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Load Frequency Control in Microgrids using target adjusted Model Predictive Control

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***Abstract*—Model Predictive Control (MPC) has been applied in multiple ways to the Load Frequency Control problem. In this paper we illustrate and compare a target adjusted MPC to a classical MPC formulation. The target adjusted approach is also posed as optimal control law. The target adjusted MPC is an alternative formulation that incorporates the system equilibrium into the control objective. The derived alternative controller can be used as alternative to classical MPCs.**

***Index Terms*—Predictive Control, Load Frequency Control, Active Power Control, Disturbance Observer, Real-Time Systems**

TABLE I
NOMENCLATURE. </> DENOTES UNSPECIFIED UNITS.

Symbol	Description	Unit
$u^{\star}$	Optimal input sequence (Control variables)	pu
$\bar{u}$	Input reference trajectory	pu
x	System state	/
$\hat{x}$	One–step prediction of x	/
$\tilde{x}$	State target	/
$\tilde{u}$	Control target	/
$\hat{d}$	Disturbance estimate	pu
$\hat{d}_e$	ϵ augmented disturbance estimate	/
y	Frequency deviation (Controlled variable)	pu
$\hat{y}$	One–step prediction of y	pu
y_m	Grid frequency measurement	Hz
ϵ	Integrated output error	Hz
$\epsilon_A, \epsilon_B, \epsilon_{B_d}$	Multiplicative model–plant errors w.r.t. corresponding model parameters	/
$\tilde{p}$	System equilibrium	/
w	State error: Wiener process	/
v	Measurement error: White noise process	Hz
θ	Model parameter vector	/
σ, h	Nonlinear model functions	/
ω	Standard Wiener process	/
T_s	Controller sampling time	s
N	Prediction horizon	/
$L_x,\ L_d$	Kalman gain w.r.t. states and w.r.t. disturbance	/
A, B, B_d, G, E, C, D	State Space System matrices	/
Φ_x	Free Markov parameters	/
Γ_u	Forced Markov parameters (controlled)	/
Γ_d	Forced Markov parameters (uncontrolled)	/
G, h	Objective inequality coefficients, bounds	/
K	Optimal Control feedback gain	/
K_{∞}	Lumped deduced disturbance gain	/
$K_{u,\infty}$	Disturbance gain to the system inputs	pu
$K_{x,\infty}$,	Disturbance gain to the system states	/
Δf	Frequency deviation with respect to nominal frequency	Hz
W_z	Output space precision penalization	/
$W_{\Delta u}$	Rate of movement penalization	/
$W_{\bar{u}}$	Input reference tracking penalization	/
β	Tuning term: Input reference tracking	/
H	Inertia based supply time	s
D	Load damping coefficient	/

I. INTRODUCTION

The increasing share of Renewable Energy Sources (RES) in the energy production mix is associated with considerable power production uncertainty. Remedies for the issue can be categorized into improvements of infrastructure and improvements of system controls. An approach enabling for the combination of the two categories is the concept of Microgrids (MGs) which makes it possible to unlock the flexibility required for integrating large shares of fluctuating RES. Coordinated control of controllable units within the MG enables for leveraging of synergies in an optimized manner, due to that the limited complexity of the confined system allows for the implementation of online optimization control strategies at a high degree of precision in the controls. An example for such control strategy is Model Predictive Control (MPC). The system operation and resilience can then be improved amongst others by inclusion of information of uncertain processes in the form of forecasts, for example with respect to the uncertainty associated with RES. Furthermore, this facilitates the use of the MG as a virtual and flexible power plant which enhances the possibilities of unlocking the flexibility required to comply with agreed market bids.

Designing controls for virtual power plants involves the setup of a control structure with consideration of complexity and system dynamics. Incorporation of predictions of stochastic processes introduce complexity due to the combinatorial explosion of manifold process outcomes. In contrary, fast system control loops require prompt decision making. Handling problem complexity in this setting and simultaneously providing sufficient sampling rates of well–posed control signals constitute two major challenges associated with the optimized control of MGs with high shares of RES. This problem complexity is usually handled by the setup of a temporal control hierarchy — the problem complexity is then managed by several specialized control routines [Sch78]. See Figure 1 as illustration of this principle. Control hierarchies are also considered in related areas such as ancillary services provision [DZ+18; Mad+14].

