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RESEARCH ARTICLE

Dynamic Car–Following Model Calibration Using SPSA and ISRES Algorithms

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Abstract

Calibration plays a fundamental role in successful applications of traffic simulation and Intelligent Transportation Systems. In this research, the calibration of car–following models is seen as a dynamic problem, which is solved at each individual time–step. The optimization of model parameters is fulfilled using the Simultaneous Perturbation Stochastic Approximation (SPSA) algorithm. The output of the optimization is a distribution of parameter values, capturing a wide range of various traffic conditions. The methodology is demonstrated via a case study, where the proposed framework is implemented for the dynamic calibration of the car–following model used in the TransModeler traffic simulation model and Gipps' model. This method results to model parameter distributions, which are superior to simply using point parameter values, as they are more realistic, capturing the heterogeneity of driver behavior. Flexibility is thus introduced into the calibration process and restrictions generated by conventional calibration methods are relaxed.

Keywords

dynamic calibration, SPSA algorithm, parameters optimization, car–following models

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1 Introduction

Over the past few decades Intelligent Transportation Systems (ITS) have matured and nowadays are widely applied to many different operational scenarios. Their successful application depends on the effectiveness of calibration and validation processes. ITS models should adequately represent reality; therefore researchers should choose each time the appropriate methodology for the efficient model calibration. The access to increasing volumes of potentially useful data is considered a significant advantage (Antoniou et al., 2011). Emerging data collection techniques (such as opportunistic sensors, found in most modern smartphones) provide richer data, which contribute to the overall optimization of processes (Antoniou et al., 2014b).

Most of the proposed calibration approaches are chosen for the calibration of a few selected model parameters for simplicity. However, they may not take advantage of the richness of the available data and may be restricted. Other researchers have taken into consideration all the required parameters, thus capturing all correspondences between them (Antoniou et al., 2007; Balakrishna, 2006). Building upon the concepts outlined in Antoniou et al. (2014b), distribution–based calibration is further explored in this research. Specific consideration was given to model parameters' distributions, as well as to the improvement of the conditions under which model's equations could represent a real phenomenon more accurately. Finally, the use of median values for the determination of constant/average values has been examined.

The remainder of this paper is structured as follows. Section 2 provides an overview of car following–models and more specifically of the Caliper (2012) TransModeler's traffic simulation model used in this research. The methodological framework based on calibration using distributions is outlined in Section 3, while a case study is used for demonstration in Section 4. Finally, Section 5 concludes the paper with a discussion for further work.

2 Overview of Car-Following Models

A car–following model (Brackstone and McDonald, 1999) controls the behavior of drivers in relation with the preceding vehicle in the same lane. A vehicle is limited by the movement of the vehicle in front of it, because driving at the desired speed can lead to a collision. When a vehicle is unrestricted by a preceding vehicle, it is assumed that it moves freely at the drivers desired speed. The actions of a vehicle which follows another are defined by the acceleration of the vehicle, even though in some models like that of Gipps' (1981), the actions of the vehicle following are based on its velocity. According to Olstam and Tapani (2004) car following models are divided into categories, such as Gazis-Herman-Rothery models (Gazis et al., 1961), safe distance models (Kometani and Sasaki, 1958; Gipps, 1981), psycho-physical models (Weidemann and Reiter, 1992) and fuzzy logic models (Kikuchi and Chakroborty, 1992; Al-Shihabi and Mourant, 2003). This classification depends on utilized logic. The GHR model uses a stimulus-response type of function in order to control the actual following behavior. In safety distance models, the driver of the following vehicle is assumed to always keep a safe distance to the preceding vehicle. Psycho-physical models use thresholds where the driver changes his/her behavior. The fuzzy logic car-following models describe driving behavior using linguistic terms and associated rules, instead of deterministic mathematical functions (Olstam and Tapani, 2004).

Some car following models describe the behavior of the drivers, only in the case that they are following some other vehicle, while they include all other situations. Every car following model must define the state of the vehicle as well as the actions performed in each situation. Therefore, there has been a shift from a single state models to multi–regime approaches. Multi-regime car following models adopt different rules under different traffic states, so that driving behavior can be best captured. This research focuses on capturing various traffic states by modifying model parameters dynamically using optimization algorithms. Gipps' model and TransModeler model have been selected for the implementation of the proposed method, as these models are used in two well-known traffic simulation softwares, AIMSUN and TransModeler respectively. Both models estimate driving behavior in more than one traffic state. Gipps' model includes freeway and car-following states, while in TransModeler car-following model different parameters are applied for acceleration and deceleration rates. However, the effectiveness of these models are closely related to the estimation of their parameters and are imposed to limitations from their formulas. For instance, using TransModeler model, when the lead and following vehicle are travelling at the same speed, the acceleration/deceleration response is zero, regardless the spacing between the two vehicles. Moreover, Gipps' model tends to be unable to reproduce unstable traffic phenomena (Punzo and Tripodi, 2007).

Most car–following models describe the behavior of drivers in various situations (Boer, 1999). They are considered as multi–agent and are defined by a system of differential equations, each of which captures a different state. Konishi et al. (2000) proposed a coupled map (CM) car-following model to describe the dynamical behavior of an open flow. Ge et al. (2014) proposed a control method to suppress two–lane traffic congestion. Considering high speed following on expressway or highway, an improved car–following model was also developed by Jia et al. (2014). They introduced the parameter of "variable safety headway distance". Therefore, it is understood that there are many researchers who try to reproduce the actual trafficconditions, changing existing models or creating new ones. Many researchers have attempted to calibrate both models, especially Gipps' model (Punzo and Tripodi, 2007; Rahka et al., 2007). Punzo and Tripodi (2007) calibrated Gipps′ model for various classes of vehicles. In this research, an alterative way of calibration is attempted using distributions. It is important to develop a comprehensive methodology that will allow quick and efficient calibration of model parameters. Antoniou et al. (2014a) provide some discussion on the need for guidelines for the calibration and validation of traffic simulation models. Calibration and validation have been widely demonstrated as optimization problems with various approaches, such as top-down approach (Dowling et al., 2004), or genetic algorithms (Cheu et al., 1998). The simultaneous multiple parameters calibration has proved to be helpful, therefore the Simultaneous Perturbation Stochastic Approximation (SPSA) could be a fairly promising algorithm.

