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APPLICATION OF A ROBUST MULTI-DATASET FREQUENCY DOMAIN DECOMPOSITION TO ESTIMATE THE GLOBAL MODAL PARAMETERS OF A HIGH-RISE BUILDING

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ABSTRACT

In multi-dataset identification with the classic Frequency Domain Decomposition (FDD) technique, the global mode shapes are obtained by identifying the mode shape parts of different patches separately. Then, the estimated mode shape parts are merged to form the global mode shapes by using the reference mode shape components. However, the presence of closely spaced or poorly excited modes can affect differently the estimation of the mode shape parts; hence, the resulting global mode shapes might not present well-defined modal configurations. This paper describes the application of a correlation spectrum based FDD technique to estimate the modal parameters of a high-rise building by identifying the global mode shape vectors in one step. This is expected to provide mode shape estimates with increased accuracy. To illustrate the performance of the correlation spectrum based FDD technique, the modal properties estimated with this technique is compared to those estimated with the classic FDD one.

Keywords: Modal Identification, Frequency Domain Decomposition, Multi-dataset Identification, Global Mode Shapes, High-rise Building

1. INTRODUCTION

The Frequency Domain Decomposition (FDD) technique is widely used in Operational Modal Analysis (OMA) to estimate the modal properties from vibration responses measured in output-only vibration tests. The classic FDD [1, 2] uses the data-driven spectra estimates - or the so-called *periodogram* or *Welch's* method [3], as it is known from scientific literature - as primary data to estimate the natural frequencies and their corresponding mode shapes. When dealing with vibration tests that involve multiple datasets, the FDD identification of the global mode shapes are carried out basically by identifying the modes shape parts from each dataset and merging the identified mode shape parts with aid of the reference mode shape components to form the estimated global mode shapes of the tested structure. A detailed description of the classic multi-dataset FDD can be found, for instance, in [4, 2].

It turns out that, in cases of non-stationary operating conditions, the tested structure tends to be excited differently over measured datasets, meaning that it is not always possible to identify the same physical modes in every measured dataset. Moreover, even in cases where the physical modes of the tested structure can be identified in all the datasets, the mode shape parts estimated from different datasets are influenced differently by the varying operating conditions, leading to not well defined global mode shape estimates. This problem is extensively discussed, for instance, in [5, 6]. In [7], an approach based on geometric projection of the individual mode shape parts is proposed to reduce the influence of non-stationary operating conditions on the estimates of the global mode shapes.

In this paper, novel approach is proposed to estimate the global mode shapes from multi-dataset output-only vibration data with FDD technique. The innovative aspects of such approach with regard to its classic FDD counterpart are related to the facts that: (i) rather than the classic *periodogram*, the correlation-driven spectral density estimates [3, 8] are used as primary data to identify the global mode shapes and the corresponding natural frequencies; and (ii) the identification of the global mode shapes is performed by taking the singular value decomposition of a global spectrum matrix which is assembled by stacking the spectrum matrix of the different datasets on the top of each other.

One of main differences of this approach with regard to the classic *periodogram*-driven FDD is that it forces the mode shape parts of the different datasets to be in the same subspace and, therefore, it is expected to provide clearer global mode shape estimates for closely spaced and/or poorly excited modes. The application of the proposed correlation spectrum based FDD is assessed and compared to the classic *periodogram*-driven FDD in the final part of the paper by means of a multi-dataset output-only vibration test of a high-rise building.

2. BACKGROUND THEORY

2.1. Correlation driven Spectrum Estimation

Rather than the *periodogram* spectrum (also known as "*Welch's*" spectrum), the multi-dataset FDD proposed in this paper uses the correlation spectrum (also known from literature as "*correlogram*" spectrum [3, 8]) as primary data. The non-parametric spectrum estimation from correlation function is accomplished by taking the discrete Fourier Transform of the correlation function

$$G_{yy}(\omega_f) = \sum_{k=-L}^{L} w_k R_k e^{-j\omega_f k\Delta t} \tag{1}$$

where $G_{yy}(\omega_f) \in \mathbb{C}^{N_o \times N_o}$ is the spectrum matrix computed at the discrete angular frequencies, ω_f, with N_o denoting the number of measured responses in the output-only vibration test; L is the maximum number of *time lags* at which the correlation matrices are estimated; w_k denotes a window function used to minimize the leakage and the statistical errors of the higher correlation *lags* on the resulting spectral matrix, Δt is the sampling interval and $j = \sqrt{-1}$ stands for the imaginary unit. The correlation-driven spectrum is widely used as primary data in parametric output-only modal identification, as seen, for instance, in [8], [9] and [10]. A detailed description of the non-parametric spectrum estimation technique based correlation functions can be found, for instance, in [3].

