, which allows the wake to just pass by downstream turbines. The last turbine column will not be altered since the wake does not interact with any turbines. Similar to induction control, the first turbine experiences a loss in power production. The loss due to yaw misalignment is higher than that due to reducing the induction factor. However,
the gain for the wake-affected turbines is highest for the wake steering strategy since the wake deficits do not impinge downstream turbines directly and the inflow to the turbines contains more kinetic energy. The power production on farm level can be improved by 15.6%.

However, in this case we have only considered one wind direction and speed and found optimal reductions of the axial induction factor and yaw misalignments. A complete wind farm optimization strategy will require this information for all inflow angles. The improvement by applying the new strategy can then evaluated if the wind speed and direction distribution is know.

The direction considered here implied the largest wake losses and the reported improvements are expected to be smaller for other wind directions (except for the symmetrical cases). To access the farm control improvements the wind direction distribution is necessary. Further, it was assumed that the wind direction is known, deterministic and not changing within the wind farm. If a stochastic behaviour is included in the optimization more conservative changes in yaw misalignment for wake steering have been found in Quick et al. [60]. Also, as mentioned previously, the estimation of the wind direction can have significant influences on the performance of a wake steering controller [56].

Lidars offer the possibility to visualize the flow behind turbines and have been used to study the behaviour of wake flows behind turbines (e.g. Bingöl, Mann, and Larsen [61] and Trujillo et al. [62]). The lidar systems are usually pointed backwards to measure the wake emitted by the turbine they are mounted on. Their high spatial and temporal resolution can yield precise information about wake center and expansion [63] and by
probing the wake at different downstream distances the movement of the wake can be tracked. Several studies utilized lidars to investigate the wake deflection due to yaw misalignments [57, 64] and a tracking procedure to follow the motion of the wake behind a turbine for wake steering purposes has been presented in Raach et al. [65]. A closed-loop controller using wake tracking information from lidar systems was introduced by Raach, Schlipf, and Cheng [66]. It was possible to track the wake center position using lidar measurements, but the definition of the wake center influenced the performance.

Nacelle-mounted lidars also have the potential of providing accurate wind direction measurements because their measurements are not influenced by the blade and the nacelle, which introduce a bias on nacelle wind vanes or sonic anemometers [67, 47]. This can be achieved by looking forwards and measuring the inflow towards the turbine. In this case lidar measurements can reduce the uncertainty of the wind direction estimation, which are input to wind farm controllers. However, lidars make assumptions that are violated within wake flows and huge biases can arise if a wake is affecting one of the lidar beams. Within this project a methodology to detect situations, where a wake flow is affecting one or more beams was presented in Held and Mann [17]. The algorithm can give information whether the turbine is affected by a half- or full-wake. This information can potentially be used as a sensor for closed-loop wake steering control of wind farms.

1.4 Project Scope & Thesis Structure

Today, there is a wide range of different design types with regard (but not limited) to detection method, laser source, mounting position and scanning type. In this work we will focus on lidar systems produced by Windar Photonics A/S that use a \textit{mono-static coherent detection method} with \textit{continuous-wave near-infrared} laser sources and are \textit{mounted on the nacelle} of wind turbines.

The remainder of this thesis is structured as follows and investigates different topic that spans from the determination of LOS speed from a Doppler spectrum to their applications for wind turbine and farm control, see Figure 1.19.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{structure.png}
\caption{Structure of the thesis, which spans from analysis of lidar data to the application of nacelle lidars to wind turbine and farm control.}
\end{figure}
At first, in Chapter 2 the influence of frequency estimators on the LOS measurements are investigated. The effect of three different methods (maximum, centroid and median of the Doppler spectrum) on the volume averaging of the lidar and the error compared to a sonic anemometer are evaluated. A careful choice of the method can lead to improvements in terms of reduction of volume averaging and accuracy compared to the sonic anemometer. The median method showed the overall lowest root mean squared error compared to the sonic anemometer and had a slight reduction of the volume averaging effect compared to the centroid method.

In Chapter 3 the usage of lidars for collective pitch control is analyzed. Here, a model for the coherence between the rotor-effective wind speed experienced by the turbine and estimated from lidar measurements is developed. The model is also validated in an experimental setup and showed improved performance compared to previous models. It can be used to optimize the position of the focus points that maximizes the coherence.

Finally, in Chapter 4 an algorithm to detect wakes in the inflow of turbines is presented. The algorithm uses the LOS-equivalent definition of turbulence intensity as a detection parameter. The standard deviation is defined as the width of a time-averaged Doppler spectrum. Its performance is assessed during an experimental campaign using 2-beam lidars at a test turbine and showed that all half- and full-wake situations could be detected. Also a correction method is proposed that utilizes the undisturbed wind direction information from a meteorological mast. Additionally, test results from a commercial wind farm as presented in Appendix A.
CHAPTER 2

Effect of Doppler spectrum frequency estimators on lidar measurements

In this chapter the influence of the frequency estimator of the Doppler spectrum is investigated. Here we consider the maximum, centroid and median methods and assess their effect on the spatial averaging of the lidar and their accuracy in comparison with two sonic anemometers. The study was motivated by numerical simulations that indicated that the maximum method, even though using the least amount of information from the Doppler spectrum, performed best at reducing the effect of the probe volume.

This fact was validated through a field experiment and more detailed simulations. However, the analysis also showed that the root mean squared error (RMSE) is increased significantly for the maximum method. Thus, from this analysis we recommend using the median method, as it shows a slight reduction of the probe volume effect and had the best RMSE of all methods. The results of the study have been published in Atmospheric Measurement Techniques [13] and the article is attached below.
Comparison of methods to derive radial wind speed from a continuous-wave coherent lidar Doppler spectrum

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Abstract. Continuous-wave (cw) lidar systems offer the possibility to remotely sense wind speed but are also affected by differences in their measurement process compared to more traditional anemometry like cup or sonic anemometers. Their large measurement volume leads to an attenuation of turbulence. In this paper we study how different methods to derive the radial wind speed from a lidar Doppler spectrum can mitigate turbulence attenuation. The centroid, median and maximum methods are compared by estimating transfer functions and calculating root mean squared errors (RMSEs) between a lidar and a sonic anemometer. Numerical simulations and experimental results both indicate that the median method performed best in terms of RMSE and also had slight improvements over the centroid method in terms of volume averaging reduction. The maximum, even though it uses the least amount of information from the Doppler spectrum, performs best at mitigating the volume averaging effect. However, this benefit comes at the cost of increased signal noise due to discretisation of the maximum method. Thus, when the aim is to mitigate the effect of turbulence attenuation and obtain wind speed time series with low noise, from the results of this study we recommend using the median method. If the goal is to measure average wind speeds, all three methods perform equally well.

1 Introduction

Remote sensing is an attractive alternative to traditional in situ measurements of wind speed. For wind turbines, light detection and ranging (lidar) devices can replace the installation of large meteorological masts hosting cup or sonic anemometers in order to meet the constantly increasing measurement height requirements. This flexibility led to a large variety of applications of lidars spanning from lidar-assisted yaw and pitch control (Schlipf, 2015) to site assessment (Sanz Rodrigo et al., 2013) and power curve validation (Borraccino et al., 2017). However, one has to keep in mind that there is one important principal difference between measurements from a lidar device and sonic or cup anemometers, namely the averaging over a rather large measurement volume.

Lidars can be operated in two different modes: laser light can either be emitted in continuous-wave (cw) or pulsed form. For a cw lidar, a laser beam is focused on the desired point in space and measures the backscattered light. The radial speed of the aerosols can be estimated from the induced Doppler shift of the backscattered light. However, this benefit comes at the cost of increased signal noise due to discretisation of the maximum method. Thus, when the aim is to mitigate the effect of turbulence attenuation and obtain wind speed time series with low noise, from the results of this study we recommend using the median method. If the goal is to measure average wind speeds, all three methods perform equally well.

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ample, Frehlich (2013) presents an investigation on the maximum likelihood (ML) algorithm and minimum mean squared error method. Both estimators had similar performance and the latter was chosen because of computational efficiency. In addition, Dolfi-Bouteyre et al. (2016) examined two simple methods (maximum and centroid), the previously mentioned ML approach, polynomial fitting and an adaptive filter method. The polynomial fitting method was found to perform best in laminar flow but it is suggested that in more complex flow, advanced estimators are needed. However, for a cw lidar these more complex methods are unavailable because they rely on an underlying model for the shape of the Doppler spectrum. According to the current knowledge of the authors no such model exists for cw lidars.

One of the measurement differences of a lidar compared to cup or sonic anemometers is its large probe volume, which leads to turbulent fluctuation attenuation. While the effect of the probe volume on turbulence attenuation can be modelled by theory for the centroid method, no theories exist for the median or maximum methods. Several studies aimed at validating the theory for the centroid method by comparing the spectral transfer function between a lidar and sonic anemometer, i.e. the ratio between the power density spectra of lidar and sonic measurements. An early study by Lawrence et al. (1972) compared a cw carbon dioxide (CO₂) lidar to a cup anemometer around 10 m above the ground. Due to the short focus distance (approximately 30 m), the Rayleigh length was less than 0.5 m and analysis of the PSD showed no effect of the lidar’s volume averaging. Another cw CO₂ lidar at focus distances up to 200 m close to the ground was used in Banakh and Smalikho (1999). High-frequency fluctuation attenuation due to volume averaging was observed. A power spectral analysis showed significant deviations between the lidar and sonic power spectrum, starting at 0.4 Hz and becoming more severe at increasing frequencies. The experimental results support the theory developed in the paper.

A cw lidar focused close to a sonic anemometer mounted 78 m above ground was used in Sjöholm et al. (2009). Data gathered at 20 Hz were used to investigate the spatial averaging, and good agreement between theory and experiment was found. However, during periods with low-level clouds the measurements were affected negatively. The same objective was followed in Angelou et al. (2012), in which the transfer function of a tower-mounted horizontally staring lidar was determined against a mast-mounted sonic anemometer allowing for horizontal measurements. The study was limited to when the lidar beam was aligned with the wind direction, and during these periods the spatial averaging effect from measurements agreed perfectly with the theory for the centroid method attenuation. Further, this study presented a method to reduce the influence of noise at high frequencies by estimating the spectral transfer function using the cross-spectrum between lidar and sonic measurements and the auto-spectrum of the sonic anemometer.

Two studies investigated the spatial averaging of a long-range pulsed lidar compared to mast-mounted sonic anemometers (Mann et al., 2009; Fuertes et al., 2014). Both studies showed the feasibility of measuring 3-D wind vectors by synchronised lidars focused at one point in space. It was also shown that the attenuation from spatial averaging can be predicted by the theory for pulsed lidar systems, which is presented in the two studies.

A slightly different approach was followed in Peña et al. (2017). In this study a cw nacelle lidar and a pulsed nacelle lidar were compared against both a cup and sonic anemometer. Turbulence statistics was calculated by fitting a spectral tensor model including a lidar volume averaging model to the sonic measurements and an average Doppler spectrum method, which was also used in Branlard et al. (2013). The first method enabled the retrieval of filtered turbulence statistics, while the second yielded measures not affected by spatial averaging. These results reiterated that predictions of the spatial averaging effect are consistent with theory.

A machine-learning approach to produce unfiltered wind speed variances from pulsed lidar signals was used in Newman and Clifton (2017). Besides a model for spatial averaging, the algorithm also includes automatic noise removal. Comparisons to sonic anemometers showed improvements when using the algorithm under all stability classes, but the results are highly dependent on the input variables and the training sets.

From the studies mentioned above it can be seen that the effect of the lidar’s spatial averaging can be predicted theoretically, which has also been confirmed experimentally. In contrast to pulsed lidars, little work has been done on the effect of how the radial wind speed is calculated from a Doppler spectrum for cw lidars. Thus, the objective of this study is to investigate the influence of using different methods of determining the dominant frequency in a lidar Doppler spectrum (maximum, median, centroid) and its influence on the volume-averaging effect of lidar measurements. This is important because the lidar’s probe volume has an attenuating effect on the measurement of turbulent fluctuations. As a consequence, estimates of wind speed variances will be biased if the lidar is used for site characterisation. Also for lidar-assisted control integration an accurate measurement of the turbulent fluctuations is important. Since no theory has been formulated for the median and maximum method yet, the study was motivated by initial numerical simulations that showed improved performance for these two methods compared to the centroid method. In this study the numerical simulations are extended and compared to data gathered during a field experiment.

2 Materials and methods

Statistically, the fluctuating part of an incompressible, homogeneous wind field \( u(x) \) can be described by the spectral ten-
sor $\Phi_{ij}(k)$, where $k$ is the wave number vector. To simulate synthetic wind fields, models for $\Phi_{ij}(k)$ have been derived, e.g. von Karman (1948) or Mann (1998). This allows us, on the one hand, to directly calculate the statistical behaviour of a point measurement (i.e., from a sonic anemometer) and a volume measurement (i.e., from a lidar) in wave number space, and on the other hand, to generate a simulated box of turbulence. These boxes consist of wind velocity values at specified grid points that are frozen in space. They can be used to simulate the lidar measurement process and calculate Doppler spectra from which the radial wind speed can be derived. Both methods will be compared to the experimental findings.

2.1 Theory

A cw lidar measurement can be modelled by the convolution of the projected radial component $n$ $u$ and a weighting function $\psi(s) = \frac{1}{\pi z_R s^2}$ (Sonnenschein and Horrigan, 1971):

$$v_r(r) = \int_{-\infty}^{\infty} \psi(s)n \cdot u(s + r)ds,$$  \hspace{1cm} (1)

where $z_R$ is the so-called Rayleigh length that characterises the probe volume, $s$ is the distance from the focus point along the beam, $n$ is the beam unit vector and $r$ is the focus position. The Rayleigh length can vary from a few centimetres at small focus distances to tens of metres at very large focus distances and varies with the focus distance squared, i.e. $z_R \propto |r|^2$ (Hu, 2016).

Equation (1) assumes the definition of the centroid of the Doppler spectrum; in the following we will refer to it as cen. Another method of determining the dominant frequency in a Doppler spectrum is by simply taking the frequency bin where the peak occurs (max) or by treating it as a probability density function (PDF) and taking the median value (med). The estimated radial wind speed using these three methods will be compared to the laser-line-projected sonic wind velocity $v_s = n \cdot u(x)$. This will be done for the numerical simulations and the experiment.

To evaluate how well lidar and sonic measurements correlate in the wave number domain, an estimation of the transfer function between the two signals is used:

$$G(k_1) = \frac{|\chi_{r,s}(k_1)|^2}{F_r(k_1)},$$  \hspace{1cm} (2)

where $\chi_{r,s}(k_1)$ refers to the cross-spectrum between the lidar and sonic signal and $F_r(k_1)$ refers to the auto-spectrum of the sonic signal. The closer $G(k_1)$ is to unity, the smaller the effect of volume averaging of the lidar is. We prefer to use the transfer function defined in Eq. (2) to the more traditional $G(k_1) = F_r(k_1)/F_s(k_1)$ because the auto-spectrum $F_s(k_1)$ may be affected by noise whereas the cross-spectrum $\chi_{r,s}(k_1)$ is not (Angelou et al., 2012), assuming the sonic measurements contain considerably less noise than the lidar measurements.

When using $F[\psi(s)](k) = \exp(-z_R|k|)$, Eq. (1) can be expressed in wave number space by

$$F_r(k_1) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_{ij}(k)\exp(-2z_R|n \cdot k|)dk_2dk_3,$$  \hspace{1cm} (3)

where $F[.]$ refers to the Fourier transformation. The integration in Eq. (3) can only be solved analytically for simple forms of $\Phi_{ij}(k)$. For the line-of-sight projected sonic measurement $v_s$, which can be approximated by a point measurement due to the small volume measurement, the exponential term in Eq. (3) drops out and we are left with

$$F_s(k_1) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_{ij}(k)dk_2dk_3.$$  \hspace{1cm} (4)

The cross-spectrum between the lidar and sonic measurements can then be written as

$$\chi_{r,s}(k_1) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_{ij}(k)\exp(-z_R|n \cdot k|)dk_2dk_3.$$  \hspace{1cm} (5)

It should be noted that $\lim_{k_1 \to 0}G(k_1) = 1$ is only true when the lidar beam is aligned with the mean wind direction. For misaligned cases $\lim_{k_1 \to 0}G(k_1) < 1$ holds (Kristensen and Jensen, 1979). Thus, for misaligned cases we do not expect the transfer function to tend towards unity for small wave numbers.

