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Bödvarsdottir, Elin Björk; Bagger, Niels-Christian Fink; Høffner, Laura Elise; Stidsen, Thomas Jacob Riis

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A comprehensive integer programming formulation of the nurse rostering problem in Denmark

Elín Björk Böðvarsdóttir (ebod@dtu.dk)
Department of Technology, Management and Economics, Technical University of Denmark

Niels-Christian Fink Bagger (nc@finkoptimization.com)
Fink Optimization

Laura Elise Høffner (laebu@regionsjaelland.dk)
Department of Data and Development Support, Region Zealand

Thomas Stidsen (thst@dtu.dk)
Department of Technology, Management and Economics, Technical University of Denmark

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1 Introduction

In this report, we present a comprehensive integer programming formulation for the nurse rostering problem (NRP). This model is the result of a collaboration between DTU Management and the Department of Data and Development Support (DU) at Region Zealand, with financial support from the Danish Ministry of Health. The work has been conducted in a close collaboration with healthcare practitioners at Danish hospitals, and the model has been iteratively built up to include all aspects that are needed to match the real-world problem that practitioners face.

This report is organized as follows: Section 2 briefly introduces the problem this report considers. Section 3 presents the model, constraint by constraint, divided into subsections based on the characteristics of the constraints. Then, Section 4 describes the generation of *conflict cliques*, which are used to reduce the number of constraints, and at last, Section 5 presents concluding remarks.

The appendices provide a summary, with a short description of the different constraints in Appendix A, an overview of the notation in Appendix B and the full model in Appendix C.

2 Problem description

As in other nurse rostering problems, the main task is to assign nurses to shifts on a given set of days (i.e., the rostering horizon). To ensure that we produce a feasible roster, we also need to consider some assignments from the previous rostering horizon, i.e., on the days leading up to the current horizon.

As the nursing staff at Danish hospitals does not only consist of educated nurses, but also assistants, we will use the more general term *employees* throughout this report. We classify the employees in two categories, those that we should assign to shifts on all days of the rostering horizon, and those that we should only consider occasionally (indicated by the input data).

For shifts, we use two terms: *Shift type* and *Shift set*, where the former refers to a single event (e.g., work shift or day off) and the latter refers to a set of events (e.g., all work shifts). We note that for various constraints it is indifferent whether we use shift types or shift sets, but for consistency we will present these constraints using the notation for shift sets throughout the report.

As the formulation is developed in collaboration with Danish hospitals, Danish legislation applies. This mainly affects the formulation of days off, as described in Section 3.2.2. In addition to various general constraints (see Appendix A), the employees also have the option to make a request for every day of the rostering horizon (e.g., to be assigned to a specific shift). Thus, the model should capture the individual wants and needs of each employee, along with satisfying numerous general requirements.

3 Model

We now introduce the problem formulation in detail, alongside an integer programming formulation used to solve it. The formulation consists of 14 hard constraints (H1-H14) and 15 soft constraints (S1-S15). Constraints H1-H10 and S1-S10 are employee-specific (i.e., ensuring the quality of all individual rosters), and the remaining constraints ensure that the ward as a whole functions according to the standards that have been set (e.g., ensuring sufficient staffing). We present the individual constraints in Section 3.2 and the ward constraints in Section 3.3.

We formulate the problem as a minimization problem with a single weighted objective, where each soft constraint (S1-S15) has an associated weight. The weight can either be positive, representing a penalty, or negative, representing a reward. We present the objective function in Section 3.4

3.1 Notation

Throughout the formulation, we use the following notation: $\mathcal{E}$ represents the set of employees that we should create a full roster for and $\mathcal{E}^{all}$ the set of all employees in the ward. An employee $e \in \mathcal{E}^{all} \backslash \mathcal{E}$ is not scheduled by the model, except when he or she specifically requests a shift. $\mathcal{D}$ represents the set of days in the rostering horizon and $\mathcal{D}^{pre}$ is a set of days from the previous rostering horizon. Additionally, we define $\mathcal{D}^{all} = \mathcal{D} \cup \mathcal{D}^{pre}$ for all days and $\mathcal{W}$ as the set of weeks of the rostering horizon, along with subsets of days $\mathcal{D}_w$ for all weeks $w \in \mathcal{W}$.

$\mathcal{S}$ represents the set of shift types where we denote the starting and ending times on day $d \in \mathcal{D}^{all}$ with $T^{start}_{s,d}$ and $T^{end}_{s,d}$, respectively. Additionally, we define subsets $\mathcal{S}^{work}$ for work shifts and $\mathcal{S}_{e,d}$ for feasible shift types for employee $e \in \mathcal{E}^{all}$ on day $d \in \mathcal{D}$, and extend it to include previous assignments for $d \in \mathcal{D}^{pre}$. We define the set of feasible assignments as $\mathcal{A} = \mathcal{E}^{all} \times \mathcal{D} \times \mathcal{S}_{e,d}$, and the extension $\mathcal{A}^{all} = \mathcal{E}^{all} \times \mathcal{D}^{all} \times \mathcal{S}_{e,d}$ also includes assignments from the previous horizon. Finally, we define the set of shift sets $\mathcal{Z}$, where we denote the relation between shift type $s \in \mathcal{S}$ and shift set $\sigma \in \mathcal{Z}$ with parameter $\alpha_{s,\sigma} \geq 0$. If $\alpha_{s,\sigma} = 0$ then the shift type does not belong to the shift set, but if $\alpha_{s,\sigma} = 1$ then the shift type fully belongs to the shift set. In addition, a relation $\alpha_{s,\sigma} \in]0,1[$ means that the shift type partly belongs to the shift set.

We introduce the assignment variables $x_{e,d,s} \in \{0,1\}$, to denote whether we assign employee $e \in \mathcal{E}^{all}$ to shift type $s \in \mathcal{S}_{e,d}$ on day $d \in \mathcal{D}$. The first soft constraint S1 corresponds to a weight $\omega^{assign}_{e,d,s}$ associated with each assignment. The weight can either be a penalty or a reward, depending on the assignment.

We let $x^{pre}_{e,d,s} \in \{0,1\}$ denote the (fixed) solution from the previous horizon, i.e., whether employee $e \in \mathcal{E}^{all}$ was assigned to shift type $s \in \mathcal{S}$ on day $d \in \mathcal{D}^{pre}$. We let $\tilde{x}_{e,d,s}$ be the extension of $x_{e,d,s}$ to $\mathcal{E}^{all} \times \mathcal{D}^{all} \times \mathcal{S}_{e,d}$ by setting $\tilde{x}_{e,d,s} = x^{pre}_{e,d,s}$ for $e \in \mathcal{E}^{all}, d \in \mathcal{D}^{pre}$ and $s \in \mathcal{S}$.

When assigning shifts we need to ensure that they do not overlap in time, and that employees get sufficient rest between them. According to the Danish legislation, employees should generally get $\rho^{full} > 0$ hours to rest between work shifts. Nonetheless, the rest may be reduced to $\rho^{reduced} > 0$ by the employee's request, where $\rho^{reduced} < \rho^{full}$.

3.2 Individual constraints

Most constraints in the model relate to the roster of each individual employee. We present these constraints in the following categories: Section 3.2.1 presents the basis for creating a feasible roster, i.e., assigning employees to shifts without any conflicting assignments. Section 3.2.2 presents the formulation for days off as required by Danish law. Section 3.2.3 and Section 3.2.4 present counter constraints and series constraints, respectively, following the categorization from previous research. Finally, Section 3.2.5 presents individual constraints that do not fit any other category.

3.2.1 Assigning shifts

The first hard constraint is H1, stating that we should assign each employee $e \in \mathcal{E}$ to at least one shift type $s \in \mathcal{S}_{e,d}$ for each day $d \in \mathcal{D}$. This constraint is given with Equation (1).

$$\sum_{s \in \mathcal{S}_{e,d}} x_{e,d,s} \geq 1, \quad \forall e \in \mathcal{E}, d \in \mathcal{D} \tag{1}$$

The second hard constraint is H2, stating that we cannot schedule any conflicting assignments. We define a pair of conflicting assignments as two assignments $a_1, a_2 \in \mathcal{A}^{all}$ where assigning both of them would lead to an infeasible roster. These conflicts can, for example, occur due to physical restrictions (e.g., employee cannot work two shifts simultaneously) or due to legislation (e.g., employees should get sufficient rest).

We define a set of *conflict cliques* Γ, where $\gamma \in \Gamma$ corresponds to a subset of assignments $\mathcal{A}_\gamma \subset \mathcal{A}^{all}$, such that every two assignments in $\mathcal{A}_\gamma$ are conflicting. Using the conflict cliques, we can express H2 using Equation (2).

$$\sum_{(e,d,s) \in \mathcal{A}_\gamma} \tilde{x}_{e,d,s} \leq 1, \quad \forall \gamma \in \Gamma \tag{2}$$

In addition to Γ, we also generate sets of *work conflict cliques* $\Gamma_{e,d}$ for each employee $e \in \mathcal{E}$ and day $d \in \mathcal{D}$. We can use these cliques to reduce the number of constraints in the model. We describe the generation of cliques in Section 4.

3.2.2 Protected days off

According to Danish law, all employees are entitled to *protected days off*, which are subject to various constraints. In this section, we describe two specific constraints for protected days off, but note that some general constraints presented in later sections also apply for protected days off. We let $s_{pf} \in \mathcal{S}$ denote the shift type for a protected day off and note that constraints H2, for conflicting assignments, ensure that if we assign $s_{pf} \in \mathcal{S}$, then it is the only shift assigned to that employee on that day. We define variables $p_{e,d,i} \in \{0,1\}$ denoting whether employee $e \in \mathcal{E}$ starts a sequence of $i \in \mathbb{N}$ protected days off on day $d \in \mathcal{D}^{all}$. We link these variables using Equations (3)-(5).

$$\sum_{\substack{d' \in \mathcal{D}^{all}, \\ i \in \mathbb{N}: \\ d-i<d' \leq d}} p_{e,d',i} = \tilde{x}_{e,d,s_{pf}}, \quad \forall e \in \mathcal{E}, d \in \mathcal{D}^{all} : s_{pf} \in \mathcal{S}_{e,d} \tag{3}$$

$$\sum_{\substack{d' \in \mathcal{D}^{all}, \\ i \in \mathbb{N}: \\ d-i<d' \leq d}} p_{e,d',i} = 0, \quad \forall e \in \mathcal{E}, d \in \mathcal{D}^{all} : s_{pf} \notin \mathcal{S}_{e,d} \tag{4}$$

$$\sum_{j \in \mathbb{N}} p_{e,d,j} + \sum_{\substack{d' \in \mathcal{D}^{all}, \\ i \in \mathbb{N}: \\ d'+i=d}} p_{e,d',i} \leq 1, \quad \forall e \in \mathcal{E}, d \in \mathcal{D}^{all} \tag{5}$$

Equations (3)-(4) ensure that the $p_{e,d,i}$ variable can only be one if we have assigned employee $e \in \mathcal{E}$ to the protected day off shift $s_{pf} \in \mathcal{S}$ on all days from $d \in \mathcal{D}^{all}$ to $d + i \in \mathcal{D}$, including both days.

Furthermore, Equations (5) ensure that two sequences of protected days off are never adjacent, but instead defined as one longer sequence.

The former constraint for protected days off is H3, stating that a day off can only count as a protected day off if it satisfies a given minimum number of hours off. This constraint represents a legal requirement, which is a minimum of either 32 or 35 hours off for a single protected day off or a minimum of $24 \cdot k + 7 \cdot \lfloor k/2 \rfloor$ hours off for k consecutive protected days off.

We let T^{prev}, and T^{fut}, denote reference points in time from previous, and upcoming, rostering horizons. When comparing to these reference values, we round on hours to two decimal places, denoted with $\lfloor \cdot \rceil_2$ in the formulation. The exact values for T^{prev} and T^{fut} are irrelevant, as they are only used as a baseline to compare different points in time. We define two variables, $\tau^{last}_{e,d} \geq 0$ and $\tau^{next}_{e,d} \geq 0$, where $\tau^{last}_{e,d}$ represents the number of hours between T^{prev} and the end of the last work shift employee $e \in \mathcal{E}$ has before day $d \in \mathcal{D}$ and $\tau^{next}_{e,d}$ represents the number of hours between T^{fut} and the start of the next work shift employee $e \in \mathcal{E}$ has after day $d \in \mathcal{D}$.