The control structure that evolved historically in the context of frequency control is split into a primary frequency control loop, a secondary frequency control loop and a tertiary frequency control loop. The primary controls hereby serve for the stabilization of the system frequency after a disturbance within delay of a few seconds. Secondary control initiates its compensation to such event in the magnitude of some seconds

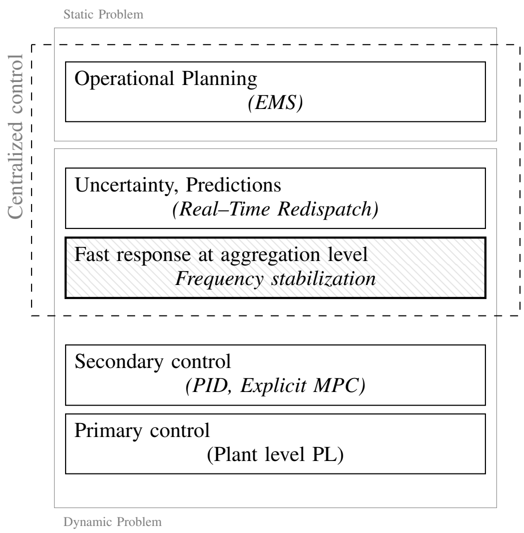

Fig. 1. Exemplary hierarchy of controllers in the centralized control scheme and their associated dominant focus. An operational planning layer takes long–term predictions and complex system requirements in the planning stage into account. The real–time redispatch adjusts the system operation to altered requirements during operation. Scope of this paper: The frequency stabilization problem at the aggregated system level (centralized controls).

to minutes. Tertiary control covers a longer temporal window [Bev14; Kun94].

TABLE II
ABBREVIATIONS.

RES	Renewable Energy Systems
LFC	Load Frequency Control
AGC	Automatic Generation Control
MG	Microgrid
MPC	Model Predictive Control
OC	Optimal Control
LQG	Linear Quadratic Gaussian regulator
CL	Closed Loop
MISO	Multiple–Input Single–Output
SDE	Stochastic Differential Equation
c0	Classic quadratic regulator formulation
c1	Target adjusted regulator formulation

[Per+17; Par+16; Han+14] are examples where the optimization problem for the aggregated MG system is posed with a sampling rate in the magnitude of multiple minutes. Consequently, they act on the tertiary control layer. Stochastic Programming formulations are typically used for such problems, due their capability to treat uncertainty associated with process predictions. [Pan+13b; Sha+09; Sha+17] provide literature overview over the topic of Load Frequency Control (LFC) and Automatic Generation Control (AGC). For the secondary control problem many solution approaches have been proposed, including optimal control [Cal72; Bar73; Fos+70; Zha+13] or adaptive controllers and robust controllers [Sir+10; Wan+94]. Often, these approaches are combined with state observers [Yam+86] and system identification techniques [Hai+00]. Proactive action in the LFC can improve its performance and MPC is a control strategy enabling for it. Examples can be found in [Ers+16; Ven+08; Shi+13]. In this paper we illustrate an MPC formulation for LFC based on [Pan+03; Gon+08]. To the best of our knowledge this controller has not yet been presented for the LFC problem. An Optimal Controller (OC) using the same approach is also presented. The control problem is stated here in the context of MG LFC but can be applied to different problems alike. This paper does not consider a thorough treatment of the underlying control theory — it can be obtained by consideration of, among others, [Pan+03; Gon+08].

II. Models

We consider the swing equation [Ben15; Bev14; Kun94] as our main state in the controller model:

$$\frac{d}{dt}\Delta f(t) = -\frac{D}{2H}\Delta f(t) + \frac{1}{2H}\Delta P_{\text{mech}}(t) \tag{1}$$

Δf denotes the frequency deviation from nominal frequency, D is the load damping coefficient and H the inertia based supply time. ΔP_{mech} is the power balance within the grid. The model maps the overall power imbalance to an angular frequency deviation from the nominal grid frequency by taking the approximated system inertia into account. The swing equation is a means to express the lumped system inertia and its parameters are both unknown and time–varying. Consequently adaptive estimation techniques [Ulb+14] should be applied in order to obtain a precise model for varying conditions.