2.2. Proposed Multi-dataset FDD

In this section, a new approach is proposed to estimate the global mode shapes from test data recorded in multi-patch output-only vibration test with the FDD technique. This approach basically consists of: (1) estimating the spectral densities of each dataset from their corresponding correlation functions; (2) stacking all the estimated spectra on the top of each other without any prior scaling; (3) perform the identification; and (4) subsequently scale the mode shape parts to yield the global mode shape estimates for the tested structure. This merging strategy has been used in parametric mult-dataset modal identification [6] and [5]. In the following, however, a novel approach to post-scale the mode shape parts with the FDD technique is proposed. Underlying idea of this approach is to scale the singular vectors, rather than the mode shapes to yield the (scaled) global mode shapes of the tested structure. The first step to compute the scaled global mode shapes with this approach is to take the SVD of the global (unscaled) spectrum matrix, $G_u(\omega_f) \in \mathbb{C}^{N_s N_o \times N_{ref}}$, as

$$G_u(\omega_f) = \begin{bmatrix} G^1_{yy_{ref}}(\omega_f) \\ \vdots \\ G^{N_s}_{yy_{ref}}(\omega_f) \end{bmatrix} = U_{G_u}(\omega) S_{G_u}(\omega_f) V^H_{G_u}(\omega_f) \tag{2}$$

where $U_{G_u}(\omega_f) \in \mathbb{C}^{N_s N_o \times N_s N_o}$ is a matrix containing the singular vectors, $S_{G_u}(\omega_f) \in \mathbb{R}^{N_s N_o \times N_{ref}}$ a matrix containing the positive real valued singular values, and $V_{G_u} \in \mathbb{C}^{N_{ref} \times N_{ref}}$ a matrix containing unitary vectors, respectively, and $G^i_{yy_{ref}}(\omega_f) \in \mathbb{C}^{N_o \times N_{ref}}$ is the spectral matrix of the i^{th} dataset; N_{ref} and N_s denote the number of responses used as reference in each dataset and the number of datasets acquired in the output-only vibration test, respectively; and $(\bullet)^H$ stands for the Hermitian (conjugate transpose of a complex matrix). The unscaled singular vectors, $U_{G_u}(\omega_f) \in \mathbb{C}^{N_{ref} \times N_s N_o}$, can be partitioned into the following block matrices

$$U_{G_u}(\omega_f) = \begin{bmatrix} U_{yy_{ref},1}(\omega_f) & U_{yy_{ref\sim},1}(\omega_f) \\ \vdots & \vdots \\ U_{yy_{ref},N_s}(\omega_f) & U_{yy_{ref\sim},N_s}(\omega_f) \end{bmatrix} = \begin{bmatrix} U_{y_{ref}y_{ref},1}(\omega_f) & U_{y_{ref}y_{ref\sim},1}(\omega_f) \\ U_{y_{mov}y_{ref},1}(\omega_f) & U_{y_{mov}y_{ref\sim},1}(\omega_f) \\ \vdots & \vdots \\ U_{y_{ref}y_{ref},N_s}(\omega_f) & U_{y_{ref}y_{ref\sim},N_s}(\omega_f) \\ U_{y_{mov}y_{ref},N_s}(\omega_f) & U_{y_{mov}y_{ref\sim},N_s}(\omega_f) \end{bmatrix} \tag{3}$$