We connect the variables $\tau^{last}_{e,d}$ using Equations (6)-(7), and $\tau^{next}_{e,d}$ using Equations (8)-(9). Finally, we let ρ^{pf}_i denote the minimum number of hours off required for $i \in \mathbb{N}$ consecutive protected days off and express H3 using Equations (10), where $\Gamma_{e,d}$ are work conflict cliques as defined in Section 3.2.1.

$$\tau^{last}_{e,d} - \sum_{\substack{s \in \mathcal{S}^{work}: \\ (e,d-1,s) \in \mathcal{A}_\gamma}} \left\lfloor T^{end}_{s,d-1} - T^{prev} \right\rceil_2 \cdot \tilde{x}_{e,d-1,s} \geq 0, \qquad \forall e \in \mathcal{E}, d \in \mathcal{D}^{all}, \gamma \in \Gamma_{e,d} \quad (6)$$

$$\tau^{last}_{e,d} \geq \tau^{last}_{e,d-1}, \qquad \forall e \in \mathcal{E}, d \in \mathcal{D}^{all} \quad (7)$$

$$\tau^{next}_{e,d} - \sum_{\substack{s \in \mathcal{S}^{work}: \\ (e,d+1,s) \in \mathcal{A}_\gamma}} \left\lfloor T^{start}_{s,d+1} - T^{fut} \right\rceil_2 \cdot \tilde{x}_{e,d+1,s} \leq \left\lfloor T^{fut} - T^{prev} \right\rceil_2, \qquad \forall e \in \mathcal{E}, d \in \mathcal{D}^{all}, \gamma \in \Gamma_{e,d} \quad (8)$$

$$\tau^{next}_{e,d} \leq \tau^{next}_{e,d+1}, \qquad \forall e \in \mathcal{E}, d \in \mathcal{D}^{all} : d+1 \in \mathcal{D}^{all} \quad (9)$$

$$\tau^{last}_{e,d} + \sum_{i \in \mathbb{N}} \rho^{pf}_i \cdot p_{e,d,i} \leq \tau^{next}_{e,d}, \qquad \forall e \in \mathcal{E}, d \in \mathcal{D}^{all} \quad (10)$$

When we assign employee $e \in \mathcal{E}$ to a work shift $s \in \mathcal{S}^{work}$ on day $d-1 \in \mathcal{D}^{all}$, then Equations (6) ensure that the counter variable $\tau^{last}_{e,d}$ is updated according to the end of the work shift. Similarly, Equations (8) ensure that the counter variable $\tau^{next}_{e,d}$ is updated according to the start of a work shift on day $d+1 \in \mathcal{D}^{all}$. Equations (7) and (9) ensure the consistency of the variables on days where we do not assign a work shift on the previous or subsequent day. Finally, Equations (10) ensure that the variable $p_{e,d,i}$ can only become one when the difference between the counters $\tau^{next}_{e,d}$ and $\tau^{prev}_{e,d}$ is more than the required minimum hours for $i \in \mathbb{N}$ protected days off.

The latter constraint for protected days off is S2, stating that the number of days between two contiguous sequences of protected days off should be below a maximum, denoted with v^{pref}. We note that this constraint categorizes as a series constraint. We define variables $v_{e,d} \in [0,1]$ denoting whether employee $e \in \mathcal{E}$ exceeds v^{pref} days without a protected day off from day $d \in \mathcal{D}$, and link them using Equations (11). To avoid violation, the objective function penalizes with weight $\omega^{pf} \geq 0$ when $v_{e,d} > 0$.

$$v_{e,d} + \sum_{\substack{d' \in \mathcal{D}: \\ d \leq d' < d+v^{pref}}} \tilde{x}_{e,d',s_{pf}} \geq 1, \quad \forall e \in \mathcal{E}, d \in \mathcal{D}^{all} : d + v^{pref} \in \mathcal{D} \quad (11)$$

3.2.3 Counter constraints

We include five counter constraints in the formulation. The first two are; H4, to not exceed a maximum when assigning an employee to certain shifts during the rostering horizon, and H5, to assign a fixed number of certain shifts to an employee during the rostering horizon. We use $M_{e,\sigma}^{total,ub}$, and $M_{e,\sigma}^{total,fix}$, to denote the maximum, and fixed, total assignments for employee $e \in \mathcal{E}$ to shift set $\sigma \in \mathcal{Z}$. These parameters are only defined for a limited subset of $\mathcal{E} \times \mathcal{Z}$, as various employees or shift sets do not have associated upper bounds or targets. We then express H4-H5 using Equations (12)-(13), respectively.

$$\sum_{\substack{d \in \mathcal{D}, \\ s \in \mathcal{S}_{e,d} \cap \sigma}} \alpha_{s,\sigma} \cdot x_{e,d,s} \leq M_{e,\sigma}^{total,ub}, \quad \forall e \in \mathcal{E}, \sigma \in \mathcal{Z} : M_{e,\sigma}^{total,ub} \text{ defined} \tag{12}$$

$$\sum_{\substack{d \in \mathcal{D}, \\ s \in \mathcal{S}_{e,d} \cap \sigma}} \alpha_{s,\sigma} \cdot x_{e,d,s} = M_{e,\sigma}^{total,fix}, \quad \forall e \in \mathcal{E}, \sigma \in \mathcal{Z} : M_{e,\sigma}^{total,fix} \text{ defined} \tag{13}$$

The third counter constraint is H6, to assign a fixed number of certain shifts to an employee in a given week. This counter constraint does not consider the rostering horizon as a whole but as separate weeks. We use $M_{e,\sigma,w}^{week,fix}$ to denote the fixed number of assignments for employee $e \in \mathcal{E}$ to shift set $\sigma \in \mathcal{Z}$ during week $w \in \mathcal{W}$, and note that these parameters are only defined for a limited subset of $\mathcal{E} \times \mathcal{Z} \times \mathcal{W}$. We then express H6 using Equations (14).

$$\sum_{\substack{d \in \mathcal{D}_w, \\ s \in \mathcal{S}_{e,d} \cap \sigma}} \alpha_{s,\sigma} \cdot x_{e,d,s} = M_{e,\sigma,w}^{week,fix}, \quad \forall e \in \mathcal{E}, \sigma \in \mathcal{Z}, w \in \mathcal{W} : M_{e,\sigma,w}^{week,fix} \text{ defined} \tag{14}$$

The fourth counter constraint is S3, stating that we should minimize the deviation between contractual hours and assigned hours. This constraint is global, i.e., not defined for a single rostering horizon but for a longer reference period. We let T_e^{target} denote the target hours, i.e., the hours we should assign to employee $e \in \mathcal{E}$ during the rostering horizon, after correcting for deficit or surplus from previous rostering horizon. In addition, we let $T_{e,d,s}^{hour}$ denote the hours an assignment to shift type $s \in \mathcal{S}$ on day $d \in \mathcal{D}$ counts towards the target for employee $e \in \mathcal{E}$. We define variables $h_{e,w} \geq 0$ for the hours assigned to employee $e \in \mathcal{E}$ during week $w \in \mathcal{W}$, and link them using Equations (15).

We formulate S3 using a two-factor penalty where variables $t_e^+ \in [0, N^+]$, and $t_e^- \in [0, N^-]$, denote the positive, and negative, deviation from the target hours within a bound N^+, and N^-. In addition, we define variables $t_e^{++} \geq 0$, and $t_e^{--} \geq 0$, denoting the deviation exceeding the bounds. We then link these variables using Equations (16).

$$\sum_{\substack{d \in \mathcal{D}_w, \\ s \in \mathcal{S}_{e,d}}} T_{e,d,s}^{hour} \cdot x_{e,d,s} = h_{e,w}, \qquad \forall e \in \mathcal{E}, w \in \mathcal{W} \tag{15}$$

$$\sum_{w \in \mathcal{W}} h_{e,w} - t_e^+ + t_e^- - t_e^{++} + t_e^{--} = T_e^{target}, \quad \forall e \in \mathcal{E} \tag{16}$$

To penalize for deviating from the contractual hours, we define weights $\omega^+ \geq 0$, and $\omega^- \geq 0$, as the penalties for the positive, and the negative, deviation from the target hours within the bounds N^+, and N^-. Additionally, we define $\omega^{++} \geq \omega^+$, and $\omega^{--} \geq \omega^-$, for penalizing the positive, and negative, deviation exceeding these bounds.

The fifth counter constraint is S4, stating that we should distribute the hours between weeks according to weekly targets for each employee. We let $T_{e,w}^{week}$ denote the target hours for employee $e \in \mathcal{E}$ during week $w \in \mathcal{W}$. Furthermore, we let variables $\lambda_{e,w}^+ \geq 0$, and $\lambda_{e,w}^- \geq 0$, denote the positive, and negative, deviation from the weekly target and link these variables using Equations (17). The objective function penalizes with weight $\omega^{week} \geq 0$ when $\lambda^{+}_{e,w}, \lambda^{-}_{e,w} > 0$.

$$h_{e,w} - \lambda^{+}_{e,w} + \lambda^{-}_{e,w} = T^{week}_{e,w}, \quad \forall e \in \mathcal{E}, w \in \mathcal{W} \tag{17}$$

3.2.4 Series constraints

In addition to S2 (for protected days off), we include eight series constraints in the formulation. The first two are H7 and S5, stating that we should not exceed a maximum in a row when assigning an employee to certain shifts. We let $M^{row}_{e,d,\sigma}$ denote the maximum number of days in a row that we can assign employee $e \in \mathcal{E}$ to shift set $\sigma \in \mathcal{Z}$, from and including day $d \in \mathcal{D}^{all}$, and note that these parameters are only defined for a limited subset of $\mathcal{E} \times \mathcal{D}^{all} \times \mathcal{Z}$.

As the constraint can either be hard or soft depending on the employee, the day and the shift, we have parameters $\beta^{row}_{e,d,\sigma}$ denoting whether it should be hard ($\beta^{row}_{e,d,\sigma} = 1$) or soft ($\beta^{row}_{e,d,\sigma} = 0$). For $\beta^{row}_{e,d,\sigma} = 0$, we define variables $\mu_{e,d,\sigma} \geq 0$ denoting whether employee $e \in \mathcal{E}$ is assigned to shift set $\sigma \in \mathcal{Z}$ more than $M^{row}_{e,d,\sigma}$ days in a row from day $d \in \mathcal{D}^{all}$, and extend it to be zero for $\beta^{row}_{e,\sigma} = 1$. We then express H7 and S5 using Equations (18). To avoid assigning too many consecutive assignments, the objective function penalizes with weight $\omega^{maxrow}_{e} \geq 0$ when $\mu_{e,d,\sigma} > 0$.

$$\sum_{\substack{d' \in \mathcal{D}^{all}, \\ s \in \mathcal{S}_{e,d} \cap \sigma: \\ d \leq d' \leq d + M^{row}_{e,d,\sigma}}} \alpha_{s,\sigma} \cdot \tilde{x}_{e,d',s} - \mu_{e,d,\sigma} \leq M^{row}_{e,d,\sigma}, \quad \forall e \in \mathcal{E}, d \in \mathcal{D}^{all}, \sigma \in \mathcal{Z} : M^{row}_{e,d,\sigma} \text{ defined} \tag{18}$$

The third series constraint is H8, stating that we cannot exceed a maximum of hours when assigning certain shifts on consecutive days. We let $H^{consec}_{e,d,\sigma}$ be the maximum hours we can assign to employee $e \in \mathcal{E}$ from shift set $\sigma \in \mathcal{Z}$ on $D^{consec}_{e,d,\sigma}$ consecutive days from day $d \in \mathcal{D}^{all}$, and note that these parameters are only defined for a limited subset of $\mathcal{E} \times \mathcal{D}^{all} \times \mathcal{Z}$. We express H8 using Equations (19), where $T^{hour}_{e,d,s}$ is as defined in Section 3.2.3.

$$\sum_{\substack{d \leq d' \leq d + D^{consec}_{e,d,\sigma} - 1, \\ s \in \sigma \cap \mathcal{S}_{e,d'}}} T^{hour}_{e,d',s} \cdot \tilde{x}_{e,d',s} \leq H^{consec}_{e,d,\sigma}, \quad \forall e \in \mathcal{E}, d \in \mathcal{D}^{all}, \sigma \in \mathcal{Z} : H^{consec}_{e,d,\sigma} \text{ and } D^{consec}_{e,d,\sigma} \text{ defined.} \tag{19}$$

The fourth and fifth series constraints are H9 and S6, referring to patterns of assignments on consecutive days that we either forbid (H9) or penalize (S6). We let $\mathcal{Y}$ denote the set of patterns and for each pattern $y \in \mathcal{Y}$ we let l_y denote the length in days and β^{pat}_{y} denote whether the pattern is forbidden ($\beta^{pat}_{y} = 1$) or penalized ($\beta^{pat}_{y} = 0$). Furthermore, each pattern $y \in \mathcal{Y}$ represents a series of shift sets $\langle \sigma_1, \ldots \sigma_{l_y} \rangle$ with $\sigma_i \in \mathcal{Z}, \ \forall i \in \{1, \ldots, l_y\}$. For $\beta^{pat}_{y} = 0$, we define variables $\pi_{e,d,y} \in [0,1]$ denoting a violation of pattern $y \in \mathcal{Y}$ for employee $e \in \mathcal{E}$ starting on day $d \in \mathcal{D}^{all}$, and extend it to be zero for $\beta^{pat}_{y} = 1$. As we can relax some patterns by request, we let $\Pi_{e,d,y}$ denote whether pattern $y \in \mathcal{Y}$ should be active for employee $e \in \mathcal{E}$ starting on day $d \in \mathcal{D}$. We then express H9 and S6 using Equations (20). The objective function penalizes with weight $\omega^{pat}_{y} \geq 0$ when $\pi_{e,d,y} > 0$.

$$\sum_{\substack{i \in \{1, \ldots, l_y\}, \\ s_i \in \sigma_i \cap \mathcal{S}_{e,d+i-1}}} \tilde{x}_{e,d+i-1,s_i} \leq l_y - 1 + \pi_{e,d,y}, \quad \forall e \in \mathcal{E}, d \in \mathcal{D}^{all}, y \in \mathcal{Y} : d + l_y - 1 \in \mathcal{D}, \Pi_{e,d,y} = 1 \tag{20}$$

The sixth series constraint is S7, stating that we should restrict the number of assignments to different shifts for an employee on consecutive days. We let $\mathcal{Z}^{diff} \subseteq \mathcal{Z}$ be the set of shift sets we restrict, and n^{diff} be the maximum of different assignments on D^{diff} consecutive days. We define variables $\kappa_{e,d,\sigma} \in [0,1]$ denoting whether we assign employee $e \in \mathcal{E}$ to $\sigma \in \mathcal{Z}^{diff}$ on D^{diff} consecutive days from day $d \in \mathcal{D}^{all}$. Furthermore, we define variables $\kappa^{more}_{e,d} \geq 0$ denoting how much we exceed n^{diff} for employee $e \in \mathcal{E}$ on D^{diff} consecutive days from $d \in \mathcal{D}^{all}$. We bind the variables using Equations (21)-(22). The objectieve function penalizes with weight $\omega^{diff} \geq 0$ when $\kappa^{more}_{e,d} > 0$.