The considered underlying processes are nonlinear and can be modeled using Stochastic Differential Equations (SDEs) such as formulated for example in [Kri+04]:

$$\mathrm{d}x_t = f(x_t, u_t, t, \theta)\mathrm{d}t + \sigma(u_t, t, \theta)\mathrm{d}\omega_t \tag{2}$$

$$y_k = h(x_k, u_k, t_k, \theta) + v_k \tag{3}$$

where t is the time variable; t_k are sampling instants; x_t is a vector of system states with the main state being the frequency deviation from nominal frequency Δf; u_t is a vector of input variables; y_k is the single output variable and equals the main state Δf; θ is a vector of parameters; f, σ and h are nonlinear functions; ω_t is a standard Wiener process and v_k is a white noise process with $v_k \in \mathcal{N}(0, S(u_k, t_k, \theta))$. See [Kri+04] for further clarifications and details of this formulation.

All used system models are linearized, enabling the application of linear control theory. The power balance is obtained by using lumped system models — groups of actors sharing dominant dynamics and requirements are hereby lumped together, resulting in a reduced order model. See in this context [Ers+16; Sax19]. The accepted loss in precision of this reduced model compared to the untreated linear system model is a design choice and has to be traded against the gained reduction in computational load in the optimization step. The linearized discrete time system model can be formulated as stated in Equation 4, see [Kri+04].

$$\frac{\mathrm{d}x_{t|j}}{\mathrm{d}t} = f_0 + A(x_t - x_j) + B(u_t - u_j) + G(d_t - d_j) + w_t \tag{4a}$$

$$y_t = Cx_t + e_t \tag{4b}$$

x is the system state; u the controlled system input, d the uncontrolled system input (disturbance). w and e are process and measurement noise respectively. This is a multiple–inputs single–output (MISO) system if more than one unit in the MG are considered.

III. Target adjusted DLQR

The feedback control law of the classical DLQR is commonly formulated as

$$u_k^\star = -K\hat{x}_{k|k} \tag{5}$$

Whereas the target adjusted DLQR can be stated as

$$u_k^\star = K(\hat{x}_{k|k} - \tilde{x}_{k|k}) - \tilde{u}_{k|k} \tag{6}$$

K is hereby found by solving the discrete–time algebraic Riccati equation [VD81; Lau78]. The equilibrium operating point of the system can be stated in terms of the input and state of the system $\tilde{p}$. $\tilde{p}$ can be linearly related to the filtered lumped disturbance $\hat{d}$:

$$\tilde{p}_{k|k} = \{\tilde{x}_{k|k}\ , \tilde{u}_{k|k}\} = K_\infty \hat{d}_{k|k} \tag{7}$$

K_∞ is a gain from a unit disturbance to one corresponding system equilibrium point. Scaling by the estimate $\hat{d}$ recovers another system equilibrium corresponding to $\hat{d}$. K_∞ can be obtained using a least–squares approximation, due to that the lumped system matrix M for the considered systems is non–symmetric in the MISO case:

$$\overbrace{\begin{bmatrix} A-I & B \\ C & 0 \end{bmatrix}}^{M} \overbrace{\begin{bmatrix} K_{x,\infty} \\ K_{u,\infty} \end{bmatrix}}^{K_\infty} = \begin{bmatrix} B_d \\ 0 \end{bmatrix} \tag{8}$$

This approach is outlined in [Mus+93; Pan+03] and related approaches have been applied e.g. in [Huu+10]. Notice that the system of equations denoted in Equation 8 has to be solved once for each model formulation. B_d hereby denotes the lumped modeled disturbance dynamics. Mismatch of B_d related to the real system dynamics lead to loss of controller performance. This loss of performance is then to be compensated for by application of appropriate robustness and adaptive control strategies which are not subject of this paper. For an ideal B_d, this regulator formulation achieves asymptotic stability in the controlled variable Δf.

