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ASSESSMENT OF SHEAR STRENGTH OF DEEP RC BEAMS AND BEAMS WITH SHORT SHEAR SPAN WITHOUT TRANSVERSE REINFORCEMENT

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Abstract

A shear model, the so-called “crack sliding model”, based on plasticity theory is used in this study of deep reinforced concrete beams and beams with short shear span. The study re-assesses the model by introducing a new effective tensile strength and new effectiveness factors as well as an additional contribution for taking into account the effect of the enhanced failure load for beams with short shear span. The modified model is compared with experimental results. The comparison shows good agreement between model and experiments. Furthermore, the modified model has been compared with the original model to show that the suggested new formulas in general are better for predicting shear strength than the original ones.

Keywords: Plasticity theory, transverse stress, shear strength, deep reinforced concrete beams and beams with short shear span.

1. Introduction

This paper on reinforced concrete beams presents new effectiveness formulas for the shear model named “the crack sliding model” and compares the model with experimental results. The crack sliding model uses empirical effectiveness factors which are determined by calibrating with tests on straight members. The reason why the crack sliding model cannot predict the shear strength of deep beams and beams with short shear span is explained. The main reasons are: the old empirical formulas are not valid for large value of the beam depth and they didn’t take into account in a proper way the difference in the behaviour of beams with long shear span and beams with short shear span. The paper presents an improved model.

2. Shear strength of non-shear reinforced beams

The following describes the theory of non-shear reinforced concrete beams based on plasticity theory. The theory assumes over-reinforcing in shear i.e. $\Phi/\nu \geq 1/2$. Here $\Phi$ is the reinforcement degree i.e. $\Phi = A_s (bh) f_y/f_c$ and $\nu$ the effectiveness factor, $A_s$ is the longitudinal reinforcement, $h$ the depth of the rectangular section considered and $b$ the width. The yield stress of the reinforcement is denoted $f_y$ and the compressive strength of the concrete if $f_c$. Only the case of simply supported beams with one or two symmetrical point loads shear is considered. As the theory will only be described briefly, it is recommended to read e.g. (Nielsen and Hoang 2011) for further details.

2.1. The classic plasticity solution

First, a short review of the classical plasticity approach for beam shear problems is given. In the classical plasticity approach, concrete is assumed a rigid plastic modified Coulomb material i.e. a Coulomb material without tensile strength. Because concrete is not really a perfectly plastic material, a so-called effectiveness factor $\nu$ is introduced. This factor is multiplied on the concrete strength to render the
effective compressive strength \( f_c \). The effectiveness factor allows for the combined effects of softening and micro-cracking. In the classical plasticity approach, the effectiveness factor also covers any possible strength reduction due to macro-cracking. Thus, the model is not capable of handling the strength reduction due to macro-cracking in a consistent way. The classic plasticity solution is illustrated in figure 1a.

![Figure 1. Beam without shear reinforcement a) Beam with a simple statically admissible stress field, b) Beam with idealised diagonal cracks.](image)

A longitudinally reinforced beam with rectangular cross section with shear span \( a \), is shown in figure 1. If the stress field in figure 1a consisting of a simple concrete strut and a tensile bar, is considered, a lower bound solution can be obtained:

\[
P_u = x_0v f_c b
\]

where \( x_0 \) is the support length, which can be calculated by:

\[
x_0 = \frac{1}{2}\left(\sqrt{a^2 + h^2} - a\right) = \frac{1}{2}h\left(1 + \left(\frac{a}{h}\right)^2 - \frac{a}{h}\right)
\]

In the solution the depth of the compression zone is half the depth. For the solution (1) to be valid for a real beam, the reinforcement must be fully anchored and placed in a position corresponding to the solution in figure 1a and the loads must be transferred along a length greater than or equal to \( x_0 \). If these conditions are satisfied, the solution is an exact plastic solution.

