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## FINITE ELEMENT MODEL COUPLED WITH LUMPED PARAMETER ELEMENTS

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This paper presents a numerical modelling technique for loudspeakers in which a finite element model is coupled with a lumped parameter model. Our approach includes the electric motor system of the loudspeaker in the numerical simulation without increasing the computational cost. This is achieved by considering the motor system as a lumped component, furthermore, parts of the mechanical system are also included in the lumped parameter model. Lastly, initial results are presented, where we illustrate the effect of including the electric motor system by comparing the proposed model with an acoustic-mechanical finite element model.

*Keywords:* Lumped parameter model; Acoustic-structure interaction; Loudspeaker design

### 1. Introduction

Moving-coil loudspeakers are a challenging task to model, due to the fact that many engineering areas of expertise are to be combined to create an accurate model of the loudspeaker. The physics that are to be computed are strongly coupled which means that the accuracy of the solution will depend on the modelling choices made.

One could include everything in a finite element (FE)-model and accept a large number of degrees of freedom (DOFs), which could cause infeasible computational times. This leads to the usage of much simpler models such as the lumped parameter models (LPMs). These types of models are 1 degree of freedom (DOF) models that represents the co-dependent physics in simple electric circuits.

The accuracy of these models decreases as the excitation frequency increases [4]. This is due to the fact that above the first mechanical resonance frequency the vibration pattern of the loudspeaker starts to deviate from the piston-like movement it exhibits below the first resonance frequency and thereby making the LPM inaccurate.

In-order to accurately capture the high frequency vibration pattern of a loudspeaker FE-models or similar computational tools are necessary. Furthermore, with a FE-model of the loudspeaker, structural optimization can be utilized [1].

The purpose of this paper is to develop a simplified generic numerical model of a loudspeaker that can be modelled with an in-house numerical framework, which in the future can be used for structural optimization of selected parts of the transducer, in this paper we focus on the diaphragm. This paper explicitly describes the necessary steps to implement and use this method. It should be noted that a similar method exist for users of the commercial simulation tool Comsol [5]. An optimization schemes is an iterative procedure, where the numerical is solved for each iteration, thus, it is desired to be able to solve it as efficiently as possible. Therefore, this paper proposes a hybrid method in which the advantages of LPM and FE-models are utilized.

## 2. Theory

This paper deals with two simplifications, the first is to consider the FE model problem as 2D-axisymmetric, this method is well known and thoroughly described in [3]. The second simplification is an approach in which the entire electrical motor system and parts of the mechanical components are lumped, thus only spending computational resources on modelling the diaphragm and the acoustic waves radiating from the diaphragm. The proposed method makes it possible to include the lumped system in the actual finite element matrices, thus no post-processing of the numerical results are required since all equations are solved at once.

Figure 1 shows a 2D sketch of a generic loudspeaker. The following sections will be used to establish the system of equations for both the FE-model and the lumped parameter model and describe how these two system can be combined.

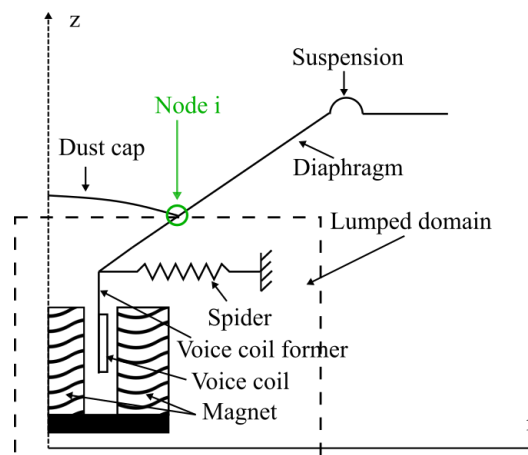


Fig. 1. sketch of the 2D axisymmetric model problem, everything inside the boxed region is contained in the lumped model, the rest is modelled with a FE-model.