where $U_{yy_{ref},i}(\omega_f) \in \mathbb{C}^{N_o \times N_{ref}}$, $U_{yy_{ref\sim},i}(\omega_f) \in \mathbb{C}^{N_o \times (N_s N_o - N_{ref})}$, $U_{y_{ref}y_{ref},i} \in \mathbb{C}^{N_{ref} \times N_{ref}}$, $U_{y_{mov}y_{ref},i}(\omega_f)$ $\in \mathbb{C}^{N_m \times N_{ref}}$, $U_{y_{ref}y_{ref\sim},i}(\omega_f) \in \mathbb{C}^{N_{ref} \times (N_s N_o - N_{ref})}$, $U_{y_{mov}y_{ref\sim},i}(\omega_f) \in \mathbb{C}^{N_m \times (N_s N_o - N_{ref})}$ are sub-matrices of the global singular vectors, $U_{G_u}(\omega_f)$, that correspond to the i^{th} patch; and the sub-matrices $U_{y_{ref}y_{ref},i}$ and $U_{y_{mov}y_{ref},i}(\omega_f)$ are sub-matrices of $U_{yy_{ref},i}(\omega_f)$ and $U_{yy_{ref\sim},i}(\omega_f)$, with N_m denoting the number of roving sensors. Once the global unscaled singular vectors are partitioned according to eqs. 3, the global (scaled) singular vectors can be computed as

$$U_{G_s}(\omega_f) = \begin{bmatrix} U_{y_{ref}y_{ref},R}(\omega_f) \\ U_{y_{mov}y_{ref},1}(\omega_f) U_{y_{ref}y_{ref},1}(\omega_f)^{-1} U_{y_{ref}y_{ref},R}(\omega_f) \\ \vdots \\ U_{y_{mov}y_{ref},N_s}(\omega_f) U_{y_{ref}y_{ref},N_s}(\omega_f)^{-1} U_{y_{ref}y_{ref},R}(\omega_f) \end{bmatrix} \tag{4}$$

where $U_{y_{ref}y_{ref},R}(\omega_f)$ is given by

$$U_{y_{ref}y_{ref},R}(\omega_f) = \frac{1}{N_s}\sum_{i=1}^{N_s} U_{y_{ref}y_{ref},i}(\omega_f) \tag{5}$$

From this point onwards, it is straightforward to estimate the global (scaled) mode shapes which is done just like as in the classic FDD approach, i.e., by picking the picks in the singular values plot and selecting the corresponding global mode shape vectors, Φ_{G_s}, from the scaled singular vectors, $U_{G_s}(\omega_f)$. The proposed procedure previously described to scale the global mode shapes after the identification with the **FDD** technique is summarized in Fig. 1.

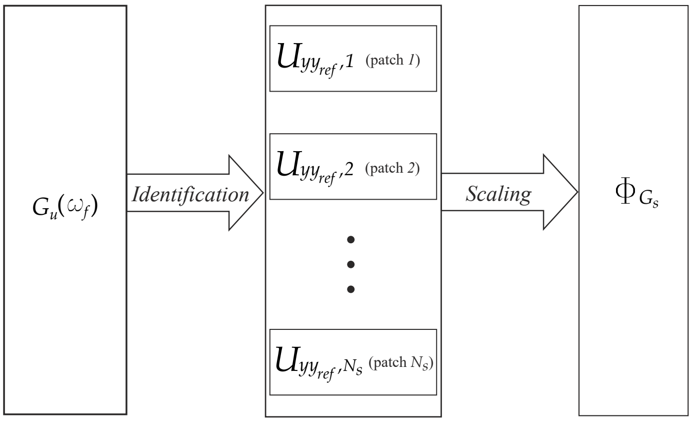


Figure 1 Identification flowchart of the scaling procedure with the proposed correlation spectrum based **FDD**

3. APPLICATION EXAMPLE

The application example consists of a multi-dataset vibration test performed on a Vancouver style high-rise building that normally resembles a podium-tower-shaped structure. This kind of structure is normally composed by a basement and low-rise buildings (2-3 storeys townhouses), and by a high-rise building that is often situated at the geometric centre of the basement. The tested Vancouver style building corresponds to the City Crest Tower depicted in Fig. 2, which is an 83.2 m tall RC building with a total of 32-storeys above the ground level. The high-rise building is located in Vancouver downtown (British Columbia, Canada), and was constructed from autumn 1992 to January 1994. The high-rise tower and the two-storey townhouses are connected structurally through the basement and two walkway bridges. The typical tower storey height is 2.6 m, except for the ground floor, where the height measures 4.0 m, in order to facilitate commercial activities.