Another measure used to evaluate the performance of the different methods is the root mean squared error (RMSE):

$$\text{RMSE}(v_r_{\text{method}}) = \sqrt{(v_r_{\text{method}} - v_s)^2},$$  \hspace{1cm} (6)

where method refers to either centroid, median or maximum and the overbar indicates averaging in time. In contrast to the transfer function estimated previously, for this measure, noise in lidar measurements will affect the performance and gives an indication of the difference between lidar and sonic measurements in the time domain.

2.2 Numerical simulations

Numerical simulations illustrate results in an environment where no noise is present. The methodology used to perform these simulations was developed in Mann (1998). First it is described how a Doppler spectrum is obtained from a simulated wind time series. We narrowed our investigation to the (horizontal) 2-D case in which the cw lidar measures horizontally only. We furthermore assumed Taylor’s frozen turbulence hypothesis:

$$u(x, y, t = 0) = u(x + U_t, y, t),$$  \hspace{1cm} (7)
where $U$ is the mean wind speed, so the wind field at any given time can be obtained by translating the wind field at $t = 0$. We did not consider any sources of noise. In this case the Doppler spectrum $S(v, t)$ can be written as

$$S(v, t) = \int_{-\infty}^{\infty} \varphi(s) \delta(v - u(s) \cdot n) \, ds,$$

where $\delta$ is the Dirac delta function. Notice that Eq. (8) is a convolution of the weighting function $\varphi$ and the delta function $\delta(v - u \cdot n)$. If $\varphi$ was disregarded, Eq. (8) could be viewed as a histogram of wind velocities. The discretisation of the histogram is chosen to match the typical velocity bin resolution of a lidar system, which in this simulation case was $0.1 \text{ ms}^{-1} \text{ bin}^{-1}$. When $\varphi$ is included, the wind velocities are weighted, such that the velocities around the focus point count most. Due to the finite length of the simulated turbulent boxes, the integration in Eq. (8) needs to be truncated. Here we chose a distance of $M = 12 \varepsilon_R$ along the beam after which the truncation is applied, where the Lorentzian weighting function has a value of $\approx 1.5 \cdot 10^{-4}$ (or 0.69 % relative to the maximum value at the focus point):

$$S(v, t) = \int_{-M}^{M} \varphi(s) \delta(v - u(s) \cdot n) \, ds.$$  

To generate the wind time series we assumed for simplicity that the turbulent fluctuations in the direction of the mean wind can be described by the model by Mann (1994). A turbulent wind box was created with a horizontal 2-D wind vector at each grid point. The dimensions of the box are $4096 \times 4096$ grid points with a separation of $0.732 \text{ m}$. A total of 20 turbulent boxes with different initial turbulence seeds have been simulated, resulting in 20 different realisations of...
the turbulent field. Note that here we only simulate the fluctuating part, and the mean wind speed is zero. An illustration of the simulation set-up and an example of a simulated Doppler spectrum including the radial wind speed estimates using the three different methods is shown in Fig. 1.

2.3 Experimental set-up

In this section the experiment conducted at Risø campus with a lidar system by Windar Photonics A/S will be presented. The WindEYE is a commercial Doppler wind lidar that uses an all-semiconductor laser source with a wavelength of 1553 nm; see Hu (2016). Because the purpose of the product is wind direction measurement, it can focus in two positions by deflecting the beam through two different lenses. The switching occurs every half second, which means that the lidar focuses on one position for 0.5 s and then a liquid crystal will bend the beam towards the second focus point for another 0.5 s. Usually the device is mounted on the nacelle of wind turbines, but for this experiment it was installed on a tower to be able to focus on the location of two sonic anemometers around 10 m above the ground; see Fig. 2. The lidar beams are aligned horizontally to measure horizontal components only. The focus distance is 90 m and the Rayleigh length $z_R$ is 14.5 m. The angle between the two beams is 60°, but since the beams are compared individually it is of no importance here.

The sonic anemometers are two USA-1 anemometers by Metek GmbH, which were mounted on a tower at the exact position of the focus points. The focus distances have been verified experimentally in an optical laboratory, and the alignment of the lidar to the sonic anemometers was checked using an infrared sensor card; see Dellwik et al. (2015). The laser beams pass approximately 1 m above the sonic devices. The sonic anemometers were sampled at 35 Hz and have a transducer distance of 0.175 m implying that the device retrievals approximate a point measurement compared to the averaging volume of the lidar. For all measurements the standard 2-D flow correction has been removed and instead a 3-D correction was used (Bechmann et al., 2009). Due to the inherent switch mechanism of the WindEYE lidar the data had to be combined to a rather low sampling frequency of 1 Hz. The experiment extended from 9 January to 23 March 2014.
but due to synchronisation problems only the periods in February and March could be used.

3 Results and discussion

In this section we first present an example of the numerical simulation and experimental results and then we will compare both to the analytical results. An example of the numerical simulation and the experiments can be found in Fig. 3.

3.1 Experimental results

At first the 10 min averages of the lidar measured wind speed component , and the 3-D sonic wind vector projected on the line of sight, , have been compared. A filter has been applied for radial components less than $2 \text{ ms}^{-1}$ because it is not possible to accurately determine below that value for a homodyne lidar system. The comparison can be seen in Fig. 4. For both beams a very good comparison can be observed; the line fits yield slopes of unity and an $R^2$ of almost 100 %. The line fits for the other methods can be found in Table 1. The theoretical results are represented by a solid black line, simulation results (Simu) by coloured solid lines and experimental results (Exp) by coloured dashed lines. The different colours stand for the three methods to derive the radial speed from the Doppler spectrum: centroid (cen), median (med) and maximum (max).

The wind rose derived from the two sonic anemometers is shown in Fig. 5. Mean wind speed and direction were calculated over 10 min periods and then the average of both anemometers was taken. Two main wind directions can be identified, of which one is aligned with the northern beam direction. Thus, more data were gathered for a misalignment of $0^\circ$ for the northern beam compared to the southern beam.

For wind lidar systems using a homodyne detection method, there is an ambiguity in the wind direction (whether the wind blows towards or away from the lidar). Further the limitation to the line-of-sight component of the wind vector leads to an ambiguity in the misalignment (whether the wind direction is misaligned towards the left or right side of the beam). In all these cases the radial wind speed measurements will be the same. For example, a case of a wind direction misaligned by $10^\circ$ is equivalent to a misalignment of 170, 190 and 350°. Thus, it is possible to reduce the full 360° to one quadrant ranging from 0 to 90°. In the following analysis the data have been binned into 10° sectors ranging from 0 to 90°.

To create numerical simulations as close as possible to the experimental conditions, the Mann spectral tensor has been fitted, following the procedure in Mann (1994), to the $u$ and $v$ spectra obtained from sonic measurements, where the mean wind speed was above $6 \text{ ms}^{-1}$. The tensor model has three parameters: $\alpha \epsilon^{2/3}$, where $\epsilon$ is the rate of viscous dissipation of turbulent kinetic energy and $\alpha$ is the spectral Kolmogorov constant; $L$, which is a length scale; and $\Gamma$, which is an anisotropy parameter. For details, see Mann (1994). This has been done sector-wise for sectors of $10^\circ$ for each beam, and the results can be seen in Table 2.

In order to reduce the computational effort, we have taken the average value of each parameter over all sectors and both beams and found the following parameter: $\alpha \epsilon^{2/3} = 0.58 \cdot 10^{-2} \text{m}^{4/3} \text{s}^{-2}$, $L = 22.3 \text{ m}$, $\Gamma = 2.26$. These parameters have been used to perform the numerical simulations mentioned in Sect. 2.2.

3.2 Theoretical, numerical and experimental results

In this section we will present the combination of experimental and simulation results together with the numerical integration of Eq. (2) (using Eqs. 3–5). In the following plots theoretical results are represented by a solid black line, simulation results (Simu) by coloured solid lines and experimental results (Exp) by coloured dashed lines. The different colours stand for the three methods to derive the radial speed from the Doppler spectrum: centroid (cen), median (med) and maximum (max).

First, we present the simplest case when the lidar beam is aligned with the wind direction. In this case Eq. (2) can be

<table>
<thead>
<tr>
<th>Centroid</th>
<th>Median</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line fit</td>
<td>$R^2$ (%)</td>
<td>Line fit</td>
</tr>
<tr>
<td>North</td>
<td>$1.001x - 0.040$</td>
<td>99.42</td>
</tr>
<tr>
<td>South</td>
<td>$1.003x - 0.028$</td>
<td>99.55</td>
</tr>
</tbody>
</table>

Figure 5. Wind rose derived from 10 min averages of sonic anemometer wind speed and direction of the 2-month-long experiment. The beam directions are indicated as dashed lines.
Table 2. Sector-wise fitted parameters of the Mann model for each beam to data obtained from sonic measurements at 10 m over 2 months.

<table>
<thead>
<tr>
<th>Misalignment</th>
<th>0°</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
<th>50°</th>
<th>60°</th>
<th>70°</th>
<th>80°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>10⁻²αϵ² (m⁴/³s⁻²)</td>
<td>1.11</td>
<td>1.02</td>
<td>1.14</td>
<td>0.38</td>
<td>0.37</td>
<td>0.26</td>
<td>0.32</td>
<td>0.41</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>North L (m)</td>
<td>5.1</td>
<td>7.5</td>
<td>10.4</td>
<td>19.0</td>
<td>26.3</td>
<td>28.3</td>
<td>35.3</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Γ(–)</td>
<td>3.33</td>
<td>2.49</td>
<td>2.03</td>
<td>1.94</td>
<td>1.97</td>
<td>2.35</td>
<td>1.88</td>
<td>1.77</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>10⁻²αϵ² (m⁴/³s⁻²)</td>
<td>0.61</td>
<td>0.65</td>
<td>0.90</td>
<td>0.40</td>
<td>0.39</td>
<td>0.32</td>
<td>0.35</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>South L (m)</td>
<td>19.9</td>
<td>14.8</td>
<td>7.1</td>
<td>36.5</td>
<td>42.1</td>
<td>31.1</td>
<td>22.8</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Γ(–)</td>
<td>1.85</td>
<td>2.43</td>
<td>3.31</td>
<td>1.60</td>
<td>1.87</td>
<td>2.32</td>
<td>2.79</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Figure 6. Transfer function $G(k_1)$ for aligned beams for the northern beam data (a) and southern beam data (b) using 20 simulations and 2 months of experimental data at a sampling rate of 1 Hz.

solved analytically because the exponential in Eqs. (3) and (5) does not depend on either $k_2$ or $k_3$ and can be moved outside the integral. This results in

$$G(k_1) = \frac{\chi_{r,s}(k_1)}{F_r} = \frac{\exp (-z_R |n \cdot k|) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_{ij}(k) dk_2 dk_3}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_{ij}(k) dk_2 dk_3} = \exp (-2z_R k_1). \quad (10)$$

Note that in Eq. (10) the radial speed is defined by the centroid method. For the aligned case the transfer function only depends on $z_R$ and not on the turbulence model parameters.

The results for the aligned case are shown in Fig. 6, where Eq. (10) is shown as the black solid line. It can be seen that both red lines (solid and dashed) agree well with the theoretical results. Some significant deviations can be observed for the simulation results at small wave numbers, which can be explained by the truncation of the Doppler spectrum in Eq. (9).

It can also be seen that the transfer functions when using the median or maximum method lie above the results for the centroid method. This indicates that the turbulence attenuation is less severe for these two methods compared to the centroid method. Thus, fluctuations which have been measured by the sonic and are attenuated when using the centroid method due to volume averaging can indeed be sensed when using the median and maximum method. The improved performance is stronger for the numerical simulation due to the absence of noise. The median method seems to perform slightly better than the centroid method, and the maximum method has an even bigger improvement.

As an example for a misaligned case, we will now focus on a misalignment of $40^\circ$; see Fig. 7. The transfer functions for the remaining misalignments can be found in the Appendix. For a misalignment of $40^\circ$, Eq. (10) is not valid anymore. However, Eq. (2) can be integrated numerically. It is shown again as the solid black line. Again we can see that the simulation results using the centroid method matches the theoretical results well for large wave numbers, but there are deviations at low wave numbers. Similar to the aligned case, it can be observed that the transfer functions for the median and maximum method lie above that of the centroid method. Again, this implies that turbulent fluctuations that were not measured using the centroid method can be sensed using the median and maximum method. The maximum method again shows the best performance.

Examples of these improvements can also be identified in the time domain when looking at Fig. 3. For the numerical
Figure 7. Transfer function $G(k_1)$ for misaligned beams for the northern beam data (a) and southern beam data (b) using 20 simulations and 2 months of experimental data at a sampling rate of 1 Hz.

Figure 8. RMSE value for the median and maximum method normalised by RMSE($v_{r,\text{cen}}$) for the experimental results (a) and the numerical simulations (b) using 20 simulations and 2 months of experimental data at a sampling rate of 1 Hz.

Simulations (panel a) the improved fluctuation measurements using the maximum method are very clear, while the median method is also able to slightly enhance the measurements compared to the centroid method, which performs worst. A similar tendency is also observed from the experiment (panel b). Just before 120 s we can note a very good agreement between sonic and maximum-method $v_r$, whereas the other two methods are not able to detect this fluctuation. In other periods the observed improvement is very small.

Next we consider the RMSE results. Since it was seen previously that both the median and maximum method outperformed the centroid method, the RMSE of the two methods normalised by the RMSE of the centroid is compared now:

$$1 - \frac{\text{RMSE}(v_r, \text{method})}{\text{RMSE}(v_{r,\text{cen}})}, \quad (11)$$

where method is either the median or the maximum method. Thus positive numbers indicate better performance compared to the centroid method and vice versa.
The results can be seen in Fig. 8 and show that the median method consistently outperforms the centroid method. Improvements of up to 4% from the experimental data was observed, while the simulations showed performance increases between 3% and 5%. In contrast, the maximum method has persistent disadvantages compared to the centroid method. The shortcoming increases with increasing misalignment and reaches values of approximately $-9\%$ for the experiments and close to $-10\%$ for the numerical simulations. This indicates that the improved performance of the maximum method in reducing the effect of spatial averaging has the consequence that more signal noise is introduced. This is the result of selecting the maximum of the Doppler spectrum since the maximum position can vary depending on the frequency bin width and Doppler spectrum noise. The other two methods are more robust against noise contamination.

4 Conclusions

In this study we compared a cw wind lidar to sonic measurements, where the sonic anemometers are mounted exactly at the focus positions of the lidar system. The lidar measurements are affected by their large probe volume, which leads to an attenuation of turbulence. The objective of the paper was to study how different methods of determining the dominant frequency in a Doppler spectrum affect wind speed measurements by a cw lidar. We used an estimation of the transfer function to evaluate the lidar’s attenuation of turbulent fluctuations and the RMSE to give a metric to the general performance of the methods. Theoretical analysis, numerical simulation and data from a 2-month-long experiment have been used, and three different methods for deriving the radial speed were applied: the centroid, median and maximum method.

The analysis was able to show that the simulations, as well as the experiments, agree well with the theoretical results for the centroid method. Further, the median and maximum methods performed better both in simulations and experiments compared to the centroid method in reducing the effect of spatial averaging. Interestingly the maximum method had the highest reduction of the effect of spatial averaging. However, it also showed the highest RMSE values out of all methods due to the discretisation of picking the maximum value of the Doppler spectrum. Thus, from this study we recommend, if one’s aim is to mitigate the effect of turbulence attenuation by the lidar and retrieve time series with low noise levels, using the median method as it shows slight improvements of reducing the volume effect compared to the centroid method and has the best RMSE performance. When comparing 10 min averages all methods performed equally well.

The method of using average Doppler spectra (typically 10 or 30 min averages) has also been studied to derive turbulence statistics (Branlard et al., 2013). However, using this approach, only statistics can be derived, namely the wind speed PDF and its statistical moments. What is presented here shows how carefully choosing the method of radial speed retrieval from a Doppler spectrum can partly alleviate the inherent volume averaging effect of lidar systems to provide time series information.