2.1 TransModeler's Car–Following model

TransModeler is a powerful and versatile traffic simulation package that simulates a wide variety of facility types, including mixed urban and freeway networks. The Caliper Transmodeler software (Caliper, 2012) includes a car–following model, which is described by the following formula:

$$A_j^{\pm}[t+\Delta t] = alpha^{\pm} \frac{V_j^{beta\pm}[t]}{D_{j,j-1}^{gamma\pm}[t]} \left(V_{j-1}[t] - V_j[t]\right) + \varepsilon_j^{CF} \qquad (1)$$

where:

$A_j^{\pm}[t+\Delta t]$ = Acceleration rate of vehicle

$V_j[t]$ = Speed of the subject vehicle

$V_{j-1}[t]$ = Speed of the front vehicle

$D_{j,j-1}^{y\pm}[t]$ = Distance between the subject and front vehicles

$alpha^{\pm}$, $beta^{\pm}$, $gamma^{\pm}$ = Model parameters

ε_j^{CF} Vehicle-specific error term for the car-following regime

The superscripts $^{\pm}$ indicate that the calculated acceleration could be positive (+, acceleration) or negative (-, deceleration). If the speed of the subject vehicle $V_j[t]$ is lower than the speed of the front vehicle $V_{j-1}[t]$ the acceleration rate will be positive (i.e. subject will accelerate). Otherwise, it will be negative (i.e. subject will decelerate).

TransModeler's users' manual (Caliper, 2012) provides some initial values for the model parameters $alpha^{\pm}$, $beta^{\pm}$, $gamma^{\pm}$, which are obviously not able to represent all traffic

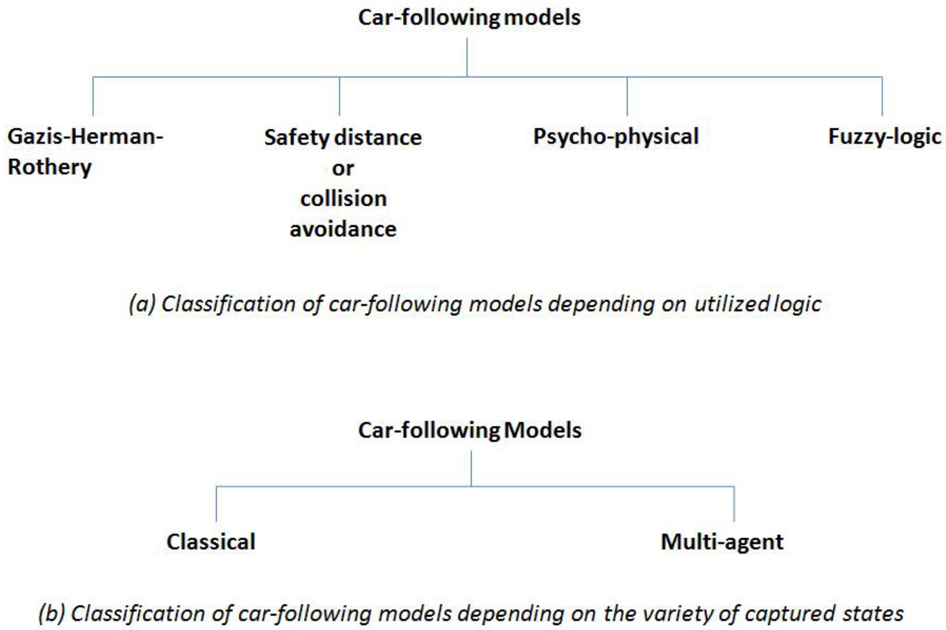


Fig. 1 Classification of car–following models

conditions and driving behaviors. Calibration is the crucial step that will estimate the appropriate values and will fit our model to the requirements of the particular research.

2.2 Gipps' model

Another car–following model that has been used in this research is a safety distance model based on the model developed by Gipps (Gipps, 1981; Olstam and Tapani, 2004; Barceló et al., 2005). The model suggests that the speed of a vehicle (n-1) is subject to three constraints (Eq. (2)). First, the vehicle speed does not exceed the driver's desired speed (V_n). Second, the vehicle accelerates rapidly until it approaches the desired speed and then the acceleration is reduced almost to zero. If two vehicles are far apart, they behave as in the free flow condition. These two conditions are summarized in the first part of Eq. (2). The third condition is taken into account, when the vehicle is constrained by the vehicle in front. It is taken for granted that the following vehicle will adjust its velocity so as to keep a safe distance from the preceding vehicle. This condition is described by the second part of Eq. (2). Overall, according to the above restrictions, the speed of vehicle n at time (t+τ) could be calculated by the following formula:

$$v_n[t+\tau] = \min \begin{Bmatrix} v_n[t] + 2.5 \cdot a_n \cdot \tau \cdot \left(1 - \frac{v_n[t]}{V_n} \cdot \sqrt{\left(0.025 + \frac{v_n[t]}{V_n}\right)}\right) \\ b_n \cdot \tau + \sqrt{(b_n \cdot \tau)^2 - b_n \cdot \left[2 \cdot \left(x_{n-1}[t] - s_{n-1} - x_n[t]\right) - v_n[t] \cdot \tau \frac{v_{n-1}^2[t]}{\hat{b}}\right]} \end{Bmatrix} \tag{2}$$

where:

a_n : the maximum acceleration that the driver of vehicle n wishes to acquire (m / s^2).

b_n : the maximum braking that the driver of vehicle n wishes to apply in order to avoid a crash, $b_n < 0$ (m / s^2).

$\hat{b}$: the estimated maximum braking that the driver of the preceding vehicle (n-1) wishes to apply (m / s^2).

$s_{n-1} = L_{n-1}$ + Safety, namely the size of the preceding vehicle (n-1) including its length and the safety distance at which vehicle n is unwilling to compromise even when at rest (m).