A. Offset free frequency tracking

In order to drive the output $f \rightarrow \bar{f}$, where $\bar{f}$ is the goal frequency and $\bar{f} = f_{\text{nom}} + \Delta f$, the control law Equation 5 can be augmented to include the integrated offset

$$\epsilon_{k+1|k} = \epsilon_{k|k} + \hat{y}_{k|k} - \bar{y}_k \tag{9}$$

$\hat{y}_{k|k}$ here is the output of the system model using the state estimate $\hat{x}_{k|k}$ and $\bar{y}_k = \Delta\bar{f}_k$, the goal frequency deviation. The target Equation 7 then becomes

$$\tilde{p}_{k|k} = \{\tilde{x}_{k|k}\ , \tilde{u}_{k|k}\} = K_\infty(\epsilon_{k|k} + \hat{d}_{k|k}) \tag{10}$$

IV. Model Predictive Regulators

A. Classic quadratic objective

The classic quadratic reference tracking objective can be stated as such:

$$\begin{aligned} \min_{u,\ k}\ J_0 = &||\Phi_x \hat{x}_{k|k} + \Gamma_u u_k + \Gamma_d \hat{d}_{k|k} - \tilde{y}_k||^2_{W_z} \\ &+ \beta||u_k||^2_{W_{\Delta u}} \\ &+ (1-\beta)||u_k - \bar{u}_{k|k}||^2_{W_{\bar{u}}} \end{aligned} \tag{11}$$

Notice that we could neglect the control action regularization term $||u_k||^2_{W_{\Delta u}}$ in the case where we use a Kalman filter as smoothing component in the control loop. β is a tuning term used to gradually move the controller from regulatory behavior *without* input reference tracking ($\beta = 1$) to regulatory behavior *with* input reference tracking ($\beta = 0$). If offset–free control in the controlled variable Δf is aimed for, $\hat{d}$ can be augmented with the integrated error in the controlled variable ϵ. Then, d_e is used instead of d. In this case, $\tilde{y} = 0$. See for example [Huu+11].

$$\hat{d}_{e,k|k} = \hat{d}_{k|k} + \epsilon_k \tag{12}$$

$$\epsilon_{k+1|k} = \epsilon_{k|k} + \hat{y}_{k|k} - \bar{y}_k \tag{13}$$

Then

$$\tilde{y}_k = 0 \tag{14}$$

Alternatively, offset–free control can be achieved by using the following integrating term in the objective function:

$$\tilde{y}_{k+1|k} = \tilde{y}_{k|k} + \hat{y}_{k|k} - \bar{y}_k \tag{15}$$

A mismatch in the disturbance–associated model dynamics Γ_d can lead to loss of performance in the controlled variables.

B. Target adjusted quadratic objective

The target adjusted approach discussed in Section III can be applied in the MPC framework using

$$\begin{aligned} \min_{u,\ k}\ J_1 = &||\Phi_x(\hat{x}_{k|k} - \tilde{x}_{k|k}) + \Gamma_u(u_k - \tilde{u}_{k|k}) - \bar{y}_k||^2_{W_z} \\ &+ \beta||u_k - u^\star_{k-1}||^2_{W_{\Delta u}} \\ &+ (1-\beta)||u_k - \bar{u}_{k|k}||^2_{W_{\bar{u}}} \end{aligned} \tag{16}$$

Again β denotes a tuning term used to switch the controller from regulatory behavior without input reference tracking to regulatory behavior with input reference tracking. Hereby, the target Equation 7 is used. A similar regulator implementation can be found in [Ban+18].

C. Constraints

Hard input constraints and ramp–rate constraints for both MPCs can be formulated as

$$u_{\min,k} \leq u_k \leq u_{\max,k} \tag{17}$$
$$\Delta u_{\min,k} \leq \Delta u_k \leq \Delta u_{\max,k} \tag{18}$$
$$G_k \; u_k \leq h_k \tag{19}$$

[Jør+11] include examples of hard input constraint and ramp–rate constraint formulations.

V. State Observer

We estimate the residual $\hat{d}$ using a Kalman filter following the formulations given in [Pan+01; Pan+03]. The augmented system model with integrating disturbance estimate and filter equations is then given by:

$$\begin{bmatrix} \hat{x}_{k+1|k} \\ \hat{d}_{k+1|k} \end{bmatrix} = \begin{bmatrix} A & B_d \\ 0 & I \end{bmatrix} \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + \begin{bmatrix} L_x \\ L_d \end{bmatrix} (y_{m,k|k-1} - C\hat{x}_{k|k-1} - C_d\hat{d}_{k|k-1}) \tag{20}$$

where y_m is the local grid frequency measurement. This is the one–step predictor of both estimated state $\hat{x}$ and disturbance $\hat{d}$. Notice that $\hat{d}$ hereby is a lumped disturbance capturing any mismatch between desired and effective input–output relation. As improvement to this approach an Extended Kalman Filter can be used as stated for example in [Kri+04], in order to achieve faster convergence and to estimate the uncertainty P_d of the disturbance as well.