It should be noted that these conclusions only apply to cw lidars and not to pulsed systems as the method of deriving radial velocities is different for the latter.

Code and data availability. The computer code to generate synthetic turbulence fields can be found at http://www.wasp.dk/weng#details_icec-turbulence-simulator (WAsP Engineering, 2018).
Appendix A: Transfer functions for the remaining misalignment directions

Figure A1.
Figure A1. Transfer function $G(k_1)$ for the remaining misalignment directions.
Author contributions. DPH performed the research work and prepared the manuscript. JM conceived the research plan and supervised the research work and the manuscript preparation.

Competing interests. The work of Dominique P. Held was partly funded by Windar Photonics A/S through an industrial PhD stipend (project number: 5016-00182).

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In the previous chapter the volume averaging effect was discussed, which is an inherent property of the lidar. If not properly corrected, it does lead to an underestimation of small-scale fluctuations. On the other, a wind turbine itself acts as a filter of turbulence due to the large rotor size. The filtering of the turbine occurs in a plane perpendicular to the advection direction, while the lidar performs averaging over its probe volume. Still, the spatial averaging effect of the lidar can be beneficial if the rotor-effective wind speed (REWS) is of interest.

In this chapter the coherence between the REWS seen by the turbine and estimated from the lidar is investigated. First in Section 3.1, the results of an experimental campaign at DTU’s test site at Risø will be presented, where a 2- and a 4-beam nacelle lidar have been compared to measurement on the turbine. The results are compared to predictions of a coherence model based on the Mann turbulence model. In Section 3.2 the model will be used to optimize the lidar focus distance and half-cone opening angle to achieve best coherence.

3.1 Lidar Estimation of Rotor-Effective Wind Speed - An Experimental Comparison

The following manuscript has been submitted to Wind Energy Science.
Lidar Estimation of Rotor-Effective Wind Speed - An Experimental Comparison

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\textbf{Abstract.} Lidar systems have the potential of alleviating structural loads on wind turbines by providing a preview of the incoming wind field to the control system. For a collective pitch controller the important quantity of interest is the rotor-effective wind speed (REWS). In this study, we present a model of the coherence between the REWS and its estimate from continuous-wave nacelle-mounted lidar systems. The model uses the spectral tensor definition of the Mann model. Model results were compared to field data gathered from a 2- and 4-beam nacelle lidar mounted on a wind turbine. The comparison shows close agreement for the coherence and the data fits better to the proposed model than to a model based on the Kaimal turbulence model, which underestimates the coherence. Inflow conditions with larger length scales led to a higher coherence between REWS and lidar estimates than inflow turbulence of smaller length scale. When comparing the two lidar systems, it was shown that the 4-beam lidar is able to resolve small turbulent structures with a higher degree of coherence. Further, the advection speed by which the turbulent structures are transported from measurement to rotor plane can be estimated by 10 minute averages of the lidar estimation of REWS. The presented model can be used as a computationally efficient tool to optimize the position of the lidar focus points in order to maximize the coherence.

\section{Introduction}

The control system is an integral part of a wind turbine and has substantial influence on its behaviour. Its aim is to maximize the power production while keeping the turbine structural loading within the design limits. In order to decrease the levelized cost of energy, several novel sensors and control strategies have been proposed. One of them is a lidar-assisted pitch controller and one of the first was introduced by Harris et al. (2006). It utilizes nacelle- or spinner-mounted lidar systems to retrieve information about the inflow. In contrast to traditional feedback (FB) control of rotor speed, disturbances in the inflow can be measured by the lidar before they affect the turbine. For collective pitch control, a simple approach is to add a feedforward (FF) pitch angle demand $\theta_{FF}$ based on lidar measurements to the FB demand $\theta_{FB}$ derived from the rotor speed deviation from its desired value $\Omega_r$, see fig. 1.

For such a controller the important information about the wind is the rotor-effective wind speed (REWS) $v_{\text{eff}}$, which can be defined in several ways (Soltani et al., 2013). One definition states that the REWS is the average longitudinal wind speed component over the entire rotor plane, which is used in this work. Alternatively, the average of the longitudinal wind speed

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig1}
\caption{Example figure}
\end{figure}
Figure 1. Block diagram of a lidar feedforward collective pitch controller to assist a traditional feedback controller.

with different weights for the hub or tip regions of the blade can be used. In the ideal case of perfect lidar measurements of $v_{eff}$ and turbine modeling, disturbances can be completely rejected and optimal rotor speed control can be achieved (Dunne et al., 2011). However, this is not achievable in reality and important shortcomings of the lidar systems are

- the contamination from lateral and vertical wind speed components,
- the spatial averaging due to the lidar’s probe volume,
- the scarcity of measurement points in the rotor plane
- and the uncertain estimation of the time delay between lidar measurement and disturbance arrival at the rotor.

Thus, it is important to optimize the measurement positions of the lidar to maximize the correlation between lidar measurement and REWS for which flexible and computationally efficient models are required.

Previously, Schlipf et al. (2013) presented an analytic correlation model in frequency domain to calculate the magnitude-squared coherence and transfer function between a lidar and a turbine using the Kaimal spectral model and an empirical exponential decay model of the longitudinal wind component for separations perpendicular to the flow. The turbulence model is defined in the IEC-61400-1 standard (IEC, 2005). The advantage of this approach compared to simulations in time domain is the reduced computational effort. However, integrating certain lidar properties becomes complicated if done analytically.

For example the spatial averaging effect of the lidar has not been integrated into the model. Therefore Schlipf et al. (2013) also proposed a semianalytic model, where properties of the lidar can be added in frequency domain and coherence and transfer function can then be calculated. The model has been extended by Haizmann et al. (2015a) to include linear rotor-effective horizontal and vertical shear estimation. Different optimal focus positions were found for REWS and shear estimations implying that a compromise needs to be found if both quantities want to be measured. Another optimization was performed in Schlipf et al. (2015), where additionally the wind evolution and constraints from the controller were considered. In Simley and Pao (2013b) a similar semianalytic method was presented to calculate the correlation between a spinner-mounted lidar and blade-effective wind speeds. The difference lies in the fact that spinner lidars rotate with the rotor and thus sample the wind field rotationally.

Comparisons between data gathered during field experiments and models were conducted in several studies. In Schlipf et al. (2013) the previously mentioned semianalytic model was compared against data gathered on NREL’s CART2 test turbine. The
measured and modeled transfer function showed very good agreement and the maximum coherent wavenumber, defined as the wave number where the coherence reaches a value of 0.5, was 0.06 rad/m for both methods. A similar comparison was performed on NREL’s CART3 turbine by Scholbrock et al. (2013), where deviations between model and measured data was observed. As a possible explanation interference of the guy wires of a close-by meteorological mast with the lidar was given. In a later experiment on the same turbine with a different lidar system Haizmann et al. (2015b) found great agreement between data and model. For this lidar, the maximum coherent wavenumber was found to be 0.03 rad/m.

The integration of lidar measurements into turbine control by suitable controllers and their associated benefits have been the topic of various analyses. FF additions to FB controllers have been studied in e.g. Laks et al. (2011); Dunne et al. (2011). A more sophisticated flatness-based controller was proposed in Schlipf and Cheng (2014), while individual pitch controllers have been considered in e.g. Dunne et al. (2012). Model predictive control approaches were examined in e.g Mirzaei et al. (2013).

To verify simulated performances, field test have been pursued. In Scholbrock et al. (2013) a pulsed lidar system was used on NREL’s CART3 turbine. A collective pitch feedforward approach (similar to fig. 1) was compared to a feedback controller only and load reduction at low frequencies (below 0.1 Hz) were observed. Damage equivalent loads (DELs) were reduced by approximately 2% and 7% for the tower fore-aft bending and the blade flapwise bending moments, respectively. A similar study was presented in Schlipf et al. (2014) using the CART2 turbine, where a reduction in the blade and tower DELs were reduced by 10%. However, periods where the lidar’s vision was obstructed by hard targets showed an increase in DELs and thus emphasizing the sensitivity of environmental conditions on lidar measurements. Another experiment on CART2 was performed by Kumar et al. (2015), where, besides adding a feedforward controller, the gains of the feedback controller has been reduced. Here the load analysis showed that a reduction was achieved after reducing the feedback gains.

In this paper we present a model of the coherence between REWS estimated from turbine and lidar measurements. The model uses the description of a turbulence field according to the model by Mann (1994), which allows to derive expressions of the auto- and cross-spectra numerically. In sec. 2 these expression are presented as well as the determination of REWS from field measurements at the turbine and from the lidar. Sec. 3 explains the test site and its characterization, while sec. 4 shows the measurement results and the comparison with the presented model. The model can be used as a computationally efficient tool to predict the auto- and cross-spectra of REWS from turbine and lidar measurements and the optimization of the lidar focus point positions.

2 Methodology

In this section we present a coherence model between nacelle lidar systems and a wind turbine. The theoretical expression to calculate the variances of turbine and lidar measurements have already been derived in Mirzaei and Mann (2016) and here we extend those to also calculate auto- and cross-spectra.

The fluctuating part of a three-dimensional (3D) wind field can be represented by the vector field \( \mathbf{u}(\mathbf{x}, t) = (u_1, u_2, u_3) \), where \( \mathbf{x} = (x_1, x_2, x_3) \) and \( \mathbf{k} = (k_1, k_2, k_3) \) refer to a 3D spatial and wavenumber domain, respectively. We assume that the vector field \( \mathbf{u}(\mathbf{x}, t) \) is frozen and the fluctuations are advected by the mean wind speed, i.e. Taylor’s frozen turbulence hypoth-
esis (Mizuno and Panofsky, 1975) applies:

\[ u(x,t) = u(x_1 - Ut, x_2, x_3), \]  

(1)

where \( U = \langle u(x_1,0,0) \rangle \) is the mean wind speed along the advection direction \( x_1 \). Thus, the dependence on time can be eliminated. The field \( u(x) \) can be written as a Fourier transform pair:

\[ u(x) = \int u(k)e^{ik \cdot x} dk \iff u(k) = \frac{1}{(2\pi)^3} \int u(x)e^{-ik \cdot x} dx, \]  

(2)

where an integral over the three-dimensional quantity, \( k \) or \( x \), means the integral from \( -\infty \) to \( \infty \) over all three components. The more rigorous Fourier-Stieltjes notation (Batchelor, 1953) was avoided due to brevity and clarity. The ensemble average of the absolute squared Fourier coefficients is the spectral tensor \( \Phi_{ij}(k) \)

\[ \langle u_i^*(k)u_j(k') \rangle = \Phi_{ij}(k)\delta(k-k'). \]  

(3)

Since \( u^*(k) = u(-k) \), eq. 3 can be written as

\[ \langle u_i(k)u_j(k') \rangle = \Phi_{ij}(k)\delta(k+k'). \]  

(4)

In this study we have used an estimation of the coherence to evaluate the correlation between models and measurements. Specifically, we were interested in the magnitude squared coherence between the REWS measured at the turbine and estimated from lidar measurements

\[ \gamma_{RL}(k_1) = \frac{|S_{RL}(k_1)|^2}{S_{LL}(k_1)S_{RR}(k_1)}, \]  

(5)

where \( S_{LL} \) and \( S_{RR} \) are the auto-spectra of the lidar and turbine estimates of REWS and \( S_{RL} \) is their cross-spectrum. From time series measurements these spectra were calculated over a 10 minute period. The resulting frequency domain is converted into wavenumber domain by the use of Taylor’s frozen turbulence hypothesis using \( k_1 = \frac{2\pi f}{U} \). The remainder of this section will present the methods to calculate the REWS and spectra. At the end the model is compared against numerical simulations to validate the implementation.

### 2.1 Rotor-effective wind speed

The REWS \( v_{\text{eff}} \) is the defined as the longitudinal wind vector component averaged over the entire rotor plane

\[ v_{\text{eff}}(x_1) = \frac{1}{4\pi R^2} \int u_1(x)dx_2 dx_3 = \frac{1}{4\pi R^2} \int \int u_1(k)e^{ik_1 x_1} dk dx_2 dx_3 \]  

(6)

\[ = \frac{1}{4\pi R^2} \int u_1(k)e^{ik_1 x_1} \int e^{i(k_2 x_2 + k_3 x_3)} dx_2 dx_3 dk \]  

(7)

\[ = \int u_1(k)e^{ik_1 x_1} \frac{2J_1(\kappa R)}{\kappa R} dk, \]  

(8)
where \( R \) is the rotor radius, \( \kappa = \sqrt{k_2^2 + k_3^2} \) and \( J_1 \) is the Bessel function of the first kind. The rotor is positioned perpendicular to the \( x_1 \)-axis, i.e. no yaw misalignment.

The auto-spectrum of \( \nu_{\text{eff}} \) can then be calculated using

\[
S_{RR}(k_1) = \int_{-\infty}^{\infty} \Phi_{11}(k) \frac{4J_2^2(\kappa R)}{\kappa^2 R^2} dk_2 dk_3.
\]  
(9)

### 2.2 REWS estimated from turbine measurements

To estimate \( \nu_{\text{eff}} \) from signals measured on the turbine the approach in Østergaard et al. (2007) was followed. It is based on using the entire rotor as an anemometer and derive the rotor-effective wind speed by considering the turbine model characteristics and several measured signals. The methods gives the magnitude of an undisturbed wind field that creates the (unique) combination of power production, rotational speed and pitch angle at the turbine. Thus, there is no need to correct for the effect of turbine induction.

The entire turbine is modelled by a simple drive train model

\[
J \dot{\Omega} = Q_a - Q_g - Q_{\text{loss}},
\]  
(10)

where \( J \) is the moment of inertia of the drive train, \( \Omega \) is the rotational speed of the rotor, \( Q_a \) is the aerodynamic torque produced by the rotor, \( Q_g \) is the generator torque and \( Q_{\text{loss}} \) is a collective term for the lost torque along the drive train. In our field experiment, torque measurements at the low-speed shaft (LSS) were performed. Thus, the measurements are taken before the gearbox and generator (where most of the losses occur) and we can replace \( Q_{LSS} = Q_g + Q_{\text{loss}} \) in eq. 10. The sampling rate of the turbine data was 1 Hz. Further a low-pass filter was used to reduce the influence of measurement noise in the estimation of \( \dot{\Omega} \). The aerodynamic torque is defined by

\[
Q_a = \frac{1}{2} \rho \pi R^2 \frac{\nu_{\text{eff}}^3}{\Omega} C_p(\beta, \lambda) = \frac{1}{2} \rho \pi R^2 \frac{R^3 \Omega^2}{\lambda^3} C_p(\beta, \lambda),
\]  
(11)

where \( \rho \) is the air density, \( \lambda = \frac{\Omega R}{\nu_{\text{eff}}} \) is the tip-speed ratio (TSR), \( C_p(\beta, \lambda) \) is the power coefficient as function of pitch angle \( \beta \) and TSR. By solving eq. 10 for \( Q_a \) and substitute it in eq. 11 we arrive at

\[
\frac{C_p(\beta, \lambda)}{\lambda^3} = \frac{2Q_a}{\rho \pi R^4 \Omega^2} = \frac{2(Q_{LSS} + J \dot{\Omega})}{\rho \pi R^4 \Omega^2}.
\]  
(12)

With a measurement of the pitch angles, a look-up with linear interpolation can be used to find \( \lambda \) that satisfies eq. 12 and by the definition of the TSR the REWS can be estimated

\[
\tilde{\nu}_{\text{eff,R}} = \frac{\Omega R \lambda}{\lambda}.
\]  
(13)

The necessary \( C_p(\beta, \lambda) \) surface can be precomputed. Details can be found in appendix A. Note that issues of non-monotony of \( C_p(\beta, \lambda) \) in eq. 12 can be avoided by performing the look-up on \( \frac{C_p(\beta, \lambda)}{\lambda^3} \) and not on \( C_p(\beta, \lambda) \). The air density has been calculated from pressure and temperature measurements on a nearby meteorological mast.
2.3 REWS estimated from lidar measurements

The measurement of a continuous-wave lidar system can be expressed mathematically as the convolution of the line-of-sight (LOS) component of the wind vector and a weighting function given by the laser light intensity along the laser beam:

\[ v_\text{LOS}(x_f) = \int_{-\infty}^{\infty} n \cdot u(sn + x_f) \varphi(s - df) ds, \quad (14) \]

where \( x_f \) is the position of the lidar focus point, \( n \) is the laser beam unit vector,

\[ \varphi(s) = \frac{z_R}{\pi z_R^2 + s^2} \quad (15) \]

is the weighting function defined by the Rayleigh length \( z_R \) and \( df \) is the focus distance. Note that the probe volume of the lidar increases with focus distance, i.e. \( z_R \propto d_f^2 \). The probe volume has an attenuating effect on the turbulent fluctuations of the wind field. Eq. 14 is assuming that the first statistical moment is used to calculate the dominant frequency of the Doppler spectrum.

