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Damping system for long-span suspension bridges

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Summary
A damping system targeting flutter instability motions of long-span suspension bridges is presented. The damping system consists of four symmetrically located and equally tuned passive devices extracting energy from the structural system, based on the relative displacement between the pylons and the main suspension cables. Each device consists of a viscous damper and a spring in parallel, connecting the pylon to the suspension cable via a pretensioned cable. The tuning of the damping system is based on the asymptotic solution to a two-component subspace approximation, using still-air modes as input. It is shown that the simple tuning approach provides accurate results and that the damping system is capable of providing a relatively high amount of damping on the modes relevant for flutter. The efficacy of the damping system is illustrated numerically on a full aeroelastic model of a single-span suspension bridge with and without midspan cable clamps, and it is found that the stability limit of both structural systems can be increased significantly. Also, the buffeting response is evaluated, and especially, the torsional response is lowered. Different device configurations are investigated by shifting the anchorage point on the pylon and on the suspension cable, and the influence of the flexible connecting cable is assessed. Finally, design considerations concerning spring deformations, pretensioning of the system, and in-service displacements and forces are discussed.

KEYWORDS
damper calibration, long-span bridges, passive damping, passive flutter control, suspension bridges

1 | INTRODUCTION

For long-span suspension bridges, the aerodynamic stability limit is an important design factor typically setting the limit for the maximum span length. The self-excited forces causing the system to become unstable are primarily generated by the mean wind flow around the bridge girder, whereby aerodynamic shaping of the girder is a key issue in the design process. However, as there is a need for longer and longer spans, installation of external damping to increase the stability limit further has continually been investigated. A general investigation of the influence of modal damping on the flutter and buffeting response was carried out by Jain et al., where it was found that higher damping would increase the...
stability limit as well as lower the buffeting response. Two main damper concepts have been widely considered in relation to long-span bridges. One is the tuned mass damper, where structural vibrations are suppressed by one or more oscillating masses connected to the structure via a suitably tuned parallel spring–damper connection. The other is the aerodynamic movable flaps or winglets that control the wind flow around the bridge girder to suppress vibrations.

The installation of passive tuned mass dampers for increasing the flutter wind speed of long-span suspension bridges was studied analytically and experimentally on a cross-sectional model by Gu et al. where it was found that a significant improvement of the bridge stability characteristics could be obtained by tuning of the dampers in the neighborhood of the flutter frequency. An improved tuning of standard tuned mass dampers in relation to flutter motion was discussed by Chen and Kareem, and nonlinear hysteretic tuned mass dampers for multimode flutter mitigation were suggested by Casalotti et al., providing the ability to improve post-flutter behavior. To address the concern of the weight penalty related to installation of tuned mass dampers in long-span bridges, a tuned mass damper with two degrees of freedom targeting mitigation of a vertical and a torsional mode simultaneously was discussed by Mokrani et al. The suggested two degrees of freedom damper was numerically and experimentally tested showing the ability to damp both torsional and vertical vibrations. However, it was concluded that installation of the damper in a real bridge with a wide deck would be challenging. The use of multiple tuned mass dampers to limit the oscillations of the top of the pylons was analyzed by Casciati and Giuliano using slightly different tuning to cover a wider frequency interval. Installation of semiactive tuned mass dampers for suppression of flutter motion was considered by Gu et al., where a tuned mass damper with adjustable frequency was found to provide efficient control, and finally, numerical and experimental investigations of active tuned mass dampers for flutter suppression were carried out by Körlin and Starossek. Semiactive and especially fully active devices are providing high control performance but require high maintenance, and failure of the system can be fatal for the structure, thus limiting their use.

Control of flutter motion via winglets or flaps has also been widely studied, including both passive and active systems. An active control system was first proposed by Kobayashi and Nagaoka where section model experiments showed that the flutter wind speed could be increased by a factor of two. The setup included two symmetrically located wings located above the main girder. An improved control algorithm was suggested by Wilde and Fujino, and a setup with control surfaces placed underneath the bridge deck clear of the bridge cable system was shown to provide a stable system at any wind speed. Since then, the concept has been further developed with more recent contributions by Li et al. providing a new theoretical framework for feedback control and by Bakis et al. with a full model implementation and a multimodal flutter evaluation. A passive control system was proposed and numerically tested by Omenzetter et al., showing that an antisymmetrical setup was able to suppress aeroelastic response efficiently. However, this system is then only fully active for wind from one particular direction. A corresponding symmetric setup showed limited effectiveness. A passive system with winglets controlled by tuned mass dampers was suggested by Starossek and Aslan, thereby avoiding installation of an active system as well as the relatively high weight of traditional tuned mass dampers on long-span bridges. Only few alternatives to tuned mass dampers and winglets for suppression of flutter motion are found in the literature. However, a study on the use of a liquid column damper was carried out by Suduo et al. showing the ability to lower the buffeting response as well as increasing the flutter wind speed.

All of the above mentioned studies showed that the concept of introducing external damping is indeed a viable method for increasing the aerodynamic stability limit of long-span bridges. However, for long-span bridges, the tuned mass damper solutions are related to concerns regarding the added mass, and the surface control systems are primarily effective with active control. This paper presents a passive damping system for long-span suspension bridges extending the results for sagging cables damped by a transverse force, where substantial damping can be obtained by applying the damper force near the cable support. The damping system consists of four identical devices located symmetrically near the supports of the two main cables at the pylons. The dampers are connected to the main cables by smaller cables, and in order to keep these in tension, a pretensioning is applied via a loaded spring in parallel with the damper. A somewhat similar configuration was considered in Preumont et al. using piezoelectric active tendon control in a model for damping of vibrations from pedestrian loads on a footbridge. The present damping system is targeting the lowest vertical and torsional modes to suppress flutter-type instability of the aeroelastic system. Tuning of the damping system is based on the explicit two-component subspace method developed by Main and Krenk, where the frequency dependence and optimal tuning to a particular mode follow in a simple way from combining the two solutions corresponding to a system without dampers and a system in which the dampers are locked. The spring reduces the efficiency of the damper by decreasing the frequency increase following from a hypothetical locking of the damper as discussed by Krenk and Høgsberg. In the present case, the frequency shift needed for the design is obtained directly from analyzing the structure with a spring as part of the structural system. The tuning of the damping system is based on the still-air modes, and
the effectiveness is illustrated numerically for a full three-dimensional aeroelastic bridge model as presented by Møller et al. Different configurations and choices regarding the tuning are discussed, and the influence on both the buffeting response and the stability limit is illustrated. Also, the influence of the configuration and flexibility of the cables used to connect the dampers to the main cables of the bridge is discussed.

2 | SUSPENSION BRIDGE DAMPING

Figure 1 shows a beam-truss element model of a typical suspension bridge. The aerodynamic stability limit, also referred to as the flutter limit, is the point where the wind–structure interaction will cause the aeroelastic system to have negative damping, which is fatal for the structure. In order to increase the stability limit, an external damper system can be applied to the system.