$$\tilde{x}_{e,d',s} \leq \kappa_{e,d,\sigma}, \quad \forall e \in \mathcal{E}, d \in \mathcal{D}^{all}, d' \in \{d, \ldots, d + D^{diff} - 1\}, \sigma \in \mathcal{Z}^{diff}, s \in \sigma : d + D^{diff} - 1 \in \mathcal{D} \tag{21}$$

$$\sum_{\sigma \in \mathcal{Z}^{diff}} \kappa_{e,d,\sigma} - n^{diff} \leq \kappa^{more}_{e,d} \quad \forall e \in \mathcal{E}, d \in \mathcal{D}^{all} : d + D^{diff} - 1 \in \mathcal{D} \tag{22}$$

The seventh series constraint is S8, stating that we should not assign work sequences (i.e., consecutive work days) shorter than a minimum number of days, denoted with m^{seq}. We define variables $\theta^{start}_{e,d} \in [0,1]$ denoting whether employee $e \in \mathcal{E}$ starts a work sequence on day $d \in \mathcal{D}^{all}$. Similarly we define $\theta^{end}_{e,d} \in [0,1]$ for the end of a work sequence. We link these variables using Equations (23)-(24), respectively, where $\Gamma_{e,d}$ are work conflict cliques as defined in Section 3.2.1. Additionally, we define variables $\delta_{e,d} \geq 0$, used to penalize for the deviation from m^{seq} if employee $e \in \mathcal{E}$ has a shorter work sequence than preferred ending on day $d \in \mathcal{D}$. We connect these variables using Equations (25). The objective function penalizes with weight $\omega^{seqlen} \geq 0$ when $\delta_{e,d} > 0$.

$$\sum_{\substack{s \in \mathcal{S}^{work}: \\ (e,d,s) \in \mathcal{A}_\gamma}} \tilde{x}_{e,d,s} - \sum_{s \in \mathcal{S}^{work}} \tilde{x}_{e,d-1,s} \leq \theta^{start}_{e,d}, \quad \forall e \in \mathcal{E}, d \in \mathcal{D}^{all}, \gamma \in \Gamma_{e,d} \tag{23}$$

$$\sum_{\substack{s \in \mathcal{S}^{work}: \\ (e,d,s) \in \mathcal{A}_\gamma}} \tilde{x}_{e,d,s} - \sum_{s \in \mathcal{S}^{work}} \tilde{x}_{e,d+1,s} \leq \theta^{end}_{e,d}, \quad \forall e \in \mathcal{E}, d \in \mathcal{D}^{all}, \gamma \in \Gamma_{e,d} : d+1 \in \mathcal{D} \tag{24}$$

$$\left(m^{seq} - d + d' - 1\right)^2 \cdot \left(\theta^{end}_{e,d} + \theta^{start}_{e,d'} - 1\right) \leq \delta_{e,d}, \quad \forall e \in \mathcal{E}, d \in \mathcal{D}, d' \in \mathcal{D}^{all} : d - m^{seq} < d' \leq d \tag{25}$$

Equations (23) force $\theta^{start}_{e,d}$ to be one if employee $e \in \mathcal{E}$ has a work shift on day $d \in \mathcal{D}^{all}$ but not on day $d-1 \in \mathcal{D}^{all}$, and similarly Equations (24) force $\theta^{end}_{e,d}$ to be one if employee $e \in \mathcal{E}$ has a work shift on day $d \in \mathcal{D}^{all}$ but not on day $d+1 \in \mathcal{D}^{all}$. Finally, Equations (25) consider work sequences shorter than preferred and bind the penalty variable $\delta_{e,d}$ to be the square of the deviation from m^{seq}, to ensure that extremely short sequences, e.g., single work days, are heavily penalized.

At last, the eight series constraint is S9, stating that the number of work sequences containing certain shifts should not exceed a maximum for an employee on consecutive days. We let $\mathcal{Z}^{seq}_e \subseteq \mathcal{Z}$ represent the set of shift sets that we restrict for employee $e \in \mathcal{E}$, and let $\chi_{e,\sigma}$ denote the maximum number of work sequences, containing shift set $\sigma \in \mathcal{Z}^{seq}_e$, that we should assign to employee $e \in \mathcal{E}$ on $D^{seq}_{e,\sigma}$ consecutive days.

We define variables $\phi^{start}_{e,d,\sigma} \in [0,1]$ denoting whether employee $e \in \mathcal{E}$ starts a work sequence containing $\sigma \in \mathcal{Z}^{seq}_e$ on day $d \in \mathcal{D}^{all}$. Similarly we define $\phi^{end}_{e,d,\sigma} \in [0,1]$ for the end of a sequence. We link these variables using Equations (26)-(27), respectively. Additionally, we define variables $\phi^{count}_{e,d,\sigma} \geq 0$ denoting the number of sequences, exceeding $\chi_{e,\sigma}$, on $D^{seq}_{e,\sigma}$ consecutive days from $d \in D^{all}$, where we assign employee $e \in \mathcal{E}$ to $\sigma \in \mathcal{Z}^{seq}_e$. We connect these variables using Equations (28). The objective function penalizes with weight $\omega^{seqconsec} \geq 0$ when $\phi^{count}_{e,d,\sigma} > 0$.

$$x_{e,d_2,s} + \theta^{start}_{e,d_1} - \sum_{\substack{d \in \mathcal{D}: \\ d_1 \leq d < d_2}} \theta^{end}_{e,d} - 1 \leq \phi^{start}_{e,d_1,\sigma}, \quad \forall e \in \mathcal{E}, d_1 \in \mathcal{D}^{all}, d_2 \in \mathcal{D}^{all}, \sigma \in \mathcal{Z}^{seq}_e,$$

$$s \in \mathcal{S}_{e,d_2} \cap \sigma : d_1 \leq d_2 \tag{26}$$

$$x_{e,d_1,s} + \theta^{end}_{e,d_2} - \sum_{\substack{d \in \mathcal{D}: \\ d_1 < d \leq d_2}} \theta^{start}_{e,d} - 1 \leq \phi^{end}_{e,d_2,\sigma}, \quad \forall e \in \mathcal{E}, d_1 \in \mathcal{D}^{all}, d_2 \in \mathcal{D}^{all}, \sigma \in \mathcal{Z}^{seq}_e,$$

$$s \in \mathcal{S}_{e,d_1} \cap \sigma : d_1 \leq d_2 \tag{27}$$

$$\phi^{end}_{e,d,\sigma} + \sum_{\substack{d' \in \mathcal{D}^{all}: \\ d < d' \leq d + D^{seq}_{e,\sigma}}} \phi^{start}_{e,d',\sigma} - \chi_{e,\sigma} \leq \phi^{count}_{e,d,\sigma}, \quad \forall e \in \mathcal{E}, d \in \mathcal{D}^{all}, \sigma \in \mathcal{Z}^{seq}_e : d + D^{seq}_{e,\sigma} \in \mathcal{D} \tag{28}$$

Equations (26) consider all days from the start and to the end of a sequence, including both days, and check whether we assign $e \in \mathcal{E}$ to shift set $\sigma \in \mathcal{Z}^{req}_e$ on any of those days. If so, then we force $\phi^{start}_{e,d,\sigma}$ to be one for the start day $d \in \mathcal{D}^{all}$ of the sequence. Equations (27) work similarly for the end of a sequence. Finally, Equations (28) count the number of sequences including shift set $\sigma \in \mathcal{Z}^{seq}_e$ that we have assigned to employee $e \in \mathcal{E}$ on $D^{seq}_{e,\sigma}$ consecutive days, and forces the penalty variable $\phi^{count}_{e,d,\sigma}$ to be positive if we exceed the preferred number $\chi_{e,\sigma}$.

3.2.5 Other individual constraints

In addition to the constraints presented in previous sections, we include two individual constraints that do not fit any category.

The first constraint is H10, stating that we can only schedule certain assignments for a given employee in combination with other assignments. We let $L^{emp,both}_{e,d_1,d_2,\sigma_1,\sigma_2} \in \{0,1\}$ denote whether we only assign employee $e \in \mathcal{E}$ to shift set $\sigma_1 \in \mathcal{Z}$ on day $d_1 \in \mathcal{D}^{all}$ in combination with assigning shift set $\sigma_2 \in \mathcal{Z}$ on day $d_2 \in \mathcal{D}^{all}$, and vice versa. Similarly, $L^{emp,one}_{e,d_1,d_2,\sigma_1,\sigma_2} \in \{0,1\}$ denotes whether we only assign employee $e \in \mathcal{E}$ to shift set $\sigma_1 \in \mathcal{Z}$ on day $d_1 \in \mathcal{D}^{all}$ in combination with assigning shift set $\sigma_2 \in \mathcal{Z}$ on day $d_2 \in \mathcal{D}^{all}$, but not necessarily the other way around. We then express H10 using Equations (29)-(30). We will present similar constraints on a ward level in Section 3.3.1.

$$\sum_{s \in \mathcal{S}_{e,d_1} \cap \sigma_1} \tilde{x}_{e,d_1,s} = \sum_{s \in \mathcal{S}_{e,d_2} \cap \sigma_2} \tilde{x}_{e,d_2,s}, \quad \forall e \in \mathcal{E}, d_1, d_2 \in \mathcal{D}^{all}, \sigma_1, \sigma_2 \in \mathcal{Z} : L^{emp,both}_{e,d_1,d_2,\sigma_1,\sigma_2} = 1 \tag{29}$$

$$\sum_{s \in \mathcal{S}_{e,d_1} \cap \sigma_1} \tilde{x}_{e,d_1,s} \leq \sum_{s \in \mathcal{S}_{e,d_2} \cap \sigma_2} \tilde{x}_{e,d_2,s}, \quad \forall e \in \mathcal{E}, d_1, d_2 \in \mathcal{D}^{all}, \sigma_1, \sigma_2 \in \mathcal{Z} : L^{emp,one}_{e,d_1,d_2,\sigma_1,\sigma_2} = 1 \tag{30}$$

The second constraint is S10, stating that employees working some periods should get the surrounding days off. We define $\mathcal{Q}_e$ as the set of periods where we should assign surrounding days off to employee $e \in \mathcal{E}$ if he or she is working during the period. Each period $q \in \mathcal{Q}_e$ is an interval of days $[q^s, q^e]$ where we should assign q^b days off before and q^a days off after. We let $\mathcal{O}_{e,q} \subset \mathcal{S}^{work} \times \mathcal{D}^{all}$ denote the set of assignments that overlap with $q \in \mathcal{Q}_e$ and let ψ_{d_1,d_2,s_1,s_2} be the number of full days between shift type $s_1 \in \mathcal{S}^{work}$ on day $d_1 \in \mathcal{D}^{all}$ and shift type $s_2 \in \mathcal{S}^{work}$ on day $d_2 \in \mathcal{D}$, where $d_1 \leq d_2$.

We define variables $B_{e,q} \geq 0$ and $A_{e,q} \geq 0$, denoting the number of days off we assign to employee $e \in \mathcal{E}$ before and after period $q \in \mathcal{Q}_e$, respectively. We bind these variables using Equations (31)-(33), and the objective function rewards with weights $\omega^b \leq 0$ when $B_{e,q} > 0$ and $\omega^a \leq 0$ when $A_{e,q} > 0$.

$$\begin{aligned} B_{e,q} + q^b \cdot (\tilde{x}_{e,d_1,s_1} + x_{e,d_2,s_2}) \leq 2q^b + \psi_{d_1,d_2,s_1,s_2}, \quad & \forall e \in \mathcal{E}, q \in \mathcal{Q}_e, d_1 \in \left[q^s - q^b, q^s - 1\right], \\ & s_1 \in \mathcal{S}_{e,d_1} \cap \mathcal{S}^{work}, d_2 \in \mathcal{D}, \\ & s_2 \in \mathcal{S}_{e,d_2} \cap \mathcal{S}^{work} : d_1 \leq d_2, \\ & (s_1, d_1) \notin \mathcal{O}_{e,q}, (s_2, d_2) \in \mathcal{O}_{e,q}, \\ & 0 \leq \psi_{d_1,d_2,s_1,s_2} < q^b \end{aligned} \tag{31}$$

$$A_{e,q} + q^a \cdot (\tilde{x}_{e,d_1,s_1} + x_{e,d_2,s_2}) \leq 2q^a + \psi_{d_1,d_2,s_1,s_2}, \quad \forall e \in \mathcal{E}, q \in \mathcal{Q}_e, d_2 \in [q^e+1, q^e+q^a], \\ s_1 \in \mathcal{S}_{e,d_1} \cap \mathcal{S}^{work}, d_1 \in \mathcal{D}^{all}, \\ s_2 \in \mathcal{S}_{e,d_2} \cap \mathcal{S}^{work} : d_1 \leq d_2, \\ (s_1,d_1) \in \mathcal{O}_{e,q}, (s_2,d_2) \notin \mathcal{O}_{e,q}, \\ 0 \leq \psi_{d_1,d_2,s_1,s_2} < q^a \tag{32}$$

$$(q^b + q^a) \cdot \sum_{\substack{d \in \mathcal{D}^{all}, \\ s \in \mathcal{S}_{e,d} \cap \mathcal{S}^{work}: \\ (s,d) \in \mathcal{O}_{e,q}}} \tilde{x}_{e,d,s} \geq B_{e,q} + A_{e,q}, \quad \forall e \in \mathcal{E}, q \in \mathcal{Q}_e \tag{33}$$

Equations (31) ensure that if we assign employee $e \in \mathcal{E}$ to shift $s_2 \in \mathcal{S}^{work}$ during the period $q \in Q_e$ and also to shift $s_1 \in \mathcal{S}^{work}$ less than q^b days before the period, then the variable $B_{e,q}$ is bounded from above by the number of full days off between the two assignments. Similarly, Equations (32) ensure that if we assign employee $e \in \mathcal{E}$ to shift $s_1 \in \mathcal{S}^{work}$ during the period $q \in Q_e$ and also to shift $s_2 \in \mathcal{S}^{work}$ less than q^a days after the period, then the variable $A_{e,q}$ is bounded from above by the number of full days off between the two assignments. Finally, Equations (33) ensure that the variables $B_{e,q}$ and $A_{e,q}$ can only be positive if we assign a work shift to employee $e \in \mathcal{E}$ during period $q \in \mathcal{Q}_e$.