Different frequency estimators can yield less turbulence attenuation (Held and Mann, 2018). The Fourier transformation of the weighting function (eq. 15) is

\[ F[\varphi(s)](k) = \exp(-z_R|k|) \]

and the auto-spectrum of the lidar measurement along a single beam can be expressed as

\[ S_{LL}(k_1) = n_i n_j \int_{-\infty}^{\infty} \Phi_{ij}(k) e^{-2z_R|k|} d\kappa_2 d\kappa_3, \quad \text{(one beam only)} \quad (16) \]

where \( n_i \) refers to the components of the laser beam unit vector \( n \) and summation of repeated indices is implied.

The typical setup of a nacelle lidar looking forward is shown in the left part of fig. 4. The lidar systems probes sequentially several focus points in front of the rotor. Due to the limitation of measuring only the LOS component of the wind vector assumptions are necessary to derive the REWS from the lidar measurements. Here we apply the following assumptions: (1) no vertical components, (2) zero turbine yaw misalignment. Based on these assumptions the REWS can be estimated from lidar measurements as the average of all \( v_\text{LOS} \) velocities:

\[ v_{\text{eff},L} = \frac{1}{b \cos \alpha} \sum_{i=1}^{b} v_{\text{LOS},i}, \quad (17) \]

where \( b \) is the number of beams and \( \alpha \) is the half-cone opening angle of the scanning cone (see left panel of fig. 4).

In wave-number domain the auto-spectrum of the REWS estimate from lidar measurement using eq. 17 and 14 can be written as:

\[ S_{LL}(k_1) = \frac{1}{b^2 \cos^2 \alpha} \sum_{i,j=1}^{b} \int_{-\infty}^{\infty} n_{ik} \Phi_{ij}(k) n_{j\ell} e^{i(d_f k \cdot n_i - n_j) - z_R(|k| n_i + |k| n_j)} d\kappa_2 d\kappa_3. \quad (18) \]

Similarly the cross-spectrum between the REWS and its estimate from lidar measurements \( v_{\text{eff},L} \) can be calculated using

\[ S_{RL}(k_1) = \frac{1}{b \cos \alpha} \sum_{i=1}^{b} \int_{-\infty}^{\infty} n_{ij} \Phi_{j1}(k) e^{i(d_f k \cdot n_i + k_i \Delta x) - z_R|k| n_i} \frac{2J_1(\kappa R)}{\kappa R} d\kappa_2 d\kappa_3, \quad (19) \]
where $\Delta x$ indicates the distance between rotor and lidar measurement plane.

When evaluating eq. 17 from field measurement two corrections are necessary. First, we used an analytic solution for the flow speed reduction and diversion around the rotor (Conway, 1995). The model assumes an actuator disk model and laminar, uniform inflow with uniform, non-rotational loading. An example of the flow around a rotor can be found in the appendix B.

The induction correction $a_c = U / U_\infty$, where $U_\infty$ is the undisturbed free steam wind speed, can be defined from the calculated flow field at the focus positions of the lidar beams. Thus, the induction correction depends on lidar parameters, i.e. the half-cone opening angle $\alpha$ and the focus distance $L_f$, and on the operational point of the turbine, i.e. the axial induction factor $a$. The induction factor is determined from the measured 10 minute mean REWS by the lidar and a steady-state thrust curve is used to look up the thrust coefficient $C_t$. Then the relation $C_t = 4a(1 - a)$ is used to calculate the induction factor. The effect of the induction is assumed to be constant over a 10 minute period.

Second, an estimation of the average turbine misalignment $\varphi$ can be derived from lidar measurements. Since eq. 17 assumed perfect turbine alignment, a correction of the 10 minute average misalignment was used by dividing by a correction factor $a_\varphi$. The beam vector for the $i$-th beam is $\mathbf{n}_i = (\cos \alpha, \sin \beta_i \sin \alpha, \cos \beta_i \sin \alpha)$, where $\beta_i$ is the azimuth angle along the measurement cone, compare tab. 1. The horizontally misaligned wind normal vector is $(\cos \varphi, \sin \varphi, 0)$ and the correction can be defined as

$$a_{\varphi,i} = \mathbf{n}_i \cdot (\cos \varphi, \sin \varphi, 0) = (\cos \alpha \cos \varphi + \sin \beta_i \sin \alpha \sin \varphi)$$

Integrating induction and misalignment corrections into eq. 17 yields

$$\hat{v}_{\text{eff},L} = \frac{1}{a_c} \sum_{i=1}^{b} \frac{v_{\text{LOS},i}}{a_{\varphi,i}}.$$ (21)

### 2.4 Model implementation and validation against simulations

For the implementation of the model a C++ code has been created to numerically solve eq. 9, 18 and 19. Adaptive cubature integration was used as an integration algorithm\(^1\). To validate the implementation numerical simulations have been performed. At first six random 3D turbulence boxes with different turbulence seeds have been created according to the Mann spectral tensor (Mann, 1998)\(^2\). The boxes had dimensions of 2800 m x 64 m x 64 m using 8192 x 32 x 32 grid points per box and contained only the turbulent part of the wind field, i.e. the mean wind speed was zero. The lidar measurements have been simulated using eq. 14, however due to the finite size of the boxes the integration has been truncated at $\pm 10 z_R$ from the focus point; details can be found in Held and Mann (2018). The two lidar systems presented in tab. 1 have been used. The rotor plane (with a diameter of 52 m) was discretized by 100 x 100 grid points.

The results of the coherence analysis can be found in fig. 2. It can be seen that the coherence for the 2-beam lidar drops at lower wavenumbers than the 4-beam lidar. This is due to the greater coverage of the rotor plane using four distinct focus locations compared to only two for the 2-beam lidar. Further, the comparison between the simulations and the model shows

\(^1\)The adaptive cubature integration scheme was written by Steven G. Johnson and is available on GitHub: https://github.com/stevengj/cubature
\(^2\)The software can be downloaded free of charge at http://www.wasp.dk/weng#details__iec-turbulence-simulator
Figure 2. Coherence between the estimation of REWS from the turbine and the lidar. The comparison between the numerical simulations (Simu) and the implementation of the model (Theo) show very good agreement.

very good agreement. Some deviations remain, which can be attributed to using only six simulations when estimating the coherence.

3 Experimental setup

3.1 Instrumentation

Field measurements have been conducted at DTU’s test site at Risø, located at the Roskilde Fjord in Denmark. The site consists of one row of wind turbines intended for testing and several meteorological masts are installed around the turbines, see fig. 3. During the experiments only a Nordtank was operative, which is located at a distance of 215 m (4.1D) at an angle of 195° (from north). In general, there is a slight positive terrain slope from the fjord towards the turbines. To the east of the Vestas V52 some buildings and vegetation exists, while towards the west the turbine is facing flat fields and the fjord.

For this experiment two continuous-wave coherent Doppler lidars manufactured by Windar Photonics A/S have been mounted on a Vestas V52 turbine. The lidar systems, a 2-beam and a 4-beam lidar, are mounted on the nacelle of the turbine and have been staring forwards to measure the inflow of the turbine. An illustration and a photo of the 4-beam lidar can be seen in fig. 4. The specifications for both lidars can be found in tab. 1. The two systems both contain one laser source located inside the nacelle and switch between the focus point sequentially. Each scan is completed in one second. Note the different Rayleigh lengths due to the different focus distances and the increased half-cone opening angle for the 2-beam system. The azimuth angle refers to the position on the scanning cone surface. The position at the top of the cone is at an azimuth angle of 0°. Hence, the 2-beam lidar consists of two horizontal beams, while the 4-beam lidar has one focus point in each quadrant of the rotor area.
**Figure 3.** Digital terrain model (DTM) of the DTU’s test site at Risø, where the Vesats V52, its meteorological mast and the Nordtank turbine are indicated. Zone 32 UTM coordinates centered at the Vestas V52 turbine were used. The DTM data was obtained from the Danish Map Supply (Agency for Data Supply and Efficiency).

![Digital terrain model](image)

**Figure 4.** *Left:* Illustration of the 4-beam lidar focus locations on a cone with apex at the turbine nacelle. *Right:* Photo of the 4-beam lidar mounted on the Vestas V52 turbine at the Risø test site.

The Vestas V52 turbine has a diameter of 52 m and a hub height of 44 m with a rated power of 850 kW. It is heavily instrumented with several mechanical strain gauges, in particular a strain gauge set-up to measure the torque on the low-speed shaft. Also a meteorological mast is located approximately $2.5D$ in front of it. To characterize the flow conditions during the experiment a Metek P2901 USA-1 3D sonic anemometer mounted at hub height was used. Further measurements from a Vaisala PTB110 air pressure sensor and a Vaisala R/H HMP 155 humidity sensor were used in addition to the temperature measurements from the sonic anemometer to calculate the air density.
Table 1. Information of lidar setup parameters and measurement periods. The azimuth angle refers to the position on the scanning cone surface with $0^\circ$ being the top of the cone.

<table>
<thead>
<tr>
<th></th>
<th>2-beam</th>
<th>4-beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focus distance $d_f$ [m]</td>
<td>37</td>
<td>62</td>
</tr>
<tr>
<td>Rayleigh length $z_R$ [m]</td>
<td>2.1</td>
<td>6.0</td>
</tr>
<tr>
<td>Half-cone opening angle $\alpha$ [°]</td>
<td>30</td>
<td>18</td>
</tr>
<tr>
<td>Azimuth angle $\beta$ [°]</td>
<td>90 and 270</td>
<td>45, 135, 225 and 315</td>
</tr>
<tr>
<td>Distance focus points - rotor $\Delta x$ [m]</td>
<td>32</td>
<td>59</td>
</tr>
<tr>
<td>Distance lidar - rotor $d_{Nac}$ [m]</td>
<td>$\approx$ 1</td>
<td>$\approx$ 1</td>
</tr>
<tr>
<td>Scan time per beam [s]</td>
<td>0.5</td>
<td>0.25</td>
</tr>
</tbody>
</table>

3.2 Site characterization

The wind rose derived from wind direction and horizontal wind speed of the sonic anemometer measurements during the periods of the experiment are presented in the left panel of fig. 5. The main wind direction is from the west with winds coming from the Roskilde Fjord.

Figure 5. Left: Wind rose gathered during the test periods at the Risø test site. Right: The three top panels show the result of fitting the Mann Model to the calculated mean spectra as function of wind direction. The bottom panel shows the number of acquired 10 minute periods as function of wind direction.

To get a clearer picture of the inflow conditions the data set was grouped into sectors of $30^\circ$ and the Mann model has been fitted to the average spectra in each sector. The fitting followed the procedure in Mann (1994) and was performed on the $u$-, $v$-, $w$-spectra and the $uw$-co-spectra. The spectra have been normalized by the mean wind speed squared. The model has
three parameters: $\alpha \epsilon^{2/3}$ where $\epsilon$ is the rate of viscous dissipation of turbulent kinetic energy and $\alpha$ the spectral Kolmogorov constant, $L$ is a length scale and $\Gamma$ is an anisotropy parameter, for details see Chougule et al. (2015). The results of the three model parameter as function of wind direction can be seen in the right panel of fig. 5. First of all, the effect of the wake from the Vestas V52 turbine onto the sonic anemometer is clearly seen at a wind direction of $90^\circ$. In this sector a very high turbulent kinetic energy dissipation rate and a low anisotropy parameter were calculated. The results from this sector were disregarded and linearly interpolated. Secondly, two wind regimes can be identified. A region spanning from $330^\circ$ to $180^\circ$ shows a length scale $L$ of approximately 20 m, while for the region from $210^\circ$ - $300^\circ$ larger length scales were fitted. Similarly the normalized dissipation rate is higher in the first region compared to the second. This is in agreement with the terrain of the test site. The inflow for the first region is characterized by obstacles like buildings and tall vegetation. The second region faces open fields and the fjord fetch. The fit has also been performed for the Kaimal turbulence model, which is defined in the IEC 61400-1 standard (IEC, 2005). This model has one characteristic length parameter $L_k$. Here only the $u$-, $v$- and $w$- spectra were fitted. The measured spectra have been been normalized by their measured variance and the frequency domain has been converted into wave number domain. Then the model was fitted to the measured spectra by minimizing the combined mean squared error of the spectra.

We separated the following analysis into two regions. The information on the two regions can be found in tab. 2 including the averaged fits to the Mann model. More 10 minute periods were obtained for region 2 due to the dominant wind direction from the west. The fitting results for the Kaimal model can also be found in tab. 2. Similar to the Mann model a larger length scale parameter was found for region 2. In Appendix C the spectra and the fitted Mann model can be found.

Table 2. Measurement sectors and fitted Mann model ($\frac{\alpha \epsilon^{2/3}}{U^2}$, $L$ and $\Gamma$) and Kaimal model ($L_k$) parameter.

<table>
<thead>
<tr>
<th></th>
<th>Region 1</th>
<th>Region 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direction</td>
<td>$330^\circ$ - $180^\circ$</td>
<td>$210^\circ$ - $300^\circ$</td>
</tr>
<tr>
<td>Nr. of 10-min periods</td>
<td>1678</td>
<td>2713</td>
</tr>
<tr>
<td>$\frac{\alpha \epsilon^{2/3}}{U^2}$ [$10^{-3}$ m$^{-1}$]</td>
<td>4.29</td>
<td>1.60</td>
</tr>
<tr>
<td>$L$ [m]</td>
<td>18.5</td>
<td>37.9</td>
</tr>
<tr>
<td>$\Gamma$ [-]</td>
<td>2.36</td>
<td>2.41</td>
</tr>
<tr>
<td>$L_k$ [m]</td>
<td>201.0</td>
<td>326.6</td>
</tr>
</tbody>
</table>

4 Results

The first step in the analysis of the results was to apply appropriate data filters. It was necessary to identify periods where the turbine was in a normal power production state. Thus, a lower threshold on the minimum power production (i.e. >0 kW), minimum rotor speed (i.e. >16 rpm) and maximum pitch angle (i.e. <23$^\circ$) in a 10 minute period were utilized. These thresholds were found by inspection of the available turbine data. This filter removed 52.8% and 53.8% of the data for the 2- and 4-beam experiments, respectively.
Figure 6. Comparison of 10-minute REWS estimates from the lidars and the turbine to the meteorological mast’s sonic anemometer. The data was taken from periods when the turbine was operational and facing the mast.

The filter applied to the lidar data consisted of a minimum number (>90% or 540 measurements) of available measurements on each beam in a 10 minute interval. Unavailable measurement have been interpolated linearly. Instances where 4 or more consecutive unavailable measurements occurred on any beam were also discarded. Whether a measurement is available or not was decided internally by the lidar system and depends on carrier-to-noise ratio and the Doppler peak shape and area. After applying the turbine availability filter, this filter for the lidar data lead to an additional discard of 2.9% and 15.3% for the 2- and 4-beam system, respectively. Since the 4-beam system was under development during the field test a higher unavailability is observed.

Additionally, inflow from all yaw position except from the wake sector (195° ± 30°) were considered because the lidar yaw misalignment measurements are biased in wake situations (Held et al., 2018). The yaw position filter lead to an additional exclusion of 6.0% and 6.3% of the data for the 2- and 4-beam periods, respectively.