The typical instability motions of a suspension bridge are the low-frequency vertical and torsional deck motions. These deck motions are directly related to the first and second mode shape of a suspended cable with sag shown in Figure 2a,b. The vertical deck motion is related to in-phase motion of the suspension cables, whereas the torsional deck motion is related to out-of-phase motion of the suspension cables. Thereby, both the antisymmetric vertical and torsional deck motions are governed by the first antisymmetric cable mode, and both the symmetric vertical and torsional deck motions are governed by the first symmetric cable mode. This observation shows that all modes of interest in relation to aerodynamic instability of the bridge can be addressed by four dampers placed in a double-symmetric setup, extracting energy from the system via viscous dampers working on the relative motion of the main suspension cables and the pylons. As the distance between the device attachment point on the pylon and on the suspension cable is relatively long, a connection via a pretensioned cable is considered. The cable connection only transfers tension forces, and thus, the viscous damper needs to be placed in parallel with a pretensioned spring.

2.1 | Local spring–damper system

A conceptual sketch of a suspension bridge with external spring–damper devices between the pylons and the suspension cables is shown in Figure 3. The damper system consists of four symmetrically placed identical devices with damping coefficient $c$ and spring stiffness coefficient $k$. The pretensioned connecting cable is not shown.

![FIGURE 1](image1.png)  
**FIGURE 1** Suspension bridge model

![FIGURE 2](image2.png)  
**FIGURE 2** Cable modes: (a) antisymmetric mode and (b) symmetric mode
The force acting on the structure due to local external damping devices as illustrated in Figure 3 is represented in the form

$$\mathbf{f}(t) = -\sum_j w_j f_j(t),$$

(1)

where $f_j(t)$ is the magnitude of the force of device $j=1,\ldots,4$ and $w_j$ is the corresponding connectivity vector providing the location of the external device in the global model. The connectivity vector contains zeros at all degrees of freedom unrelated to the device attachment nodes, whereby

$$w_j^T = \begin{bmatrix} \ldots, w_{A_j}^T, \ldots, w_{B_j}^T, \ldots \end{bmatrix}.$$

(2)

The dots indicate zeros, and index $A$ and $B$ refer to the connected nodes as shown in Figure 3, whereby $w_A$ and $w_B$ are nodal connectivity vectors with unit length providing the direction of the damper force in the particular structural node. The damper force at the points $A$ and $B$ are of opposite sign, and the direction can be found by the vector connecting the two points $x_{AB}$ whereby the nodal connectivity vectors are found as

$$w_A = -w_B = \frac{x_{AB}}{||x_{AB}||}.$$

(3)

The force–displacement relation of the the spring–damper device in the frequency domain is given as

$$f_j(\omega) = H_j(\omega)q_j(\omega),$$

(4)

where $H_j(\omega)$ is the frequency response function of device $j$ and $q_j(\omega)$ is the actuator displacement found as the relative displacement of node $A_j$ and $B_j$, whereby it can be expressed in terms of the connectivity vector and the global displacement vector $\mathbf{q}$ as $q_j(\omega) = w_j^T \mathbf{q}$.

The global frequency domain force–displacement relation of the structure due to external damping devices can now be found as the sum of the contribution of each device

$$\mathbf{f}(\omega) = -\sum_j H_j(\omega)w_j w_j^T \mathbf{q}(\omega).$$

(5)

For a parallel spring–damper device, the frequency function is a sum of a contribution from the spring and from the damper, whereby the frequency function takes the form

$$H_j(\omega) = k_j + i\omega c_j.$$

(6)

Introducing this into Equation (5), each term $H_j(\omega)w_j w_j^T$ appears as an additional stiffness and damping matrix from the corresponding device attached to the structure,

$$\mathbf{C}_d = \sum_j c_j w_j w_j^T, \quad \mathbf{K}_d = \sum_j k_j w_j w_j^T.$$

(7)

This result implies that the damper design consists of two steps: a decision on the positioning of the device giving the connectivity vector $w_j$ followed by a tuning process to obtain the device parameters $k_j$ and $c_j$. 

---

**FIGURE 3** External device positioning (distorted geometry)
2.2 | Two-component subspace approximation

In the following, the device damping parameters are taken to be equal, \( c_j = c \), and similarly for the device stiffness parameters, \( k_j = k \). The calibration of the damping and stiffness parameters \( c \) and \( k \) can then be done based on the two-component subspace method.\(^\text{19}\) This method is applicable when the implemented devices do not introduce substantial changes to the considered modes. The displacement is assumed to be described by a linear combination of the relevant free vibration mode shape vector \( u_0 \) and a modified shape vector \( u_\infty \) corresponding to clamped dampers,

\[
q(t) = u_0 \xi_0(t) + u_\infty \xi_\infty(t) = S \xi(t).
\]  

The response variables \( \xi_0 \) and \( \xi_\infty \) determine the magnitude of the contribution from each of the two modes combined in the matrix \( S = [u_0, u_\infty] \). The representation is illustrated for the first antisymmetric cable mode in Figure 4.

Making use of the subspace assumption, the additional stiffness from the external spring–damper devices are counted as part of the original structure, which is assumed to be without structural damping in the present context. Hence, the equation of motion of the structural system for assumed harmonic load and response takes the form

\[
\begin{bmatrix}
K - \omega^2 M + \sum_j H_j(\omega) w_j w_j^T
\end{bmatrix} \mathbf{q} = \mathbf{f}.
\]  

Introducing the two-component subspace assumption into Equation (9) and premultiplying by the transpose of the real valued shape vector \( S^T \xi \), the following reduced equation of motion is obtained

\[
\begin{bmatrix}
S^T K S - \omega^2 S^T M S + \sum_j H_j(\omega) S^T w_j w_j^T S
\end{bmatrix} \xi = S^T f.
\]  

It is seen that each term within the square brackets is a two-by-two matrix with components obtained by premultiplication and postmultiplication with the vectors \( u_0 \) and \( u_\infty \).

In the present context, it is convenient to normalize the mode shape vectors \( u_0 \) and \( u_\infty \) such that they correspond to unit modal mass,

\[
\begin{align*}
\sum_{i} u_i^T M u_i &= 1, & u_i^T K u_i &= \omega_i^2, \\
\sum_{i} u_i^T M u_i &= 1, & u_i^T K u_i &= \omega_i^2,
\end{align*}
\]  

whereby \( \omega_0 \) and \( \omega_\infty \) are the real valued eigenfrequency of the free and clamped system, respectively. The off-diagonal terms in the two structural matrices in (10) are represented by the nondimensional coefficient

\[
\kappa = \sum_i u_i^T M u_i = \omega_0^2 u_i^T K u_i,
\]  

where the last equality follows from premultiplication of the original undamped eigenvalue problem by \( u_i^T \).