Figure 2 The City Crest Tower, South-East (left) and South-West (right).

The City Crest Tower is situated in an earthquake-prone area with increased seismicity and its central concrete core has been designed to undertake the lateral forces induced by the earthquake excitations. The columns located in the perimeter of the slab is the essential part of the load-bearing gravity system. The columns are replaced by 204 mm thin walls from the 30th storey onwards. The construction method used for high-rise tower was the conventional in situ-cast of beams, columns, walls and slabs. The concrete core measures 10.3 m by 7.6 m (Fig. 3), and the wall thickness varies with the tower height, from 457 mm at the basement to 356 mm at the top floors. Figs. 2 and 3 illustrate how the floor plan layouts change significantly throughout the height of the tower. A detailed description of the City Crest Tower and its structural components can be found, for instance, in [11, 12] and the references cited therein.

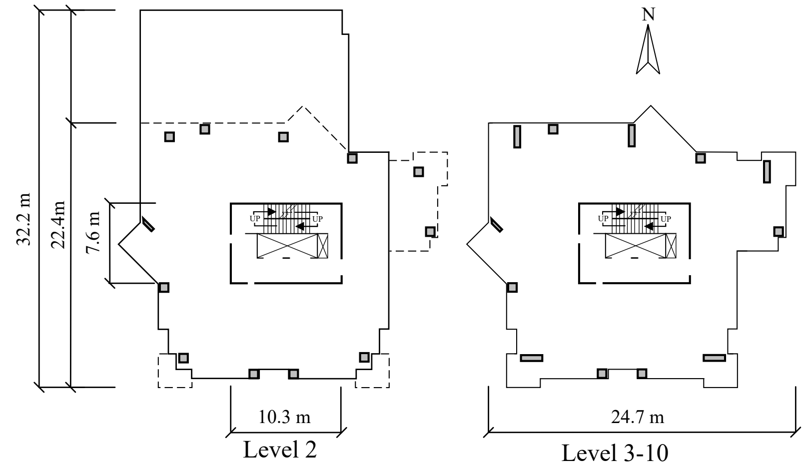


Figure 3 Level 2 (left) and level 3-10 (right) floor layout of the City Crest Tower.

3.1. Output-only Vibration Test

The vibration test campaign was performed over course of two days in the summer of 1993 with Kinematrix Inc. force balance accelerometers, each bolted to the concrete slab. The raw signals ware amplified, filtered and converted from analogue to digital on site.

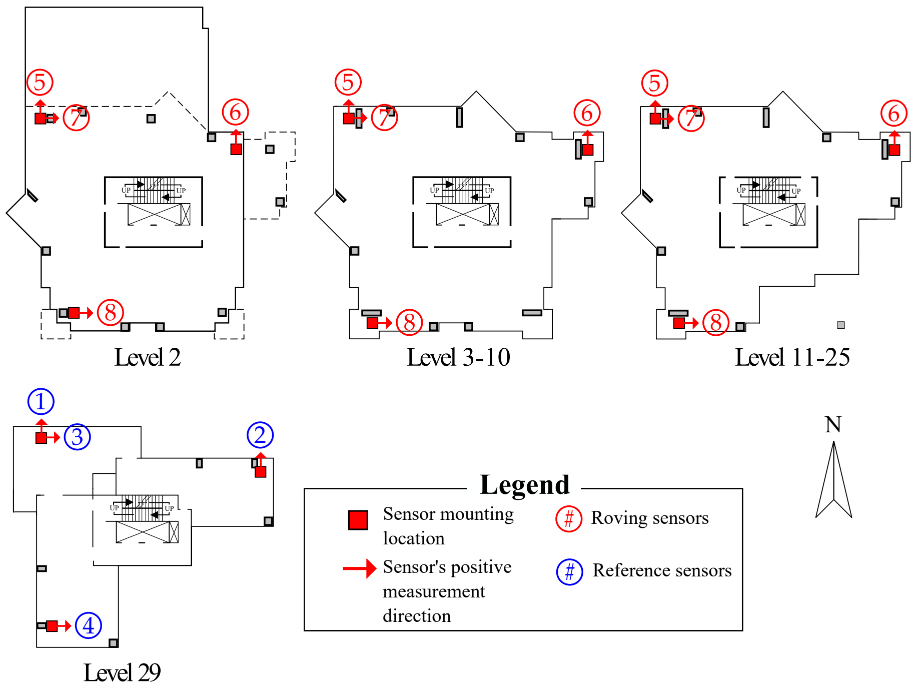
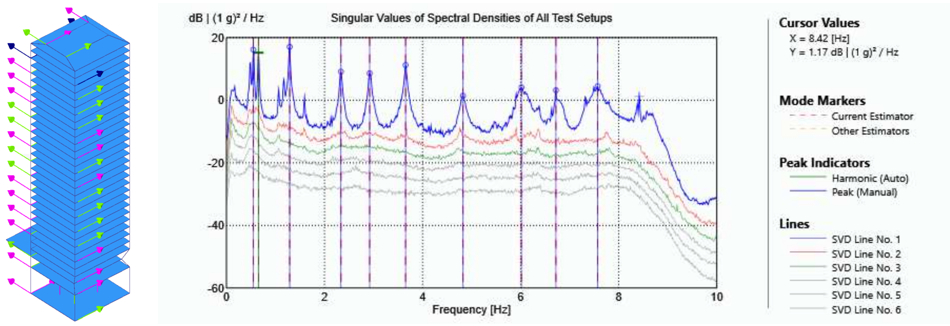