The performance of this filter affects the control performance. See [Huu+10] for additional applications.

VI. Predictions

The classical MPC formulation stated in Equation 11 incorporates the disturbance prediction sequence $\hat{d}_{k+N|k}$ via the disturbance impulse response coefficients Γ_d. The discussed target adjusted DLQR and MPC formulations, Equation 5 and Equation 16, do not have this capability. However, they can incorporate an expected future state of the disturbance process $\hat{d}_{k+j|k}$. The predictive performance of using this approach versus consideration of the full disturbance prediction sequence consequently is lower in most cases.

VII. Tuning

For the discussed controllers — as generally for OC and MPC — a multitude of tuning opportunities do exist. Tuning is then most commonly a recursive process in which the controlled parameters are adjusted such as to comply for example with network standards [Eur13].

Soft output constraints are one means to adjust the CL performance. In the context of the LFC problem, soft output constraints allow, for example, for tailoring of the objective in order to more aggressively aim for frequency stabilization outside of the specified frequency band. For soft output constraints and a MISO system, a set of $2N$ slack variables are introduced into the optimization problem, N being the prediction horizon in the presented control objectives. This leads to a computationally more demanding formulation. See e.g. [Gur+09].

Another important CL system property for the LFC problem is the capability to balance between variance in the controlled variable and variance of the control variables. The latter is often referred to as control effort. The control effort hereby is to be tuned in order to distribute the regulatory share and balance the wear and tear in the set of system actors, see e.g. [Huu+10]. As discussed in [Ers+16], the control effort tuning can be augmented to include economical weights — prices which inform the control law about how to distribute the control effort. Due to the mixing of operational and economical considerations in the resulting objective, this is a sub–optimal treatment of economical aspects.

When using controllers within a control hierarchy, input reference tracking is required. The corresponding tracking precision selection is adjusted by tuning of the penalization matrix $W_{\bar{u}}$. Both $W_{\Delta u}$ and $W_{\bar{u}}$ are hereby selected by some tuning method: Genetic Algorithms (GA) [Pan+13a] is an example for such method.

As generally in context of MPC, online system identification techniques, incorporation of adaptive measures and robustness considerations should be considered in order to compensate for unmodeled uncertainty. Such methods may be applied for the discussed target adjusted controller as well.

VIII. Simulations

Consider the test system as shown in Equation 21 and the constraints given in Equation 22. It consists of

- Swing equation parameterized with $D = 1.5$ and $H = 6.0$ as stated in Equation 1
- Actors (control inputs `U0, U1, U2` respectively):
  - Tie–line dynamics
  - Two generators including turbine and governor dynamics

then Control inputs are chosen based on the documented control laws. The system exposes the poles and zeroes illustrated in Figure 2. The dynamics are selected in order to reflect a simple multi–actor system with a reasonable range of dynamics, see Table III.

$$A = \begin{bmatrix} 0.7165 & 0.0265 & 0.0165 & 0.2146 & 0.0087 & 0.1125 & 0.0775 \\ 0.0 & 0.7165 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -0.0066 & -0.0825 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.066 & 0.8253 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -0.0036 & -0.0454 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0727 & 0.9085 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.1353 \end{bmatrix} \tag{21a}$$

$$B = \begin{bmatrix} 0.0297 & 0.0173 & 0.0089 \\ 1.7008 & 0.0 & 0.0 \\ 0.0 & 0.066 & 0.0 \\ 0.0 & 0.1397 & 0.0 \\ 0.0 & 0.0 & 0.0727 \\ 0.0 & 0.0 & 0.1464 \\ 0.0 & 0.0 & 0.0 \end{bmatrix} \tag{21b}$$

$$C = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \tag{21c}$$

$$D = \begin{bmatrix} 0.0 & 0.0 & 0.0 \end{bmatrix} \tag{21d}$$

$$\begin{bmatrix} -0.5 \\ 0.0 \\ 0.0 \end{bmatrix} \leq u \leq \begin{bmatrix} 0.5 \\ 0.2 \\ 0.3 \end{bmatrix} \tag{22a}$$

$$\begin{bmatrix} -0.02 \\ -0.01 \\ -0.01 \end{bmatrix} \leq \Delta u \leq \begin{bmatrix} 0.08 \\ 0.005 \\ 0.0025 \end{bmatrix} \tag{22b}$$

TABLE III
TEST SYSTEM STEP–RESPONSE CHARACTERISTICS OBTAINED USING MATLAB.