The clamping condition corresponds to \( w_i^T u_\infty \), and thus, the damping contribution in (10) is expressed in terms of the single scalar frequency function

\[
H(\omega) = \sum_j H_j(\omega) u_j^2, \quad u_j = w_j^T u_0,
\]  

where \( u_j \) denotes the displacement across device no. \( j \), described entirely by the original undamped mode \( u_0 \).

With this notation, the frequency equation of the two degrees of freedom system can be determined from the determinant of (10) as
\[
\begin{vmatrix}
\omega_0^2-\omega^2 + H(\omega) & \kappa(\omega_0^2-\omega^2) \\
\kappa(\omega_0^2-\omega^2) & \omega_\infty^2-\omega^2
\end{vmatrix} = 0.
\]

(14)

It is easily verified that when introducing the difference between the two normalized mode shapes \(\Delta \mathbf{u}_\infty = \mathbf{u}_\infty - \mathbf{u}_0\), the coefficient \(\kappa\) can be expressed in the form

\[
\kappa = 1 - \frac{1}{2} \Delta \mathbf{u}_\infty^T \mathbf{M} \Delta \mathbf{u}_\infty.
\]

(15)

This formula suggests that when the locked mode shape vector \(\mathbf{u}_\infty\) constitutes only a modest modification of the original mode shape vector \(\mathbf{u}_0\), the parameter \(\kappa\) is close to unity. Thus, it is assumed in the following that \(\kappa=1\), a representative assumption. With this assumption, the determinant Equation (14) can be rearranged into the form

\[
\frac{\omega^2-\omega_0^2}{\omega_\infty^2-\omega_0^2} = \frac{H(\omega)/(\omega_\infty^2-\omega_0^2)}{1 + H(\omega)/(\omega_\infty^2-\omega_0^2)}.
\]

(16)

It is seen that introducing damping into the system will cause the frequency \(\omega\) to become complex valued, corresponding to a decaying vibration with damping ratio \(\zeta = \text{Im}[\omega]/|\omega|\).

This formula may also be used to estimate the frequency increase caused by introducing a spring at the location of the damping device. In this case, the frequency shift before mounting the damper is given in terms of the nondimensional parameter

\[
\xi = \frac{H(\omega)}{\omega^2_\infty-\omega_0^2} = \frac{\sum k_j u_j^2}{\omega^2_\infty-\omega_0^2}
\]

(17)
as

\[
\frac{\omega^2-\omega_0^2}{\omega_\infty^2-\omega_0^2} \simeq \frac{\xi}{1 + \xi}.
\]

(18)

Naturally, the frequency increase from introducing a spring at each device location can also be calculated from an additional eigenvalue problem. The calibration of the dampers can be carried out either by including the spring component in the system determining the undamped frequency \(\omega_0\) or by an eigenvalue analysis of the system without the spring, followed by introducing the correction of \(\omega_0\) given by (18).

### 2.3 Asymptotic results

For design of external damper devices, it is convenient to consider the asymptotic result. This result has been shown\(^{19}\) to provide good accuracy for small perturbation where the frequency shift imposed by a locked device is small relative to the original system frequency \(\Delta \omega_\infty \ll \omega_0\), where \(\Delta \omega_\infty = \omega_\infty - \omega_0\). The left hand side of the result presented in (16) can be approximated as

\[
\frac{\omega^2-\omega_0^2}{\omega_\infty^2-\omega_0^2} \simeq \frac{\Delta \omega}{\omega_\infty} + \omega_0 \simeq \frac{\Delta \omega}{\omega_\infty},
\]

(19)

where \(\Delta \omega = \omega_0 - \omega_0\) is the frequency change imposed by the damper. This gives the linearized relation between the damped complex frequency of the system and the devise frequency function

\[
\frac{\Delta \omega}{\Delta \omega_\infty} \simeq \frac{i \eta}{1 + i \eta},
\]

(20)

with the nondimensional damping parameter
\[ \eta = \frac{1}{1 - \frac{\omega}{\omega_\infty}}. \]  

(21)

For viscous dampers, the damping parameter can be written in terms of the damping coefficients \( c_j \) as

\[ \eta = \frac{\omega}{\omega_\infty + \omega_0} \frac{\sum c_j u_j^2}{\Delta \omega_\infty} \approx \frac{\sum c_j u_j^2}{2 \Delta \omega_\infty}. \]  

(22)

The asymptotic results are characterized by the frequency trace forming a semicircle in the complex plan as illustrated in Figure 5a and the corresponding damping curve shown in Figure 5b. Here, the dashed line represents the results obtained considering an external damper, and the solid line represents the results obtained with a parallel spring–damper device, where the stiffness of the spring is resulting in a 10% reduction of the frequency shift \( \Delta \omega_\infty \).

The semicircular shape of the frequency trace yields a high utilization of the damping potential of a particular mode for \( \text{Re}[\Delta \omega]/\Delta \omega_\infty \in [0.25; 0.75] \), and from geometrical consideration, the maximal damping can be approximated as

\[ \zeta_{\text{max}} \approx \frac{\omega_\infty - \omega_0}{\omega_\infty + \omega_0}, \]  

(23)

The damping ratio for modes with nonoptimal damping can be estimated as \( \zeta = \text{Im}[\omega]/|\omega| \approx \text{Im}[\omega]/\omega_0 \), and expressing the frequency as in (20) yields

\[ \zeta \approx \frac{\Delta \omega_\infty}{\omega_0} \frac{\eta}{1 + \eta^2}. \]  

(24)

A simple tuning procedure for damping of several modes with identically tuned dampers is to target optimal damping of a single mode and subsequently check the utilization of the damping potential of the other modes.

### 3 | AEROELASTIC MODEL

A three-dimensional aeroelastic finite element model is used for numerical evaluation of the effectiveness of the damper system. The bridge model is a typical single-span suspension bridge here exemplified by a model of a 3000 m suspension bridge proposed for crossing Sulafjorden in Norway as part of the Coastal Highway E39 project. The bridge model is shown in Figure 1 and is implemented with beam elements for the towers and nonlinear Green-strain truss elements for the cables to ease calibration, whereafter a linearized model is used for response calculations. The deck is modeled with aeroelastic beam elements where the aerodynamic self-excited forces are included via additional state variables and the model size is reduced by quasi-steady reduction. A full description of the aeroelastic implementation has been presented by Møller et al.\textsuperscript{21}
A detailed description of the geometry and structural properties is given by Rambøll\textsuperscript{22} and is summarized here: The distance between the pylons is 3000m, the sag to span ratio is 1:10, and the clearance height of the bridge deck is 74m. The distance between the suspension cables is 40m, and the distance between the hangers is 30m. The bridge is considered without and with a midspan cable clamp. The cable clamp is a stiff connection between the main suspension cables and the bridge girder and is a well-established method to increase the stability limit of a suspension bridge, as it prevents lateral displacement of the cable relative to the girder that significantly increases the frequency of the first antisymmetric cable mode and thereby the frequency of the first heave and torsional mode of the bridge. A picture of a cable clamp is shown in Figure 6a, and the corresponding model implementation is shown in Figure 6b where the clamp is modeled as a rigid connection between the girder and main cables.