Figure 4 Sensors layout at different levels

Four roving and four reference sensors were included in the 19 different test setups adopted in the ambient vibration test, with the 29th floor being chosen as the reference floor. Fig. 4 illustrates the sensor locations throughout the 19 acquired datasets. The vibration responses in acceleration were acquired for 13 minutes and 39.2 seconds with a sampling rate of 40 Hz ensuring a Nyquist frequency of 20 Hz.

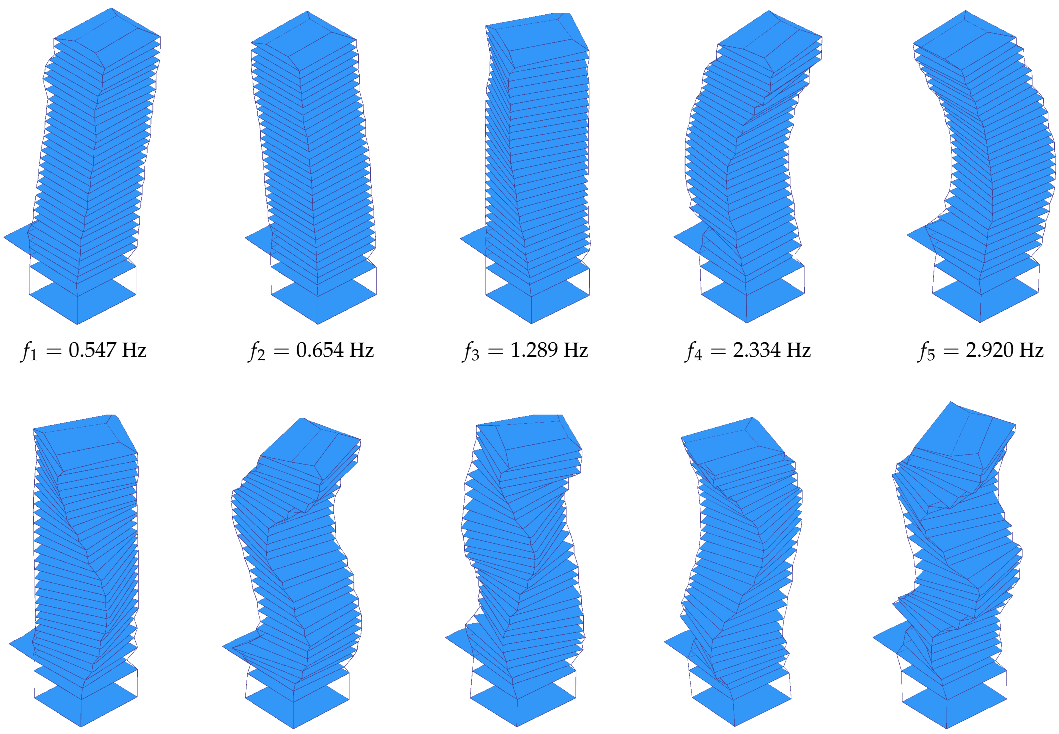
3.2. Identification with Classic FDD technique

The classic multi-dataset FDD technique [4] was applied to identify the modal properties of the City Crest Tower using the power spectral densities estimated with the *periodogram* approach from the measured acceleration responses. The identification was performed with the commercial software ARTeMIS Pro Modal (v4) [13], by making use of the geometrical model of the high-rise structure illustrated in Fig. 5a.