V_n : the speed at which the driver of vehicle n wishes to travel (m/s).

x_n (t), x_{n-1} (t) : the location of the front side of the respective vehicle (n or n-1) at time t (m)

v_{n-1} (t) : the speed of the preceding vehicle (n-1) at time t (m/s)

v_n (t) : the speed of the following vehicle (n) at time t (m/s)

τ : the apparent reaction time (a constant for all vehicles) (s)

3 Calibration Framework

Traffic simulation models are significant tools for traffic analysis, since they are an effective approach for quantifying the benefits and limitation of different alternatives. When their basic format is known, and there is sufficient data available, calibration is the procedure that estimates the parameters value that will lead to results, as close as possible to the observed ones in the field.

3.1 Overview

Car–following models include several parameters that need to be calibrated. These parameters are arguably constantly changing, therefore guidelines with various approaches, such as top-down approach (Dowling et al., 2004) and genetic algorithms (Cheu et al., 1998; Lee et al., 2001; Kim and Rilett, 2004) should be followed (Antoniou et al., 2014a). Most of the problems are highly non–linear, therefore the desired outcome is usually a result of multiple algorithms' and models' processes.

For the calibration procedure, either optimization algorithms or state-space representations could be used, depending on the nature and the requirements of the problem. Optimization algorithms can be classified into pattern search, path search, and random search techniques (Ashok, 1996). The last category includes the simultaneous perturbation methods. Their mathematical representation is the minimization (or maximization) of some scalar-valued objective function with respect to a vector of adjustable parameters (Spall, 1998).

3.2 Simultaneous Perturbation Stochastic Approximation (SPSA)

The Simultaneous Perturbation Stochastic Approximation (SPSA) algorithm (Spall, 1992; 2012) is an optimization method that has attracted considerable international attention. Its essential feature is the underlying gradient approximation that requires only two objective function measurements per iteration, regardless of the dimension of the optimization problem. Its methodology may reduce significantly the run times of large–scale problems from weeks or days to hours or minutes, compared to other applicable algorithms.

For the two–sided SP gradient approximation, the following equation is its main feature:

$$\hat{g}(\theta^i) = \frac{z(\theta^{i+}) - z(\theta^{i-})}{2c^i} \begin{bmatrix} \Delta_{i1}^{-1} \\ \Delta_{i2}^{-1} \\ \vdots \\ \Delta_{iK}^{-1} \end{bmatrix} \tag{3}$$

where each element from the mean–zero K–dimensional random perturbation vector Δ_{iK} is drawn from a probability distribution that is symmetrically distributed around zero, and satisfies the conditions that both $|\Delta_{iK}|$ and $|\Delta_{iK}^{-1}|$ are bounded above by constants. In this research, the Bernoulli ±1 with equal probability for each component of Δ_{iK} has been used. It is a simple and popular distribution that satisfies the inverse moments condition.

The loss function is evaluated at two points, by obtaining two measurements based on the simultaneous perturbation on “either side” of θ^i. These points correspond to $\theta^{i+} = \theta^i + c^i\Delta_i$ and $\theta^{i-} = \theta^i - c^i\Delta_i$. Each point is checked if it is between the lower and upper bound constraints before function evaluation. The iteration or termination of the algorithm is decided, based on the convergence criterion. Convergence is declared when θ^i and the corresponding function value z(θ^i) stabilize across several iterations.

During each iteration, two important step sizes are calculated as:

$$a^i = a/(i+1+A)^\alpha \tag{4}$$

$$c^i = c/(i+1)^\gamma \tag{5}$$

where:
A, a, c, γ, α: non–negative coefficients, whose values are selected according to the nature of the problem.

The performance of SPSA depends on the choice of these gain sequences to a considerable extent. There are great chances that the method will not converge to the optimal solution, if a wrong combination of system parameters (A, a, c, γ, α) has been assigned. In general, SPSA can easily be entrapped in a local minimum, and not depart from it, in order to approach the optimal solution.

Spall (1998) highlights some details, that someone should take into account for the efficient implementation of SPSA. Balakrishna (2006) noted, that if a too large value is chosen, then the SPSA may overlook a nearby solution and venture too far away. The parameter c should be set at a level approximately equal to the standard deviation of the measurement noise in $y(\theta)$ in order to keep the K elements of $\widehat{g_K}(\widehat{\theta_K})$ form getting excessivelylarge in magnitude. A large c may lead parameters components to their bounds really fast, thus rendering the gradient approximations invalid (Balakrshna, 2006). It is also noted, that the values of α and A can be chosen together to ensure that the algorithm will perform effectively (Spall, 1998).

Researchers have presented several modifications of the basic SPSA algorithm, in attempt to overcome some of its limitations. The first–order SPSA (1SPSA) is related to the Kiefer-Wolfowitz (K-W) stochastic approximation (SA) method (Spall, 1992), whereas the second–order SPSA (2SPSA) is a stochastic analogue of the deterministic Newton-Raphson algorithm (Spall, 2000). Lu et al. (2015) and Antoniou et al. (2015) proposed an enhanced SPSA algorithm for large–scale dynamic traffic assignment applications, called Weighted SPSA (W-SPSA), which incorporates the information of spatial and temporal correlation in a traffic network to limit the impact of noise and improve convergence and robustness. Tymbakianaki et al. (2015) proposed the c–SPSA algorithm, a modification which applies the simultaneous perturbation approximation of the gradient within a small number of carefully constructed “homogeneous” clusters one at a time, as opposed to all elements at once.

3.3 Calibration using distributions

Most of the proposed calibration approaches choose to calibrate a few selected model parameters for simplicity. However, they may not take advantage of the richness of the available data and may be restricted. In emergency situations, where the behavioral parameters of drivers present high variation (Prionisti and Antoniou, 2012), it is important to depart from point values of surveillance data and restrict the necessary assumptions by dealing with distributions (Antoniou et al., 2014b).