3.3 Ward constraints

The remaining constraints consider the interaction between employees required to make the ward function as a whole. We present these constraints in the following three categories: Section 3.3.1 presents coverage constraints and Section 3.3.2 presents constraints that balance the workload between employees. Finally, Section 3.3.3 presents constraints for chaperoning and employees that should be working together.

3.3.1 Coverage constraints

We let $\mathcal{C}$ denote the set of coverage constraints, ensuring that we assign employees to work shifts according to pre-defined staffing requirements. In general, we have two types of coverage constraints; H11, stating that we should assign a minimum number of employees to a given coverage, and H12, stating that we can not exceed a maximum number of employees for a given coverage. For a coverage $j \in \mathcal{C}$ and day $d \in \mathcal{D}$ we denote the minimum with $c_{j,d}^{min}$, and the maximum with $c_{j,d}^{max}$, and note that at least one needs to be defined for each coverage constraint. Without loss of generality we assume that every coverage $j \in \mathcal{C}$ has an associated shift set $\sigma_j \in \mathcal{Z}$.

We let β_j^{float} denote whether we allow ($\beta_j^{float} = 1$) or forbid ($\beta_j^{float} = 0$) float nurses for coverage $j \in \mathcal{C}$. For $\beta_j^{float} = 1$, we define variables $f_{j,d} \in \mathbb{N}_0$ denoting the number of float nurses we assign to coverage $j \in \mathcal{C}$ on day $d \in \mathcal{D}$, and extend it to be zero for $\beta_j^{float} = 0$. The objective function penalizes with weight $\omega_j^{float} \geq 0$ for $f_{j,d} > 0$.

Additionally, we define Ξ_j as the set of skills with the competences required for coverage $j \in \mathcal{C}$, where ξ_e represents the skill of employee $e \in \mathcal{E}^{all}$. We then express H11-H12 using Equations (34)-(35), respectively.

$$\sum_{\substack{e \in \mathcal{E}^{all}, \\ s \in \sigma_j: \\ \xi_e \in \Xi_j}} \alpha_{s,\sigma_j} \cdot x_{e,d,s} + f_{j,d} \geq c_{j,d}^{min}, \quad \forall j \in \mathcal{C}, d \in \mathcal{D} : c_{j,d}^{min} \text{ defined} \tag{34}$$

$$\sum_{\substack{e \in \mathcal{E}^{all}, \\ s \in \sigma_j: \\ \xi_e \in \Xi_j}} \alpha_{s,\sigma_j} \cdot x_{e,d,s} + f_{j,d} \leq c_{j,d}^{max}, \quad \forall j \in \mathcal{C}, d \in \mathcal{D} : c_{j,d}^{max} \text{ defined} \tag{35}$$

An additional constraint for the coverage is H13, stating that we can only schedule certain assignments in combination with other assignments. This constraint is similar to H10 from Section 3.2.5.

We let $L^{ward,both}_{d_1,d_2,\sigma_1,\sigma_2} \in \{0,1\}$ denote whether we only assign shift set $\sigma_1 \in \mathcal{Z}$ on day $d_1 \in \mathcal{D}^{all}$ in combination with assigning shift set $\sigma_2 \in \mathcal{Z}$ on day $d_2 \in \mathcal{D}^{all}$, and vice versa. Similarly, $L^{ward,one}_{d_1,d_2,\sigma_1,\sigma_2} \in \{0,1\}$ denotes whether we only assign shift set $\sigma_1 \in \mathcal{Z}$ on day $d_1 \in \mathcal{D}^{all}$ in combination with assigning shift set $\sigma_2 \in \mathcal{Z}$ on day $d_2 \in \mathcal{D}^{all}$, but not necessarily the other way around. We then express H13 using Equations (36)-(37).

$$\sum_{\substack{e\in\mathcal{E}^{all},\\ s\in\mathcal{S}_{e,d_1}\cap\sigma_1}} \tilde{x}_{e,d_1,s} = \sum_{\substack{e\in\mathcal{E}^{all},\\ s\in\mathcal{S}_{e,d_2}\cap\sigma_2}} \tilde{x}_{e,d_2,s}, \quad \forall d_1,d_2 \in \mathcal{D}^{all}, \sigma_1,\sigma_2 \in \mathcal{Z} : L^{ward,both}_{d_1,d_2,\sigma_1,\sigma_2} = 1 \tag{36}$$

$$\sum_{\substack{e\in\mathcal{E}^{all},\\ s\in\mathcal{S}_{e,d_1}\cap\sigma_1}} \tilde{x}_{e,d_1,s} \leq \sum_{\substack{e\in\mathcal{E}^{all},\\ s\in\mathcal{S}_{e,d_2}\cap\sigma_2}} \tilde{x}_{e,d_2,s}, \quad \forall d_1,d_2 \in \mathcal{D}^{all}, \sigma_1,\sigma_2 \in \mathcal{Z} : L^{ward,one}_{d_1,d_2,\sigma_1,\sigma_2} = 1 \tag{37}$$

In addition to the constraints presented in this section, we have one soft constraint related to the coverage constraints. The constraint is S12, stating that for some coverage constraints we should penalize for not reaching the maximum number of employees allowed. The objective function penalizes with weight $\omega^{nonmax}_j \geq 0$ for the difference between the maximum number of employees allowed and the actual number that is assigned.

3.3.2 Balancing constraints

We include two constraints that balance the work load between employees. First is S13, stating that we should balance the excess we assign to a given coverage throughout the rostering horizon. This constraint only considers coverage constraints that include all set of skills, i.e., we only balance the number of employees but not the distribution between different types of employees.

We let variables $c^+_j \geq 0$, and $c^-_j \geq 0$, denote the highest, and the lowest, number of employees exceeding $c^{min}_{j,d}$ assigned to coverage $j \in \mathcal{C}$ on any day $d \in \mathcal{D}$. If a coverage $j \in \mathcal{C}$ either has a maximum number of employees $c^{max}_{j,d}$ defined for all days $d \in \mathcal{D}$ or has a restriction regarding skills (i.e., $\Xi_j \neq \bigcup_{e\in\mathcal{E}^{all}} \xi_e$), then we set c^+_j and c^-_j to be zero. In other cases, we link the variables using Equations (38)-(39). The objective function penalizes with a weight $\omega^{bal} \geq 0$ for the difference between c^+_j and c^-_j.

$$\sum_{\substack{e\in\mathcal{E}^{all},\\ s\in\sigma_j}} \alpha_{s,\sigma_j} \cdot x_{e,d,s} + f_{j,d} - c^+_j \leq c^{min}_{j,d}, \quad \forall j \in \mathcal{C}, d \in \mathcal{D} : \Xi_j = \bigcup_{e\in\mathcal{E}^{all}} \xi_e \quad c^{min}_{j,d} \text{ defined, } c^{max}_{j,d} \text{ not defined} \tag{38}$$

$$\sum_{\substack{e\in\mathcal{E}^{all},\\ s\in\sigma_j}} \alpha_{s,\sigma_j} \cdot x_{e,d,s} + f_{j,d} - c^-_j \geq c^{min}_{j,d}, \quad \forall j \in \mathcal{C}, d \in \mathcal{D} : \Xi_j = \bigcup_{e\in\mathcal{E}^{all}} \xi_e \quad c^{min}_{j,d} \text{ defined, } c^{max}_{j,d} \text{ not defined} \tag{39}$$

Second is S14, stating that we should spread the assignments to some shifts evenly between the employees, relative to employee specific restrictions. We let $\tilde{M}^{total,ub}_{e,\sigma}$ be the extension of $M^{total,ub}_{e,\sigma}$ as defined with (40), where T^{target}_e is the total hours we should assign to employee $e \in \mathcal{E}$ (as defined in Section 3.2.3) and l^{gen} is the standard length of a shift type. Therefore, $\tilde{M}^{total,ub}_{e,d}$ represents a maximum number of assignments for all employees $e \in \mathcal{E}$ and shift sets $\sigma \in \mathcal{Z}$.

$$\tilde{M}^{total,ub}_{e,\sigma} = \begin{cases} M^{total,ub}_{e,\sigma} & \text{if } M^{total,ub}_{e,\sigma} \text{defined,} \\ \left\lfloor \frac{T^{target}_e}{l^{gen}} \right\rceil & \text{otherwise.} \end{cases} \tag{40}$$

We let $\mathcal{Z}^{spread} \subseteq \mathcal{Z}$ represent the set of shift sets where we prefer to spread the assignments evenly between the employees. We define variables $\zeta_\sigma^{max} \geq 0$, and $\zeta_\sigma^{min} \geq 0$, denoting the maximum, and minimum, assignments to $\sigma \in \mathcal{Z}^{spread}$ for any employee $e \in \mathcal{E}$ relative to $\tilde{M}_{e,\sigma}^{total,ub}$. We bind the variables using Equations (41)-(42), respectively. The objective function penalizes with a weight $\omega^{spread} \geq 0$ for the difference between ζ_j^+ and ζ_j^-.

$$\sum_{\substack{d\in\mathcal{D},\\ s\in\sigma}} \alpha_{s,\sigma} \cdot x_{e,d,s} - \tilde{M}_{e,\sigma}^{total,ub} \cdot \zeta_\sigma^{max} \geq 0, \quad \forall e \in \mathcal{E}, \sigma \in \mathcal{Z}^{spread} \tag{41}$$

$$\sum_{\substack{d\in\mathcal{D},\\ s\in\sigma}} \alpha_{s,\sigma} \cdot x_{e,d,s} - \tilde{M}_{e,\sigma}^{total,ub} \cdot \zeta_\sigma^{min} \leq 0, \quad \forall e \in \mathcal{E}, \sigma \in \mathcal{Z}^{spread} \tag{42}$$

3.3.3 Chaperoning constraints

This section includes two constraints related to employees in training (referred to as trainees). We define the *chaperoning shift*, $s_c \in \mathcal{S}$, as a shift type dedicated to planning and evaluating the progress of a trainee's education.

The first constraint for chaperoning is H14, stating that we can only assign the chaperoning shift to a chaperone when we also assign it to a corresponding trainee, and vice versa. We define the set of chaperones $\mathcal{E}^{chap} \subset \mathcal{E}^{all}$ and the set of trainees for each chaperone $\mathcal{E}_c^{train} \subseteq \mathcal{E}^{all}$, $\forall c \in \mathcal{E}^{chap}$. Then we express H14 using Equations (43).

$$\sum_{e\in\mathcal{E}_c^{train}} x_{e,d,s_c} = x_{c,d,s_c}, \quad \forall c \in \mathcal{E}^{chap}, d \in \mathcal{D} \tag{43}$$

The second constraint is S15, stating that we prefer some employees to work together. We define $\mathcal{E}^G \subset \mathcal{E} \times \mathcal{E}^{all}$ as the set of employees we prefer to assign work together and let $\mathcal{Z}^{together} \subseteq \mathcal{Z}$ denote the set of shift sets where we consider employees to be working together. We define variables $g_{e_1,e_2,d,\sigma} \in [0,1]$ denoting how much employees $(e_1, e_2) \in \mathcal{E}^G$ work together when assigned to $\sigma \in \mathcal{Z}^{together}$ on day $d \in \mathcal{D}$. We link the variables using Equations (44)-(45), which ensure that $g_{e_1,e_2,d,\sigma}$ can only be positive if we assign both employees $(e_1, e_2) \in \mathcal{E}^G$ to shift set $\sigma \in \mathcal{Z}$ on day $d \in \mathcal{D}$. The objective function then rewards with weight $\omega_{e_1,e_2}^{together} \leq 0$ for $g_{e_1,e_2,d,\sigma} > 0$.

$$g_{e_1,e_2,d,\sigma} \leq \sum_{s\in\sigma} \alpha_{s,\sigma} \cdot x_{e_1,d,s}, \quad \forall (e_1, e_2) \in \mathcal{E}^G, d \in \mathcal{D}, \sigma \in \mathcal{Z}^{together} \tag{44}$$

$$g_{e_1,e_2,d,\sigma} \leq \sum_{s\in\sigma} \alpha_{s,\sigma} \cdot x_{e_2,d,s}, \quad \forall (e_1, e_2) \in \mathcal{E}^G, d \in \mathcal{D}, \sigma \in \mathcal{Z}^{together} \tag{45}$$

3.4 Objectives

The objective function is a a linear combination of penalties and rewards related to the violation or satisfaction of soft constraints S1-S15, as presented with Equation (46). The weights for different constraints should represent their relative importance.