(a) **(b)**

Figure 5 Geometry and measured DOFs in ARTeMIS software for the Operational Modal Analysis (a) and the FDD singular values of spectral densities (b)



$f_1 = 0.547$ Hz $f_2 = 0.654$ Hz $f_3 = 1.289$ Hz $f_4 = 2.334$ Hz $f_5 = 2.920$ Hz

$f_6 = 3.652$ Hz $f_7 = 4.824$ Hz $f_8 = 6.016$ Hz $f_9 = 6.719$ Hz $f_{10} = 7.568$ Hz

Figure 6 Natural frequencies and corresponding mode shapes of the City Crest Tower identified with the classic multi-dataset FDD technique

The vibration raw data signals of the 19 acquired datasets were filtered with a band pass filter defined by a bandpass frequency range of 0.1-10.0 Hz. Next, the power spectral densities were computed from the filtered data signals with a resolution of 2048 frequency lines. Once these all the spectra were computed, they were used as primary data by the FDD technique to identify the natural frequencies and the corresponding global mode shapes of the City Crest Tower.

The variations of the singular values computed from spectral densities of the reference sensors over frequency lines are shown in Fig. 5. From this plot, 10 vibration modes were clearly identified by simply picking the peaks in the singular values plot. The identification of the natural frequencies with classic FDD consists of determining the frequencies associated to each picked peak. The identification of the global mode shapes, on the other hand, is basically carried out in two steps: (1) by extracting the mode shape parts from the singular vectors of each dataset; and (2) by re-scaling the extracted mode shape parts using the reference mode shape components to yield the global mode shapes associated to the picked peaks. The identification results obtained with the classic multi-dataset FDD for City Crest Tower are summarized in Fig. 6.

3.3. Identification Results with Proposed FDD technique

The application of the correlation spectrum based FDD described in section 2.2. is illustrated by also using of the vibration data of the City Crest Tower. To this end, the correlation-driven spectrum of each dataset was estimated by means of eq. (1) and used as primary data in the identification with the FDD technique. These power spectral densities were estimated from the filtered data signals with a resolution of the 2048 frequency lines by making use of an exponential window with a decay constant of 99.9% to minimize the leakage.

Once the correlation-driven spectrum matrices of all the 19 acquired datasets were estimated, they were stacked on the top of each other to form the global spectrum matrix, $G_u(\omega)$. Afterwards, the singular value decomposition was applied to this matrix. The obtained singular values at the 2048 frequency lines are plotted in Fig. 7. Similarly to the classic *periodogram*-driven FDD, the natural frequencies were simply identified by picking the peaks in the singular values plot. The identification of the global mode shapes, however, was accomplished by following the strategy described in section 2.2., i.e., by scaling the singular vectors corresponding to each identified natural frequencies.