The distribution–based calibration approach assumes as input a set of measured distributions of the appropriate measures of effectiveness (e.g. speeds or accelerations as extracted from clustering analysis). The data need to be appropriately pre–processed, to ensure they are not susceptible to measurement or equipment error, and that they comply with the experiment requirements. The specific approach presents significant challenges, regarding the statistics that could be used.

Distributions of calibrated model parameter values have been used in some off–line calibration studies. Chiabaut et al. (2010) investigate driver heterogeneity in microscopic traffic modeling through a paired analysis of vehicle trajectories. Kim and Mahmassani (2011) and Kim et al. (2013) sampled parameters under the assumption that they follow the multivariate normal distribution.

In Antoniou et al. (2014b), distributions of measured data have been used and the optimal set of model parameters for the whole dataset has been investigated. In contrary, in this research point values of surveillance data (accelerations) have been used, but a distribution of values for each parameter has been defined. For each observation the optimal set of parameters may be different and therefore a default value for each model parameter is inappropriate. SPSA algorithm identifies the optimal combination of parameters values for each observation. Therefore, the optimal value of each parameter is defined for each observation. Taking into account SPSA iterations for all the observations, a distribution of values for each parameter is provided, capturing a wide range of traffic conditions and driving behavior patterns.

This distribution could be used as a prior distribution for the calibration, e.g. by sampling from it for initial parameter values. Therefore, in subsequent applications of the algorithm, the initial value of the parameters would not be fixed, but it would be drawn from these prior distributions. This selection could be based on e.g. statistics or on a Monte–Carlo sampling method (Robert and Casella, 1999).

4 Case Study

A car following model includes several parameters that need to be calibrated in order to determine vehicle's acceleration or deceleration rate. A function of the speed and the relative position of the preceding vehicle is model's main component. It is based on the idea that each driver controls a car under the stimuli from the preceding car, which can be expressed by the function of headway distance or the relative velocity of two successive cars.

4.1 Data

For the calibration of car–following models, data from a series of experiments were used. They were conducted in the streets surrounding the city of Naples, Italy (Punzo et al., 2005); they represent real traffic conditions (including congestion) in October 2002. All the necessary data were collected from 4 vehicles, which were moved in series under different traffic conditions. All data were collected from the same platoon, namely the same four drivers by the same vehicles (vehicles 1, 2, 3, 4) moving in the same sequence (vehicle 1 as the leader, followed by vehicles 2, 3 and 4, which was the last vehicle), but from different driving sessions. The driving routes and traffic conditions were differentiated among the datasets. The participants were aware of the route they would follow, but they did not know the aim of the experiment. The leader protected the platoon from intrusions of extraneous vehicles by allowing them to proceed. Regarding the type of road, datasets with index A and C were recorded in urban roads, while datasets with index B refer to an extraurban highway (Fig. 2). All these roads are undivided and have one lane per direction, allowing for the possibility of overtaking (through entering the opposite direction). However, it is noted that during the data-collection process, when intrusions happened, data were discarded. Therefore the driving behavior is unaffected by lane changing and overtaking. All vehicles were equipped with GPS receivers that tracked the location of each vehicle per 0.1 s. Specifically, they were equipped with dual-frequency devices GPS + GLONASS with nominal horizontal accuracy 10 mm + 1.0 ppm and elevation accuracy 15 mm + 1.0 ppm.

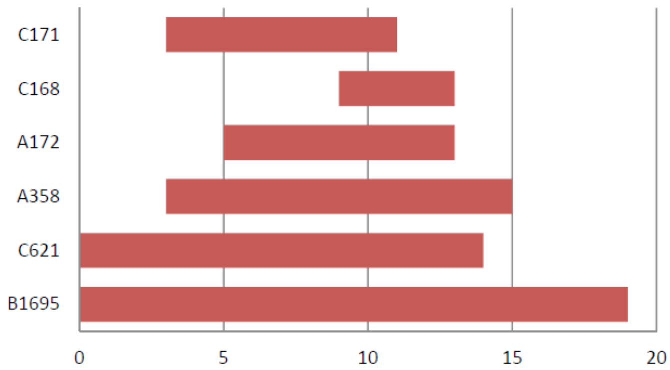


Fig. 2 Range of speeds (m/s)

Due to environmental conditions, there were gaps in the above data, i.e. for some intervals of the experiment there were no recorded measurements. However, for the purpose of this research, it was preferred to cut the whole data package in smaller pieces, in order to have files with continuous actual measurements. The usage of some linear or polynomial interpolation method for the evaluation of missing measurements was not preferred. A more detailed description of the available data could be found in Punzo et al. (2005).

The data packages include location records of each vehicle (coordinates x, y, z and time) per 0.1 s for all the vehicles. Using the above information the distances between vehicles, the distance traveled per 0.1 s for each vehicle, and their respective speeds were calculated. The size and speed ranges of each data package are shown in Table 1 and in Fig. 2.

4.2 Model calibration using SPSA

For the experiment, the file B1695 was used, because it is the most extensive data file and it includes a wider range of velocities. With this choice, the creation of a more representative model could be accomplished. The calibration process has been implemented in a Matlab interface. Following the guidelines of Spall (1998), the performance is initially analyzed using the parameters A = 20, α = 0.602, c = 1, a = 0.027 and γ = 0.101. The algorithm does not require complex and time–consuming calculations, so it is chosen to implement a total of 1000 iterations, in order to illustrate SPSA's behavior.

Table 1 Characteristics of data packages

a/a	No. Observations	Duration of measurements (s)
B1695	1695	169.4
C621	621	62.0
A358	358	35.7
A172	172	17.1
C168	168	16.7
C171	171	17.0

The root mean square normalized error (RMSN) was used to assess the performance of SPSA in replicating the initial correct data:

$$RMSN = \frac{\sqrt{S\sum_{i=1}^{S}\left(y_i - y_i\right)^2}}{\sum_{i=1}^{S} y_i} \tag{6}$$

where y_i is the i^{th} observed measurement (in this case, the "true" parameter value), and y_i the corresponding simulated (in this case estimated) quantity.