$$\begin{aligned} \min \quad & \sum_{(e,d,s)\in\mathcal{A}} \omega_{e,d,s}^{assign} \cdot x_{e,d,s} + \sum_{\substack{e\in\mathcal{E},\\ d\in\mathcal{D}^{all}}} \omega^{pf} \cdot v_{e,d} \\ & + \sum_{e\in\mathcal{E}} \left(\omega^+ \cdot t_e^+ + \omega^- \cdot t_e^- + \omega^{++} \cdot t_e^{++} + \omega^{--} \cdot t_e^{--}\right) \end{aligned}$$

$$
\begin{aligned}
&+ \sum_{\substack{e\in\mathcal{E},\\ w\in\mathcal{W}}} \omega^{week} \cdot \left(\lambda^{+}_{e,w} + \lambda^{-}_{e,w}\right) + \sum_{\substack{e\in\mathcal{E},\\ d\in\mathcal{D}^{all},\\ \sigma\in\mathcal{Z}}} \omega^{maxrow}_{e} \cdot \mu_{e,d,\sigma} \\
&+ \sum_{\substack{e\in\mathcal{E},\\ d\in\mathcal{D}^{all},\\ y\in\mathcal{Y}}} \omega^{pat}_{y} \cdot \pi_{e,d,y} + \sum_{\substack{e\in\mathcal{E},\\ d\in\mathcal{D}^{all}}} \omega^{diff} \cdot \kappa^{more}_{e,d} + \sum_{\substack{e\in\mathcal{E},\\ d\in\mathcal{D}}} \omega^{seqlen} \cdot \delta_{e,d} \\
&+ \sum_{\substack{e\in\mathcal{E},\\ d\in\mathcal{D}^{all},\\ \sigma\in\mathcal{Z}^{seq}_{e}}} \omega^{seqconsec} \cdot \phi^{count}_{e,d,\sigma} + \sum_{\substack{e\in\mathcal{E},\\ q\in\mathcal{Q}}} \left(\omega^{b} \cdot B_{e,q} + \omega^{a} \cdot A_{e,q}\right) + \sum_{\substack{j\in\mathcal{C},\\ d\in\mathcal{D}}} \omega^{float}_{j} \cdot f_{j,d} \\
&+ \sum_{\substack{j\in\mathcal{C},\\ d\in\mathcal{D}}} \omega^{nonmax}_{j} \cdot \left(c^{max}_{j,d} - \sum_{\substack{e\in\mathcal{E}^{all},\\ s\in S_{e,d}\cap\sigma_j}} \alpha_{s,\sigma_j} \cdot x_{e,d,s} \right) + \sum_{j\in\mathcal{C}} \omega^{bal} \cdot \left(c^{+}_{j} - c^{-}_{j}\right) \\
&+ \sum_{\sigma\in\mathcal{Z}^{spread}} \omega^{spread}_{\sigma} \cdot \left(\zeta^{max}_{\sigma} - \zeta^{min}_{\sigma}\right) + \sum_{\substack{(e_1,e_2)\in\mathcal{E}^{G},\\ d\in\mathcal{D},\\ \sigma\in\mathcal{Z}^{together}}} \omega^{together}_{e_1,e_2} \cdot g_{e_1,e_2,d,\sigma} \qquad (46)
\end{aligned}
$$

4 Clique generation

This section describes the automatic generation of cliques, which are used to reduce the size of the model. We represent each assignment as a triple (e, d, s) of an employee, a day and a shift type. We let $\mathcal{A}$ denote the set of feasible assignments for the rostering horizon and let $\mathcal{A}^{all}$ be the natural extension to previous rostering horizon.

To generate the set of conflict cliques Γ, we construct a *conflict graph* where each node represents an assignment $(e, d, s) \in \mathcal{A}^{all}$. For each conflicting pair of assignments $((e_1, d_1, s_1), (e_2, d_2, s_2)) \in \mathcal{A}^{all} \times \mathcal{A}$, we add an edge to connect the two corresponding nodes. We choose the set Γ to be a clique cover of the graph, meaning that for every conflicting pair of assignments at least one clique contains both assignments. To generate the cliques, we use the heuristic by Kou et al. (1978), producing a clique cover while trying to minimize $|\Gamma|$.

In addition to Γ, we also generate sets of work conflict cliques $\Gamma_{e,d}$ for each employee e and day d of the rostering horizon. We employ these cliques in constraints H3 and S8, both reducing the number of constraints needed and strengthening the formulation.

We generate these cliques using Algorithm 1, where we first iterate over all $\gamma \in \Gamma$ to create cliques $\Gamma_{e,d}$ with all work assignments in $\mathcal{A}_\gamma$ that correspond to $e \in \mathcal{E}$ and $d \in \mathcal{D}$. Afterwards, we try to reduce $|\Gamma_{e,d}|$ by iterating through $\Gamma^{desc}_{e,d}$, defined as a descending ordering of the cliques $\gamma \in \Gamma_{e,d}$ w.r.t. the number of conflicting assignments $|\mathcal{A}_\gamma|$. For $d \in \mathcal{D}^{pre}$, we define one $\gamma \in \Gamma_{e,d}$ such that $\mathcal{A}_\gamma$ only includes the assignment that employee $e \in \mathcal{E}$ got on day $d \in \mathcal{D}^{pre}$.

To further clarify the generation of work conflict cliques we provide a simple example in Table 1, where we generate work conflict cliques for day $d \in \lceil$. We let s_D, s_E and s_N denote day, evening and night shifts, respectively. Furthermore, s_c (the chaperoning shift) is categorized as a work shift, while s_{pf} and s_{off} are not. At last, $d-1$ indicates that the shift is positioned on the day before, and $d+1$ on the day after, as a clique $\gamma \in \Gamma$ does not have to be confined to a single day.

Table 1 presents the first step in generating the work conflict cliques, namely identifying the relevant cliques (γ_1-γ_4) and extracting the work assignments. In each column, the blue shift types make up an item in $\Gamma_{e,d}$ after line 6 (Algorithm 1).

We sort these cliques in a descending order, namely γ_3, γ_2, γ_1, γ_4. After initially marking all conflicting pairs of assignments as uncovered (lines 7-11), we go through the cliques in the descending order. In γ_3 we have four shift types, corresponding to six conflicting pairs that we mark as covered.

Algorithm 1 Generation of work conflict cliques

$\Gamma_{e,d} = \emptyset \quad \forall e \in \mathcal{E}, d \in \mathcal{D}$
for $\gamma \in \Gamma$ **do**
  for $e \in \mathcal{E}, d \in \mathcal{D}$ **do**
    $\Gamma_{e,d} \leftarrow \Gamma_{e,d} \bigcup \left\{(e', d', s) \in \mathcal{A}_\gamma \,|\, e' = e \wedge d' = d \wedge s \in \mathcal{S}^{work}\right\}$
  end for
end for
for $(e_1, d_1, s_1), (e_2, d_2, s_2) \in \mathcal{A}$ **do**
  if $\exists \gamma \in \Gamma : \{(e_1, d_1, s_1), (e_2, d_2, s_2)\} \subseteq \mathcal{A}_\gamma$ **then**
    $((e_1, d_1, s_1), (e_2, d_2, s_2))$ marked as *uncovered*
  end if
end for
for $e \in \mathcal{E}, d \in \mathcal{D}$ **do**
  for $\gamma \in \Gamma^{desc}_{e,d}$ **do**
    if $\exists (e_1, d_1, s_1), (e_2, d_2, s_2) \in \mathcal{A}_\gamma$: $((e_1, d_1, s_1), (e_2, d_2, s_2))$ marked *uncovered* **then**
      for $(e_1, d_1, s_1), (e_2, d_2, s_2) \in \mathcal{A}_\gamma$ **do**
        $((e_1, d_1, s_1), (e_2, d_2, s_2))$ marked as *covered*
      end for
    else
      $\Gamma_{e,d} \leftarrow \Gamma_{e,d} \backslash \{\gamma\}$
    end if
  end for
end for

Table 1: The first step in generating work conflict cliques.

γ_1	γ_2	γ_3	γ_4
s_E $(d-1)$	s_N $(d-1)$	s_{off}	$\mathbf{s_N}$
$\mathbf{s_D}$	s_{pf}	s_{pf}	s_{pf} $(d+1)$
$\mathbf{s_c}$	$\mathbf{s_D}$	$\mathbf{s_D}$	
	$\mathbf{s_c}$	$\mathbf{s_c}$	
	$\mathbf{s_E}$	$\mathbf{s_E}$	
		$\mathbf{s_N}$	

When moving through the remaining cliques, all pairs have been marked as covered and thus we can remove those cliques from the set of cliques. As a result, only one clique remains in the set of work conflict cliques, namely $\Gamma_{e,d} = \{\gamma_3\}$

5 Concluding remarks

This report has introduced an integer programming formulation for the nurse rostering problem under Danish legislation. The model has been developed in collaboration with practitioners, with the focus of practical applicability. Thus, the model is very comprehensive in the constraints that it includes and simultaneously flexible regarding their usage. Currently, the model has replaced manual scheduling in two wards in two Danish hospitals, and we aspire to spread it to a larger scale. As a supplement to this report, Böðvarsdóttir et al. (2019) presents 12 datasets from these two wards.

References

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LT Kou, LJ Stockmeyer, and CK Wong. Covering edges by cliques with regard to keyword conflicts and intersection graphs. *Communications of the ACM*, 21(2):135–139, 1978. ISSN 00010782. doi: 10.1145/359340.359346.

A Constraints

This Appendix summarizes the constraints presented in the report. The hard constraints are as follows:

H1: For each day of the rostering horizon, we should assign at least one shift type to each employee.

H2: For any conflicting pair of assignments (e.g., due to physical infeasibility), we can at most assign one of them.

H3: A protected day off needs to fulfill some minimum number of hours off between the adjacent work shifts.

H4: The total assignments for an employee to given shifts must be below an upper bound during the rostering horizon.

H5: The total number of assignments for an employee to given shifts should be fixed during the rostering horizon.

H6: The total number of assignments for an employee to given shifts should be fixed for a given week.

H7: The number of assignments in a row for an employee to given shifts must be below an upper bound.

H8: The assignments on consecutive day for an employee to certain shifts cannot exceed a maximum of hours.

H9: We forbid some patterns of shift assignments on consecutive days. This constraint is general, and each pattern can be represented as a series of shift sets $\langle \sigma_1, \ldots \sigma_{l_y} \rangle$, where assigning all shift sets in the given order on consecutive days is forbidden.

H10: For a given employee, we can only schedule some assignments in combination with another assignment. This constraint is general, and can be represented with two tuples $(d_1, \sigma_1), (d_2, \sigma_2)$ of days and shift sets, where the employee cannot be assigned to one tuple without the other (and sometimes vice versa).

H11: We require a minimum number of employees for a given coverage constraint, in correspondence to pre-defined staffing requirements.

H12: We allow a maximum number of employees for a given coverage constraint, in correspondence to pre-defined staffing requirements.

H13: For the ward as a whole, we can only assign schedule some assignments in combination with another assignment. This constraint is similar to H10, but is represented with two tuples $(e_1, d_1, \sigma_1), (e_2, d_2, \sigma_2)$ of employees, days and shift sets.

H14: We can only assign a chaperoning shift to a chaperone at the same time as a corresponding trainee, and vice versa.

Furthermore, the soft constraints are as follows:

S1: Every assignment has an associated weight, reflecting how desired or unwanted it is.

S2: The number of days between two contiguous sequences of protected days off for an employee should be below an upper bound.

S3: The hours we assign to an employee should not deviate from their contractual hours.

S4: The hours we assign to an employee each week should be close to their weekly target.

S5: The number of assignments in a row for an employee to given shifts should be below a given upper bound. This constraint is a soft version of H7.

S6: We penalize some patterns of shift assignments on consecutive days. This constraint is a soft version of H9.

S7: An employee should not exceed a maximum number of different shifts on consecutive days.

S8: The length of a *work sequence* (i.e., the number of consecutive work days) for an employee be above a lower bound.

S9: The number of work sequences containing given shifts for an employee should be above an upper bound on consecutive days.

S10: An employee working in a given period of the rostering horizon should get the time off on the days surrounding the period. Each period can be represented as an interval of days $[q^s, q^e]$ where we should assign q^b days off before and q^a days off after.

S11: We should minimize the number of float nurses assigned to meet the minimal staffing requirements.

S12: We penalize for not reaching the maximum number of employees allowed for some coverage constraints.

S13: We should balance the number of employees exceeding the minimum required throughout the rostering horizon.

S14: We should spread the assignments to some shifts evenly between the employees.

S15: We should assign some employees to work together.

B Notation

This Appendix summarizes the notation we have presented in the report, in the order that it appears.