Step–response characteristic / unit	Swing	Tie–line	Gen. 1	Gen. 2
Rise time (s)	18.00	12.00	22.00	44.00
Settling time (s)	32.00	24.00	40.00	80.00
Settling min. (p.u.)	0.6119	0.9030	0.9086	0.9089
Settling max. (p.u.)	0.6667	1.00	1.00	0.9999
Overshoot (%)	0.00	0.00	0.00	0.00
Undershoot (%)	0.00	0.00	0.00	0.00
Peak (p.u.)	0.6667	1.00	1.00	0.9999
Peak time (s)	138.00	138.00	158.00	198

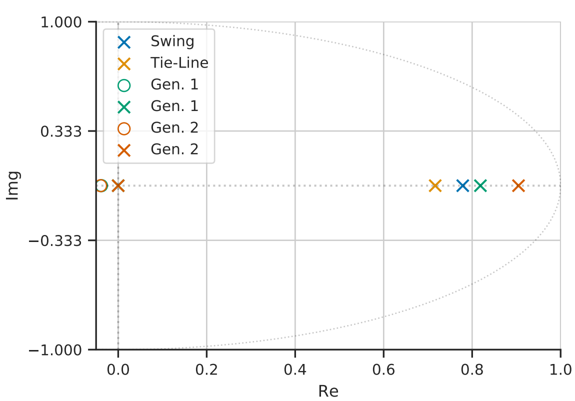

Fig. 2. Poles–Zeroes map of the considered test system (swing equation `Swing`, tie–line dynamics `Tie-Line`, generator dynamics 1 and 2, `Gen. 1` and `Gen. 2` respectively). Both `Gen. 1` and `Gen. 2` have a pole at the origin and a zero in the left–half plane.

All systems are discretized using the zero–order hold approach with sampling rate of 2 seconds. The sampling rate here is chosen arbitrarily. The online optimization problems are solved using the GUROBI solver [Gur18]. The solution to the least-squares problem is obtained using the LAPACK GELSD driver [And+99]. Load data time–series is obtained from [Ope]. The classical MPC stated in Section IV-A is in the following referred to as `c0`, the target adjusted MPC stated in Section IV-B is referred to as `c1`.

We aim to test for:

- Whether `c1` is capable of stabilizing the system frequency
- Whether constraints and penalization matrices have the desired effect on the CL system for `c1`
- How `c1` compares to `c0` in terms of sensitivity to model uncertainties

Notice that in all presented simulations no disturbance predictions are used. Given the presence of uncertainty compensated predictions, the response characteristics can be improved as a result to the proactive action of the two discussed controllers.

A. Disturbance rejection

`c1` is applied to the test system Equation 21 with control effort penalization $\tilde{W}_{\Delta u}$ as stated in Equation 23 below and without enabled input reference tracking term. $\tilde{W}_{\Delta u}$ are the first $n_u \times n_u$ elements of $W_{\Delta u}$.

$$\tilde{W}_{\Delta u} = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 0.05 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \tag{23}$$

The second generator (`U1`) resulting from $W_{\Delta u}$ is penalized the least and is consequently most active in terms of control effort. See Figure 3. This penalization is exemplary for a real application where the operation of selected units is to be maintained mostly constant. Unit `U1` saturates in the up–ramp event on the ramp–rate constraints and partly on the hard input bounds.