As is known, there is no picture of the correct parameter values which will ultimately yield the correct acceleration results for the particular place and time. As a result, the car following model parameters **alpha, beta, gamma** took as initial values the basic ones from the TransModeler traffic simulator (see Fig. 3).

After several executions of SPSA, it was found that the SPSA algorithm is not able to find the optimal car–following model parameters for this particular set of measurements. The RMSN values increase exponentially. Probably, this is due to the fact that the values of the vector θ are quite small, and the a^i and c^i coefficients of the algorithm influence more than necessary the requested vector. Other measures of goodness–of–fit might be able to overcome this limitation, and this is being explored.

The experiment was repeated, initializing the SPSA with smaller values. The basic parameters A, α, c, a and γ were divided by 100 in order to adjust their order of magnitude with the data of this application. The new results were again not satisfactory. The RMSN was still within an unacceptable range of values (greater than 1).

From several executions of the SPSA algorithm, it was also noticed that the results could be completely different from time to time. This phenomenon occurs due to the stochasticity of the SPSA algorithm.

After several efforts to improve the existing results, it was decided to limit the vector parameters of θ from three to one. The new θ will include only the TransModeler parameter **alpha**, which will once again be set at the default value from the TransModeler simulator. The parameter **alpha** was chosen, because it is not an exponent, and it could be managed more easily at the first steps of car-following model calibration.

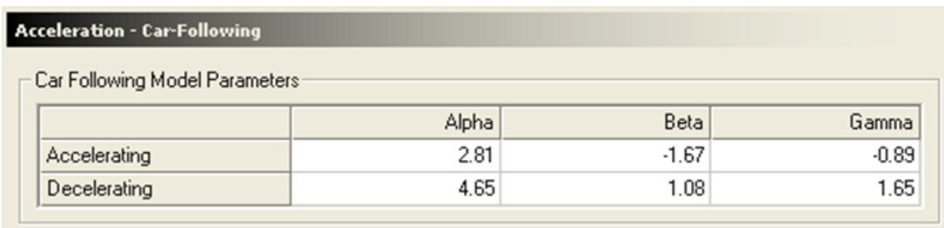
Additionally, for the full understanding of SPSA's final results, the **alpha** values that fit the available observations were estimated (single stopping criterion for SPSA was $RMSN = 0$). In Fig. 4, the histogram of values' range is presented. Each included value represents the optimum value of parameter **alpha** for a particular recording of the data package. It is observed, that distributions of the alpha values are similar both for the acceleration and deceleration case.

It is obvious that SPSA would never be able to converge to a certain value, as the histogram displays a wide range of fitted values per observation. Therefore, it was chosen to implement SPSA per observation. The RMSN value and the maximum number of iterations were the main criteria for the process termination (see Fig. 5).

The algorithm successfully managed to reach the optimal solution – the fitted value of **alpha**. Fig. 6 shows how many sets of 50 iterations SPSA needs to terminate. The larger percentage of observations requires only one set. SPSA algorithm managed to calibrate quickly and efficiently most records.

In order to gain some further insight, the parameters **beta** and **gamma** were also examined. Two different implementations of SPSA were applied, using TransModeler's parameter **beta** and **gamma** correspondingly as a single parameter in vector θ. The final calibration results are shown in Fig. 7 and Fig. 8. Both parameters show different behavior in acceleration and deceleration scenarios. It becomes apparent that **beta** and gamma do not converge to a certain value for the whole set of measurements. Therefore, the presentation of results in distributions is considered important.

The overall picture shows that the algorithm behaves correctly. The parameters seem to affect significantly drivers'


Acceleration - Car-Following

Car Following Model Parameters

	Alpha	Beta	Gamma
Accelerating	2.81	-1.67	-0.89
Decelerating	4.65	1.08	1.65

Fig. 3 Initial values of car following model parameters

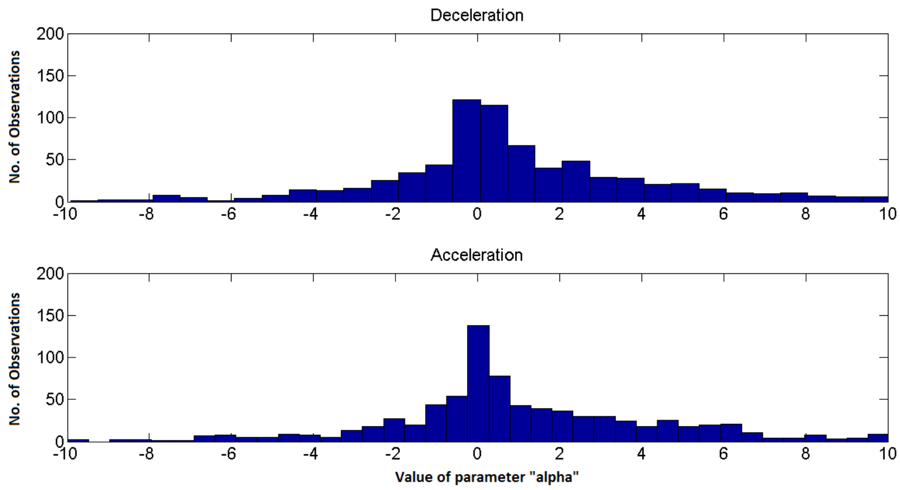


Fig. 4 Fitted values of car following model parameter “alpha”

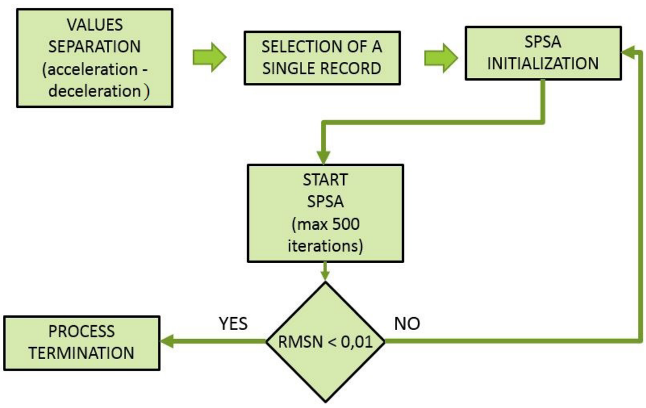


Fig. 5 Fitted values of car following model parameter “alpha”

behavior representation, therefore the correct choice of their values is important. Having obtained the distributions of the car-following parameters, the calibration procedure becomes easier for new data files and the researcher obtains a more complete picture about parameter range.