Next, the 10 minute average REWS estimates of lidar and turbine are compared to the sonic anemometer mounted on the meteorological mast. The comparisons for the 2 lidar systems and the turbine can be found in fig. 6. Besides the previously mentioned data filters, only yaw positions, where the turbine was facing the meteorological mast (i.e. a yaw heading of 289° ± 20°) have been considered. It can be seen that the both lidar systems agree well with the mast’s sonic anemometers; linear least-square fitting results in a slope close to unity with no significant bias. This indicates that the correction for turbine misalignment and induction are working as intended. Similarly, the correlation between mast and turbine also shows very good accordance with no systematic error.

Corresponding comparisons are performed between the REWS estimated from the lidar systems and the turbine, respectively. The correlation plots can be found in fig. 7. Analogous to the comparisons to the mast, theses comparisons also show that there is no systematic error between the two signals. Both linear fits show unity slopes and very small offsets. They are slightly worse than the comparisons to the meteorological mast, which can be explained by model inaccuracies when estimating the REWS when using turbine and lidar data.
Figure 7. Comparison of 10 minute REWS estimates between the lidars and the turbine. Data from the wake sector and nonoperational periods of the turbine were removed.

For illustrative purposes, the next plots in fig. 8 show a single time series result for the lidar and turbine estimate of REWS. Both signals have a sampling rate of 1 Hz. In general, it can be seen that the fluctuations in REWS that were sensed by the rotor can also be measured by the lidar systems. For the 2-beam lidar larger deviations can be observed since this lidar probes the incoming wind field only at two locations. Fluctuations that occur at the top or bottom parts of the rotor can not be measured. In case of the 4-beam system a measurement in each quadrant is performed and gives a better estimate of the wind speed effecting the entire rotor. Further, the preview ability of the lidar systems also becomes apparent. Fluctuations can be measured before they affect the rotor.

The effect of probing two versus four focus locations is now studied in wavenumber domain by comparing the squared coherences. The experimental data is also compared to two models: the model based on the Mann turbulence model introduced in sec. 2 and the Kaimal turbulence model used in previous studies (Schlipf et al., 2013). As mentioned in sec. 3.2, the analysis was split into two regions, of which one is disturbed by buildings or trees (region 1) and the other has an undisturbed inflow over open fields or the fjord’s fetch (region 2).

The coherence analysis for region 1 can be found in fig. 9. At first, it can be seen that the coherence of the 2-beam lidar drops at lower wavenumbers than the coherence of the 4-beam lidar indicating that small fluctuations can be sensed more accurately by the 4-beam system. Secondly, the measured data agrees very well with the Mann turbulence model coherence. The Kaimal model on the other hand seems to give a slight underestimation of the coherence. The wavenumber at which the measured coherence dropped to the level of 0.5 is 0.027 rad/m for the 2-beam and 0.051 rad/m for the 4-beam. These wavenumbers have been defined as the smallest detectable eddy size (Schlipf et al., 2018) and can be interpreted as the size of the eddy that is captured with an accuracy of 50%. They are approximately 219.4 m (4.2D) for the 2-beam and 122.5 m (2.6D) for the 4-beam system, where the number in the brackets are normalized by the rotor diameter. Thus, by adding two additional focal points to a 2-beam nacelle lidar system the smallest detectable eddy size can be reduced by 44%.
Figure 8. Time series example of lidar and turbine estimates of REWS. A high degree of similarity between the signals can be seen. Also the preview ability of the lidar system is evident.

Figure 9. Squared coherence between the REWS estimation of the lidar and the turbine for region 1. The two models are also included in the plot.
Figure 10. Squared coherence between the REWS estimation of the lidar and the turbine for region 2. The two models are also included in the plot.

The results for region 2 are presented in fig. 10. Here very similar observations can be made. The coherence for the 2-beam lidar drops at lower wavenumbers compared to the 4-beam. The wavenumbers at $\gamma_{RL} = 0.5$ are 0.032 rad/m and 0.056 rad/m and the smallest detectable eddy sizes are 198.8 m ($3.8D$) and 111.4 m ($2.1D$), respectively. This demonstrates once more a reduction of 44% in the smallest detectable eddy size. Comparing these numbers to the results of region 1 shows that flow having larger length scale parameter is beneficial for lidar systems as the coherence drops at higher wavenumbers.

Equivalently to region 1, the Mann turbulence model fits very well to the measured data. There are however some slight deviations for both lidars in the region of 0.01 to 0.1 rad/m. When comparing the experimental data to the Kaimal model, a larger mismatch is observed compared to region 1. These deviations could stem from the lack of the Kaimal model to represent 3D turbulent structures. It is a 1D model, which has been extended to represent 3D turbulence by applying an empirical exponential lateral coherence model with completely independent velocity components, while the Mann model defines a full 3D tensor model.

Next, the delay between lidar and turbine estimations of REWS are analyzed. The delay stems from the perpendicular distance between the rotor plane and the measurement plane $\Delta x$. It depends on the advection speed

$$\Delta t = \frac{\Delta x}{U_{adv}} - \frac{t_{Scan}}{2},$$

where $t_{Scan}$ is the time to perform one full scan, which is 1 s for both lidars and $\Delta x = d_f \cos \alpha - d_{Nac}$, where $d_{Nac} \approx 1$ m is the distance between lidar mounting position and rotor. $U_{adv}$ is the advection speed of the turbulent fluctuations, which is estimated by the 10 minute average of the lidar estimated REWS: $U_{adv} \approx \overline{\hat{v}}_{eff,L}$.

Since the experiment was performed with very good time synchronization (with a maximum time delay of a few $\mu$s), it is also possible to calculate the delay between the two signals and compare it with expected delays based on the advection speed. The delay between the two signal has been calculated using the information theoretical delay estimator presented in Moddemeijer (1988). This method is based on splitting the two input signal into two parts: the past and the future and calculating the
mutual information of two signals. By shifting the signals relative to each other, the time delay which minimizes the mutual information is found. We have found that this method performs better than other delay estimators, namely the maximum index of the cross-correlation and the slope of the cross-spectrum. Due to the sampling rate of 1 Hz calculated delays are discretized in steps of 1 s.

The result can be seen in fig. 11. For the 2-beam lidar shorter delays are expected due to the smaller focus distance and larger half-cone opening angle. Still, the results for both lidars show great overlap between the measured delay and the delay expected from advection speed and the lidar geometry. Towards high wind speeds the available preview time provided by the lidar becomes smaller. Also, the required filter time is shown for the two lidar setups. Low-pass filtering the lidar system is crucial to reject high-frequent fluctuations that are sensed by the lidar but not experienced by the rotor and if not filtered would cause detrimental pitch actuation. In this study a first-order Butterworth filter is used following the approach presented in Schlipf (2015), though different filters have been proposed, e.g. a Wiener filter (Simley and Pao, 2013a). The delay of the filter is nonlinear but can be approximated by the delay at a certain frequency of interest $\omega_{\text{delay}} = 2\pi f_{\text{delay}}$ (Schlipf, 2015):

$$t_{\text{filt}} = \frac{\arctan\left(\frac{\omega_{\text{delay}}}{\omega_{\text{cutoff}}}\right)}{\omega_{\text{delay}}}$$

where $f_{\text{delay}} = 0.433$ Hz was chosen as the frequency of one rotation at the turbine’s rated rotor speed since the collective pitch controller aims at reducing loads up to this frequency. The cutoff frequency was determined from the coherence analysis presented previously at the point where $\gamma_{RL} = 0.5$. The average over region 1 and 2 was computed for both lidar systems and $\omega_{\text{cutoff}} = U_{\text{adv}} k |_{\gamma_{RL} = 0.5}$. This implies that the cutoff frequency changes with advection speed and an adaptive filter is required. It can be seen that the expected preview time provided by the lidar system is sufficient for the low-pass filtering of both lidars. For both lidar systems expected and observed delays show that there is ample preview time to perform the filtering.
5 Conclusions

In this study we presented a model of the coherence between REWS estimated from turbine and lidar measurements. The underlying model of the 3D turbulent field is the Mann spectral tensor and allows the direct calculation of auto- and cross-spectra of REWS estimations for lidar and turbine. It is compared to field data obtained from two continuous-wave lidar systems mounted on top of the nacelle of a wind turbine. To retrieve the turbulence model parameters, measured spectra from a sonic anemometer have been fitted to the spectral tensor. The comparisons of squared coherence show that the presented model fits the field data better than previously used models, which are based on the Kaimal model defined in IEC standard. Thus, this study gives confidence that the proposed model can accurately represent the important lidar properties and it can be used to optimize the lidar focus point positions to maximize the coherence between lidar and turbine. A common parameter used in the lidar optimization is the wavenumber where the coherence drops to a value of 0.5 (Dunne et al., 2014), which can be calculated precisely by the model.

We have found that larger turbulence length scales led to higher coherences between REWS estimated of turbine and lidar compared to inflow turbulence of smaller length scale. It was also shown that the smallest detectable eddy size can by reduced by almost 50% when using the 4-beam compared to the 2-beam system. Further, the advection speed by which the turbulent structures are transported from measurement to rotor plane can be estimated from 10 minute averages of REWS from lidar measurements. This is important information for the correct timing of the measured fluctuations of the lidar systems. There is also enough preview provided by the lidar to perform the necessary low-pass filtering.

Since some of the physical mechanisms have not been modelled, future work includes additions to both the lidar and turbulence modelling. First of all, the evolution of turbulence as it travels from measurement to rotor plane has been neglected. An amendment of turbulence evolution to the Mann model has been proposed in de Maré and Mann (2016). The evolution will have most influence on the small-scale fluctuations (Bossanyi, 2013) and including the effect will reduce the coherence. Hence, the model presented here can be considered an idealized case. On the other hand, only small differences were observed between data and model implying that the evolution effect is small. For larger turbine, which require larger focus distances, this effect could be more severe. Secondly, the stability of the atmosphere was not considered, i.e. a neutral stratification was assumed. Extensions to the Mann model have been proposed to include effect of the atmospheric stability, e.g. Segalini and Arnqvist (2015) or Chougule et al. (2018). It should be noted that the discrete scanning of the lidar system and possible blade blockage effects have not been integrated into the model. Also, environmental conditions like the aerosol concentration, fog or precipitation have been disregarded.

The presented model in the current form can be applied to nacelle-mounted cw lidar and by modifying the spatial averaging of the lidar it can be extended to nacelle-mounted pulsed lidars as well. To cover spinner-mounted lidar systems the rotational sampling effect of the lidar as it rotates with the rotor needs to be modelled.
Appendix A: Calculation of the power coefficient surface

For the calculation of the $C_p(\beta, \lambda)$ surface an aerodynamic model of the Vestas V52 turbine was used. Aero-elastic simulations in HAWC2 over a domain of several pitch angles and TSR have been performed. A homogeneous, constant wind speed of 8 m/s was used and constant pitch angles and rotational speeds of the rotor were set during the simulation. Stiff tower and blades were used to avoid dynamic effects and calculate quasi-steady state $C_p$ values. The resulting surface can be found in fig. A1.

Figure A1. Calculated $C_p(\beta, \lambda)$ surface for the Vestas V52 turbine. For illustrative purposes negative $C_p$ values have been replaced with zero.

Appendix B: Induction Correction

In this appendix an example of the flow field around a rotor operating at the aerodynamic optimum according to the model of Conway (1995) is shown. The focus positions of the 4-beam lidar are indicated in red.

Appendix C: Comparison between average fitted Mann model parameter and average spectra

In section 3.2 the analysis was split into two distinct region, of which region 1 was disturbed by buildings and vegetation, while region had a more undisturbed inflow over field and the fjord. Previously, the average spectra were fitted to the Mann model for sectors of 30° width. The fitted parameter were then averaged per region to obtain a representative set of parameters for each region. Here the averaged parameter are compared to the average spectra for region 1 and 2. In the process each individual $u$-, $v$- and $w$- spectrum and the $uw$ co-spectrum has been normalized by the 10 minute mean wind speed squared and the average over all spectra for each region was taken. The result can be seen for region 1 in the left graph of fig. C1 and for region 2 in...
Figure B1. Example of the flow speed reduction and diversion around the rotor of a wind turbine. The red lines indicate the laser beam and the red dots show the focus points.

Figure C1. Average spectra and the corresponding Mann model fits from tab. 2 for region 1 (left) and region 2 (right).

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Agency for Data Supply and Efficiency: DHM/Terræn (0.4 m grid), available at: https://download.kortforsyningen.dk/content/dhmterr%C3%A6n-04-m-grid, last accessed: 06 Dec 2018.


3.2 Optimization of the Coherence

In this section the coherence model developed in the previous section is used to optimize the 2- and 4-beam nacelle lidars by Windar Photonics. The properties of the lidar can be seen in Table 1.1. The optimization target used here is the maximum coherent wave number, which has been introduced in Schlipf et al. [68], and is defined as the wave number at which the coherence drops to a value of 0.5. The position of the focus points is influenced by changing the focus distance and the half-cone opening angle of the lidar. The azimuth location of the beam on the cone is not changed, i.e. the 2-beam lidar is measuring in the horizontal plane and the 4 beams of the 4-beam lidar are location in each quadrant of the rotor. The turbine is the Vestas V52 and the turbulence model parameters are the same also used in the previous section. MatLab’s \texttt{fminsearch} was used as an optimization algorithm.

The results of the optimization can be found in Figure 3.1 for both the 2- and 4-beam lidar. The current design used by Windar Photonics is indicated as black dots and the optimal solution found from the coherence model is shown as red dots.

For both optimizations a region of high maximum coherent wave number can be identified. It can be seen that for the 2-beam lidar the optimal positions are achieved by reducing the opening angle significantly and slightly increasing the focus distance. The optimal position is achieved at a focus distance of 41.2 m and a half-cone opening angle of 13.2°. In case of the 4-beam lidar the focus distance should be reduced to 34.7 m and the half-cone opening angle should be increased to 22.4°.

Next, the influence of changing the turbulence model parameters and the rotor radius on the maximum coherent wave is investigated, see Figure 3.2. Note that the turbulent kinetic energy dissipation rate, $\epsilon$, does not have an influence on the coherence.

Changes in the turbulence length scale show for both lidars that the maximum coherent wavenumber increased with increasing length scale. This can be explained by the presence of larger flow structures, which can be estimated better by measurements of the lidar at discrete focus points. The variation with anisotropy parameter is small. An increase in anisotropy led to a decrease of the wave number because the contamination from the lateral and vertical components increased. Lastly, the change of turbine size has a large effect. The maximum coherent wave number decreases significantly for both lidars. This implies that the optimal lidar design strongly depends on the turbine size.

The changes in the optimal design if either the turbulence parameters or the turbine size is varied is studied in the figs. 3.3 and 3.4 for the 2- and 4-beam lidar, respectively. Increases in the turbulence length scale show that the optimal focus distance should be reduced and the half-cone opening angle should be slightly increased to achieve optimal preview. Again, the effect of the anisotropy parameter is small and the turbine size has the biggest effect on the design. The required focus distance and half-cone opening angle as function of rotor radius increases linearly for both systems. At a rotor radius of 80 m, which is used for large offshore wind turbines, the optimal focus distance is approximately 80 m with an opening angle of roughly 20° for the 2-beam lidar and a similar focus distance, but an half-cone opening angle of above 30° for the 4-beam lidar. These focus distances can be reached with the current technology of Windar Photonics’
lidars.

The coherence model does not include the effect of turbulence evolution between measurement and rotor plane because Taylor’s frozen turbulence hypothesis is assumed. Including the evolution into the coherence model would reduce the maximum coherent wave numbers found previously and suggest a placement of the focus points closer to the turbine rotor.
3.2 Optimization of the Coherence

Figure 3.1: Optimization results of the focus point position that maximizes the maximum coherent wave number. The initial design refers to the current design choice of Windar Photonics and acts as a starting point of the optimization. The optimal design is shown as red dots. Results for the 2-beam lidar can be seen on the top plot and for the 4-beam lidar on the bottom.
3.2 Optimization of the Coherence

Figure 3.2: Sensitivity analysis of the maximum coherent wave number to the turbulence length scale (top), anisotropy parameter (center) and rotor radius (bottom). The vertical dotted lines indicate the settings used in the previous optimization.