### 2.1. Finite element model

The finite element model consists of a structural mechanics part and an acoustic domain. The governing equation for a mechanical elastic body subject to a time harmonic force is

$$(\mathbf{K} - \omega^2 \mathbf{M}) \mathbf{u} = \mathbf{f}, \quad (2.1)$$

where the capital bold letters indicate that it is a matrix, small bold letters are vectors,  $\omega$  is the excitation frequency,  $\mathbf{M}$  is the structural mass matrix,  $\mathbf{K}$  is the structural stiffness matrix  $\mathbf{u}$  is the solution vector containing the structural displacements and  $\mathbf{f}$  is the externally applied time harmonic force. Damping is included as a structural loss factor  $\eta$ , such that  $\mathbf{K} = \mathbf{K}(1 + j\eta)$ , where  $j$  is the imaginary number.

In the acoustic domain we solve the Helmholtz equation to obtain the nodal pressure. The loudspeaker is radiating into free-space. Perfectly Matched Layers (PMLs) are therefore used [2]. This results in a frequency dependent matrices in the acoustic domain

$$(\mathbf{K}_a(\omega) - \omega^2 \mathbf{M}_a(\omega)) \mathbf{p} = \mathbf{f}_a. \quad (2.2)$$

Here  $\mathbf{M}_a$  is the acoustic mass matrix,  $\mathbf{K}_a$  is the acoustic stiffness matrix,  $\mathbf{p}$  is the solution vector containing the nodal pressures and  $\mathbf{f}_a$  is the vector containing acoustic sources, which will be regarded as a zero vector for the remainder of this paper.

The coupling between the two domains is defined such that when the mechanical structure vibrates it acts as an acoustic source, furthermore the pressure acting on the mechanical structure is accounted for. This is included in the coupling matrix  $\mathbf{S}$ , the entire system of equations for the FE-model are [3]

$$\left( \begin{bmatrix} \mathbf{K} & -\mathbf{S}^T \\ \mathbf{0} & \mathbf{K}_a(\omega) \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \rho_a \mathbf{S} & \mathbf{M}_a(\omega) \end{bmatrix} \right) \begin{Bmatrix} \mathbf{u} \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{0} \end{Bmatrix}. \quad (2.3)$$

We choose to write eq. (2.3) in compact form

$$(\tilde{\mathbf{K}} - \omega^2 \tilde{\mathbf{M}}) \begin{Bmatrix} \mathbf{u} \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{0} \end{Bmatrix}. \quad (2.4)$$

### 2.2. Coupled system

As shown on Fig. 1. parts of the mechanical system and the entire electric motor system are lumped. The lumped systems are represented by simple electrical circuits as shown in Fig. 2. The left circuit corresponds to the electrical part of the motor system, in which  $e_g$  is the applied voltage,  $i_c$  is the current,  $R_E$  is the resistance of the voice coil,  $L_E$  is the inductance of the voice coil and  $e_c$  describes the back induced voltage from the mechanical system. In the expression for  $e_c$  the product of the force factor,  $Bl$ , and the velocity,  $\dot{u}$ , appears from the mechanical structure in the coupling node. The right circuit corresponds to the partly lumped mechanical system, the force on the mechanical system,  $F^{elek}$  is proportional to the electrical current,  $M_{lump}$  is the mass of the lumped components,  $R_{lump}$  is

the damping coefficient of the lumped system,  $C_{lump}$  is the compliance of the mechanical components and finally the finite element model is included as an equivalent impedance.

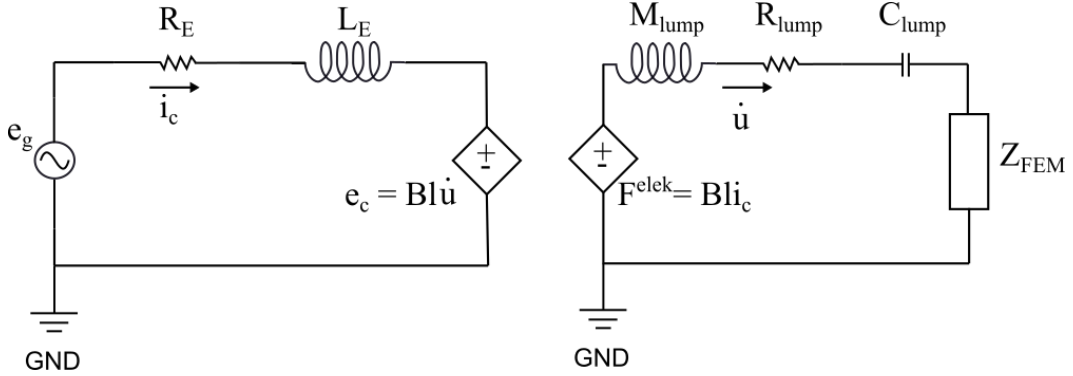


Fig. 2. The lumped system model, including the contribution from the FE-model as an impedance. The scalar equation cast in the frequency domain for the electrical motor system in Fig. 2.

$$e_g = R_E i_c + j\omega L_E i_c + j\omega B l u , \quad (2.5)$$

and this equation is to be included in the finite element equation.