Table 2: Sets used in compact model

Set	Description
$\mathcal{E}$	Employees for whom we should create a full schedule.
$\mathcal{E}^{all}$	All employees in the ward, including those that the model generally does not schedule.
$\mathcal{D}$	Days of the planning horizon.
$\mathcal{D}^{pre}$	Days from the previous horizon.
$\mathcal{D}^{all}$	All days $\mathcal{D}^{all} = \mathcal{D} \cup \mathcal{D}^{pre}$ that we consider,
$\mathcal{W}$	Weeks of the planning horizon.
$\mathcal{D}_w$	Days $\mathcal{D}_w \subseteq \mathcal{D}$ of week $w \in \mathcal{W}$.
$\mathcal{S}$	Shift types.
$\mathcal{S}^{work}$	Work shift types $\mathcal{S}^{work} \subset \mathcal{S}$.
$\mathcal{S}_{e,d}$	Shift types $\mathcal{S}_{e,d} \subseteq \mathcal{S}$ that are feasible for employee $e \in \mathcal{E}^{all}$ on day $d \in \mathcal{D}$, or the shift types that were assigned if $d \in \mathcal{D}^{pre}$.
$\mathcal{A}$	Feasible assignments for the planning horizon $\mathcal{A} = \mathcal{E}^{all} \times \mathcal{D} \times \mathcal{S}_{e,d}$.
$\mathcal{A}^{all}$	Feasible assignments including those that were assigned during previous horizon $\mathcal{A}^{all} = \mathcal{E}^{all} \times \mathcal{D}^{all} \times \mathcal{S}_{e,d}$.
$\mathcal{Z}$	Shift sets.
Γ	Conflict cliques.
$\mathcal{A}_\gamma$	Subset of assignments $\mathcal{A}_\gamma \subset \mathcal{A}$ belonging to conflict clique $\gamma \in \Gamma$.
$\Gamma_{e,d}$	Work conflict cliques for employee $e \in \mathcal{E}$ on day $d \in \mathcal{D}^{all}$
$\mathcal{Y}$	Patterns that we forbid or penalize (see constraints H9 and S6).
$\mathcal{Z}^{diff}$	Subset of shift sets $\mathcal{Z}^{diff} \subseteq \mathcal{Z}$ which we consider when restricting the assignments to different shift sets on consecutive days (see constraint S7).
$\mathcal{Z}^{seq}_e$	Subset of shift sets $\mathcal{Z}^{seq}_e \subseteq \mathcal{Z}$ where we restrict the number of sequences on consecutive days containing certain shifts (see constraint S9).
$\mathcal{Q}_e$	Periods where we prefer to assign surrounding days off if employee $e \in \mathcal{E}$ is working in the period (see constraint S10).
$\mathcal{O}_{e,q}$	Assignments $\mathcal{O}_{e,q} \subset \mathcal{S}^{work} \times \mathcal{D}^{all}$ that overlap with period $q \in \mathcal{Q}_e$ for employee $e \in \mathcal{E}$.
$\mathcal{C}$	Coverage constraints (see constraints H11 and H12).
Ξ_j	Positions with competences for coverage $j \in \mathcal{C}$.
$\mathcal{Z}^{spread}$	Subset of shift sets $\mathcal{Z}^{spread} \subseteq \mathcal{Z}$, for which we should spread the assignments evenly among the employees.
$\mathcal{E}^{chap}$	Chaperons $\mathcal{E}^{chap} \subset \mathcal{E}^{all}$.
$\mathcal{E}^{train}_c$	Trainees $\mathcal{E}^{train}_c \subset \mathcal{E}^{all}$ with chaperone $c \in \mathcal{E}^{chap}$.
$\mathcal{E}^G$	Pairs of employees $\mathcal{E}^G \subset \mathcal{E} \times \mathcal{E}^{all}$ that we should assign work together.
$\mathcal{Z}^{together}$	Subset of shift sets $\mathcal{Z}^{together} \subseteq \mathcal{Z}$ where employees assigned to the same set are considered working together.
$\Gamma^{desc}_{e,d}$	An descending ordering of the cliques $\gamma \in \Gamma_{e,d}$ with respect to the number of conflicting assignments $\lvert\mathcal{A}_\gamma\rvert$.

Table 3: Parameters used in compact model

Parameter	Description
$T^{start}_{s,d}$	The starting time of shift type $s \in \mathcal{S}$ on day $d \in \mathcal{D}^{all}$.
$T^{end}_{s,d}$	The ending time of shift type $s \in \mathcal{S}$ on day $d \in \mathcal{D}^{all}$.
$\alpha_{s,\sigma}$	The relation between shift set $\sigma \in \mathcal{Z}$ and shift type $s \in \mathcal{S} \cap \sigma$.
$x^{pre}_{e,d,s}$	A binary parameter denoting whether employee $e \in \mathcal{E}^{all}$ had shift type $s \in \mathcal{S}$ on day $d \in \mathcal{D}^{pre}$ in the previous schedule.

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Table 3: Parameters used in compact model

Parameter	Description
ρ^{full}	The minimum time required for full rest between assignments.
ρ^{red}	The minimum time required for reduced rest between assignments.
T^{prev}	A reference point in time from previous planning horizon.
T^{fut}	A reference point in time from upcoming planning horizon.
ρ_i^{pf}	The minimum number of hours off for $i \in \mathbb{N}$ consecutive protected days off.
υ^{pref}	The maximum distance between two contiguous protected days off.
$M_{e,\sigma}^{total,ub}$	The maximum number of assignments employee $e \in \mathcal{E}$ should get from shift set $\sigma \in \mathcal{Z}$ during the planning horizon.
$M_{e,\sigma}^{total,fix}$	The fixed number of assignments employee $e \in \mathcal{E}$ should get from shift set $\sigma \in \mathcal{Z}$ during the planning horizon.
$M_{e,\sigma,w}^{week,fix}$	The fixed number of assignments to shift set $\sigma \in \mathcal{Z}$ employee $e \in \mathcal{E}$ should get in week $w \in \mathcal{W}$.
T_e^{target}	The number of hours we should assign to employee $e \in \mathcal{E}$ during the planning horizon.
$T_{e,d,s}^{norm}$	The number of hours that assigning shift type $s \in \mathcal{S}$ on day $d \in \mathcal{D}$ counts towards the target for employee $e \in \mathcal{E}$.
N^+	The maximum positive deviation from the target hours before penalizing heavier in two-factor penalty.
N^-	The maximum negative deviation from the target hours before penalizing heavier in two-factor penalty.
$T_{e,w}^{week}$	The number of hours we should assign to employee $e \in \mathcal{E}$ during week $w \in \mathcal{W}$.
$M_{e,d,\sigma}^{row}$	The maximum number of days in a row from day $d \in \mathcal{D}^{all}$ we should assign employee $e \in \mathcal{E}$ to shift set $\sigma \in \mathcal{Z}$.
$\beta_{e,d,\sigma}^{row}$	A binary parameter denoting whether the maximum assignments in a row from day $d \in \mathcal{D}$ for employee $e \in \mathcal{E}$ to shift set $\sigma \in \mathcal{Z}$ should be a hard or a soft constraint.
$H_{e,d,\sigma}^{consec}$	The maximum number of hours we can assign employee $e \in \mathcal{E}$ to shift set $\sigma \in \mathcal{Z}$ on $D_{e,d,\sigma}^{consec}$ consecutive days from day $d \in \mathcal{D}$.
$D_{e,d,\sigma}^{consec}$	The number of consecutive days when restricting the number of hours assigned to employee $e \in \mathcal{E}$ from shift set $\sigma \in \mathcal{Z}$ from day $d \in \mathcal{D}$.
l_y	The length of pattern $y \in \mathcal{Y}$ in days.
β_y^{pat}	A binary parameter denoting whether pattern $y \in \mathcal{Y}$ should be a hard or a soft constraint.
$\Pi_{e,d,y}$	A binary parameter denoting whether pattern $y \in \mathcal{Y}$ should be active for employee $e \in \mathcal{E}$ from day $d \in \mathcal{D}$.
n^{diff}	The maximum number of different shift sets from $\mathcal{Z}^{diff}$ we should assign on D^{diff} consecutive days.
D^{diff}	The number of consecutive days when restricting the number of different shift sets.
m^{seq}	The minimum length of a work sequence in days.
$\chi_{e,\sigma}$	The maximum number of work sequences including shift set $\sigma \in \mathcal{Z}_e^{seq}$ we should assign to employee $e \in \mathcal{E}$ on $D_{e,\sigma}^{seq}$ consecutive days.
$D_{e,\sigma}^{seq}$	The number of consecutive days when restricting the number sequences including the same shift set.
$L_{e,d_1,d_2,\sigma_1,\sigma_2}^{emp,both}$	A binary parameter denoting whether we can only assign employee $e \in \mathcal{E}$ to shift set $\sigma_1 \in \mathcal{Z}$ on day $d_1 \in \mathcal{D}$ in combination with assigning him to $\sigma_2 \in \mathcal{Z}$ on day $d_2 \in \mathcal{D}$, and vice versa.
$L_{e,d_1,d_2,\sigma_1,\sigma_2}^{emp,one}$	A binary parameter denoting whether we can only assign employee $e \in \mathcal{E}$ to shift set $\sigma_1 \in \mathcal{Z}$ on day $d_1 \in \mathcal{D}$ in combination with assigning him to $\sigma_2 \in \mathcal{Z}$ on day $d_2 \in \mathcal{D}$, but not necessarily the other way around.
q^s	The first day of period $q \in \mathcal{Q}_e$ for employee $e \in \mathcal{E}$.
q^e	The last day of period $q \in \mathcal{Q}_e$ for employee $e \in \mathcal{E}$.
q^b	The number of days employee $e \in \mathcal{E}$ should have off before working in period $q \in \mathcal{Q}_e$.

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Table 3: Parameters used in compact model

Parameter	Description
q^a	The number of days employee $e \in \mathcal{E}$ should have off after working in period $q \in \mathcal{Q}_e$.
ψ_{d_1,d_2,s_1,s_2}	The number of full days between shift type $s_1 \in \mathcal{S}^{work}$ on day $d_1 \in \mathcal{D}$ and shift type $s_2 \in \mathcal{S}^{work}$ on day $d_2 \in \mathcal{D}$, where $d_1 \leq d_2$.
$c^{min}_{j,d}$	The minimum number of employees required for coverage $j \in \mathcal{C}$ on day $d \in \mathcal{D}$.
$c^{max}_{j,d}$	The maximum number of employees allowed for coverage $j \in \mathcal{C}$ on day $d \in \mathcal{D}$.
β^{float}_j	A binary parameter denoting whether a float nurse is allowed for coverage $j \in \mathcal{C}$.
ξ_e	The position of employee $e \in \mathcal{E}^{all}$.
$L^{ward,both}_{d_1,d_2,\sigma_1,\sigma_2}$	A binary parameter denoting whether we can only assign any employee to shift set $\sigma_1 \in \mathcal{Z}$ on day $d_1 \in \mathcal{D}$ in combination with assigning any employee to $\sigma_2 \in \mathcal{Z}$ on day $d_2 \in \mathcal{D}$, and vice versa.
$L^{ward,one}_{d_1,d_2,\sigma_1,\sigma_2}$	A binary parameter denoting whether we can only assign any employee to shift set $\sigma_1 \in \mathcal{Z}$ on day $d_1 \in \mathcal{D}$ in combination with assigning any employee to $\sigma_2 \in \mathcal{Z}$ on day $d_2 \in \mathcal{D}$, but not necessarily the other way around.
l^{gen}	The standard length of a shift type in hours.
$\tilde{M}^{total,ub}_{e,\sigma}$	An extension of $M^{total,ub}_{e,\sigma}$, defined for all $e \in \mathcal{E}$ and $\sigma \in \mathcal{Z}$

Table 4: Variables used in compact model

Variable	Description
$x_{e,d,s} \in \{0,1\}$	Denotes whether we assign employee $e \in \mathcal{E}^{all}$ to shift type $s \in \mathcal{S}_{e,d}$ on day $d \in \mathcal{D}$, i.e. whether we schedule assignment $(e,d,s) \in \mathcal{A}$.
$\tilde{x}_{e,d,s} \in \{0,1\}$	Extension of $x_{e,d,s}$ that includes the (fixed) assignments $x^{pre}_{e,d,s}$ from previous horizon.
$p_{e,d,i} \in \{0,1\}$	Denotes whether we assign employee $e \in \mathcal{E}$ to $i \in \mathbb{N}$ consecutive protected days off starting at day $d \in \mathcal{D}$.
$\tau^{last}_{e,d} \geq 0$	The number of hours from the reference point T^{prev} to the end of the last work shift employee $e \in \mathcal{E}$ has before day $d \in \mathcal{D}$.
$\tau^{next}_{e,d} \geq 0$	The number of hours from the start of the first work shift employee $e \in \mathcal{E}$ has after day $d \in \mathcal{D}$ to the reference point T^{fut}.
$v_{e,d} \in [0,1]$	Denotes whether employee $e \in \mathcal{E}$ has no protected day off from day $d \in \mathcal{D}^{all}$ to day $d + v^{allow} \in \mathcal{D}$, including both days.
$h_{e,w} \geq 0$	The number of hours we assign to employee $e \in \mathcal{E}$ during week $w \in \mathcal{W}$.
$t^+_e \in [0, N^+]$	The number of hours we assign to employee $e \in \mathcal{E}$ above the target hours, within a bound N^+.
$t^{++}_e \geq 0$	The number of hours we assign to employee $e \in \mathcal{E}$ above the target hours, exceeding N^+.
$t^-_e \in [0, N^-]$	The number of hours we assign to employee $e \in \mathcal{E}$ below the target hours, within a bound N^-.
$t^{--}_e \geq 0$	The number of hours we assign to employee $e \in \mathcal{E}$ below the target hours, exceeding N^-.
$\lambda^+_{e,w} \geq 0$	The number of hours we assign to employee $e \in \mathcal{E}$ during week $w \in \mathcal{W}$ above the weekly target.
$\lambda^-_{e,w} \geq 0$	The number of hours we assign to employee $e \in \mathcal{E}$ during week $w \in \mathcal{W}$ below the weekly target.
$\mu_{e,d,\sigma} \geq 0$	If $\beta^{row}_{e,d,\sigma} = 0$, this denotes whether we assign employee $e \in \mathcal{E}$ to shift set $\sigma \in \mathcal{Z}$ on more than $M^{row}_{e,d,\sigma}$ days in a row, starting on day $d \in \mathcal{D}^{all}$.
$\pi_{e,d,y} \in [0,1]$	If $\beta^{pat}_y = 0$, this denotes whether we violate pattern $y \in \mathcal{Y}$ starting on day $d \in \mathcal{D}^{all}$ for employee $e \in \mathcal{E}$.
$\kappa_{e,d,\sigma} \in [0,1]$	Denotes whether we assign employee $e \in \mathcal{E}$ to shift set $\sigma \in \mathcal{Z}^{diff}$ on any day $d' \in \{d, \dots d + D^{diff} - 1\}$ for $d \in \mathcal{D}^{all}$.