The still-air modal frequencies are given in Table 1 for a system without and with a cable clamp at mid span - System 1 and 2, respectively. The mode types are assigned abbreviations with A and S denoting antisymmetric and symmetric, respectively, and D, H, and T denoting drag, heave, and torsion, respectively. It is seen that the frequencies of the first antisymmetric heave and torsional modes are significantly higher for System 2 including a cable clamp, whereas other modes have similar frequencies for the two different systems.

The bridge deck is designed as a twin box girder, but at the current state, the wind–structure interaction is treated by the Theodorsen theory for flat plates\textsuperscript{23} with a reference width of the plate to account for the slotted hole.\textsuperscript{24} The aerelastic forces are implemented with a rational function approximation including a single exponential term to account for noninstantaneous effects. The aerodynamic coefficients of the bridge deck are set to $C_D = 1.20$, $C_L = -0.15$, and $C_M = 0.30$, and their first derivative with respect to the deck angle is $C_D' = 0$, $C_L' = 6.3$, and $C_M' = 1.0$. The height and the reference width of the bridge girder are $D = 2.5$m and $B = 13$m. The diameter of the suspension cables and hanger cables is $D_{sc} = 1.19$m and $D_h = 0.10$m, and the drag coefficient of the cables is set to 0.8.

The structural damping of long-span bridges is very low and at the same time difficult to determine. In the current study, it is of interest to investigate the effect of external dampers on different modes, different systems, and at different aerelastic states. To allow for a direct comparison of the results, modal damping has been implemented with a damping coefficient of $\zeta = 0.32\%$ suggested by Hansen et al.\textsuperscript{26} The still-air mode shapes of main interest in relation to aerodynamic stability of the considered structure are shown in Figure 7a–d being the first antisymmetric and symmetric heave mode and the first antisymmetric and symmetric torsional mode. Figure 7e,f shows the shape of the critical mode at the point

### Table 1 Still-air frequencies

<table>
<thead>
<tr>
<th>System 1</th>
<th>System 2</th>
<th>System 1</th>
<th>System 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$ [rad/s]</td>
<td>Type</td>
<td>$\omega$ [rad/s]</td>
<td>Type</td>
</tr>
<tr>
<td>0.233</td>
<td>SD1</td>
<td>0.233</td>
<td>SD1</td>
</tr>
<tr>
<td>0.473</td>
<td>AD1</td>
<td>0.477</td>
<td>AD1</td>
</tr>
<tr>
<td>0.476</td>
<td>AH1</td>
<td>0.572</td>
<td>SH1</td>
</tr>
<tr>
<td>0.572</td>
<td>SH1</td>
<td>0.605</td>
<td>AH1</td>
</tr>
<tr>
<td>0.743</td>
<td>AT1</td>
<td>0.756</td>
<td>SD2</td>
</tr>
</tbody>
</table>

Abbreviations: D/H/T, drag/heave/torsion; S/A, symmetric/antisymmetric.
of zero damping, and it is seen that these modes appear primarily as combinations of the displayed pairs of still-air modes.

The evolution of the modal properties of the four modes is now considered. Figure 8a shows the frequencies as function of the normalized mean wind speed for System 1 without a midspan cable clamp. The heave modes are represented with dashed lines, and the torsional modes are represented with solid lines. Symmetric and antisymmetric modes are red

![Mode shapes](image)

**FIGURE 7** Mode shapes: (a) antisymmetric heave, (b) symmetric heave, (c) antisymmetric torsion, (d) symmetric torsion, (e) antisymmetric flutter, and (f) symmetric flutter

![System 1](image)

**FIGURE 8** System 1: (a) frequencies and (b) damping ratios.  - AH1;  - SH1;  - AT1;  - ST1
and blue, respectively. Near the critical wind speed, the frequencies of the torsional modes are decreasing, whereas the frequencies of the heave modes are more constant. In accordance with the still-air modal frequencies presented in Table 1, the antisymmetric modes have the lowest frequencies. Figure 8b shows the damping of the four modes as function of the normalized wind speed. Near the critical wind speeds, the damping of the original heave modes is very large, whereas the damping of the original torsional modes is low. At the mean wind speed $U = U_{cr,1}$, where $U_{cr,1} = 57.4\, \text{m/s}$ is the critical wind speed, the antisymmetric torsional mode has zero damping, whereas the symmetric torsional mode becomes unstable at a mean wind speed 20% higher than the antisymmetric torsional mode.

The evolution of the modal frequencies and the modal damping of System 2 with cable clamps is shown in Figure 9a, b. In contrast to System 1 without cable clamps, the symmetric modes now have the lowest frequencies. Also, the damping curves have shifted, so that now, the damping ratio of the first symmetric torsional mode crosses the zero damping line first at $U_{cr,2} = 1.2U_{cr,1}$, whereas the instability limit of the antisymmetric modes is shifted to a wind speed around 40% higher than the critical wind speed of the symmetric mode.

### 3.1 Damper design

The configuration investigated in detail in the following consists of four parallel spring–damper devices placed symmetrically at the middle cross beam of the pylons and connected to the main suspension cables at the third hanger connection by a cable, as shown in Figure 10a. The devices are attached to the center of the pylons to ensure that they are free of the hanger cables. The height of the attachment point to the pylon is chosen based on the existing pylon geometry, and the horizontal distance to the suspension cable attachment corresponds to 3% of the span length, which for a cable with sag would provide approximately 3% damping on the first antisymmetric cable mode. Figure 10b shows a sketch of the spring–damper–cable system. Initially, the cable connecting the parallel spring–damper device to the main
suspension cable is considered infinitely stiff in comparison with the spring, whereby the the clamped frequency \( \omega_\infty \) is found by inserting an infinitely stiff bar element between the pylon and the suspension cable.

Table 2 shows the still-air natural frequencies \( \omega_0 \) and \( \omega_\infty \) of the model with free and clamped devices, respectively. The frequencies are shown for the four most relevant modes in relation to stability of the suspension bridge and for the system without and with midspan cable clamps, referred to as System 1 and 2, respectively. These frequencies are the basis of the design of the spring–damper system together with the structural frequency \( \omega_k \) found when the stiffness of the spring is included in the model. In the design, the stiffness of the spring is determined to provide a 10% reduction of the frequency interval \( \omega_\infty - \omega_0 \) of the mode chosen for design. This corresponds to the case with asymptotic solution illustrated in Figure 5.