### 2.2. The crack sliding model

In figure 1b, a beam with an idealised diagonal crack is shown. The basic assumption in the crack sliding model is that in many cases the sliding failure takes place in a former tensile crack which has much lower strength than a sliding failure in concrete with no macro-cracks. It is assumed that the yield line is straight with a vertical relative displacement forming an angle \( \alpha \) with respect to the straight yield line. The horizontal projection of the yield line is \( x \). Analysing the beam in figure 1b, the ultimate shear strength can be established. An upper bound solution for the shear capacity may be written as:

\[
P_u = \frac{1}{2}v f_c bh\left(\sqrt{1 + \left(\frac{x}{h}\right)^2} - \frac{x}{h}\right) \quad \text{for } \Phi \geq \frac{1}{2}v
\]

If \( x \) in the upper bound solution (3) is replaced by \( a \), the lower bound solution (1) is obtained. To get agreement with tests Jin Ping Zhang introduced an effectiveness factor \( v = v_0 \psi \), (Nielsen and Hoang 2011):

\[
v_0 = \frac{0.88}{\sqrt{f_c}}\left(1 + \frac{1}{\sqrt{h}}\right)(1 + 26\rho)
\]

Here \( h \) is in metres, \( f_c \) is in MPa, \( \rho \) is the reinforcement ratio \( A_s/(bh) \) and the so-called sliding reducing factor \( \psi = 0.5 \).
The new effectiveness factor can according to (Kragh-Poulsen, unpubl.) be taken as:

\[ \nu = \frac{5.0}{f_c^2} \left( 1 + \frac{0.35h}{16 + a_g} \right)^{-0.62} \left( 1 + 32.2\rho \right) \]  

(4)

where \( f_c \) is in MPa, \( h \) is the depth in millimetres and \( a_g \) is the maximum aggregate size also in millimetres. Equation (4) is valid in range \( 80 \text{ mm} \leq h \leq 4000 \text{ mm} \) and \( 8 \text{ mm} \leq a_g \leq 32 \text{ mm} \).

The cracking load \( P_{cr} \) may be derived from the equivalent plastic stress distribution shown in figure 2. The solution is:

\[ P_{cr} = \frac{1}{2} f_{tef} b \left( x^2 + h^2 \right) \]

(5)

where according to Zhang (Nielsen and Hoang 2011) the effective tensile strength is:

\[ f_{tef} = 0.156 f_c^2 \left( \frac{h}{0.1} \right)^{-0.3} \]  

(6)

where \( f_c \) is in MPa and \( (h/0.1)^{0.3} \) with \( h \) in metres is a size effect factor.

The new effective tensile strength \( f_{tef} \) of concrete may according to (Kragh-Poulsen, unpubl.) be taken as:

\[ f_{tef} = 0.301 f_c^2 \left( 1 + \frac{0.35h}{16 + a_g} \right)^{-0.62} \]  

(7)

Here \( f_c \) is in MPa and \( h \) and \( a_g \) in millimetres.

Figure 2. Equivalent plastic stress distribution in concrete at the formation of a crack. The influence of the reinforcement is here neglected.

Figure 3. Cracking load \( P_{cr} \) and shear capacity \( P_u \) versus the horizontal projection of a diagonal crack \( x \).

Figure 3 shows schematically the shear capacity \( P_u \) and cracking load \( P_{cr} \) versus the horizontal projection \( x \) of the diagonal crack. Figure 3 may explain the shear failure mechanism: The cracks normally start to
develop vertically in the mid-span. When the applied load $P$ increases further, the idealised cracks like the dotted lines in figure 1b may appear.

Sliding failure along these cracks cannot occur as long as the applied load $P$ is less than the sliding strength $P_u$, i.e. as long as the $P_{cr}$ curve lies below the $P_u$ curve. Sliding failure will occur when the cracking load $(P_{cr})$ curve intersects with the crack sliding capacity $(P_u)$ curve, i.e. when the shear capacity of the beam has been reached.

For calculating the ultimate shear strength, the equations (3) and (5) are set equal to each other and the horizontal projection $x$ of the critical crack can be calculated. If $x$ is larger than $a$, then $x = a$ must be inserted into (3) to calculate the ultimate shear strength.

Furthermore, sliding is only possible for $x/h \geq \tan \varphi = 0.75$, $\varphi$ being the angle of friction. This is due to the constraint imposed by the normality condition of the theory of plasticity on the angle between the relative displacement and the yield line when plane strain conditions are assumed.

2.3. Effect of the transverse stress

From figure 5.32 in (Nielsen and Hoang 2011) it appears that the plastic solution has difficulties in predicting the shear strength of a beam with $a/h \leq 2$. The difference in behaviour for long and short shear span may be illustrated by the simple strut and tie systems shown in figure 4.