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Table 4: Variables used in compact model

Variable	Description
$\kappa^{more}_{e,d} \geq 0$	The number of shift sets from $\mathcal{Z}^{diff}$ we assign to employee $e \in \mathcal{E}$ on any day $d' \in \{d, \dots d + D^{diff} - 1\}$ for $d \in \mathcal{D}^{all}$ exceeding the maximum n^{diff}.
$\theta^{start}_{e,d} \in [0, 1]$	Denotes whether employee $e \in \mathcal{E}$ has a work sequence starting on day $d \in \mathcal{D}^{all}$.
$\theta^{end}_{e,d} \in [0, 1]$	Denotes whether employee $e \in \mathcal{E}$ has a work sequence ending on day $d \in \mathcal{D}^{all}$.
$\delta_{e,d} \geq 0$	The square of the violation of minimum work sequence when employee $e \in \mathcal{E}$ has a sequence below the minimum that ends on day $d \in \mathcal{D}$.
$\phi^{start}_{e,d,\sigma} \in [0, 1]$	Denotes whether employee $e \in \mathcal{E}$ starts a work sequence including shift set $\sigma \in \mathcal{Z}^{seq}_e$ on day $d \in \mathcal{D}^{all}$.
$\phi^{end}_{e,d,\sigma} \in [0, 1]$	Denotes whether employee $e \in \mathcal{E}$ ends a work sequence including shift set $\sigma \in \mathcal{Z}^{seq}_e$ on day $d \in \mathcal{D}^{all}$.
$\phi^{count}_{e,d,\sigma} \geq 0$	The number of work sequences exceeding $\chi_{e,\sigma}$ that we assign to employee $e \in \mathcal{E}$ on $D^{seq}_{e,\sigma}$ consecutive days from $d \in \mathcal{D}^{all}$, that include $\sigma \in \mathcal{Z}^{seq}_e$.
$B_{e,q} \geq 0$	The number of days off we assign to employee $e \in \mathcal{E}$ before period $q \in \mathcal{Q}$ if the employee is assigned work during the period.
$A_{e,q} \geq 0$	The number of days off we assign to employee $e \in \mathcal{E}$ after period $q \in \mathcal{Q}$ if the employee is assigned work during the period.
$f_{j,d} \in \mathbb{N}_0$	If $\beta^{float}_j = 1$, this denotes the number of float nurses assigned to coverage $j \in \mathcal{C}$ on day $d \in \mathcal{D}$.
$c^+_j \geq 0$	The highest number of employees exceeding $c^{min}_{j,d}$ that we assign to coverage $j \in \mathcal{C}$ on any day $d \in \mathcal{D}$.
$c^-_j \geq 0$	The lowest number of employees exceeding $c^{min}_{j,d}$ that we assign to coverage $i \in \mathcal{C}$ on any day $d \in \mathcal{D}$.
$\zeta^{max}_\sigma \geq 0$	The maximum assignments to shift set $\sigma \in \mathcal{Z}^{spread}$ for any employee, relative to employee specific restrictions.
$\zeta^{min}_\sigma \geq 0$	The minimum assignments to shift set $\sigma \in \mathcal{Z}^{spread}$ for any employee, relative to employee specific restrictions.
$g_{e_1,e_2,d,\sigma} \in [0, 1]$	Denotes how much employees $(e_1, e_2) \in \mathcal{E}^G$ assigned to shift set $\sigma \in \mathcal{Z}^{together}$ on day $d \in \mathcal{D}$ are working together.

Table 5: Weights used in compact model

Weight	Constraint	Description
$\omega^{assign}_{e,d,s}$	S1	Penalty or reward for assigning employee $e \in \mathcal{E}$ to shift $s \in \mathcal{S}$ on day $d \in \mathcal{D}$.
ω^{pf}	S2	Penalty for exceeding the maximum distance between two contagious protected days off.
ω^{+}	S3	Penalty for a positive deviation from the target hours, within the bound N^+.
ω^{++}	S3	Penalty for a positive deviation from the target hours, exceeding N^+ .
ω^{-}	S3	Penalty for a negative deviation from the target hours, within the bound N^-.
ω^{--}	S3	Penalty for a negative deviation from the target hours, exceeding N^-.
ω^{week}	S4	Penalty for deviation from the weekly targets.
ω^{maxrow}_e	S5	Penalty for exceeding the number of maximum assignments in a row for employee $e \in \mathcal{E}$.
ω^{pat}_y	S6	Penalty for assigning non-preferred pattern $y \in \mathcal{Y}$.
ω^{diff}	S7	Penalty for the number of shift sets exceeding the preferred we assign on D^{diff} consecutive days.
ω^{seqlen}	S8	Penalty for violating the minimum sequence.
$\omega^{seqconsec}$	S9	Penalty for exceeding the maximum number of sequences including a certain shifts on consecutive days.

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Table 5: Weights used in compact model

Weight	Constraint	Description
ω^b	S10	Reward for scheduling surrounding days off before a period where it is preferred.
ω^a	S10	Reward for scheduling surrounding days off after a period where it is preferred.
ω_j^{float}	S11	Penalty for assigning a float nurse to coverage $j \in \mathcal{C}$.
ω_j^{nonmax}	S12	Penalty for not hitting the maximum target for coverage $j \in \mathcal{C}$.
ω^{bal}	S13	Penalty for not balancing the excess personnel.
ω_σ^{spread}	S14	Penalty for not spreading assignments to shift set $\sigma \in \mathcal{Z}^{spread}$ relatively equally between the employees.
$\omega_{e_1,e_2}^{together}$	S15	Reward for assigning employees $(e_1, e_2) \in \mathcal{E}^G$ work together.

C Compact model

This Appendix gives a compact overview of the model, using the same numbers for each equation as in the report.

$$
\begin{aligned}
\min \quad & \sum_{(e,d,s)\in\mathcal{A}} \omega^{assign}_{e,d,s} \cdot x_{e,d,s} + \sum_{\substack{e\in\mathcal{E},\\ d\in\mathcal{D}^{all}}} \omega^{pf} \cdot v_{e,d} + \sum_{e\in\mathcal{E}} \left(\omega^{+} \cdot t^{+}_{e} + \omega^{-} \cdot t^{-}_{e} + \omega^{++} \cdot t^{++}_{e} + \omega^{--} \cdot t^{--}_{e}\right) \\
& + \sum_{\substack{e\in\mathcal{E},\\ w\in\mathcal{W}}} \omega^{week} \cdot \left(\lambda^{+}_{e,w} + \lambda^{-}_{e,w}\right) + \sum_{\substack{e\in\mathcal{E},\\ d\in\mathcal{D}^{all},\\ \sigma\in\mathcal{Z}}} \omega^{maxrow}_{e} \cdot \mu_{e,d,\sigma} + \sum_{\substack{e\in\mathcal{E},\\ d\in\mathcal{D}^{all},\\ y\in\mathcal{Y}}} \omega^{pat}_{y} \cdot \pi_{e,d,y} \\
& + \sum_{\substack{e\in\mathcal{E},\\ d\in\mathcal{D}^{all}}} \omega^{diff} \cdot \kappa^{more}_{e,d} + \sum_{\substack{e\in\mathcal{E},\\ d\in\mathcal{D}}} \omega^{seqlen} \cdot \delta_{e,d} + \sum_{\substack{e\in\mathcal{E},\\ d\in\mathcal{D}^{all},\\ \sigma\in\mathcal{Z}^{seq}_{e}}} \omega^{seqconsec} \cdot \phi^{count}_{e,d,\sigma} \\
& + \sum_{\substack{e\in\mathcal{E},\\ q\in\mathcal{Q}}} \left(\omega^{b} \cdot B_{e,q} + \omega^{a} \cdot A_{e,q}\right) + \sum_{\substack{j\in\mathcal{C},\\ d\in\mathcal{D}}} \omega^{float}_{j} \cdot f_{j,d} + \sum_{\substack{j\in\mathcal{C},\\ d\in\mathcal{D}}} \omega^{nonmax}_{j} \cdot \left(c^{max}_{j,d} - \sum_{\substack{e\in\mathcal{E}^{all},\\ s\in S_{e,d}\cap\sigma_j}} \alpha_{s,\sigma_j} \cdot x_{e,d,s} \right) \\
& + \sum_{j\in\mathcal{C}} \omega^{bal} \cdot \left(c^{+}_{j} + c^{-}_{j}\right) + \sum_{\sigma\in\mathcal{Z}^{spread}} \omega^{spread}_{\sigma} \cdot \left(\zeta^{max}_{\sigma} - \zeta^{min}_{\sigma}\right) + \sum_{\substack{(e_1,e_2)\in\mathcal{E}^{G},\\ d\in\mathcal{D},\\ \sigma\in\mathcal{Z}^{together}}} \omega^{together}_{e_1,e_2} \cdot g_{e_1,e_2,d,\sigma}
\end{aligned} \tag{46}
$$

$$\text{s.t.} \quad \sum_{s\in\mathcal{S}_{e,d}} x_{e,d,s} \geq 1, \qquad \forall e\in\mathcal{E}, d\in\mathcal{D} \tag{1}$$

$$\sum_{(e,d,s)\in\mathcal{A}_{\gamma}} \tilde{x}_{e,d,s} \leq 1, \qquad \forall \gamma\in\Gamma \tag{2}$$

$$\sum_{\substack{d'\in\mathcal{D}^{all},\\ i\in\mathbb{N}:\\ d-i<d'\leq d}} p_{e,d',i} = \tilde{x}_{e,d,s_{pf}}, \qquad \forall e\in\mathcal{E}, d\in\mathcal{D}^{all} : s_{pf}\in\mathcal{S}_{e,d} \tag{3}$$

$$\sum_{\substack{d'\in\mathcal{D}^{all},\\ i\in\mathbb{N}:\\ d-i<d'\leq d}} p_{e,d',i} = 0, \qquad \forall e\in\mathcal{E}, d\in\mathcal{D}^{all} : s_{pf}\notin\mathcal{S}_{e,d} \tag{4}$$

$$\sum_{j\in\mathbb{N}} p_{e,d,j} + \sum_{\substack{d'\in\mathcal{D}^{all},\\ i\in\mathbb{N}:\\ d'+i=d}} p_{e,d',i} \leq 1, \qquad \forall e\in\mathcal{E}, d\in\mathcal{D}^{all} \tag{5}$$

$$\tau^{last}_{e,d} - \sum_{\substack{s\in\mathcal{S}^{work}:\\ (e,d-1,s)\in\mathcal{A}_{\gamma}}} \left\lfloor T^{end}_{s,d-1} - T^{prev} \right\rceil_{2} \cdot \tilde{x}_{e,d-1,s} \geq 0, \qquad \forall e\in\mathcal{E}, d\in\mathcal{D}^{all}, \gamma\in\Gamma_{e,d} \tag{6}$$

$$\tau^{last}_{e,d} \geq \tau^{last}_{e,d-1}, \qquad \forall e\in\mathcal{E}, d\in\mathcal{D}^{all} \tag{7}$$

$$\tau^{next}_{e,d} - \sum_{\substack{s\in\mathcal{S}^{work}:\\ (e,d-1,s)\in\mathcal{A}_{\gamma}}} \left\lfloor T^{start}_{s,d+1} - T^{fut} \right\rceil_{2} \cdot \tilde{x}_{e,d+1,s} \leq \left\lfloor T^{fut} - T^{prev} \right\rceil_{2}, \qquad \forall e\in\mathcal{E}, d\in\mathcal{D}^{all}, \gamma\in\Gamma_{e,d} \tag{8}$$

$$\tau^{next}_{e,d} \leq \tau^{next}_{e,d+1}, \qquad \forall e\in\mathcal{E}, d\in\mathcal{D}^{all} : d+1\in\mathcal{D}^{all} \tag{9}$$

$$\tau^{last}_{e,d} + \sum_{i\in\mathbb{N}} \rho^{pf}_{i} \cdot p_{e,d,i} \leq \tau^{next}_{e,d}, \qquad \forall e\in\mathcal{E}, d\in\mathcal{D}^{all} \tag{10}$$

$$v_{e,d} + \sum_{\substack{d'\in\mathcal{D}:\\ d\leq d'<d+v^{pref}}} \tilde{x}_{e,d',s_{pf}} \geq 1, \qquad \forall e\in\mathcal{E}, d\in\mathcal{D}^{all} : d+v^{pref}\in\mathcal{D} \tag{11}$$

$$\sum_{\substack{d\in\mathcal{D},\\ s\in\mathcal{S}_{e,d}\cap\sigma}} \alpha_{s,\sigma}\cdot x_{e,d,s} \leq M^{total,ub}_{e,\sigma}, \quad \forall e\in\mathcal{E},\sigma\in\mathcal{Z}: M^{total,ub}_{e,\sigma} \text{ defined} \tag{12}$$

$$\sum_{\substack{d\in\mathcal{D},\\ s\in\mathcal{S}_{e,d}\cap\sigma}} \alpha_{s,\sigma}\cdot x_{e,d,s} = M^{total,fix}_{e,\sigma}, \quad \forall e\in\mathcal{E},\sigma\in\mathcal{Z}: M^{total,fix}_{e,\sigma} \text{ defined} \tag{13}$$

$$\sum_{\substack{d\in\mathcal{D}_w,\\ s\in\mathcal{S}_{e,d}\cap\sigma}} \alpha_{s,\sigma}\cdot x_{e,d,s} = M^{week,fix}_{e,\sigma,w}, \quad \forall e\in\mathcal{E},\sigma\in\mathcal{Z},w\in\mathcal{W}: M^{week,fix}_{e,\sigma,w} \text{ defined} \tag{14}$$