The asymptotic result of the two-component subspace approximation is shown for the four modes of interest in Figure 11 for System 1. The dashed semicircle represents the approximated frequency trace in the situation where the devices consist solely of a viscous damper, whereas the solid semicircle is the approximated frequency trace of the modes including a spring with stiffness determined via the frequency shift of the first antisymmetric torsional mode. The axes are normalized by the corresponding frequency \( \omega_0 \) of the undamped system. The plots clearly reveal that the relative frequency increment from zero to infinite damping varies between modes. The normalization also entails that the y-axis is now an approximated damping ratio as \( \xi = \text{Im}[^{\omega}] / |^{\omega|} \approx \text{Im}[^{\omega}] / \omega_0 \). From the plots, it is seen that the damping potential is largest for the antisymmetric modes, as expected because these modes are governed by the first antisymmetric cable modes where the relative displacement of the cable at the damper connection point is larger.

### Table 2

<table>
<thead>
<tr>
<th>System</th>
<th>State</th>
<th>( \omega_0 )</th>
<th>( \omega_\infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Free</td>
<td>0.476</td>
<td>0.572</td>
</tr>
<tr>
<td>1</td>
<td>Clamped</td>
<td>0.495</td>
<td>0.594</td>
</tr>
<tr>
<td>2</td>
<td>Free</td>
<td>0.605</td>
<td>1.053</td>
</tr>
<tr>
<td>2</td>
<td>Clamped</td>
<td>0.613</td>
<td>1.060</td>
</tr>
</tbody>
</table>

Abbreviations: H/T, heave/torsion; S/A: symmetric/antisymmetric.

**FIGURE 11** Complex frequency locus: (a) AH1, (b) SH1, (c) AT1, and (d) ST1
The devices are now calibrated to provide optimal damping of a single mode based on the asymptotic result of the two-component subspace method presented in (22). The stiffness and damping coefficients providing optimal damping of each of the considered modes are presented in Table 3. The damping coefficient that is optimal for the first antisymmetric torsional mode is used throughout this section to illustrate the modal damping of all the four modes of interest.

Figure 12 shows the normalized design curves including the estimated frequencies of the four considered modes marked with blue circles. The estimated frequencies are found for the damping coefficient optimal for the first antisymmetric torsional mode, mode III, whereby the estimated frequency of this particular mode is seen to be at $\eta=1$ and accordingly at the top of the frequency trace semicircle. Furthermore, the results clearly show that the other three modes exhibit damping relatively close to their optimal potential. This shows that four equally tuned and symmetrically located dampers are indeed capable of efficient damping of all the four modes.

The actual damping ratios of the four modes are found by evaluating the complex frequencies of the bridge model including the external devices with the design tuning parameters. Figure 13 shows the asymptotically estimated frequencies marked with blue circles and the actual frequencies from the full model marked with red crosses. The frequency intervals are here normalized by the undamped frequency providing an estimate of the modal damping ratio of the considered modes. It is seen that the damping ratios obtained for each of the four modes vary considerably due to the

<table>
<thead>
<tr>
<th>System</th>
<th>Parameter</th>
<th>AH1 (I)</th>
<th>SH1 (II)</th>
<th>AT1 (III)</th>
<th>ST1 (IV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$k \times 10^6$ [N/m]</td>
<td>0.950</td>
<td>0.990</td>
<td>1.091</td>
<td>1.135</td>
</tr>
<tr>
<td>1</td>
<td>$c \times 10^6$ [kg/s]</td>
<td>19.772</td>
<td>17.173</td>
<td>14.556</td>
<td>12.366</td>
</tr>
<tr>
<td>2</td>
<td>$k \times 10^6$ [N/m]</td>
<td>0.886</td>
<td>1.040</td>
<td>0.976</td>
<td>1.247</td>
</tr>
<tr>
<td>2</td>
<td>$c \times 10^6$ [kg/s]</td>
<td>14.546</td>
<td>18.103</td>
<td>9.268</td>
<td>13.681</td>
</tr>
</tbody>
</table>

![FIGURE 12](image1.png) Individually normalized design curves: (a) frequency trace and (b) damping curve

![FIGURE 13](image2.png) Common normalization of design curves: (a) frequency trace and (b) damping curve
dependence of the real part of the modal frequency as well as the frequency shift between a undamped mode and a
locked mode. It is observed that as the damping of a particular mode is defined by its particular curve, the chosen tuning
of the device will only shift the damping along its own curve. Tuning of a damper system to more than a single mode is
therefore a trade-off between optimizing the damping of the mode with the highest damping potential or to provide
more equal damping at more modes.

3.2 Effect of dampers on the stability limit

The damping coefficients are set according to the estimated optimal values shown in Table 3, and the spring stiffnesses
are chosen based on a frequency shift of 10% of the same mode. Figure 14a,b shows the angular frequency and damping
ratio of the first antisymmetric torsional mode (III) as function of the normalized mean wind speed of the model of Sys-

tem 1 without midspan cable clamps. The dashed black curve indicates the reference system without external damping,
and the gray and blue curves represent the damped system with different tuning of the devices. The frequency plot
shows the expected frequency increase due to the external devices. The damping plot shows a significant increase of
the damping ratio that results in an increase of the stability limit of around 41% to 43% depending on the chosen
damping parameters. The blue curve corresponds to optimal damping of the first symmetric heave mode (II) and pro-
vides the best stability conditions of the considered system. However, optimal tuning of each of the four modes results
in an almost equal stability improvement.

Figure 15a,b shows the frequency and damping evolution for increasing wind speeds of the first symmetric torsional
mode (IV) for System 2 including midspan cable clamps. Again, the dashed line indicates the system without external

![Figure 14](image1.png)

**Figure 14** System without cable clamp: (a) frequency and (b) damping ratio of mode AT1 (III) as function of mean wind speed. — undamped; -- damped

![Figure 15](image2.png)

**Figure 15** System with cable clamp: (a) frequency and (b) damping ratio of mode ST1 (IV) as function of mean wind speed. — undamped; -- damped
damping, whereas the red and gray curves represent the system with external damping with damper tuning as shown in Table 3. The expected shifts of the frequency and the damping are seen together with an increase of the stability limit of around 27% to 28%. This is lower than the increase for the system without a midspan cable clamp, corresponding well with the lower damping potential of symmetric modes by the particular damping configuration. However, the stability increase is still substantial. The red curve represents results obtained for optimal tuning of the first antisymmetric mode (I), but again, the chosen tuning has only slight influence on the stability limit.

### 3.3 Device configuration

The position of the device in terms of attachment to the pylon and to the main cable will determine the effectiveness of the device. Figure 16a shows three different configurations in which the device is connected to hanger position 2, 3, and 4, respectively. Figure 16b shows three alternative configurations of the external dampers, now shifting the attachment at the pylon plus/minus 30m from the original location.