Figure 3.3: Changes in the optimal design as function of the turbulence parameters and the rotor radius for the 2-beam lidar. Optimal values for the focus distance are shown on the left and for the half-cone opening angle on the right.
Figure 3.4: Changes in the optimal design as function of the turbulence parameters and the rotor radius for the 4-beam lidar. Optimal values for the focus distance are shown on the left and for the half-cone opening angle on the right.
CHAPTER 4

Detection of Wakes in the Inflow of Turbines Using Nacelle Lidars

As mentioned previously, wakes from neighboring turbines violate the horizontally homogeneous inflow assumption that lidars use to derive inflow characteristics. In this chapter we study the effect of wakes onto the yaw misalignment estimates. We also present a detection algorithm that is able identify wake in the inflow by measuring the spectral broadening due to small-scale turbulence generated by wakes. Finally, identified wake situations are corrected using an empirical relationship between spectral broadening and undisturbed wind direction.

The results demonstrate that nacelle lidars can be used to estimate yaw misalignment even in the presence of wakes if corrections are applied. They can also overcome the disturbing effect that the blades and the nacelle has on the traditional nacelle anemometry and estimate the free flow wind direction. The detection result can potentially also be applied to wind farm wake steering algorithms.

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Detection of wakes in the inflow of turbines using nacelle lidars

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Abstract. Nacelle-mounted lidar systems offer the possibility of remotely sensing the inflow of wind turbines. Due to the limitation of line-of-sight measurements and the limited number of focus positions, assumptions are necessary to derive useful inflow characteristics. Typically, horizontally homogeneous inflow is assumed which is well satisfied in flat, homogeneous terrain and over sufficiently large time averages. However, it is violated if a wake impinges the field of view of one of the beams. In such situations, the turbine yaw misalignment measurements show large biases which require the detection and correction of these observations. Here, a detection algorithm is proposed based on the spectral broadening of the Doppler spectrum due to turbulence within the probe volume. The small-scale turbulence generated within wake flows will typically lead to a significantly larger broadening than in the ambient flow. Thus, by comparing the spectral widths at several locations, situations, where a wake is impinging the field of view of one or more beams can be identified. The correction method is based on an empirical relationship between the difference in turbulence levels at distinct beams and the difference in wind direction derived from the lidar and the real wind direction. The performance of the algorithm is evaluated in a field experiment identifying all wake situations, and thus, correcting the lidar derived wind direction.

1 Introduction

Modern wind turbines usually follow an upwind turbine design where the rotor is installed upstream of the tower. This design requires an active yaw angle control which traditionally uses nacelle-mounted sonic anemometer or wind vane misalignment measurements to align the turbine with the wind. The yaw motor actuation is slow to avoid excessive wear on its components and temporary yaw misalignment is unavoidable (Burton et al., 2011). On the other hand, a systematic error in yaw tracking leads to production losses and should be prevented.

One source of yaw tracking error stems from the misalignment measurements provided by the nacelle sonic anemometer or wind vane which are heavily disturbed by the flow around the rotor blades and the nacelle. These yaw sensors can be calibrated against undisturbed reference wind direction sensors, for example mast mounted sonic anemometers and wind vanes or vertically profiling lidars. However, differences in inflow and site conditions between calibration site and actual installation site of the turbine are not reflected in this calibration and might introduce biases in the measurements. A method to detect and correct the degradation of the sensors over time is presented in Mittelmeier and Kühn (2018). Additionally, field experiments showed that wakes from neighboring turbines can lead to yaw offsets from the dominant wind direction for downstream turbines (Schepers, 2009; McKay et al., 2013).
Several alternative sensors have been proposed to sense yaw misalignment. In Pedersen et al. (2015) the use of spinner anemometry is suggested to overcome the flow distortions from blades and the nacelle. Three 1-dimensional sonic anemometers were mounted at the spinner to measure horizontal wind speed, yaw misalignment and flow inclination. Calibration of the sensor is necessary to remove measurement biases (Demurtas and Janssen, 2016).

A wind state observer for vertical and horizontal shear, yaw misalignment and flow inclination based on flap- and edgewise blade bending moment measurements was formulated and tested through simulations in Bertelè et al. (2017) and validated on a scaled experimental turbine in Bertelè et al. (2018). In Bottasso et al. (2018) a similar observer using flapwise blade bending moments was developed to estimate the average wind speed over four azimuthal sector of the rotor plane. By comparing upper with lower and left with right sector vertical shear and impinging wakes from upstream turbines could be detected. For the detection of wakes the derived sector-wise mean wind speed and turbulence intensity were used.

Nacelle-mounted light detection and ranging (lidar) devices have also been suggested to estimate yaw misalignment (Scholbrock et al., 2015; Fleming et al., 2014). Due to their remote sensing capabilities, it is possible to measure the inflow in front of the turbine. Measurements at close distance from the rotor are affected by the induction of the turbine but the severe influences of blades and nacelle behind the rotor can be avoided. However, the limitation of measuring only the line-of-sight (LOS) component along the laser beam requires the lidar systems to probe the wind field at several position over the rotor plane and to make assumptions of the incoming flow field to derive the yaw misalignment of the turbine. As a consequence, nacelle lidars are able to steer the laser light towards several focus positions. The most simple setup is a 2-beam system which scans two horizontal positions at the hub height of the turbine, see fig. 1.

![Figure 1](image.png)

**Figure 1.** Illustration of a 2-beam nacelle-lidar mounted on a wind turbine to measure yaw misalignment. Misaligned horizontally homogeneous flow will affect the LOS measurements at the two focus positions (left). A wake in the field of view on one of the beams will affect the LOS measurement in a similar way as a misaligned flow (right). From lidar measurements these two situations can not be distinguished.

To derive the yaw misalignment from nacelle lidar measurements two common assumptions are made:

1. No vertical wind vector component
2. Horizontally homogeneous flow

In case of a horizontally scanning lidar the measured vertical component will always be very small since only the LOS can be measured. Further, the horizontally homogeneous flow assumption is well satisfied if the terrain is flat and homogeneous and if appropriate averages over time (commonly one to 10 minute averages) are taken. Note that by changing the second assumption to a perfect yaw alignment, the measurement of the lidar system can be interpreted as horizontal shear. This can pose a problem if the lidar is used for yaw alignment and cyclic pitch control (Schlipf, 2015).

The assumption of horizontal homogeneity is violated if a wake from an upstream turbine is impinging the field of view of one of the beams. Such a situation is illustrated in the right panel of fig. 1. The left half of the rotor is exposed to a lower wind speed due to the wake and thus the measurement at the left LOS, \( v_{\text{LOS},1} \), is reduced. This is similar to a reduction of one of the LOS speed due to misaligned flow and cannot be distinguish from lidar measurements, see left illustration in fig. 1. Thus, an impinging wake in the field of view of one of the beams will lead to a yaw misalignment measurement even though the flow is aligned. If the lidar should supply yaw misalignment measurements to the yaw controller, these situations need to be identified and, if possible, the influence of the wake needs to be corrected for.

In general, scanning lidar systems, where the laser follows a trajectory rather than probes discrete points, are able to measure large scale flow phenomena due to their high spatial and temporal resolution. The first field experiment of measuring wakes behind turbines by a scanning continuous-wave (cw) lidar was conducted by Bingöl et al. (2010) and Trujillo et al. (2011). A cw lidar was mounted on the nacelle of a turbine looking downstream and the derived flow information was sufficient to identify and and characterize the meandering of the wake. More recently, a nacelle-mounted cw lidar with a fast scanning head was used to provide a detailed image of wake behind a turbine and track its movement under different atmospheric conditions (Herges et al., 2017). Long-range pulsed lidars positioned on the ground (Iungo et al., 2013; Smalikho et al., 2013; Bodini et al., 2017) or in offshore wind farms (Krishnamurthy et al., 2017) have also been used to measure wakes. The advantage of pulsed systems is their increased range (of several kilometers) compared to cw systems which allows to visualize the wakes of multiple turbines in a wind farm.

However, the limited amount of information gathered by commercially available nacelle lidars makes the detection and characterization of wakes challenging. Currently, commercially available systems include Windar Photonics’ 2- and 4-beam cw lidars with fixed focus distance, Leosphere’s 4-beam pulsed lidar and ZX Lidars’ cw lidar with 50 focus points distributed on a cone at variable focus distances\(^1\).

In this study Windar Photonics’ 2-beam nacelle lidar will be used to investigate the effect of wakes on the turbine yaw misalignment measurements. The aim is to detect half-wake situations that lead to biased misalignment measurements and correct the measurements so that they can be used to perform turbine yaw alignment. First, we will propose an algorithm that utilizes the spectral broadening of Doppler peaks from small-scale turbulence to detect wakes in the turbine inflow (sec. 2). We will then present the results of the algorithm (sec. 4) gathered during a field experiment described in sec. 3. Finally, an empirical relationship between the spectral width measurements and the difference in wind direction between lidar and a mast-mounted sonic anemometer is used to correct the erroneous misalignment measurements.

\(^1\)More information can be found on the company websites: Windar Photonics A/S, Leosphere and ZX Lidars.
2 Methodology

2.1 Measurement principles of nacelle lidars

The fluctuating part of a 3-dimensional (3D) wind field can be described by the vector field \( \mathbf{u}(x, t) = (u_1, u_2, u_3) \), where \( x \) and \( t \) refer to a position in space and time, respectively. Lidar systems can sense wind speed by measuring the Doppler frequency shift of the backscattered laser light from aerosols suspended in the atmosphere. Cw systems emit light focused on a point in space continuously, while pulsed systems transmit parcels of light and measure the backscatter as the light progresses in space. In this study we will focus on cw lidars.

The measurement process of a cw lidar can be represented mathematically as the convolution between the projected LOS component \( \mathbf{n} \cdot \mathbf{u} \) and a weighting function \( \varphi(s) \) based on the laser intensity:

\[
v_{\text{LOS}}(r) = \int_{-\infty}^{+\infty} \varphi(s) \mathbf{n} \cdot \mathbf{u}(sn + r) ds, \tag{1}
\]

where \( \mathbf{n} \) is the laser beam unit directional vector, \( r \) is the focus position and \( s \) is distance from the focus position along the laser line (Mann et al., 2010). The weighting function of a cw lidar can be approximated by (Sonnenschein and Horrigan, 1971)

\[
\varphi(s) = \frac{1}{\pi z_R^2 + s^2}, \tag{2}
\]

where \( z_R \) is the Rayleigh length that characterizes the probe volume of the lidar. It depends on the focus distance squared, \( |r|^2 \), and can vary between a few centimeters to tens of meters. The probe volume has an attenuating effect on the turbulent fluctuations in \( \mathbf{u} \) and variances (and thus turbulence intensities) will be underestimated. The attenuated turbulent fluctuations will cause a broadening of the Doppler spectrum. The Doppler spectrum \( S(v, r) \) as function of LOS component wind speed can be defined by

\[
S(v, r) = \int_{-\infty}^{\infty} \varphi(s) \delta(v - \mathbf{n} \cdot \mathbf{u}(sn + r)) ds, \tag{3}
\]

where \( \delta(.) \) is the Dirac delta function. The delta function implies that the Doppler spectrum is an integration (or summation) of the values of \( \varphi(s) \), where \( \mathbf{n} \cdot \mathbf{u}(sn + r) = v \). The LOS speed \( v_{\text{LOS}} \), as defined by eq. 1, is the first statistical moment of \( S(v, r) \):

\[
v_{\text{LOS}}(r) = \int_{-\infty}^{\infty} v S(v, r) dv, \tag{4}
\]

where \( \int_{-\infty}^{\infty} S(v, r) dv = 1 \) is assumed. The second central statistical moment of \( S(v, r) \) is the variance and a measure of the width of \( S(v, r) \):

\[
\sigma_{\text{LOS}}^2(r) = \int_{-\infty}^{\infty} (v - v_{\text{LOS}}(r))^2 S(v, r) dv. \tag{5}
\]
Based on the inflow assumption mentioned in sec. 1, the LOS speed measurements at the two positions of a 2-beam lidar can be used to derive the yaw misalignment of the turbine (see also fig. 1). The following set of equations are used, where the hat notation (\(\hat{\cdot}\)) is used to indicate estimations from the lidar:

\[
\begin{align*}
v_{\text{LOS},1} &= U \cos \alpha - V \sin \alpha \\
v_{\text{LOS},2} &= U \cos \alpha + V \sin \alpha \\
\hat{U} &= \frac{v_{\text{LOS},2} + v_{\text{LOS},1}}{2 \cos \alpha} \\
\hat{V} &= \frac{v_{\text{LOS},2} - v_{\text{LOS},1}}{2 \sin \alpha} \\
\hat{U}_h &= \sqrt{\hat{U}^2 + \hat{V}^2} \\
\hat{\phi} &= \tan^{-1} \left( \frac{\hat{V}}{\hat{U}} \right)
\end{align*}
\]

where \(U\) and \(V\) are longitudinal and lateral wind speed, respectively, and \(\alpha\) is the angle between the shaft axis and the laser beam. In the case of the 2-beam lidar by Windar Photonics, \(\alpha = 30^\circ\). Yaw misalignment is defined as the angle the turbine needs to yaw in the clockwise direction (seen from above) to align itself with the flow (see fig. 1).

An impinging wake in the field of view of one of the beams will lead to a bias in the lidar estimated quantities (eqs. 8 to 11). Here, we will analyze the case, where LOS 1 is affected by a reduction in the streamwise wind speed component due to an upstream wake deficit \(u_w\), i.e. \(v_{\text{LOS},1} = (U - u_w) \cos \alpha - V \sin \alpha\). The influence of \(u_w\) on \(\hat{\phi}\) and \(\hat{U}_h\) for different turbine misalignments \(\phi\) is presented in fig. 2.

![Figure 2](image.png)

**Figure 2.** Influence of a streamwise flow speed reduction due to a wake \(u_w\) at LOS 1 on the lidar estimates of turbine yaw misalignment \(\hat{\phi}\) and horizontal wind speed \(\hat{U}_h\). The differently colored lines represent different turbine misalignments \(\phi\).

It can be seen that an increasing wake deficit imposes a positive bias onto \(\hat{\phi}\) since \(v_{\text{LOS},1}\) is reduced. The effect is strongest for negative turbine misalignments, where the flow is aligned more towards LOS 1. The kinks that appear for the negative turbine misalignments occur once the wake deficit reduces the LOS 1 component to negative values. However, since the lidar uses a homodyne detection method only the magnitude, not the sign, of LOS 1 component can be sensed and the measured
misalignment ranges between $-60^\circ$ and $60^\circ$. The effect of $u_w$ on $\hat{\bar{U}}_h$ leads to an over- and underestimation of the horizontal wind speed. Again, the negative turbine misalignments are effected more severely than positive misalignments.

2.2 Wake Detection Algorithm

The wake detection algorithm is based on the following two principles:

1. Within wind turbine wakes turbulence with a smaller length scale than the ambient flow is generated.

2. The small-scale turbulence will lead to a broadening of the Doppler spectrum due to the large probe volume of the lidar system.

The generation of small-scale turbulence has been investigated through simulations, scaled wind tunnel experiments and field tests. The primary aim was to validate mean wake deficit profiles, but some studies also investigated turbulence profiles and applied spectral analysis.

For example, in Troldborg et al. (2007) large eddy flow simulations were conducted with an actuator line model representing the wind turbine. Enhanced levels of small-scale turbulence inside the wake were found. It was also found that the turbulence inside a wake is more isotropic than the ambient flow. Similar results were found in scaled wind tunnel tests. Singh et al. (2014) showed that wind turbines extract energy from mean and large-scale structures while increasing small-scale turbulence. In the near-wake a clear influence of the rotor is visible (e.g. spikes at tower and blade frequencies in the turbulence spectra), but in the far-wake these effects merged into a range of amplified small-scale turbulence (Chamorro et al., 2012).

Field experiments using mast anemometry also compared power spectra between wake and ambient flow. In Højstrup (1999) input of energy to high frequencies was observed, which was detectable up to 14.5 rotor diameters behind the turbine. Similar result were obtained in Högström et al. (1988) where the high frequency components of the streamwise component increased fourfold in the wake. Iungo et al. (2013) analyzed wake observations gathered with a pulsed lidar system and also found increased turbulence, but more turbulence was created at the top of the rotor compared to the bottom part.