In order to look at the data in more depth and address the limitations of dealing with heterogeneous data, an attempt was made to find the theoretical distribution that best describes our results. Fig. 9 shows three histograms of the TransModeler’s parameter **beta**, that were created after SPSA applications to the data of three different drivers. Traffic conditions were similar for all drivers, as they participated in the same experiment. Using the programming language R (R Core Team, 2018) and the package fitdistrplus (Delignette-Muller et al., 2015) it was found that the **beta** values can be described by a normal distribution. It is also noticed, that the mean value of the distribution varies from driver to driver. Therefore, it is concluded that drivers behavior can not be always represented by a single parameter value.

Distribution’s characteristic parameters could capture a wider range of traffic conditions and driving behavior patterns. A possible shift of the mean value may help the representation of values’ range under different traffic conditions or driving behaviors. When new observations arise, a value from parameter’s alpha distribution could be picked and initialize the on-line calibration.

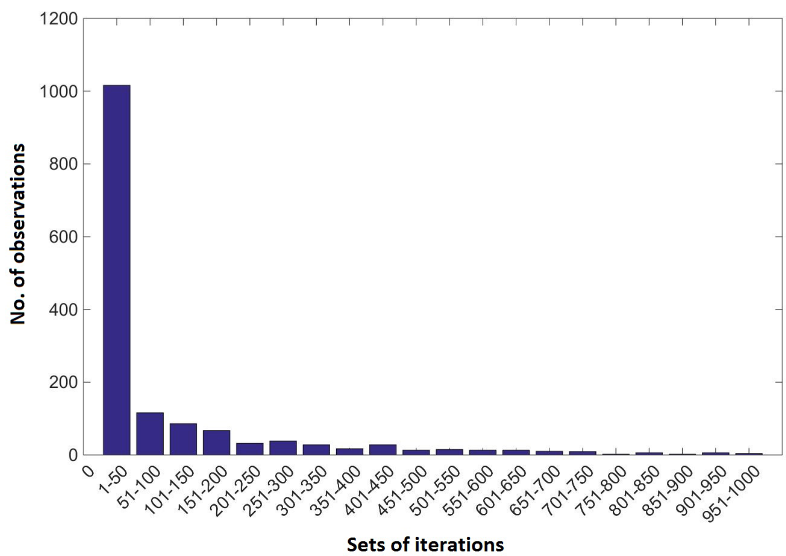


Fig. 6 Number of necessary iterations sets for SPSA termination

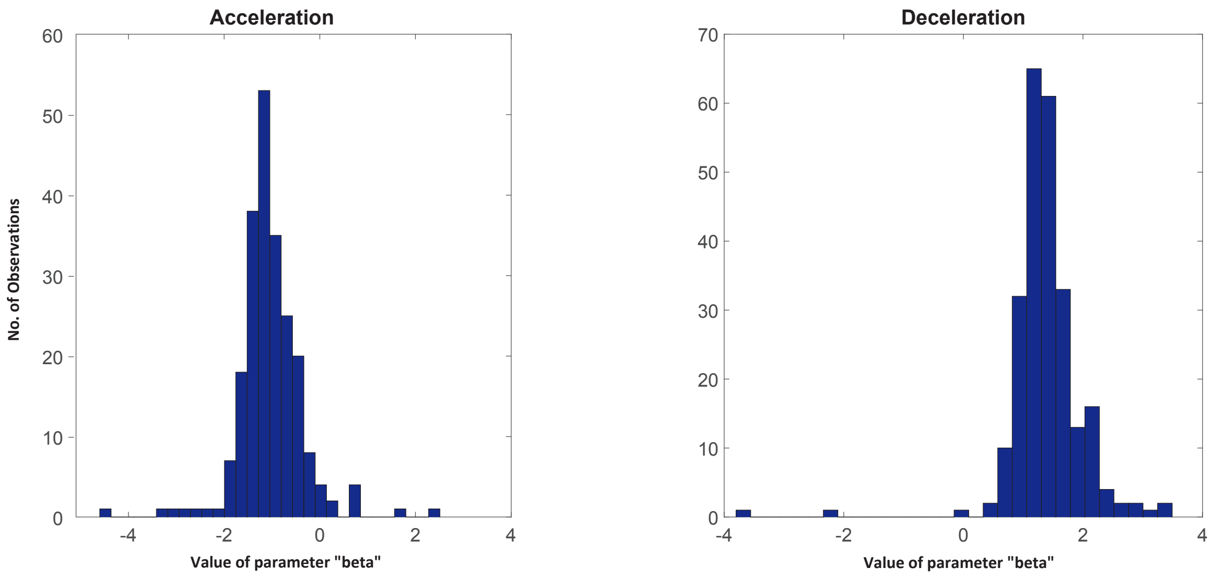


Fig. 7 Calibration results of car following model parameter “beta”