The equation for the partly lumped mechanical system, including the FE-model is

$$B l i_c(t) = -\omega^2 M_{lump} u + j\omega R_{lump} u + \frac{1}{C_{lump}} u + j\omega u Z_{FEM} , \quad (2.6)$$

where  $Z_{FEM}$  is the impedance of the mechanical part of the finite element model, which is utilized to provide an expression for the mechanical force in the FE-model

$$Z_{FEM} = \frac{F_i}{j\omega u_i} \quad (2.7)$$

where sub  $i$  refers to the value in the coupling node  $i$  of a FE vector. Inserting into eq. (2.6) yields

$$B l i_c(t) = -\omega^2 M_{lump} u + j\omega R_{lump} u + \frac{1}{C_{lump}} u + F_i . \quad (2.8)$$

This addition to the FE-model in node  $i$  is most easily done by indicator matrices as shown in eq. (2.9). Let  $\mathbf{I}_{iz}$  be a zero matrix with size of the entire system of equations with an entry of value 1 in its diagonal corresponding to the DOF for the r-direction at node  $i$  and similarly, let  $\mathbf{I}_{iz}$  have one entry in the DOF for the z-direction of node  $i$ .  $\mathbf{J}_{iz}$  is a zero matrix with one unit entry in the bottom row in the column corresponding to the DOF for the z-direction in node  $i$ , finally  $\mathbf{L}$  is defined as a zero matrix with one unit entry in the last DOF of the diagonal.

$$\begin{aligned}
\mathbf{I}_{ir} &= \begin{bmatrix} 0 & \cdots & \mathbf{0} & \cdots & 0 \\ \vdots & \ddots & & & \vdots \\ \mathbf{0} & & i_{dofr} & & \mathbf{0} \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & \mathbf{0} & \cdots & 0 \end{bmatrix}, \mathbf{I}_{iz} = \begin{bmatrix} 0 & \cdots & \mathbf{0} & \cdots & 0 \\ \vdots & \ddots & & & \vdots \\ \mathbf{0} & & i_{dofz} & & \mathbf{0} \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & \mathbf{0} & \cdots & 0 \end{bmatrix}, \\
\mathbf{J}_{iz} &= \begin{bmatrix} 0 & \cdots & \mathbf{0} & \cdots & 0 \\ \vdots & \ddots & & & \vdots \\ \mathbf{0} & & \mathbf{0} & & \mathbf{0} \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & i_{dofz} & \cdots & 0 \end{bmatrix}, \mathbf{L} = \begin{bmatrix} 0 & \cdots & \mathbf{0} & \cdots & 0 \\ \vdots & \ddots & & & \vdots \\ \mathbf{0} & & \mathbf{0} & & \mathbf{0} \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & \mathbf{0} & \cdots & 1 \end{bmatrix}.
\end{aligned} \tag{2.9}$$

Adding one equation to the FE equations of eq. (2.4) and the indicator matrices from eq. (2.9) the system of equations for the hybrid FE-LPM model is

$$\begin{aligned}
&\left( \begin{bmatrix} \tilde{\mathbf{K}} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} + \frac{1}{C_{lump,x}} \mathbf{I}_{ir} + \frac{1}{C_{lump,y}} \mathbf{I}_{iz} + R_E \mathbf{L} - B \mathbf{L} \mathbf{J}_{iz}^T \right) \\
&\quad + j\omega (R_{lump} \mathbf{I}_{iz} + B \mathbf{L} \mathbf{J}_{iz} + L_E \mathbf{L}) \\
&\quad - \omega^2 \left( \begin{bmatrix} \tilde{\mathbf{M}} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} + M_{lump} \mathbf{I}_{iz} \right) \begin{Bmatrix} \mathbf{u} \\ \mathbf{p} \\ i_c \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \\ e_g \end{Bmatrix},
\end{aligned} \tag{2.10}$$