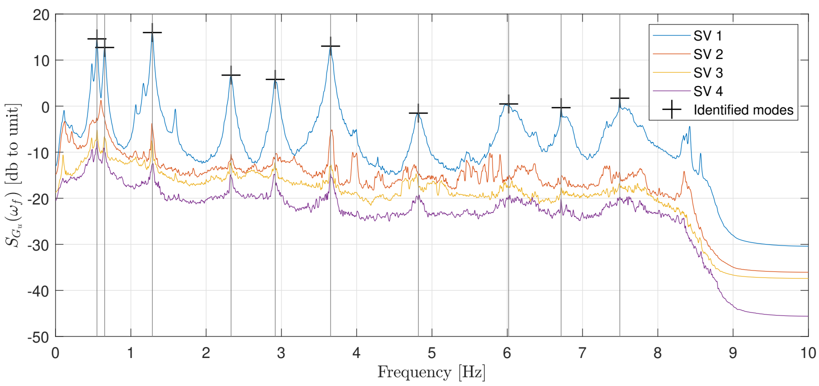


Figure 7 Singular values of the correlation-driven spectral densities

The results, in terms of natural frequencies and global mode shapes, obtained with the proposed multi-dataset FDD driven by correlation power spectrum densities is summarized in Figs. 8. Comparing these results to those shown in Fig. 6, one verifies that they are in very good agreement, both in terms of natural frequencies and global mode shapes, with the results obtained with classic multi-dataset FDD technique. The good agreement, in terms of global mode shapes, between the classic and proposed multi-dataset FDD approaches, is also corroborated by the Modal Assurance Criterion (MAC) values shown in Fig. 9.

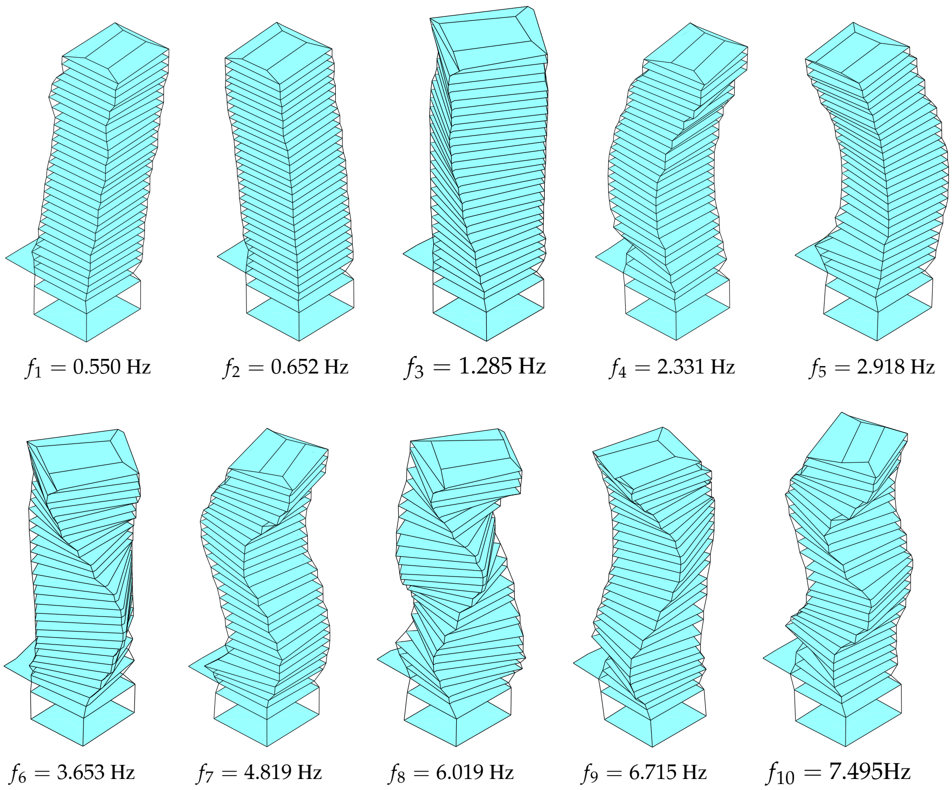


Figure 8 Natural frequencies and corresponding mode shapes of the City Crest Tower identified with the correlation spectrum FDD technique

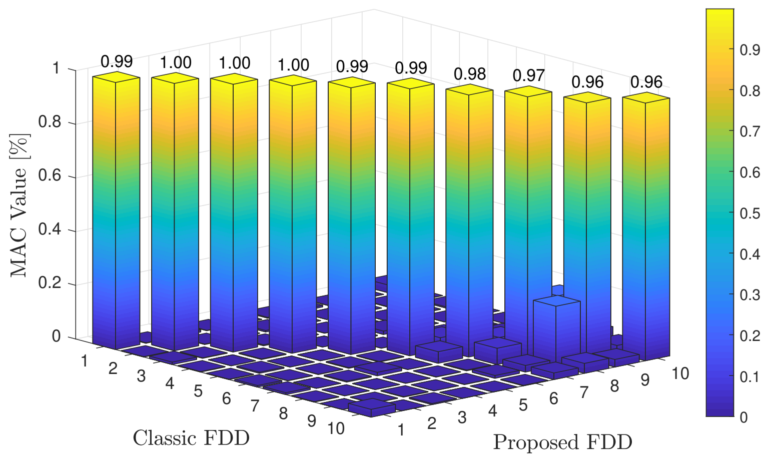


Figure 9 Modal Assurance Criterion (MAC) between the global mode shapes estimated with the classic multi-dataset FDD and those estimated with the proposed multi-dataset FDD driven by the correlation spectrum