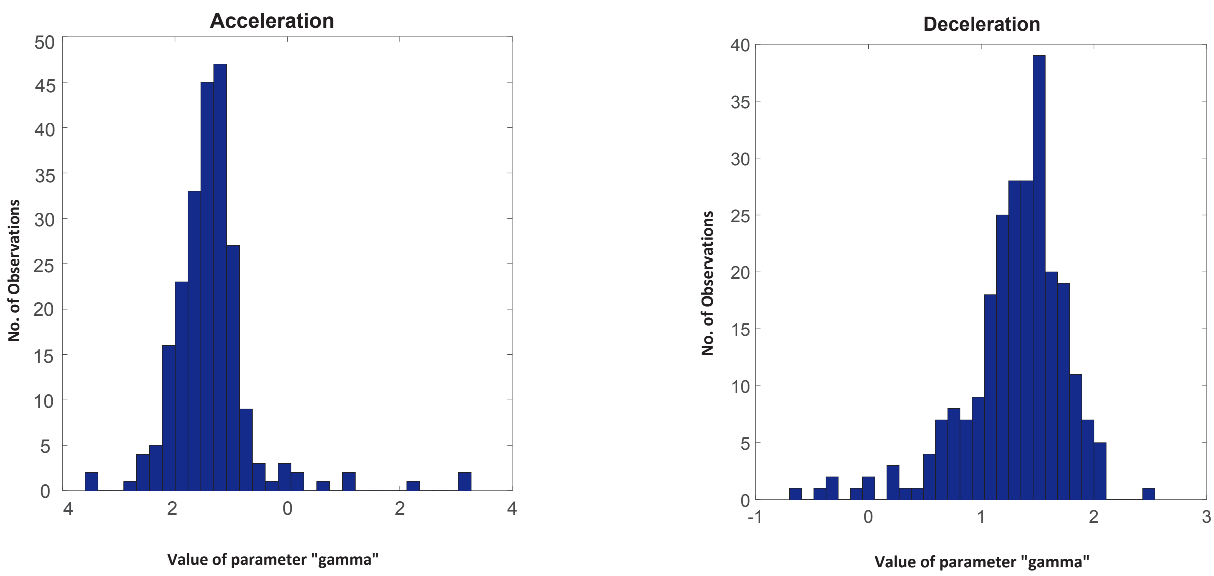


Fig. 8 Calibration results of car following model parameter “gamma”

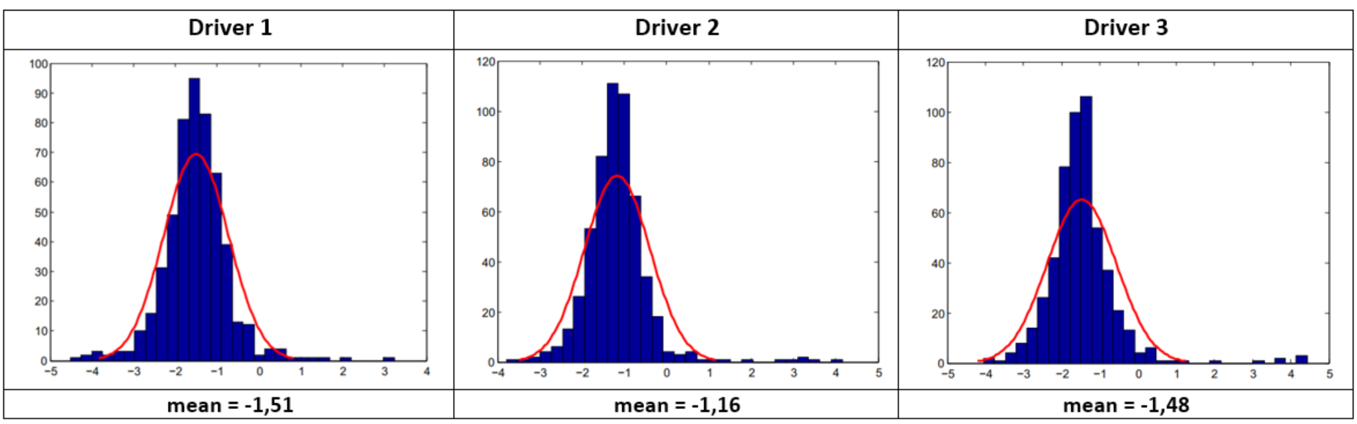


Fig. 9 Distributions of parameter beta for different drivers

4.3 Exploration of calibrated parameter values of Gipps' model

The overall results presented in the previous subsection eloquently illustrate distributions greater capacity to represent different driver behavior patterns. In this subsection, we explore parameter values of Gipps' model in order to gain some further insight.

Firstly, a static calibration is illustrated in order to be used as a reference benchmark for comparison with the proposed method. The longest data series was once again used for model calibration. For the whole dataset B1695 the optimization process has converged to the optimal set of parameters after 10000 iterations. The optimal values are presented in Table 2, where "initial values" refers to the model parameter values obtained by Papathanasopoulou and Antoniou (2015) and "optimal values" refers to the parameters obtained from the static calibration. The minimum value of the objective function, namely the RMSN, that was achieved with these optimal values of parameters was 2.2%.

Table 2 Optimization of model parameters using ISRES algorithm

Parameters of Gipps' model	Parameters range	Initial values	Optimal values
a (m/s^2)	[0.8, 2.6]	0.8	0.8
b (m/s^2)	[-5.2, -1.6]	-5.2	-3.2
V (m/s)	[10.4, 29.6]	14	14
s (m)	[5.6, 7.5]	5.6	5.9
$\hat{b}$ (m/s^2)	[-4.5, -3.0]	-3	-3.1
τ (s)	[0.4, 3.0]	0.4	0.4

Thereafter we proceeded to the dynamic calibration. Both for static and dynamic calibration an Improved Stochastic Ranking Evolution Strategy (ISRES algorithm) was used (Johnson, 2014). It is an algorithm for nonlinearly constrained global optimization. Fig. 10 presents the densities of the obtained parameters for the considered model and data–set. It is noted that, since for each time instant only one of the two equations is critical, the parameter values for that equation are considered at each time point. In each figure, the value of the static calibration is also indicated with a vertical dashed line. It becomes apparent that the dynamic values are not distributed symmetrically around the statically obtained value. This could have several implications. One question could be whether the static calibration is not really optimal. To check for this, we repeated the estimation and prediction using constant parameter values; however, this time, instead of using the value obtained from the static calibration, we used the median from the densities obtained from the dynamic calibration (i.e. the distributions shown in Fig. 10). In that case, the estimation RMSN ended up actually being inferior to that obtained from the static calibration results (with an RMSN of 3.4% instead of 2.2%). Therefore, it seems that there is something different going on, and that indeed the median values from the distributions cannot be used as best values for the determination of constant/ average values. The explanation for this may come from the nature of the Gipps model, i.e. the fact that there are two different equations, and at each given time the parameters of a single one are in effect considered. Therefore, while during the dynamic calibration the model steers only these parameters towards their desired values, using the available information, in the case of a static calibration one needs to determine single values that are relevant for all observations.