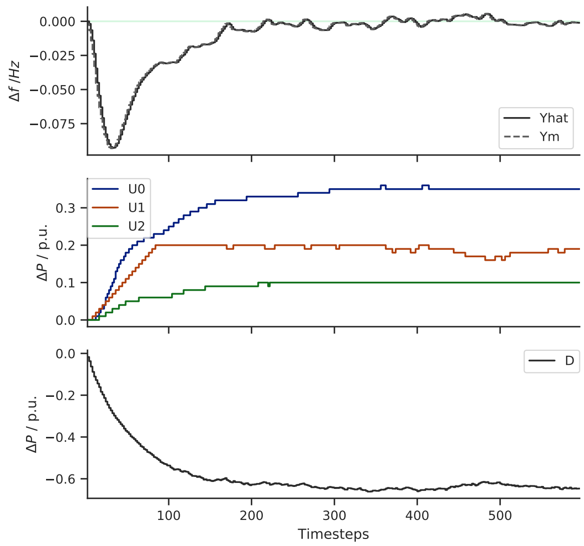

Fig. 3. Target adjusted MPC (`c1`). Uppermost graph: Frequency deviation from reference (main state and single output), middle graph: Control input deviations from reference, lowermost graph: disturbance deviation from reference (grid load). The control effort penalization depicted in Equation 23 lead to the stronger utilization of `U1`. Saturation on the ramp–rate constraints and hard bounds can be observed for `U1`.

B. Input reference tracking

Another important property for the LFC problem is the tracking of input–references, illustrated in Figure 4.

All three units receive individual input trajectories with step changes applied at two time instances throughout the experiment. As disturbance the trajectory depicted in Figure 3 is used. For the time–range `350-600` an ill–posed trajectory is given to the controller: the summation of active power injection requests does not match the actual load. The controlled variable is nevertheless maintained close to its reference due to the chosen tracking penalization term $W_{\bar{u}} = 1e^{-2}$. For increasing $W_{\bar{u}}$, the tracking precision increases. Consequently, the performance in Δf deteriorates stronger for ill–posed input reference trajectories with increasing $W_{\bar{u}}$.

C. Parametric system model mismatch

In Figure 5, `c0` and `c1` trajectories in Δf are compared for different multiplicative model–plant errors ϵ_A, ϵ_B, and ϵ_{B_d}. The disturbance trajectory given in Figure 3 without noise term is used to excite the system. It is to be noted that the sensitivity in the control effort penalization term $W_{\Delta u}$ differs for the two controllers. Additional differences in the sensitivity to available means to tuning apply. Accordingly, trajectories should be considered and compared in qualitative rather than quantitative manner. Furthermore, only a selection of lumped multiplicative parametric mismatches are considered here in order to exhibit some differences in the two considered controllers. For all considered experiments the control laws remain asymptotically stable in Δf.

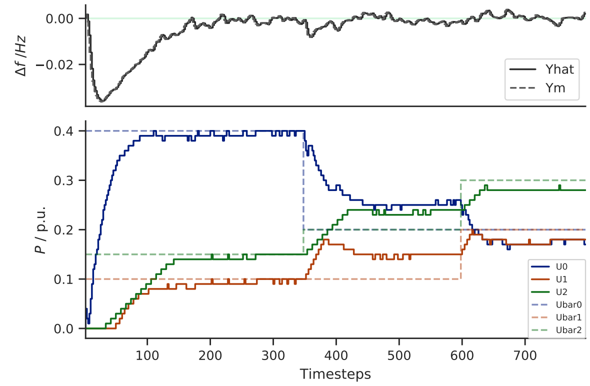

Fig. 4. Input reference tracking with `C1` and $W_{\bar{u}} = 1e^{-2}$. The reference trajectories given from timestep `350-600` are a mismatch to the actual disturbance. As disturbance the trajectory given in Figure 3 is used.

For $\epsilon_A = 1.0$ the response of `c1` is less aggressive. The opposite is true for $\epsilon_A = 0.9$. `c1` oscillates for $\epsilon_A = 0.8$, the control law is then not sufficiently damped. The stabilization of `c0` is slower for most of the corresponding trajectories, see the central graph in Figure 5. A dedicated plot for the mismatch in G is neglected here, due to that the resulting response characteristics are similar as for the already given multiplicative mismatch in B in the central graph. For $\epsilon_{B_d} = 0.5$, `c0` overshoots. When the lumped filter disturbance dynamics B_d exceed the system disturbance dynamics G by a factor of 1.5 as shown in the lower–most graph, controller `c1` exhibits a faster response.

IX. Discussion

The target adjusted MPC `c1` based on LQG is an alternative solution to the LFC control problem. It exposes different properties compared to the classical MPC formulation `c0`. The response to perturbations in form of disturbance steps is comparatively damped; a characteristic that is non–desirable. As shown in Figure 5, tuning of the Kalman Filter can alter the response characteristics and lead to a more pronounced response in comparison to the classical MPC formulation `c0`. `c1` in all considered simulations stabilizes Δf unidirectional, that is, asymptotically from a single deviation direction. `c0`, at least for the non–mismatch scenario, exposes the slight overshoot typical to OC and MPC — an often desirable property.