4. CONCLUSIONS

In this paper, a novel multi-dataset FDD driven by correlation spectral densities is introduced. The underlying idea of this approach is to identify the natural frequencies and global mode shapes of vibration from a global spectral matrix containing the spectral matrices of all datasets stacked on the top of each other. In order to assess the robustness of proposed FDD, the technique was applied to the vibration data acquired in a multi-dataset output-only vibration test of a high rise building. By comparing the results obtained with such approach to those provided by the classic multi-dataset FDD for this particular application example, it was verified that the former are in very good agreement with the latter.

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REFERENCES

[1] Brincker, R. and Zhang, L. and Andersen, P. (2001) Modal identification of output-only systems using frequency domain decomposition. *Journal of Smart Materials and Structures*, 10, (pp. 441-445).

[2] Brincker, R. and Zhang, L. (2009) Frequency Domain Decomposition Revisited. *In proceedings of the the 5th International Operational Modal Analysis Conference (IOMAC)*, Portonovo, Italy.

[3] Mitra, S.K. (1998) Digital Signal Processing - A Computer-based Approach. *McGraw-Hill, New York, USA*.

[4] Brincker, R. and Ventura, C. (2015) Introduction To Operational Modal Analysis. *John Wiley and Sons*, 1st ed.

[5] Reynders, E., Magalhães, F., De-Roeck, G. and Cunha, A. (2009) Merging Strategies for Multi-Setup Operational Modal Analysis: Application to the Luiz I steel Arch Bridge. *In proceedings of the 27th International Modal Analysis Conference (IMACXXVII)*, 36(2), (pp. 115-127).

[6] Parloo, E., Guillaume, P. and Cauberghe, B. (2003) Maximum likelihood identification of non-stationary operational data. *Journal of Sound and Vibration*, 268(5) (pp. 971–991).

[7] Juul, M.Ø.Ø., Olsen, P., Balling, O., Amador, S.D.R. and Brincker, R. (2018) Comparison of two (geometric) algorithms for auto OMA. *In proceedings of International Modal Analysis Conference (IMAC) 2018*, Orlando, FL, USA.

[8] Peeters, B., Van-Der-Auweraera, H., Vanhollebeck, F. and Guillaume, P. (2007) Operational modal analysis for estimating the dynamic properties of a stadium structure during a football game. *Journal of Shock and Vibration*, 14(1), (pp. 283-303).

[9] Amador, S.D.R., Magalhães, F., Martins, N., Caetano, E. and Cunha, A. (2013) High Spatial Resolution Operational Modal Analysis of a Football Stadium Suspension Roof. *In proceedings of the the 5th International Operational Modal Analysis Conference (IOMAC)*, Guimarães, Portugal.

[10] Amador, S.D.R., Magalhães, F., Cunha, A. and Caetano, E. (2017) High spatial resolution modal identification of a stadium suspensionroof: Assessment of the estimates uncertainty and of modal contributions. *Journal of Engineering Structures*, 135, (pp 117-135).

[11] Pao, B. (1992) Structural drawings of the City Crest Tower. *Bogdonov Pao Associates Ltd*.

[12] Savnik, N.J., Katsanos, E.I., Amador, S.D.R., Ventura, C.E. and Brincker, R. (2018) Influence of diaphragm modelling on the dynamic performance of a reinforced concrete high-rise building. *In proceedings of the 28th International Conference on Noise and Vibration Engineering (ISMA)*, Leuven, Belgium.

[13] Structural Vibration Solutions A/S. (2017) ARTeMIS Pro Modal Version 4. Aalborg, Denmark, 2017. www.svibs.com.