where  $\mathbf{J}_{iz}^T$  is the transpose of  $\mathbf{J}_{iz}$ . One should notice that the compliance has a contribution in the stiffness matrix in both r- and z-direction, whereas all other lumped parameters are assumed to only have a contribution in the direction of the applied force. The stiffness is assumed to be equal size, so in the next section  $C_{lump}$  refers to the compliance in both r- and z-direction.

### 3. Numerical Modelling

Figure 4 shows a sketch of the model problem that is used to test the implementation. Essentially, the sketch shows a flat piston ( $\Omega$ ) connected to a rigid wall via a rubber surround ( $\Omega_s$ ). The piston is connected to the lumped system in node  $i$ . The piston radiates sound into free field, which is the domain  $\Omega_F$ , the PMLs are present in  $\Omega_A$  regions.

Table 1 and 2 shows the data used in the simulations.

Tab. 1. Simulation data used in the FE-model

Mechanical FE-Data				
	E [GPa]	$\rho$ [ $kg/m^3$ ]	$\nu$	$\eta$
Surround	0.03	1400	0.45	0.25
Cone/Dustcap	10	1300	0.3	0.3

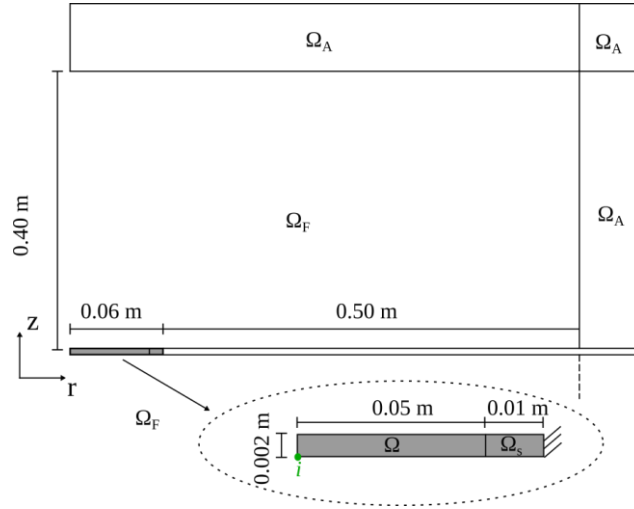


Fig. 3. Sketch of the model problem which is a piston with a radius of 5 cm connected to a rigid wall (infinite baffle) via a 1 cm rubber surround.

Tab. 2. Simulation data used in the LPM

Lumped Model Data						
$M_{lump}[kg]$	$C_{lump}[m/N]$	$Bl [Tm]$	$L_E[mH]$	$R_{lump}[N * s/m]$	$R_E[\Omega]$	$e_q[V]$
0.004	$369 * 10^{-6}$	7.0	1.22	1.85	3.3	2.83

#### 4. Numerical Results

This section will showcase the initial results obtained with the proposed FE-LP method. This section will highlight the benefits of the approach by comparing the results to a regular FE-model without LPM i.e. only a mechanical structure excited by a frequency independent time harmonic force. The compared data is obtained in node  $i$  as shown on Fig. 3.

Figure 4 compares the FE-LPM model to a FE-model excited by a constant time harmonic force. It can be observed that at the first mechanical resonance frequency there is a clear discrepancy between the two models. That is due to the fact that for the FE-LPM the total electrical impedance increases at the resonance frequency, this is opposed to the FE-model in which the force acting on the structure is constant which amplifies the resonant behavior of the structure.

The effect of the inductance is mostly observed for higher frequencies when using the FE-LPM. The displacement shows a 2<sup>nd</sup> order decay which means that the acceleration of the loudspeaker is almost constant at higher frequencies. For the model without a LPM we observe that the acceleration is not constant at higher frequencies which implies that the pressure in the far-field will increase with frequency. One could include this high frequency

behavior in a regular acoustic-mechanical FE-model by scaling the force inversely with frequency, this approach would however not capture the behavior around the mechanical resonances, to capture this effect one must consider the electrical motor system.