The stability analysis of System 2 with tuning based on each of the four modes showed that optimal tuning of the first antisymmetric heave mode (I) led to the highest stability limit increase. The current investigation is therefore carried out for optimal tuning of mode I. Table 4 shows the design parameters for each of the six damper configurations. It is noted that configurations a2 and b2 are identical and that the free vibration frequency $\omega_0$ is identical for all configurations as this refers to the vibration modes of the original structure without any external device. Going from configuration a1 to a3, the cable is attached further away from the pylon on the suspension cable. This leads to a higher clamped frequency $\omega_\infty$, whereby a higher damping ratio can be attained. The spring stiffness $k$ is set to provide a reduction of the frequency shift of 10%, and it is found that less stiffness is required when the damper connection is moved further away from the pylon. Finally, the damping coefficient also becomes smaller going from configuration a1 to a3. Changing the location of the damper connection on the pylon and thereby changing the angle of the damper relative to the cable motion only appear to have a small influence on the damping parameters. However, a tendency is seen where a smaller angle between the cable and the pylon provides a more direct connection and thereby a higher clamped frequency.

The damping ratios of the four modes estimated using (24) are shown in Table 5 for all of the six damper configurations together with the similar parameters obtained from the full numerical model given in normalized form. As expected, the damping of all four modes is increased going from configuration a1 to a3, and only slight differences are seen going from configuration b1 to b3. It is especially noted that the damping of the antisymmetric modes is increased, whereas the damping of the symmetrical modes is decreased going from configuration b1 to b3. The estimated

---

**TABLE 4** System with cable clamp: Design parameters for optimal damping of mode I

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$\omega_0$ [rad/s]</th>
<th>$\omega_k$ [rad/s]</th>
<th>$\omega_\infty$ [rad/s]</th>
<th>k [10^6 N/m]</th>
<th>c [10^4 kg/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>0.6053</td>
<td>0.6058</td>
<td>0.6100</td>
<td>1.394</td>
<td>22.917</td>
</tr>
<tr>
<td>a2</td>
<td>0.6053</td>
<td>0.6061</td>
<td>0.6129</td>
<td>0.886</td>
<td>14.546</td>
</tr>
<tr>
<td>a3</td>
<td>0.6053</td>
<td>0.6064</td>
<td>0.6160</td>
<td>0.723</td>
<td>11.847</td>
</tr>
<tr>
<td>b1</td>
<td>0.6053</td>
<td>0.6061</td>
<td>0.6126</td>
<td>0.946</td>
<td>15.526</td>
</tr>
<tr>
<td>b2</td>
<td>0.6053</td>
<td>0.6061</td>
<td>0.6129</td>
<td>0.886</td>
<td>14.546</td>
</tr>
<tr>
<td>b3</td>
<td>0.6053</td>
<td>0.6061</td>
<td>0.6131</td>
<td>0.863</td>
<td>14.170</td>
</tr>
</tbody>
</table>
The modal damping ratio of the first antisymmetric torsional mode (IV) is plotted as a function of the mean wind speed for configurations a1–a3 in Figure 17a and for configurations b1–b3 in Figure 17b. The graphs are color coded according to Figure 16a,b. The dashed black curve is the reference damping ratio of the system without external damping. It is seen that the stability limit is significantly affected by shifting the location of the damper on the main cable changing from a stability increase of around 21% to 34%. Shifting the damper position on the tower is seen to have no effect on the stability limit.

4 | DAMPED RESPONSE

The response due to turbulent wind loading is now evaluated. The wind field is represented as stretched isotropic turbulence.27 The along-wind component is represented by the von Kármán spectral density with the integral length scale $\lambda_x = 300\text{m}$ and the two transverse length scales $\lambda_y = \lambda_z = 0.5\lambda_x$. The turbulence intensity is $I_u = 0.135$, and the standard deviation of the along-wind turbulence component is found as $\sigma_u = I_u / U$ where $U$ is the mean wind speed. Direct time simulations of the three-dimensional wind field are carried out by the conditional mean field method.27 The method is an auto-regressive procedure using nonuniform spacing of the regression terms and direct evaluation of the coefficients from conditional correlation functions. A basic step size $h = 0.6\text{s}$ is used and an exponential memory layout with time intervals scaled by $k = [2^0, 2^1, \ldots, 2^5]$, providing a rather accurate representation of the statistical properties of the wind field. Numerical integration of the aeroelastic system is performed using a momentum-based, second-order method.21

A 5-min steady-state time record of the structural response of System 1 without a midspan cable clamp is shown in Figure 18a–c. The time records are shown at quarter span for a mean wind speed of 90% of the critical wind speed of the system without external damping. The blue curve represents the system without external damping, and the red curve represents the system with external damping with device coefficients optimal for the first symmetric heave mode, mode

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$\xi_{II}$ [%]</th>
<th>$\xi_{II}/\xi_{II}^{est}$</th>
<th>$\xi_{III}/\xi_{II}^{est}$</th>
<th>$\xi_{IV}/\xi_{III}^{est}$</th>
<th>$\xi_{IV}$ [%]</th>
<th>$\xi_{IV}/\xi_{IV}^{est}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>0.35</td>
<td>1.001</td>
<td>1.12</td>
<td>1.013</td>
<td>0.16</td>
<td>0.986</td>
</tr>
<tr>
<td>a2</td>
<td>0.56</td>
<td>1.003</td>
<td>1.69</td>
<td>1.020</td>
<td>0.27</td>
<td>0.987</td>
</tr>
<tr>
<td>a3</td>
<td>0.79</td>
<td>1.005</td>
<td>2.22</td>
<td>1.028</td>
<td>0.38</td>
<td>0.987</td>
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<tr>
<td>b1</td>
<td>0.53</td>
<td>1.003</td>
<td>1.66</td>
<td>1.019</td>
<td>0.24</td>
<td>0.987</td>
</tr>
<tr>
<td>b2</td>
<td>0.56</td>
<td>1.003</td>
<td>1.69</td>
<td>1.020</td>
<td>0.27</td>
<td>0.987</td>
</tr>
<tr>
<td>b3</td>
<td>0.58</td>
<td>1.003</td>
<td>1.69</td>
<td>1.020</td>
<td>0.29</td>
<td>0.986</td>
</tr>
</tbody>
</table>
II. The drag response, $q_y$, is unaffected by the external damping, the heave response, $q_z$, is slightly affected, whereas the torsional response, $q_\theta$, is significantly decreased by the added damping.

**FIGURE 18**  Response at quarter span of system without cable clamp. --- undamped; – damped

**FIGURE 19**  Response at quarter span of system without cable clamp. --- undamped; – damped

II. The drag response, $q_y$, is unaffected by the external damping, the heave response, $q_z$, is slightly affected, whereas the torsional response, $q_\theta$, is significantly decreased by the added damping.
Figure 19a–c shows the standard deviation of the drag, heave, and torsional response $\sigma_y$, $\sigma_z$, and $\sigma_\theta$ as function of the mean wind speed for System 1 without a midspan cable clamp. The black dashed curves are results for the system without external damping, and the blue curves are results with external damping and device coefficients optimal for the first symmetric heave mode, mode II. As for the time records shown in Figure 18, the external dampers have no effect on the drag response, only modest effect on the vertical response and significant effect on the torsional response. It is seen that the vertical asymptomatic value of the torsional is shifted according to the stability limit increase discussed in relation to Figure 14.