5 Conclusion

The availability of sufficiently accurate macroscopic and microscopic models is important for the design and the testing of modern freeway traffic management and control strategies. Their performance is largely independent of network's initial condition, data that could be incorporated in a model through different parameter values or small methodology changes. The proposed approach is computationally and conceptually attractive, because it allows the researcher to study simultaneously various driving behavior patterns, selecting each time the appropriate parameter value from a distribution of parameter values. This is useful to study one drivers driving behaviors under different traffic conditions or different drivers behaviors under common circumstances.

In this research, we have motivated the use of dynamic calibration of traffic simulation models. The experiments were implemented on the car–following model used by the TransModeler traffic simulator, as well as on the Gipps' model. Through the stochastic approximation, and more specifically through the SPSA algorithm, it was found that a single parameter value is not able to represent all observations. The optimization algorithm was not able to converge to a single parameter value for all data sets. Therefore, the procedure results in distributions of the model parameters, instead of more restrictive point values. It is noteworthy, that distributions differ, depending on the acceleration rate of the vehicle and the driver.

It was found that SPSA's basic parameters affect significantly its convergence. Even small changes in their values could lead to the ultimate failure of the algorithm. Many iterations will once again allow the convergence of the algorithm to the optimal value, however the necessary time would be much higher, a conclusion not permitted to complex models and large data volume. We concluded that even the parameters of SPSA should not remain constant, since a specific set of values is not able to give satisfactory results in any application.

The car–following model parameters could be represented by their distributions. Future research includes the modeling of the empirical distribution of the parameters, obtained in this research, by suitable theoretical distributions. Therefore, the researcher would be able to obtain a more formal representation of the algorithm behavior (and the underlying driver behavior).

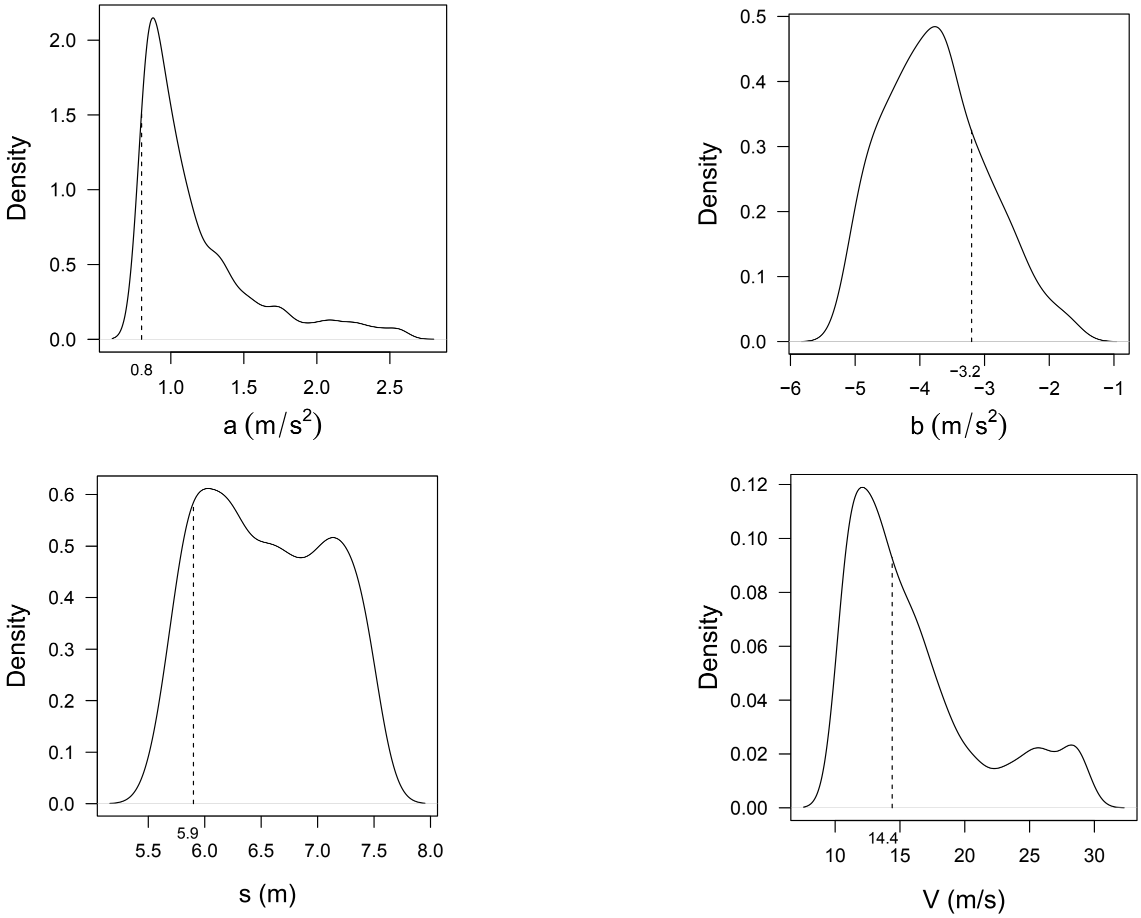


Fig. 10 Distributions of dynamically calibrated parameters. Notes: (i) dashed line: static calibration values, (ii) for each subfigure, only values from instances in which the corresponding equation is critical, are used

The calibration is now accompanied by certain rules, regarding the range of the model's values. Distributions allow us to exploit the wealth of data and relax the unrealistic assumptions that are requested using the common procedure.

Future research also includes the application of the methodology to more real settings, including incorporation of the distribution of model parameter values into a running model, and comparing the resulting calibration results to those obtained from the use of point measurements. The errors identified in the present application should be explained through further extensive experiments. Moreover, the suggested approach should be tested in a more complex situation (higher congestion, traffic lights, incidents etc.). Vehicle dynamics and the correlation between different parameter values should also be taken into account.

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