In the left figure the strut and tie system of figure 1a, is re-illustrated. Here the load is mainly carried by a compression strut from force to support and a tensile bar. This model reflects the behaviour of a beam with short shear span. In the right figure is shown a beam with long shear span. Instead of the exact plastic solution in the left figure the beam prefers to carry the load as a cracked beam with a shorter shear span. Then a tensile bar to transfer, suspend, the load to the top flange is needed. In reality this force must of course be carried by tensile stresses in the concrete. Experiments seem to show the effect of that these tensile stresses on the failure load may be modelled through the effectiveness factor $s$.

Similar considerations have been advanced by (Lee et al. 2018) with reference to (Mau and Hsu 1987). In other words for long shear span the load-carrying capacity is strongly dependent on the tensile strength of the concrete while the influence of the tensile strength for short shear span is decreasing for decreasing shear span. Of course, both cases deal with micro-cracked concrete. To account for this difference in behaviour, an additional contribution to the failure load for $a/h \leq 2$ has been suggested, (Kragh-Poulsen, unpubl.):

$$\Delta \tau_u = 0.397 f'_c \left(1 + \frac{0.35 h}{16 + a_g}\right)^{-0.62} \left(2 - \frac{a}{h}\right) \quad \frac{a}{h} \leq 2$$

Units as above. Notice that the additional contribution is proportional to the effective tensile strength (7). Thereby the shear strength (3), where $x$ is replaced with $a$, should be combined with (8):
\[
\tau_u = \frac{1}{2} \nu f_c \left( \sqrt{1 + \left( \frac{a}{h} \right)^2} - \frac{a}{h} \right) + \Delta \tau_u \quad \Phi \geq \frac{1}{2} \nu
\] (9)

The shear strength according to this formula should be restricted to twice the first term in the formula. This is in agreement with the Jin-Ping Zhang version of the crack sliding theory.

### 3. Comparison with tests

#### 3.1. Requirements for tests

In each test, the following requirements have been fulfilled: The shear span depth ratio \( a/h \leq 2 \), the compressive strength \( f_c \) shall be \( 10 \text{ MPa} \leq f_c \leq 60 \text{ MPa} \), the longitudinal reinforcement is assumed sufficiently strong to prevent flexural failure \( M_{sd}/M_u < 1 \) and the beam has to be over-reinforced for shear, \( \Phi/\nu \geq 0.5 \). The moment \( M_{su} \) at which shear failure occurs is:

\[
M_{su} = P_E (a + x_0)
\] (10)

The ultimate moment capacity is taken as:

\[
M_u = A_s f_y d \left( 1 - \frac{1}{2} \Phi_m \right)
\] (11)

where \( \Phi_m = A_s/(bd)f_c/f_c \).

#### 3.2. Selected tests

Test results satisfying the above conditions in section 3.1 have been compared with (9) and (1). In (1), the effectiveness factors \( \nu = v_0 \nu_s \) and \( \nu = v_0 \) have been used. The reason for using both \( \nu = v_0 \nu_s \) and \( \nu = v_0 \) is that Zhang (Nielsen and Hoang 2011) wanted to emphasize the difference between a yield line in concrete without macro-cracking with effectiveness factor \( v_0 \) and a yield line in cracked concrete with effectiveness factor \( v_0 \nu_s \) with \( v_s \) equal to 0.5. This distinction has been avoided in the formulation of the new effectiveness factors. In figure 5 it can be seen that the modified model (9) gives much better results than the old one. The improvements are due to a strongly modified treatment of the size effect of the depth and an improved handling of beams with short shear span.

![Figure 5](image-url)

**Figure 5.** Estimated ultimate shear strength vs. experimental strength. a) Modified model (9), b) Shear strength (1) calculated with \( \nu = v_0 \nu_s \) and c) Shear strength (1) calculated with \( \nu = v_0 \).
4. Conclusions

The modified crack sliding model may be used for assessing the shear strength of deep beams and beams with short shear span. It has been shown that the modification has increased the precision of the calculated shear strength compared with the original model. The comparison has been carried out with selected experiments and shows good agreement.

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