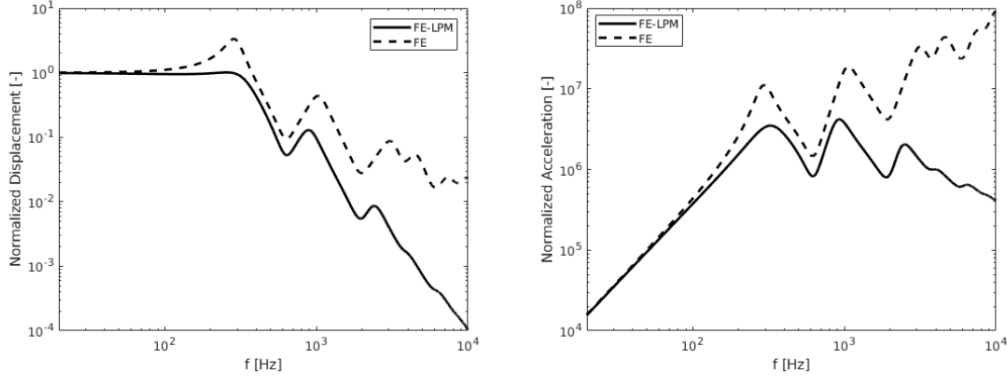


Fig. 4 (top left) normalized displacement for the FE-LPM and FE-model, (top right) normalized acceleration for the FE-LPM.

Another advantage of using the proposed FE-LPM model is that the impedance of the voice coil can now easily be obtained. This quantity can be used to compare the numerical model of a transducer or an optimized transducer with datasheets from bookshelf loudspeakers

$$Z_{VC} = \frac{e_g}{i_c}. \quad (4.1)$$

Figure 5 shows the impedance of the voice coil for the simple test example in Fig. 3, which essentially shows the influence of the loudspeaker on the electrical motor system. From the impedance curve it is possible to identify the first and second mechanical resonance,

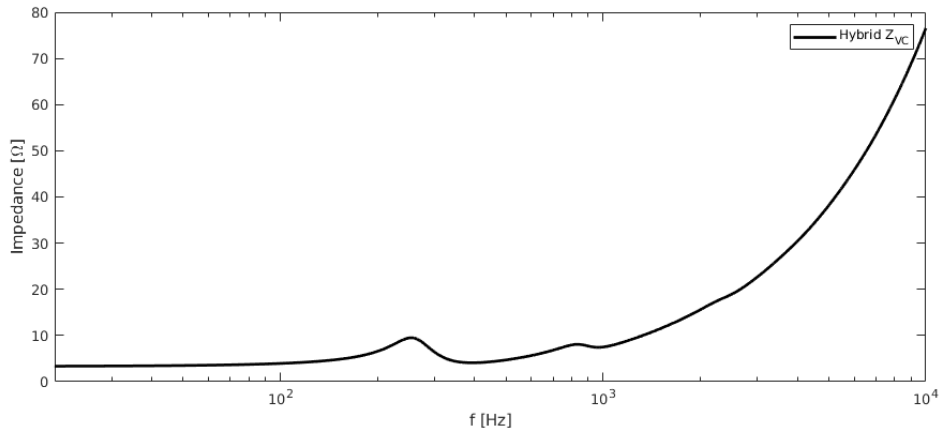


Fig. 5. The electrical impedance of the voice coil as a function of frequency.



furthermore, one can observe the impact of the inductance when the excitation frequency increases.

## 5. Conclusion

A method containing the advantages of FE and LPM has been exploited to create a hybrid FE-LPM numerical model. Modelling of the diaphragm with a FE-model ensures accuracy for high frequency excitations. By considering the electromagnetic system of the loudspeaker it is possible to fully capture the physics in the loudspeaker.

By including the LPM in the FE equations by the proposed manner the implementation can readily be used for optimization of the mechanical components of the loudspeaker, as a significant speed up is achieved by lumping parts of the transducer.

It is the intention of the authors to extend the LPM to include losses in the electric motor system, which implies that the resistance and the inductance of the voice coil should be frequency dependent. It should be noted that the frequency dependency also can be included in the mechanical components such that viscoelastic effects can be accounted for.

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