A vortex-based tip/smearing correction for the actuator line

Alexander Meyer Forsting, Georg Pirrung, Néstor Ramos-García
Overview

1. The actuator line tip correction paradox
2. Tip/smearing correction
3. Results
4. Conclusion
The actuator line technique

Sørensen and Shen (2002)

\[
f_e(x) = \sum_{i=1}^{3} \int_{0}^{R} f_{2D}(r) \eta_e(|x-r e_i|)dr
\]

\[
\eta_e(r) = \frac{1}{\epsilon^3 \pi^{3/2}} \exp\left[-\left(\frac{r}{\epsilon}\right)^2\right]
\]

The actuator line – tip correction paradox

NREL 5MW 8 m/s

- $F_n \times 10^3$ [N/m]
- $F_t \times 10^2$ [N/m]

$r/R \ [\cdot]$
The actuator line – tip correction paradox

NREL PhaseVI 7 m/s (Jha et al. 2014)

Force smearing = vortex smearing

\[ \Gamma_s = \frac{1}{2} \sqrt{v_s^2 + w_s^2 C_l(\alpha)c} \]

\[ v_\theta(r) = \frac{\Gamma}{2\pi r} \]
Force smearing = vortex smearing

\[ \frac{\Gamma}{2\pi r} \]


\[ v_\theta(r) = \frac{\Gamma}{2\pi r} \left[ 1 - \exp \left( -\frac{r^2}{\epsilon^2} \right) \right] \]
Viscous core correction for rotors

\[ u^* = \int_{0}^{\infty} f_\epsilon \delta \tilde{u} \, dl \]

\[ \delta \tilde{u} = \frac{\Delta \Gamma}{4\pi} \frac{\delta l \times x}{|x|^3} \]

\[ f_\epsilon = \exp \left( -\frac{(x \hat{e}_\perp)^2}{\epsilon^2} \right) \]

Near-wake model with convection

Viscous core correction for rotors

\[ u^* = \int_0^\infty f_\epsilon \delta \tilde{u} \, dl \]

\[ \delta \tilde{u} = \frac{\Delta \Gamma}{4\pi} \frac{\delta l \times x}{|x|^2} \]

\[ f_\epsilon = \exp \left( -\frac{(x \hat{e}_\perp)^2}{\epsilon^2} \right) \]

\[ x \hat{e}_\perp = \frac{\delta l}{|\delta l|} \times x = r \cos \phi \]

\[ \tan \phi \left( \beta \cos \beta - \sin \beta \right) \]

\[ -\tan \phi \left( -1 + \frac{h}{r} + \cos \beta + \beta \sin \beta \right) \]

\[ -1 + (1 - \frac{h}{r}) \cos \beta \]

Near-wake model with convection
Actuator line vs Lifting line
Actuator-line vs Lifting-line

- Actuator-line = Lifting-line with viscous core
- Actuator-line with smearing correction = Lifting-line

NREL 5MW $V = 8 \text{m/s}$

$N_s = 19, \Delta x = R/40, \varepsilon = R/10$
Actuator-line vs Lifting-line

- Actuator-line = Lifting-line with viscous core
- Actuator-line with smearing correction = Lifting-line

NREL 5MW

\[ N_s = 9, \Delta x = R/10, \epsilon = 0.2R \]
Smearing vs Tip correction
Sensitivity of blade loading to grid size

**Smearing correction**

**NREL 5MW +SC 8 m/s**

\[ N_v = 10, \Delta x = R/20; \epsilon = 2\Delta x \]

\[ 2\Delta x = R/10, \epsilon = R/5 \]

\[ 2N_v = 20 \]
Sensitivity of blade loading to grid size

**Smearing correction**

*NREL 5MW +SC 8 m/s*

\[ N_0 = 10, \Delta x = R/20; \epsilon = 2\Delta x \]

\[ 2\Delta x = R/10, \epsilon = R/5 \]

\[ 2N_v = 20 \]
**Sensitivity of blade loading to grid size**

**Smearing correction**

**NREL 5MW +SC 8 m/s**

\[ N_0 = 10, \Delta x = R/20; \epsilon = 2\Delta x \]

\[ 2\Delta x = R/10, \epsilon = R/5 \]

\[ 2N_y = 20 \]
Sensitivity of blade loading to grid size

NREL 5MW \(8 \text{ m/s}\)

\[N_v = 10, \Delta x = R/20; \epsilon = 2\Delta x\]

Smearing correction

Tip correction
Speed-up
Influence of simplifications

NREL 5MW

$V = 8 \text{ m/s}$

$V = 25 \text{ m/s}$
Influence of simplifications

Performance

<table>
<thead>
<tr>
<th>V [m/s]</th>
<th>orig. $\theta_{\text{max}} = 2\pi$</th>
<th>orig. $\theta_{\text{max}} = \pi/2$</th>
<th>cut loops</th>
<th>fixed $x_\perp = 2\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thrust</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>8</td>
<td>405 kN</td>
<td>-</td>
<td>0.02%</td>
<td>-0.04%</td>
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<tr>
<td>14</td>
<td>466 kN</td>
<td>-</td>
<td>0.01%</td>
<td>-0.18%</td>
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<tr>
<td>25</td>
<td>286 kN</td>
<td>0.004 %</td>
<td>-0.02%</td>
<td>-0.56%</td>
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<tr>
<td>Power</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>8</td>
<td>2.11 MW</td>
<td>-</td>
<td>0.04%</td>
<td>-0.14%</td>
</tr>
<tr>
<td>14</td>
<td>5.43 MW</td>
<td>-</td>
<td>0.02%</td>
<td>-0.27%</td>
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<tr>
<td>25</td>
<td>5.47 MW</td>
<td>0.007%</td>
<td>-0.02%</td>
<td>-0.80%</td>
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<tr>
<td>CPU time</td>
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</tr>
<tr>
<td>8</td>
<td>4.35e-02 s</td>
<td>-</td>
<td>-63%</td>
<td>-99%</td>
</tr>
<tr>
<td>14</td>
<td>4.45e-02 s</td>
<td>-</td>
<td>-64%</td>
<td>-98%</td>
</tr>
<tr>
<td>25</td>
<td>4.37e-02 s</td>
<td>-83%</td>
<td>-63%</td>
<td>-99%</td>
</tr>
</tbody>
</table>
Conclusion
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• Validated fully coupled, tuning-free smearing correction
• Actuator line forces with smearing correction largely independent of grid size
• Possibility for accurate low resolution actuator line simulations in wind farm studies
Future work

• Fast smearing correction publication

• Open-source release
Thanks

Alexander Meyer Forsting, Georg Pirrung, Néstor Ramos-García