Figure 20a shows the standard deviation of the torsional response at midspan for System 2 including midspan cable clamps. The response is shown for the reference system without external damping marked with a dashed black curve and for the damper configurations a1–a3 as shown in Figure 16a. The damping coefficients are set according to Table 4, and the graphs are colored as in Figure 16a. Figure 20b shows the standard deviation of the torsional response at midspan for the same system for results obtained with device configurations b1–b3 as shown in Figure 16b. As before, the black dashed graph marks the results obtained with the reference model without external damping. It is seen that the torsional response obtained going from configuration a1 to a3 is decreasing and that the asymptotic value at the instability limit is shifted toward higher wind velocities. The torsional response is virtually unaffected by shifting the position of the device on the pylon, obtained for configurations b1–b3. These results correspond well with the damping results shown in Figure 17.

4.1 | Cable flexibility

The flexibility of the cable connecting the spring-damper device to the suspension cable as illustrated in Figure 10b is now considered. As a first step, the tension force in the cable $T$ is determined as the sum of the damper force $\dot{q}_d$ and the spring force $kq_d + \ddot{T}$, where $q_d$ is the damper displacement for a rigid connecting cable; $\dot{q}_d$ is the first time derivative of the relative displacement, and $\ddot{T}$ is the pretension force. As basis for the cable design, the oscillating part of the cable tension force $T - \ddot{T}$ is determined at a mean wind speed of $90\%$ of the critical wind speed of the considered damped system. A five-minute time record of the force in an upstream connection and a downstream connection is shown in Figure 21a,b for configuration a2. The aeroelasticity introduces an asymmetry in the system, whereby the force in the downstream connection is higher. Comparing the standard deviation of the response of a 20-hours time series, the downstream cable force is found to be $18.2\%$ higher, and the damper system should therefore be designed based on a downstream device.

For the damper system to be working, the cable should be in tension at all times, corresponding to a positive cable force. Furthermore, the cross-section area of the cable as well as the tension force should be sufficient to provide a high axial cable stiffness relative to the spring stiffness to ensure effectiveness of the damper system. The cable stiffness $k_c$ can be estimated from the so-called Dischinger formula$^{28,29}$
where $A_c$ is the cross-section area of the cable, $L$ is the length of the cable, and $E_{eq}$ is the equivalent stiffness modulus of the cable taking into account the cable weight per unit length $w$, the horizontal span length $L_h$, and the tension force in the cable $T$. The stiffness modulus is $E_c=195\text{GPa}$.

A cable design matching the damping and stiffness coefficients of the different cable configurations in Figure 16 is carried out. A consistent design approach is applied in two steps: First, the cross-section area of the cable is determined to provide a stiffness ratio between the spring and the cable of $k_c/k=20$ assuming $E_{eq}=E_c$. Then the pretension, and thereby the initial deformation of the spring, is determined to provide a ratio $E_{eq}/E_c=0.95$ for the minimum tension occurring in the cable $T_{min} = \bar{T} - \Delta T$, where $\Delta T$ is the expected peak value of the oscillating part of the cable force. This ensures a high cable stiffness within the operating tension range. The cable design parameters are shown in Table 6. The parameter $\Delta x$ is the peak elongation/compression of the spring–damper device determined based on the structural response. The peak response is estimated as the standard deviation of the response multiplied by a peak factor $p=4$. $\bar{x}$ is the initial spring deformation, $D_c$ is the cable diameter, and $\sigma_{max}/f_y$ is the utilization ratio of the cable, where $\sigma_{max}$ is the maximum stress and $f_y=1640\text{MPa}$ is the yield stress. Finally, the stiffness ratio $k_c/k$ is provided, where $k_c$ is the cable stiffness at $x=\bar{x}$.

Considering the cable design for configurations a1 to a3, the cable length is only increasing slightly whereas the horizontal length is increased significantly. Also, it is seen that the required pretension of the cable and the peak value of the oscillating part of the cable force are increased slightly, whereas the initial displacement and the in-service elongation increase significantly. This can be explained by the decreasing stiffness and damping coefficients shown in Table 4. The stiffness decrease also explains the decreasing cross-section diameter needed to achieve the same stiffness ratio between the cable and the spring. For configuration a3, it is seen that the utilization ratio exceeds 1. This factor is

$$k_c = \frac{E_{eq} A_c}{L}, \quad E_{eq} = \frac{E_c}{1 + w^2 L_h A_c E_c / (12 T^3)}.$$  

\textbf{TABLE 6} Cable parameters

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$L$ [m]</th>
<th>$L_h$ [m]</th>
<th>$\Delta T$ [MN]</th>
<th>$\Delta x$ [m]</th>
<th>$\bar{T}$ [MN]</th>
<th>$\bar{x}$ [m]</th>
<th>$D_c$ [m]</th>
<th>$\sigma_{max}/f_y$</th>
<th>$k_c/k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>153.8</td>
<td>62.8</td>
<td>8.16</td>
<td>0.62</td>
<td>12.54</td>
<td>9.0</td>
<td>0.167</td>
<td>0.57</td>
<td>19.95</td>
</tr>
<tr>
<td>a2</td>
<td>158.1</td>
<td>91.6</td>
<td>8.64</td>
<td>1.04</td>
<td>12.31</td>
<td>13.9</td>
<td>0.135</td>
<td>0.89</td>
<td>19.97</td>
</tr>
<tr>
<td>a3</td>
<td>168.7</td>
<td>120.9</td>
<td>9.69</td>
<td>1.44</td>
<td>13.51</td>
<td>18.7</td>
<td>0.126</td>
<td>1.13</td>
<td>19.97</td>
</tr>
<tr>
<td>b1</td>
<td>134.8</td>
<td>91.6</td>
<td>8.75</td>
<td>1.00</td>
<td>12.10</td>
<td>12.8</td>
<td>0.129</td>
<td>0.97</td>
<td>19.98</td>
</tr>
<tr>
<td>b2</td>
<td>158.1</td>
<td>91.6</td>
<td>8.64</td>
<td>1.04</td>
<td>12.31</td>
<td>13.9</td>
<td>0.135</td>
<td>0.89</td>
<td>19.97</td>
</tr>
<tr>
<td>b3</td>
<td>183.4</td>
<td>91.6</td>
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<td>1.06</td>
<td>12.77</td>
<td>14.8</td>
<td>0.144</td>
<td>0.80</td>
<td>19.96</td>
</tr>
</tbody>
</table>
lowered by choosing a larger cross-section diameter. From the damping plots in Figure 17a, it was evident that a larger increase of the stability limit was obtained by moving the device attachment point on the suspension cable further away from the pylon. The results presented in Table 6 show that this is also the configuration that requires the largest initial deformation of the spring as well as the largest in-service deformations. From the cable design for configurations b1 to b3, it is seen that a shorter cable requires less initial deformation of the spring as well as slightly lower in-service deformations of the spring-damper device. From the stability results shown in Figure 17b, it was seen that the stability limit increase was similar for the three configurations, and a shorter cable is then to be preferred. For all of the configurations, the stiffness ratio $k_c/k$ is seen to be very close to 20, used as design input. Figure 22 shows the equivalent stiffness modulus as function of the tension force. The red cross marks the stiffness at $T = T_2$, and the black crosses mark the stiffness at $T \pm \Delta T$ for configuration a2. For the design choices applied, the cable stiffness is almost constant in the operating range.