$$\sum_{\substack{d\in\mathcal{D}_w,\\ s\in\mathcal{S}_{e,d}}} T^{hour}_{e,d,s}\cdot x_{e,d,s} = h_{e,w}, \quad \forall e\in\mathcal{E},w\in\mathcal{W} \tag{15}$$

$$\sum_{w\in\mathcal{W}} h_{e,w} - t^{+}_{e} + t^{-}_{e} - t^{++}_{e} + t^{--}_{e} = T^{target}_{e}, \quad \forall e\in\mathcal{E} \tag{16}$$

$$h_{e,w} - \lambda^{+}_{e,w} + \lambda^{-}_{e,w} = T^{week}_{e,w}, \quad \forall e\in\mathcal{E},w\in\mathcal{W} \tag{17}$$

$$\sum_{\substack{d'\in\mathcal{D}^{all},\\ s\in\mathcal{S}_{e,d}\cap\sigma:\\ d\leq d'\leq d+M^{row}_{e,d,\sigma}}} \alpha_{s,\sigma}\cdot\tilde{x}_{e,d',s} - \mu_{e,d,\sigma} \leq M^{row}_{e,d,\sigma}, \quad \forall e\in\mathcal{E},d\in\mathcal{D}^{all},\sigma\in\mathcal{Z}: M^{row}_{e,d,\sigma} \text{ defined} \tag{18}$$

$$\sum_{\substack{d\leq d'\leq d+D^{consec}_{e,d,\sigma}-1,\\ s\in\sigma\cap\mathcal{S}_{e,d'}}} T^{hour}_{e,d',s}\cdot\tilde{x}_{e,d',s} \leq H^{consec}_{e,d,\sigma}, \quad \forall e\in\mathcal{E},d\in\mathcal{D}^{all},\sigma\in\mathcal{Z}: H^{consec}_{e,d,\sigma} \text{ and } D^{consec}_{e,d,\sigma} \text{ defined.} \tag{19}$$

$$\sum_{\substack{i\in\{1,\dots,l_y\},\\ s_i\in\sigma_i\cap\mathcal{S}_{e,d+i-1}}} \tilde{x}_{e,d+i-1,s_i} \leq l_y - 1 + \pi_{e,d,y}, \quad \forall e\in\mathcal{E},d\in\mathcal{D}^{all},y\in\mathcal{Y}: d+l_y-1\in\mathcal{D},\Pi_{e,d,y}=1 \tag{20}$$

$$\tilde{x}_{e,d',s} \leq \kappa_{e,d,\sigma}, \quad \forall e\in\mathcal{E},d\in\mathcal{D}^{all},\sigma\in\mathcal{Z}^{diff}, d'\in\{d,\dots,d+D^{diff}-1\}, s\in\sigma: d+D^{diff}-1\in\mathcal{D} \tag{21}$$

$$\sum_{\sigma\in\mathcal{Z}^{diff}} \kappa_{e,d,\sigma} - n^{diff} \leq \kappa^{more}_{e,d} \quad \forall e\in\mathcal{E},d\in\mathcal{D}^{all}: d+D^{diff}-1\in\mathcal{D} \tag{22}$$

$$\sum_{\substack{s\in\mathcal{S}^{work}:\\ (e,d,s)\in\mathcal{A}_\gamma}} \tilde{x}_{e,d,s} - \sum_{s\in\mathcal{S}^{work}} \tilde{x}_{e,d-1,s} \leq \theta^{start}_{e,d}, \quad \forall e\in\mathcal{E},d\in\mathcal{D}^{all},\gamma\in\Gamma_{e,d} \tag{23}$$

$$\sum_{\substack{s\in\mathcal{S}^{work}:\\ (e,d,s)\in\mathcal{A}_\gamma}} \tilde{x}_{e,d,s} - \sum_{s\in\mathcal{S}^{work}} \tilde{x}_{e,d+1,s} \leq \theta^{end}_{e,d}, \quad \forall e\in\mathcal{E},d\in\mathcal{D}^{all},\gamma\in\Gamma_{e,d}: d+1\in\mathcal{D} \tag{24}$$

$$\left(m^{seq}-d+d'-1\right)^2\cdot\left(\theta^{end}_{e,d}+\theta^{start}_{e,d'}-1\right) \leq \delta_{e,d}, \quad \forall e\in\mathcal{E},d\in\mathcal{D},d'\in\mathcal{D}^{all}: d-m^{seq}<d'\leq d \tag{25}$$

$$x_{e,d_2,s} + \theta^{start}_{e,d_1} - \sum_{\substack{d\in\mathcal{D}:\\ d_1\leq d<d_2}} \theta^{end}_{e,d} - 1 \leq \phi^{start}_{e,d_1,\sigma}, \quad \forall e\in\mathcal{E},d_1\in\mathcal{D}^{all},d_2\in\mathcal{D}^{all}, \sigma\in\mathcal{Z}^{seq}_{e},s\in\mathcal{S}_{e,d_2}\cap\sigma: d_1\leq d_2 \tag{26}$$

$$x_{e,d_1,s} + \theta^{end}_{e,d_2} - \sum_{\substack{d\in\mathcal{D}:\\ d_1<d\leq d_2}} \theta^{start}_{e,d} - 1 \leq \phi^{end}_{e,d_2,\sigma}, \quad \forall e\in\mathcal{E}, d_1\in\mathcal{D}^{all}, d_2\in\mathcal{D}^{all}, \sigma\in\mathcal{Z}^{seq}_e, s\in\mathcal{S}_{e,d_1}\cap\sigma : d_1\leq d_2 \tag{27}$$

$$\phi^{end}_{e,d,\sigma} + \sum_{\substack{d'\in\mathcal{D}^{all}:\\ d<d'\leq d+D^{seq}_{e,\sigma}}} \phi^{start}_{e,d',\sigma} - \chi_{e,\sigma} \leq \phi^{count}_{e,d,\sigma}, \quad \forall e\in\mathcal{E}, d\in\mathcal{D}^{all}, \sigma\in\mathcal{Z}^{seq}_e : d + D^{seq}_{e,\sigma}\in\mathcal{D} \tag{28}$$

$$\sum_{s\in\mathcal{S}_{e,d_1}\cap\sigma_1} \tilde{x}_{e,d_1,s} = \sum_{s\in\mathcal{S}_{e,d_2}\cap\sigma_2} \tilde{x}_{e,d_2,s}, \quad \forall e\in\mathcal{E}, d_1,d_2\in\mathcal{D}^{all}, \sigma_1,\sigma_2\in\mathcal{Z} : L^{emp,both}_{e,d_1,d_2,\sigma_1,\sigma_2} = 1 \tag{29}$$

$$\sum_{s\in\mathcal{S}_{e,d_1}\cap\sigma_1} \tilde{x}_{e,d_1,s} \leq \sum_{s\in\mathcal{S}_{e,d_2}\cap\sigma_2} \tilde{x}_{e,d_2,s}, \quad \forall e\in\mathcal{E}, d_1,d_2\in\mathcal{D}^{all}, \sigma_1,\sigma_2\in\mathcal{Z} : L^{emp,one}_{e,d_1,d_2,\sigma_1,\sigma_2} = 1 \tag{30}$$

$$B_{e,q} + q^b\cdot\left(\tilde{x}_{e,d_1,s_1} + x_{e,d_2,s_2}\right) \leq 2q^b + \psi_{d_1,d_2,s_1,s_2}, \quad \forall e\in\mathcal{E}, q\in\mathcal{Q}_e, d_1\in\left[q^s - q^b, q^s - 1\right], d_2\in\mathcal{D}, s_1\in\mathcal{S}_{e,d_1}\cap\mathcal{S}^{work}, s_2\in\mathcal{S}_{e,d_2}\cap\mathcal{S}^{work} : (s_1,d_1)\notin\mathcal{O}_{e,q}, (s_2,d_2)\in\mathcal{O}_{e,q}, d_1\leq d_2, 0\leq\psi_{d_1,d_2,s_1,s_2} < q^b \tag{31}$$

$$A_{e,q} + q^a\cdot\left(\tilde{x}_{e,d_1,s_1} + x_{e,d_2,s_2}\right) \leq 2q^a + \psi_{d_1,d_2,s_1,s_2}, \quad \forall e\in\mathcal{E}, q\in\mathcal{Q}_e, d_2\in\left[q^e + 1, q^e + q^a\right], d_1\in\mathcal{D}^{all}, s_1\in\mathcal{S}_{e,d_1}\cap\mathcal{S}^{work}, s_2\in\mathcal{S}_{e,d_2}\cap\mathcal{S}^{work} : (s_1,d_1)\in\mathcal{O}_{e,q}, (s_2,d_2)\notin\mathcal{O}_{e,q}, d_1\leq d_2, 0\leq\psi_{d_1,d_2,s_1,s_2} < q^a \tag{32}$$

$$(q^b + q^a)\cdot \sum_{\substack{d\in\mathcal{D}^{all},\\ s\in\mathcal{S}_{e,d}\cap\mathcal{S}^{work}:\\ (s,d)\in\mathcal{O}_{e,q}}} \tilde{x}_{e,d,s} \geq B_{e,q} + A_{e,q}, \quad \forall e\in\mathcal{E}, q\in\mathcal{Q}_e \tag{33}$$

$$\sum_{\substack{e\in\mathcal{E}^{all},\\ s\in\sigma_j:\\ \xi_e\in\Xi_j}} \alpha_{s,\sigma_j}\cdot x_{e,d,s} + f_{j,d} \geq c^{min}_{j,d}, \quad \forall j\in\mathcal{C}, d\in\mathcal{D} : c^{min}_{j,d} \text{ defined} \tag{34}$$

$$\sum_{\substack{e\in\mathcal{E}^{all},\\ s\in\sigma_j:\\ \xi_e\in\Xi_j}} \alpha_{s,\sigma_j}\cdot x_{e,d,s} + f_{j,d} \leq c^{max}_{j,d}, \quad \forall j\in\mathcal{C}, d\in\mathcal{D} : c^{max}_{j,d} \text{ defined} \tag{35}$$

$$\sum_{\substack{e\in\mathcal{E}^{all},\\ s\in\mathcal{S}_{e,d_1}\cap\sigma_1}} \tilde{x}_{e,d_1,s} = \sum_{\substack{e\in\mathcal{E}^{all},\\ s\in\mathcal{S}_{e,d_2}\cap\sigma_2}} \tilde{x}_{e,d_2,s}, \quad \forall d_1,d_2\in\mathcal{D}^{all}, \sigma_1,\sigma_2\in\mathcal{Z} : L^{ward,both}_{d_1,d_2,\sigma_1,\sigma_2} = 1 \tag{36}$$

$$\sum_{\substack{e\in\mathcal{E}^{all},\\ s\in\mathcal{S}_{e,d_1}\cap\sigma_1}} \tilde{x}_{e,d_1,s} \leq \sum_{\substack{e\in\mathcal{E}^{all},\\ s\in\mathcal{S}_{e,d_2}\cap\sigma_2}} \tilde{x}_{e,d_2,s}, \quad \forall d_1,d_2\in\mathcal{D}^{all}, \sigma_1,\sigma_2\in\mathcal{Z} : L^{ward,one}_{d_1,d_2,\sigma_1,\sigma_2} = 1 \tag{37}$$

$$\sum_{\substack{e\in\mathcal{E}^{all},\\ s\in\sigma_j}} \alpha_{s,\sigma_j}\cdot x_{e,d,s} + f_{j,d} - c^{+}_{j} \leq c^{min}_{j,d}, \quad \forall j\in\mathcal{C}, d\in\mathcal{D} : c^{min}_{j,d} \text{ defined,}$$

$$c_{j,d}^{max} \text{ not defined} \tag{38}$$

$$\sum_{\substack{e\in\mathcal{E}^{all},\\ s\in\sigma_j}} \alpha_{s,\sigma_j} \cdot x_{e,d,s} + f_{j,d} - c_j^{-} \geq c_{j,d}^{min}, \quad \forall j \in \mathcal{C}, d \in \mathcal{D} : c_{j,d}^{min} \text{ defined,}$$

$$c_{j,d}^{max} \text{ not defined} \tag{39}$$

$$\sum_{\substack{d\in\mathcal{D},\\ s\in\sigma}} \alpha_{s,\sigma} \cdot x_{e,d,s} - \tilde{M}_{e,\sigma}^{total,ub} \cdot \zeta_{\sigma}^{max} \geq 0, \quad \forall e \in \mathcal{E}, \sigma \in \mathcal{Z}^{spread} \tag{41}$$

$$\sum_{\substack{d\in\mathcal{D},\\ s\in\sigma}} \alpha_{s,\sigma} \cdot x_{e,d,s} - \tilde{M}_{e,\sigma}^{total,ub} \cdot \zeta_{\sigma}^{min} \leq 0, \quad \forall e \in \mathcal{E}, \sigma \in \mathcal{Z}^{spread} \tag{42}$$

$$\sum_{e\in\mathcal{E}_c^{train}} x_{e,d,s_c} = x_{c,d,s_c}, \quad \forall c \in \mathcal{E}^{chap}, d \in \mathcal{D} \tag{43}$$

$$g_{e_1,e_2,d,\sigma} \leq \sum_{s\in\sigma} \alpha_{s,\sigma} \cdot x_{e_1,d,s}, \quad \forall (e_1,e_2) \in \mathcal{E}^G, d \in \mathcal{D}, \sigma \in \mathcal{Z}^{together} \tag{44}$$

$$g_{e_1,e_2,d,\sigma} \leq \sum_{s\in\sigma} \alpha_{s,\sigma} \cdot x_{e_2,d,s}, \quad \forall (e_1,e_2) \in \mathcal{E}^G, d \in \mathcal{D}, \sigma \in \mathcal{Z}^{together} \tag{45}$$