The effect spectral broadening of the Doppler spectrum due to turbulence and Doppler spectrum width measurements has been considered in several studies, for an overview see Sathe and Mann (2013). For example, Smalikho (1995) described the broadening process theoretically and proposed a method to measure the dissipation rate of turbulent kinetic energy. Branlard et al. (2013) showed that long time averages of the Doppler spectrum (in this case 10 and 30 minutes) approaches the probability density function of a sonic anemometer and can be used to improve variance measurements. The same method to derive unfiltered variance was used for vertical profiling lidars (Mann et al., 2010) and nacelle lidars (Peña et al., 2017).

From both properties mentioned above, we will define the LOS-equivalent turbulence intensity, $T_{ILOS}$, to characterize the small-scale turbulence contained in the probe volume of the lidar:

$$T_{ILOS}(r) = \frac{\sigma_{LOS}(r)}{\bar{v}_{LOS}(r)}$$  \hspace{1cm} (12)

and the difference in $T_{ILOS}$ between LOS 1 and LOS 2:

$$\Delta T_{ILOS} = T_{ILOS}(r_{LOS1}) - T_{ILOS}(r_{LOS2}),$$  \hspace{1cm} (13)
where the $r_{LOS1}$ and $r_{LOS2}$ refer to the focus positions of beam 1 and 2. The turbulence intensities and their differences will be calculated from one minute average spectra. If a beam is exposed to a wake inflow we expect an increased $TI_{LOS}$ compared to a beam exposed to the ambient flow. Considering turbulence intensities allows to compare different wind speed regimes.

The wake detection algorithm is designed to treat the one minute spectra consecutively and will compare the values of the detection parameters $TI_{LOS}(r_{LOS,1})$, $TI_{LOS}(r_{LOS,2})$, $\Delta TI_{LOS}$ and $\hat{\phi}$ to their values from ambient, wake-free conditions. Based on threshold values on the detection parameters the algorithm will then decide whether one or both beams are affected by a wake. For example, if beam 1 is affected by a wake, $v_{LOS,1}$ will be reduced compared to $v_{LOS,2}$ even though the misalignment of the turbine will not change. Simultaneously, the turbulence intensity on beam 1 will increase due to the small-scale turbulence within the probe volume and a detection signal will be visible on $\Delta TI_{LOS}$. The influence of the wake can also be seen on the yaw misalignment estimation $\hat{\phi}$ which will deviate from its mean value in ambient flow. A reversed situation appears when a wake is affecting beam 2. At the initialization the algorithm requires some observations to establish correct values of the running averages. From our experience a few hours of wake-free data is sufficient.

3 Experimental setup

In this study, two field experiments executed at DTU’s test site at the Risø Campus were analyzed. The site consists of a row of turbines, of which two have been operative during the experiment: a Vestas V52 turbine with 850 kW rated power, a hub height of 44 m and a diameter of 50 m and a smaller Nordtank turbine with 500 kW rated power, a hub height of 36 m and a diameter of 41 m. Further, data from a meteorological mast at a distance of $120 \text{ m} = 2.3D_V$ from the Vestas V52 turbine, where $D_V = 52 \text{ m}$. The distance between the Vestas and the Nordtank turbine is $215 \text{ m}$ ($5.2D_N$ or $4.1D_V$) at an angle of 195° (clockwise from north), where $D_N = 41 \text{ m}$. A picture of the site can be found in fig. 3.

The Vestas V52 turbine was equipped with a 2-beam nacelle lidar by Windar Photonics during two distinct periods. Initially, one system was mounted between 5th December 2015 and 12th January 2016 and a second system was mounted between 29th March 2016 and 4th May 2016. The systems have identical properties. The opening angle between shaft axis and beam was 30° and the focus distance was 37 m (implying that $z_R = 2.1 \text{ m}$). The objective of the experiment was to obtain yaw misalignment measurements from the lidars with special focus on the influence of the wake from the Nordtank turbine onto the Vestas V52. The data acquisition system of the turbine was logging turbine, mast and lidar data at a sampling rate of 35 Hz. For the analysis the data has been downsampled to 1 Hz.

The wind rose during the two experimental periods derived from a sonic anemometer mounted at the Vestas V52 hub height on the meteorological mast is shown in fig. 3. The direction of wakes from Nordtank onto V52 is indicated as a dashed black line. Two dominant direction can be identified; the waked wind direction only represents a small share of the data.
4 Results

In this study data is compared during normal power production of the turbines. To achieve this, lower thresholds on the minimum power production (i.e. > 0 kW), minimum rotor speed (i.e. > 16 rpm) and maximum pitch angle (i.e. < 23°) in each 10 minute period were applied for the Vestas V52 turbine. Since the Nordtank turbine is stall regulated a lower threshold on minimum power production (i.e. > 0 kW) and minimum rotor speed (i.e. > 27 rpm) have been applied. The filtering removed 53.3% of the data for both experiments. Additionally a filter was applied on the lidar data. Instances where the LOS speed could not be determined from the Doppler spectra have been summed within a 10 minute period. If more than 10% of the data (or 60 measurements) was missing, the 10 minute period was rejected. Instances where four or more consecutive unavailable measurements occurred on any of the two beams were also discarded. Whether a measurement is available or not was decided internally by the lidar system and depends on carrier-to-noise ratio and the Doppler peak shape and area. This lead to a discard of an additional 1.8% of the data.

Next, the influence on yaw misalignment estimates of a wake emitted by the Nordtank turbine onto the Vestas V52 is investigated. The derived 10 minute mean misalignment $\tilde{\phi}$ as function of yaw angle of the turbine can be seen in fig. 4.

The vertical black line indicates the position of the Nordtank turbine. It can be seen that outside the wake sector the misalignment ranges around a mean misalignment of 5°. Inside the wake sector large deviations from the mean misalignment can be observed. In situations where the right half of the rotor (at yaw position < 195°) LOS 2 will be affected by a reduced wind speed. Thus, a negative influence on the misalignment estimates by the lidar can be observed. Deviations of up to 30° can be
Figure 4. Lidar estimated 10 minute mean yaw misalignment $\hat{\phi}$ as function of turbine yaw position. The vertical black line indicates the position of the upstream Nordtank turbine. A clear influence of the wake onto the misalignment estimates around the wake sector can be seen. Similar behaviour can be recognized when the wake affects the left part of the rotor (at yaw position $> 195^\circ$). Here, the deviation is smaller, but a clear influence is still visible. An example of the two spectra are shown in fig. 5. The red spectrum was measured in the free flow and shows a higher wind speed and a slender peak compared to the blue spectrum which is affected by a wake. The reduced wind speed and increased turbulence inside the wake leads to a lower LOS speed and a larger width of the Doppler spectrum compared to the red spectrum.

Figure 5. Example of two lidar Doppler spectra. The red spectrum is measuring in the free flow, while the blue spectrum is affected by a wake. For the blue spectrum, the reduced wind speed and increased spectral width can clearly be seen. The spectra are averaged over one minute.

A time series result of the detection parameters over three days can be found in fig. 6. The top two panels show $T_{LOS}$ and $\Delta T_{LOS}$, while the bottom panel shows the yaw position of the two turbines. The horizontal black line and the gray shading indicate the wake sector. In general, it can be seen that if both turbines are positioned outside the wake sector, $T_{LOS}$ on both
beams is approximately equal, i.e. $\Delta T_{ILOS} \approx 0$. Thus, no significant difference of the turbulence levels within the lidar probe volumes is detected. As soon as both turbines yaw into a wake situation deviations of $T_{ILOS}$ between beam 1 and 2 can be detected. For example, on 23 December the wind direction changes such that the Vestas V52 is in a half wake situations, where the left half of the rotor is affected based on their yaw positions. In this case a spike in $T_{ILOS1}$ can be identified. This corresponds to increased small-scale turbulence originating from the wake flow within the probe volume of beam 1 which is not present on beam 2. Hence, a positive increase in $\Delta T_{ILOS}$ can be observed. Similarly, the half-wake situations on the right half of the rotor on 24 December lead to higher $T_{ILOS2}$ than $T_{ILOS1}$.

![Figure 6](image.png)

**Figure 6.** Example time series of $T_{ILOS}$ (top), $\Delta T_{ILOS}$ (center) and the yaw positions of the two turbines (bottom). The black horizontal line and the gray shading in the bottom plot indicate the wake position (i.e. $195^\circ$) and a band of $195^\circ \pm 20^\circ$, respectively.

An overview of the measured $T_{ILOS}$ and $\Delta T_{ILOS}$ against the yaw position of the Vestas V52 turbine can be seen in fig. 7. Here, parallel observations to the previous figure can be made. If the turbine is yawing at angles corresponding to half-wake conditions on the right half of the rotor (i.e. $< 195^\circ$) high values for $T_{ILOS2}$ and negative $\Delta T_{ILOS}$ can be observed. This case reverses for half-wake situations on the left half of the rotor where high $T_{ILOS2}$ and positive $\Delta T_{ILOS}$ can be noticed. For the full-wake case increased $T_{ILOS}$ on both beams appears and $\Delta T_{ILOS}$ shows values that are comparable to non-wake, ambient flow conditions. It can also be seen that for non-wake cases $\Delta T_{ILOS}$ is slightly positive. This is due to the mean misalignment which reduces the LOS speed on beam 1, see left illustration in fig. 1. Thus, the ratio $T_{ILOS1}$ is slightly increased compared to $T_{ILOS2}$.
Figure 7. Lidar estimated 10 minute mean yaw misalignment \( \hat{\phi} \) as function of turbine yaw position. Top: \( \text{TI}_{\text{LOS1}} \), center: \( \text{TI}_{\text{LOS3}} \) and bottom: \( \Delta \text{TI}_{\text{LOS}} \). The vertical black line indicates the position of the upstream Nordtank turbine (at 195\(^\circ\)).

The next figure presents the results of the detection algorithm, see fig. 8. Again, the misalignment derived from the lidar measurements is plotted against the yaw position of the Vestas turbine. Periods where a wake is detected in the inflow are shown as colored dots and distinguished as left half-wakes (beam 1 is affected by a wake), right half-wakes (beam 2 is affected) and full-wakes where both beams measure a wake influence. It can be seen that all detection results are clustered around the position of the upstream Nordtank turbine where a wake is expected. Also, all negative deviations from the mean misalignment are identified as right half-wakes and all positive deviations as left half-wakes. The recognized full-wakes lie very close to the vertical black line and show no significant discrepancy from the mean misalignment. However, only three full-wake situations were identified. This can be explained by the different rotor sizes and the position of the focus points of the lidar, which are located at towards the edges of the Vestas V52 rotor.

Next, the detected results will be analyzed in detail and the misalignment will be corrected with the help of the undisturbed wind direction measurement from a sonic anemometer mounted on the close-by meteorological mast at the hub height of the Vestas V52 turbine, see fig. 3. First, the correlation between the wind direction estimation from the lidar and the measurement from the sonic will be compared. The absolute wind direction can be obtained by adding the lidar misalignment measurement to the yaw position of the turbine. Both the turbine and the sonic anemometer have been carefully aligned to north. The resulting correlation can be seen in fig. 9.
Figure 8. Lidar estimated 10 minute mean yaw misalignment $\hat{\phi}$ as function of turbine yaw position. The vertical black line indicates the position of the upstream Nordtank turbine. Detected waked situations are shown as colored dots.

Figure 9. 10 minute mean wind direction correlation between the meteorological mast sonic anemometer and the wind direction estimate from the lidar, i.e. lidar misalignment + yaw position of the turbine. Outside the wake sector a high degree of similarity can be observed. The identified wake situations lead to a bias. The line fit is performed on wake-free data only.

It should be noted that outside the wake sector the two signals show a high degree of similarity. The fitted line, which has been fit using the wake-free observations only, shows a unity slope with a very small offset of $0.5^\circ$. The root mean squared error is $2.53^\circ$. However, the measurements which are affected by a wake show the characteristic deviations that could also be seen in fig. 4. This suggest that the flow assumption that are used to derive the lidar misalignment are valid if no wakes are impinging the field of view, but that a correction is necessary for wake situations. If the correlation, which is observed for non-wake observations, also holds for wake cases, the difference between the wind direction from sonic anemometer and lidar can only originate from the influence of the wake.
Figure 10. Difference between absolute wind direction estimated from the lidar (i.e. lidar misalignment + turbine yaw position) and sonic anemometer versus the detection parameter $\Delta TI_{LOS}$. Only the observations where a wake has been identified are shown. The line fit indicates that the data fits well to a linear regression model.

Thus, we suggest to use the wind direction measurements from the sonic anemometer to obtain an empirical relationship between the detection parameter $\Delta TI_{LOS}$ and the bias in the wind direction measurements from the lidar and the sonic anemometer. This relationship is shown in fig. 10. It can be observed that detected half wake situations on the right side of the rotor show persistent negative $\Delta TI_{LOS}$ and vice versa. The identified full-wakes both have a wind direction error and $\Delta TI_{LOS}$ close to zero.

A linear line has been fitted to the data which shows that the data follows approximately a linear relationship. The derived equation is: $1.39\Delta TI_{LOS} + 0.13^\circ$. This equation can now be used to correct the misalignment measurements by the lidar that are affected by a wake. Wake-free observation remain unchanged. The result of the correction can be seen in fig. 11 and by comparing this figure with fig. 8 the performance of the correction method can be evaluated. After the correction it is seen that the misalignment estimates, which are affected by wakes and previously showed large deviations from the mean misalignment, are now within the range of the wake-free observations. It was possible to adjust both the positive and negative spikes stemming from half-wakes on the left and right side of the rotor. The full-wake situation are only affected slightly because their difference in $TI_{LOS}$ is small.

Finally, an overview of the misalignment measurement is compared to the misalignment measured by the sonic anemometer at the meteorological mast. The misalignment versus wind speed is shown in fig. 12. It can be seen that both the lidar and sonic measurements follow a similar trend. At low wind speeds the misalignment is centered around $5^\circ$ and as the wind speed increases the misalignment reduces. The influence of wakes on the lidar measurements has been identified and corrected as previously described. Slightly lower wind speed measurements from the sonic anemometer can be observed which are caused by the wake of the Vestas V52 turbine on the meteorological mast (at yaw angles of 289°).
Figure 11. Lidar estimated 10 minute mean yaw misalignment $\hat{\phi}$ as function of turbine yaw position. The vertical black line indicates the position of the upstream Nordtank turbine. Detected wake situations have been corrected according to the relationship found in fig. 9.

Figure 12. Comparison between the 10 minute mean yaw misalignment estimated from the lidar and the sonic on the meteorological mast. The observations affected by wakes are shown in blue and the corrected measurements in red.

5 Conclusions

In this study we investigate how nacelle-mounted cw lidar systems can be used to estimate wind turbine misalignment even in inflow with a wake. Lidars offer the possibility to remotely sense the inflow of turbines and avoid the flow disturbance caused by the blades and the nacelle usually encountered by nacelle wind vanes or sonic anemometers. Correct alignment of turbines is important because power production losses can be mitigated.

Nevertheless, due to the limitation of measuring only the LOS component of the approaching wind, a lidar system needs to measure at several positions in front of the wind turbine and assumptions about the flow field need to be made to derive quantities of interest. A common assumption is that of horizontal homogeneity of the inflow which states that at positions of...
the same height the wind vector is equal. This assumption is usually satisfied if the terrain is flat and homogeneous and if appropriate averages over time, commonly one to 10 minute averages, are taken. It can, however, be violated if a wake from a neighboring turbine is impinging the inflow and the reduced wind speed inside a wake leads to a bias on the horizontal wind speed and wind direction.

Here, this influence is evaluated experimentally from two measurement campaigns, where a 2-beam cw nacelle lidar was mounted on a Vestas V52 turbine which is exposed to a wake from a slightly smaller neighboring turbine. It was shown that within the wake sector the influence of wakes induces biases on the the misalignment measurement as large as $30^\circ$ from the mean misalignment outside the wake sector. Half-wake situations on the right side of the rotor lead to negative deviations, while half-wakes on the left side result in positive bias. This implies that if lidars were to be used for turbine yaw alignment the observations that are affected by wake interaction must be identified and corrected.