The effect of including cable flexibility in the model is evaluated by including an additional node between the structural attachment points, whereby the spring-damper device is now connecting the tower to the intermediate point and a linear bar element with stiffness $k_c$ is connecting the intermediate point to the suspension cable. The tuning of the damper is reevaluated as the frequency for locked dampers is now lower due to the flexibility of the cable. The spring stiffness is not updated, giving tuning parameters of $k=0.886 \times 10^6 \text{N/m}$ and $c=9.586 \times 10^6 \text{kg/s}$. Figure 23a,b shows the frequency traces and the damping curves of configuration a2 including cable flexibility. The blue circles indicate the frequencies predicted based on the asymptotic solution for the two-component subspace method, and the red crosses mark the frequencies obtained with the numerical model. The design results correspond well to the frequencies and damping ratios obtained with the numerical model.

The change of the natural frequency and damping for increasing wind speeds is shown for the first symmetric torsional mode for configuration a2 in Figure 24a,b. The dashed black line is the reference result for the system without external damping, the blue curves mark results obtained with the idealized model assuming an infinitely stiff cable, and the red curves are results obtained with the model including cable flexibility. It is seen that the modal frequency is smaller when the cable flexibility is included. Also, the modal damping ratio is smaller, whereby the stability limit

![FIGURE 22](image1)

**FIGURE 22** Cable stiffness modulus

![FIGURE 23](image2)

**FIGURE 23** Design curves for configuration a2: (a) frequency trace and (b) damping curve
is reduced by 7.4% relative to the idealized case. The stiffness of the cable is a design choice, and by accepting a larger initial deformation of the spring, a stiffer cable can be used, providing a more effective damper system.

Finally, the effect of including the cable flexibility is considered in relation to the buffeting response. Figure 25a,b shows the vertical and torsional response at midspan for System 2. The black dashed curves are results obtained with the reference system without external damping, the blue curves are results obtained with the idealized system with an infinitely stiff cable, and the red curves are results including the cable flexibility. The dampers are placed according to configuration a2. It is seen that both the heave and the torsional response are higher when the cable flexibility is included.

The cable force can now be evaluated as the cable stiffness multiplied by the elongation of the bar element connecting the parallel spring–damper to the main suspension cable. Hereby, the peak value of the oscillating part of the cable force is found to $\Delta T=5.33\text{MN}$ which is 38% lower than the cable force obtained for the idealized system. Repeating the design approach, the initial deformation becomes $\bar{x} = 9.8\text{m}$, and the utilization ratio is $\sigma_{\text{max}}/f_y=0.66$. The initial deformation of the spring appears large but should be compared with the depth of the pylon leg that is around 16m. The initial deformations can advantageously be conducted as compression by attaching the cable to the far end of the spring and thereby gaining a compact design.

### 4.2 | Damper operation characteristics

The in-service damper operation characteristics are now considered for the structure with a midspan cable clamp and external dampers in configuration a2 including cable flexibility. The response is evaluated at a mean wind speed equal
to 90% of the critical wind speed of the damped system. Figure 26a shows a five-minute steady-state displacement time record of the parallel spring–damper device evaluated as the relative displacement of the device end nodes. A downstream device is considered. Figure 26b shows the force in the viscous damper \( f_d = c \dot{q}_d \), \( c = 9.586 \times 10^6 \text{kg/s} \). The standard deviation of displacement and damping force is found based on a 20-hour time simulation. Multiplication by a peak factor of 4 gives the peak values \( \Delta q_d = 0.811 \text{m} \) and \( \Delta f_d = 5.18 \text{MN} \).

A five-minute time record of the energy dissipation rate \( P_d = f_d \dot{q}_d \) in the device is shown in Figure 27. The red line marks the mean value at \( P_d = 0.18 \text{MW} \). Requirements to energy dissipation are typically part of a damper specification. This is for the design of the thermal capacity of the damper, as the mechanical energy is dissipated by transformation into heat. Location of the damper at the outside of the tower indicates that a relatively compact damper design may be necessary, i.e. concentrated heat buildup may be an issue. However, at this location, effective wind cooling can be assumed. An alternative may be to locate the damper inside the tower, where space is plenty to distribute the energy dissipation over more devices. Another benefit would be improved accessibility for inspection and maintenance. Drawbacks would be the absence of wind cooling and the need for a more complicated mechanism to transform the relative displacements of the main cable and the tower into displacements over the damper units.

5 | CONCLUSIONS

A damping system for long-span suspension bridges has been presented, and the performance has been numerically tested on a full aeroelastic model with emphasis on the aerodynamic stability limit. The tuning of the dampers is based on the asymptotic results to a two-component subspace approximation with still-air modes as calibration input, and the
simple procedure was seen to provide very accurate results. Also, it was shown that the system of four symmetrically located identical dampers is able to damp all modes related to instability of the bridge effectively. The performance of the damping system was illustrated for two suspension bridge concepts: a single-span suspension bridge without cable clamps, where unstable flutter motions are governed primarily by the antisymmetric modes, and a similar single-span suspension bridge with a midspan cable clamp, for which flutter motions are governed primarily by the symmetric modes. It was found that under the assumption of an infinitely stiff connecting cable, the system is able to provide an increase of the critical wind speed of around 40% for the system without cable clamps and an increase of around 28% for the system with midspan cable clamps, when the device is attached to the suspension cable at a distance from the pylon of 3% of the bridge span length. It is also shown that the modal damping ratio, and thereby the critical wind speed, can be increased further by shifting the device attachment point on the suspension cable further away from the pylon. However, this comes at a cost of larger device deformations and larger required pretension forces in the damping system. Shifting the anchorage point of the device on the pylon was seen to have little effect. The damping system was shown to significantly reduce the torsional buffeting response, while having little impact on the lateral and vertical response. When including the flexibility of the pretensioned cable connecting the spring–damper device to the structure, it was found that a cable with a stiffness of 20 times the spring stiffness would lower the critical wind speed with 7.4% relative to the critical wind speed of the damped system with an infinitely stiff connecting cable. Damper operation characteristics relevant for device design were shown for a realistic case, considering stroke, force, and energy dissipation rate.

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