`c1` has only predictive capabilities by using the expected disturbance considering the optimization horizon $E(\hat{d}_{k+N|k})$. `c0` in contrast evaluates a potentially available disturbance prediction sequence $\hat{d}_{k+N|k}$ directly within the objective function and consequently can achieve higher precision in the control decisions. Input reference tracking is successfully demonstrated in Figure 4, convergence to the imposed references hereby can be achieved with a chosen precision using the input reference tuning term $W_{\bar{u}}$.

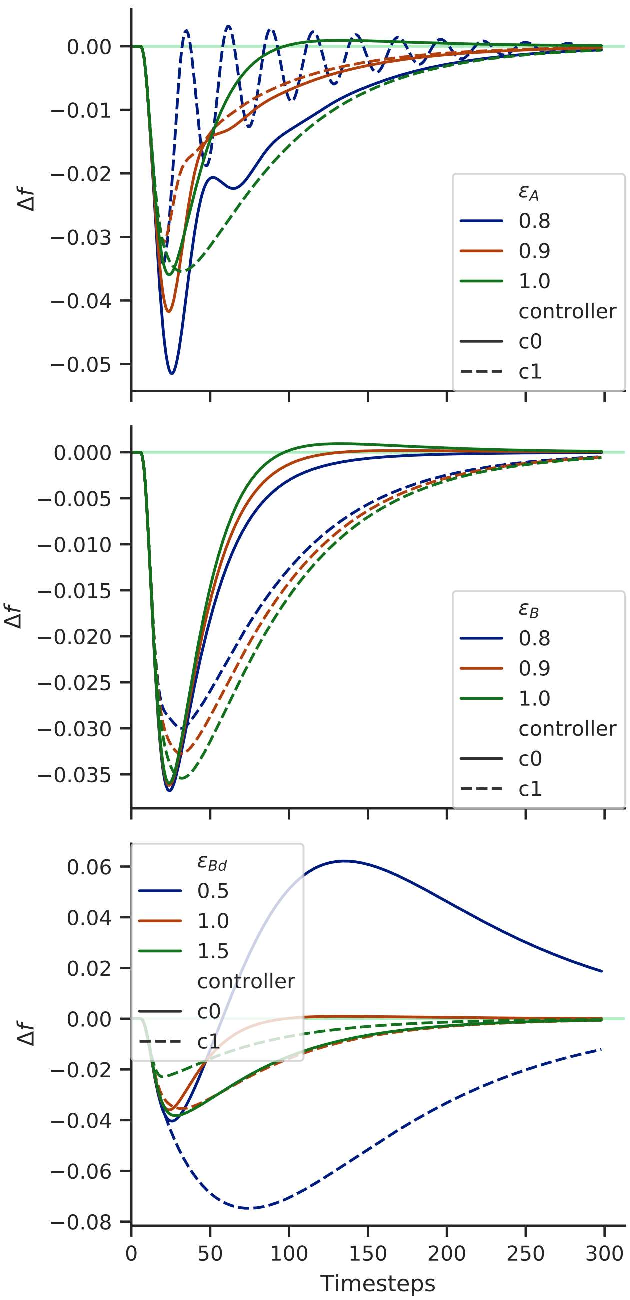

Fig. 5. Comparison of multiplicative lumped parametric mismatches of the free system response coefficients A (ϵ_A), forced system response coefficients B (ϵ_B) and lumped filter disturbance response coefficients B_d (ϵ_{B_d}). The disturbance trajectory given in Figure 3 without noise term is used.

X. Conclusion

We present an alternative optimal control and model predictive control formulation for the LFC problem. To the best knowledge of the author, these control law formulations are applied in this control problem for the first time. The formulation is compared to a classical MPC. The approaches incorporate an approximated system equilibrium into the controller objective and gain from an estimated lumped disturbance. We show that the derived MPC controller can be used to stabilize the frequency using a three–actor system and that it can be used to track input references. The proposed MPC formulations may be utilized within existing control hierarchy concepts.

It is shown that the proposed formulation does not expose advantages compared to the classical MPC. However, it can be considered an alternative in approaching the problem and means to comparison of different regulator formulations and associated properties.

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