The wake detection algorithm presented here is based on the spectral broadening effect of the lidar Doppler spectrum because of small-scale turbulence within the probe volume. Since lidar systems perform measurements over a rather large measurement volume, high frequent turbulent fluctuations are attenuated and are not visible in the LOS speed signal, but lead to a widening of the Doppler spectrum. The small-scale turbulence generated inside wake flows will lead to more broadening than in the ambient flow. Thus, by comparing the spectral width of the Doppler spectrum at different focus locations, wakes that affect the field of view of one or both beams can be detected. The detection parameter used in this study is the LOS-equivalent turbulence intensity $T_{LOS} = \sigma_{LOS}^2 v_{LOS}$.

The performance of the algorithm is presented in fig. 8 and shows that all lidar observations that measure the wind direction wrongly were identified. Only very few full-wake situations were observed due to the size difference of the turbines. To correct the wind direction measurements affected by wakes an empirical relationship between lidar turbulence measurements and the deviation of the lidar wind direction from the true direction measured by a sonic on a nearby mast was established. It was shown that the absolute wind direction measurements from both sensors show a high degree of correlation for non-wake cases. A linear relationship between the difference in $T_{LOS}$ between the two beam and the difference in wind direction between the lidar and the sonic was found. Applying this relationship to the measurements affected by wakes yielded a correction of the large misalignment deviations experienced during half-wake situations.

Thus, we have shown how the detrimental effect of wake on nacelle lidar measurements can be mitigated. For the correction it was however necessary to utilize the undisturbed wind direction measurements from a mast-mounted sonic anemometer. The turbines that have been used in this experiment are smaller than current utility-scale turbines. Hence, it was possible to use short lidar focus distances which result in a narrow probe volume and the spectral widening is due to turbulence of very small length scale. In this experiment average spectra of one minute were sufficient to detect the wake-induced turbulence. If the lidar is installed on larger turbine, where larger focus distances are required, the probe volume will increase and shorter averaging times might be necessary to only detect turbulence generated within the wake. Shorter averaging times lead to increased signal noise and can have an effect on the estimation of the Doppler spectrum variance.
Author contributions. Dominique P Held performed the research work and prepared the manuscript. Jakob Mann conceived the research plan, supervised the research work and the manuscript preparation.

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In this work the usability of nacelle-mounted forward-looking continuous-wave lidar systems for wind turbine inflow measurements was investigated. Two lidar systems have been considered: a 2-beam horizontally sensing system and a 4-beam system measuring at two different heights. Both systems have been installed on a test turbine at DTU’s test site at Risø. Measurements from the lidars were compared against meteorological mast anemometry and the turbine itself. Additionally, the 2-beam lidar was setup on a 10 m high tower and two sonic anemometers have been installed at the focus points. A total of seven months of field data was acquired.

The experiment of the 2-beam lidar performed close to the ground with two sonic anemometers at the focus points produced a very close comparison between lidar and anemometer. It was possible to analyze the LOS component directly and the performance of different frequency estimators of the Doppler spectrum (maximum, median and centroid method) was assessed, see Chapter 2. The median method achieved the best performance and was able to reduce the probe volume averaging compared to the centroid method slightly and also had the overall best root mean squared error against the sonic measurements.

Derived inflow characteristics were compared to the meteorological mast for the nacelle lidars installed on the test turbine. Due to the limitation of measuring only one component of the wind vector by the lidar, multiple scan locations and inflow assumptions are necessary to derive wind characteristics. Here, mean wind speeds, wind speed standard deviation, yaw misalignment, vertical shear and gusts were considered; results can be found in Section 1.2.3. In general, a high degree of similarity was found between the lidar and mast measurements emphasizing the validity of inflow assumptions. The only observed discrepancy existed between the 4-beam lidar wind speed estimation at the upper measurement height and sonic anemometer measurements. Further investigations are necessary to find the underlying reason.

For lidar-assisted pitch control the fundamental wind information is the rotor-effective wind speed. Lidar systems can estimate this quantity and provide a preview to the pitch controller, which can then achieve load reductions. Within this project a model has been developed to predict the coherence between turbine and lidar estimations of REWS. It was compared experimentally with the 2- and 4-beam lidar and showed improved performance against previously used models, see Chapter 3. Subsequently, the model is used to optimize the focus point positions to maximize the coherence.

Wakes in the inflow of turbines violate the horizontally homogeneous flow assumption that is used to derive wind characteristics from lidar measurements. Their impact on turbine yaw misalignment measurements are studied in Chapter 4, where large biases due to half-wake situations were identified. An algorithm was proposed to detect and correct
the erroneous measurements. It is based on the spectral broadening of the Doppler spectrum due to small-scale turbulence, which is generated within wake flows. Field experiments with 2-beam lidars show that all half- and full-wake situations could be identified and the correction yielded misalignment estimations comparable to undisturbed conditions.

During the course of this project several new ideas and future work was identified:

1. The wake detection algorithm has been tested in flat terrain, where the ambient turbulence levels are low and contribute little to the spectral broadening of the Doppler spectra. In complex terrain higher levels of ambient turbulence levels can be expected that could potentially influence the detection results. However, a confidential experiment in a forested site showed promising results, where no substantial deterioration of the detection performance was observed. Future test in complex terrain will show whether this is true for other wind farm sites as well.

2. Currently, the wake detection algorithm has been tested with 2-beam lidar systems since they are the most commonly used lidar instrument to estimate the yaw misalignment. The 4-beam lidar offers the possibility to detect wake at each quadrant of the rotor and allows for a more precise tracking of the wake. As a first step, the upper and lower two beams of the lidar can be considered as 2-beam lidars and the same algorithm can be applied. Additional advantages of a 4-beam system should be investigated.

3. The developed model of the coherence of the REWS can be used to optimized the lidar system. To begin with, in this work the focus positions have been optimized to maximize the coherence. However, turbine and controller design have not been considered in the optimization process. An example of how they can be intergrated is given in Schlipf et al. [68]. Servo-aeroelastic simulation can be used to access the controller performance when utilizing the lidar preview information. Further, the coherence model can be extended to represent more realistic flows, e.g. by including turbulence evolution [69] or stability effects [70].

4. Finally, potential application of the wake detection results to wake steering can be investigated. In Raach, Schlipf, and Cheng [66] a closed-loop controller for backward-looking nacelle lidars to track the wake behind each turbine was presented. This work could be extended to rather use forward-looking lidars and measure incoming wakes instead.
APPENDIX A

Performance of the Wake Detection Algorithm at the Tjæreborg wind farm
Wake detection in the turbine inflow using nacelle lidars

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Abstract. In this study we present the ability to detect wakes in the inflow of turbines using
nacelle-mounted continuous-wave lidar systems. Wake flows generate small-scale turbulence,
which has significantly smaller length-scales than ambient turbulence. Due to the lidars large
probe volume this turbulence is attenuated and will not be visible in the lidar’s measurements.
One approach to retrieve information about small-scale turbulence is by measuring the lidar
Doppler spectrum width. Here we present an wake detection algorithm based on these
measurements at two distinct locations in front of the turbine. By comparing the line-of-sight
turbulence intensity and considering the instantaneous misalignment it is possible to detect half-
and full-wakes. This has been tested during a 4.5 month long experiment and results show that
situations where the wake affects the lidar measurements can be removed.

1. Introduction
Coherent Doppler wind lidars are a novel anemometry devices, which offer the possibility of
remotely sensing wind speeds. These devices are especially attractive for wind energy
applications where wind measurements within the first few 100 m of the boundary layer are
required. Their spatial and temporal resolution allows measurements and flow characterization
around utility-scale wind turbines [1, 2, 3]. Investigations of lidars mounted on top of the
nacelle [4] or inside the spinner [5] to retrieve information about the turbine inflow have also
been conducted. This led to a variety of proposed applications spanning from lidar-assisted yaw

One of the problems associated with nacelle-mounted lidar systems is the limitation of
only measuring the line-of-sight (LOS) component of the wind velocity. To obtain inflow
information useful to control applications (like rotor-effective wind speed, yaw misalignment or
vertical shear) reconstruction methods with appropriate wind models are necessary [10]. Usually
the assumption of horizontal homogeneity is made. While being reasonable well satisfied in
homogeneous, flat terrain, this assumption is heavily violated in complex terrain or when wakes
impinge the field of view.

Thus it is necessary to detect flow situations where the assumptions of horizontal homogeneity
is violated. An example of the severe influence of wakes on the misalignment measurement by
a lidar system can be seen in fig. 1 and 2.
In fig. 1 three situations are illustrated. On the left horizontally homogeneous flow, which is aligned with the turbine, is shown. In this case the LOS components at each focus point are equal and the measured misalignment $\varphi = 0$. Once the flow is misaligned (the case of $\varphi > 0$ is shown in the center panel), the LOS component at the left focus point is reduced, while the right LOS component is increased. This leads to a measurement of $\varphi > 0$.

If the assumption of horizontal homogeneity is violated the wind vectors have different magnitudes even though the flow is aligned. A simplified situation is shown in the right panel of fig. 1, where a reduced speed is shown on the left side of the inflow. This can be caused by a wake, for example. Similar to the center panel this will cause a reduction of the LOS component on the left focus point and thus to a measured misalignment of $\varphi > 0$ despite an aligned flow.

Figure 1. *Left:* Homogeneous inflow aligned with the turbine. *Center:* Homogeneous inflow misaligned with the turbine. *Right:* Impinging wake on the left part of the wind field.

The influence of wakes on experimental misalignment measurements can be seen in fig. 2. Here the 10 minute mean values are presented. The wake position of another upstream turbine is shown as a black vertical line. Outside the wake sector an average misalignment of around $+5^\circ$ can be observed. Inside the wake however, it can be seen that the misalignment varies substantially, reaching misalignments of up to $\pm35^\circ$. This implies that it is crucial to detect waked situations, because the biased lidar measurements can lead to incorrect turbine alignment and thus energy losses.

Figure 2. Turbine misalignment versus met mast wind direction; the vertical black line indicates a wake situation.
2. Methodology
In this study we want to show the possibility of detecting wake in the inflow of turbines using nacelle-mounted continuous-wave (cw) lidar systems. The LOS wind speed can be expressed as a convolution of the LOS component of the wind vector and a weighting function $\varphi$:

$$v_r(r) = \int_{-\infty}^{\infty} \varphi(s)n \cdot u(sn + r)ds.$$  

Here $n$ is the beam unit vector, $r$ is a vector pointing to the focus position and $u$ is the 3-D wind vector field and $s$ is the distance from the focus point along the beam. For a focused, cw, coherent lidar the weighting function (at large focus distances) can sufficiently be approximated by a Lorentzian distribution [11]:

$$\varphi(s) = \frac{1}{\pi} \frac{z_R}{z_R^2 + s^2},$$

where $z_R$ is the so-called Rayleigh length. The wake detection methodology is based on two principles:

(i) The length-scale of wake-generated turbulence is of significant smaller scale than ambient turbulence. This has been demonstrated by [12], where it was shown that the wake-added turbulence can be modeled by synthetic turbulence with a much smaller length scale than ambient turbulence. The models fit experimental observations well.

(ii) Small-scale turbulence is responsible for increasing the width of a Doppler spectrum. Due to the large probe volume of a cw lidar small-scale fluctuations will be attenuated. The filtered turbulence will widen the Doppler spectrum. This has been experimentally verified by [13].

Combining these two principles, it is possible to define a wake detection parameter. Here we use the second centralized statistical moment $v_s^2$ of the Doppler peak, which is the spectrum variance, to estimate its width

$$v_s^2(t) = \frac{\int (v - v_r(t))^2 W(t, v)dv}{\int W(t, v)dv},$$

where $v_r(t)$ is the wind speed measured by the lidar (see eq. 1), $W(t, v)$ is the Doppler spectrum as a function of time and LOS speed and $v$ is an integration dummy.

To facilitate the detection the spectra standard deviation $v_s$ is normalized by the LOS speed $v_r$ to arrive at the LOS TI. By comparing these values from the two different beams, wakes in the inflow can be detected. The sign of the difference in LOS TI shows which half of the rotor is impinged by the wake. When high instantaneous LOS TI values compared to the averaged LOS TI are encountered a full wake is flagged.

An example of two different spectra, where one laser beam is measuring inside a wake, is shown in 3. Both spectra were obtained within 5 s. It can be seen that the blue spec has a much larger width indicating the presence of a significant amount of small-scale turbulence.

3. Experimental setup
A cw nacelle-mounted lidar by Windar Photonics is mounted on a Vestas V80 turbine. The device is a short-range lidar with a range of 80 m and can focus sequentially at two different focus locations with a angle of 60° in between. The measurement period spanned from beginning of November 2014 to mid of March 2015. A total of approximately 10,000 10-minute periods have been acquired. The site can be seen in fig. 4, where the turbine with the lidar systems is indicated by the marker. The surrounding turbine can also be seen.
Figure 3. Example of two spectra where the blue spectrum show an enhanced peak width due to a measurement inside a wake.

Figure 4. Google earth screen shot of the test site

4. Results
The results for the experimental campaign mentioned in the previous section can be found in fig. 5 and 6. The measurements are taken at a focus distance of 80 m. Firstly, the LOS TI as a function of yaw angle are shown. Here we have considered the wakes of the 5 closest turbines to the lidar-equipped turbine.

It can be observed from fig. 5 that the LOS TI is substantially higher at yaw angles when the turbine is pointing towards one of the surrounding turbines. Further when rotating clockwise, it can be seen that beam 2 will be affected first by the wake as this beam points to the right as seen from the turbine (compare fig. 1 again). So by comparing the difference in LOS TI can give information which side of the rotor is affected by the wake. Also, the LOS TI values for beam 2 have in general a lower value as for beam 1. The reason lies in the fact that the turbine is on average misaligned. The LOS wind speed on beam 2 is higher and thus reduces the LOS TI.


**Figure 5.** Map illustration of the LOS TI for beam 1 and 2. Solid points indicate the positions of the turbines.

Next, the performance of the wake detection algorithm is presented in fig. 6. The top panel shows the 10-minute mean misalignment of the turbine against its yaw position. The sinusoidal curves indicate the influence of wakes affecting the lidar measurements (compare to fig. 2). The surrounding turbines are shown as colored vertical lines and the sinusoidal influences of the wake are centered around them. Closer turbine tend to give more severe wake effects as the wind speed deficit has been recovered less compared to turbines further away.

The red, magenta and black scatter points show detected half- and full-wake situations, respectively. Here *Wake 1* refers to wake on the left half of the rotor (seen from the lidar) and *Wake 2* on the right half. It can be seen that the respective wake situations can be captured and a distinction between half- and full-wake is possible. There are some outliers, especially for the half-wake cases. It can probably attributed to non-wake sources of additional turbulence, like buildings or trees.

The bottom panel show the misalignment signal after all wake situations have been removed. Here it can be seen that it was able to remove the sinusoidal wake influences. Such a signal can be used for wind turbine yaw alignment.

### 5. Conclusion

Here we presented a methodology to detect wake situations in the inflow of turbines using 2-beam nacelle lidars. This is of importance since the assumptions made to derive wind field information from lidar measurements imply that a wake in one of the two beams is interpreted as a large misalignment. The Doppler peak width can be used to measure the small-scale turbulence found in wake flows. We showed results of a measurement campaign, where a wake detection algorithm has been tested. The results are promising and imply that it is possible to detect wake-affected turbine inflow. Thus, nacelle lidars can be used as turbine misalignment sensors. Future work
includes the analysis of different ambient turbulence levels. Interest lies in testing the algorithm in situations of high ambient turbulence, where the difference in peak width might not be as significant. Also we will investigate the possibility of not only detecting wake but to estimate the wake deficit simultaneously.

The learning objectives can be summarized as follows:

- Nacelle-mounted lidars can measure wind turbine misalignment, but wake situations include a bias in the measurements.
- Wake turbulence is increasing the Doppler peak width of lidar measurements.
- By comparing the lidar Doppler peaks second centralized statistical moment wake flows can be detected in the inflow of turbines.
- A detection algorithm can be used to flag inflows affected by wakes and thus allowing a nacelle lidar to be used as a turbine misalignment sensor.

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