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# Shape and topology optimization using CutFEM

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**Summary.** This work presents a CutFEM shape and topology optimization methodology based on well-known techniques from density based topology optimization. That is, the design field representation, the use of projection filters, the sensitivity analysis as well as the design update scheme is identical to those used in standard density based methods. The only noticeable differences are the finite element analysis, which here employs a CutFEM approach to achieve crisp and well-defined material interfaces, as well as the localization of sensitivity information due to the lack of intermediate densities (ersatz material) in the design domain. The numerical examples includes both 2D and 3D solid mechanics as well as multiphysics problems from vibro-acoustics.

*Key words:* Topology optimization, Shape optimization, CutFEM, Solid mechanics, Vibroacoustics

## Introduction

In structural optimization, density methods are often the weapon of choice due to their simplicity when compared to level set methods i.e. generalized shape optimization [1]. That is, density methods are easy to implement, regularization techniques are well established and robust, and due to the ersatz material model, the sensitivity information is global to the design domain [2]. This is in contrast to generalized shape optimization in which the sensitivity is confined to the material interface and regularization schemes, namely length scale control, still poses an unsolved problem in general. However, the family of shape optimization methods that operate with crisp interface representations, e.g. XFEM and CutFEM, have an extremely attractive quality in that they are capable of resolving and imposing complex interface conditions, which is not the case for density methods in general. Hence, it is reasonable to combine the best from both optimization disciplines in order to develop new and more flexible design tools in engineering.

## Methods

The main contribution of the presented work is the combination of a density design representation with a crisp interface immersed boundary finite element method. That is, the design field is introduced as a scalar nodal field which is filtered, projected and mapped to a convenient range using the classical robust formulation, i.e.

$$x_{phys} = M(H(F(x))) \tag{1}$$

Here $x$ is the design variables, $F(x)$ refers to a convolution filter, $H(x)$ is a smooth Heaviside filter and $M(x)$ is a linear mapping of the 0-1 design field onto a mesh size dependent interval. The latter is included to make the usual 0-1 scaling of the design field robust to mesh refinement. The lhs $x_{phys}$ is constructed using several projection parameters, exactly as done for density methods in [2], and for the minimum compliance problem we use the so-called poor mans approach such that only a single finite element analysis is needed.

The employed immersed boundary method is best described as a simplified CutFEM method with the main difference of omitting ghost penalties [3], since all our numerical experiments showed that these were not needed. Therefore the method consists of cutting the physical design field $x_{phys}$ at a specified level using marching squares (2D) or marching cubes (3D). The cut elements are then triangulated or tetrahedralized and the sub element Gauss points are mapped back into the parent quadrilateral or hexahedral element in which the integration is performed. Similar to density methods, the void regions are given an artificial material parameter to ensure that the system can be solved on the full domain without any renumbering of the dofs.

Thus, the resulting finite element systems are very alike those encountered in standard density methods, i.e. heterogeneous with high contrast, and we are therefore able to use the same type of preconditioners as presented in [4]. The optimization problem is solved using gradient based methods and the sensitivities are obtained using discrete adjoint analysis. The only difference between density based sensitivity calculations and those used here, is the computation of the system matrix differentiated with respect to the physical design variable. To simplify the implementation the term is obtained by a simple finite difference, i.e.

$$\frac{\partial K_e}{\partial x} \approx \frac{K_e^{pert} - K_e}{h} \tag{2}$$

The proposed design method is implemented in Matlab for 2D problems and using the TopOpt in PETSc [3] framework for 3D problems.

## Numerical example

The developed design method is demonstrated on examples from solid mechanics, through a number of minimum compliance and mechanism design problems in both 2D and 3D. One such result for a stiffness optimized 3D cantilever beam is seen in Figure (1), which shows the initial guess (top left) and the final optimized design (bottom right). The design domain is discretized into 525.000 elements and a solid block of material using 75% volume is chosen as the starting guess. The evolution of the design  in Figure(1) shows how the volume constraint is first made feasible and afterwards that holes appear, although no hole insertion strategy is used, i.e. holes are growing in from the sides. It should be noted that although the appearance of new holes is common in 3D, it is hardly ever seen in 2D, and thus we apply the topological derivative for 2D problems. However, the main problem here is that it generally is hard to provide feasible *and* meaningful starting guesses for generalized shape optimization problems. Therefore a noticeable amount of computational effort is devoted to making the problem feasible, and special care must be taken with the initial scaling such the constraints and objective are reasonably balanced. For the design problem shown in Figure (1), the extra computation effort amounted to 42 design cycles. The easiest way to alleviate this problem is to only use the proposed design method as a post processing tool for standard density methods. This is easily done since both methods uses the exact same design representation and is therefore a good choice for problems that can easily be solved by density methods. But, this does not apply to problems for which density methods have problems. This is often the case for problems that involves complicated interface conditions and/or problems with multiple physics.

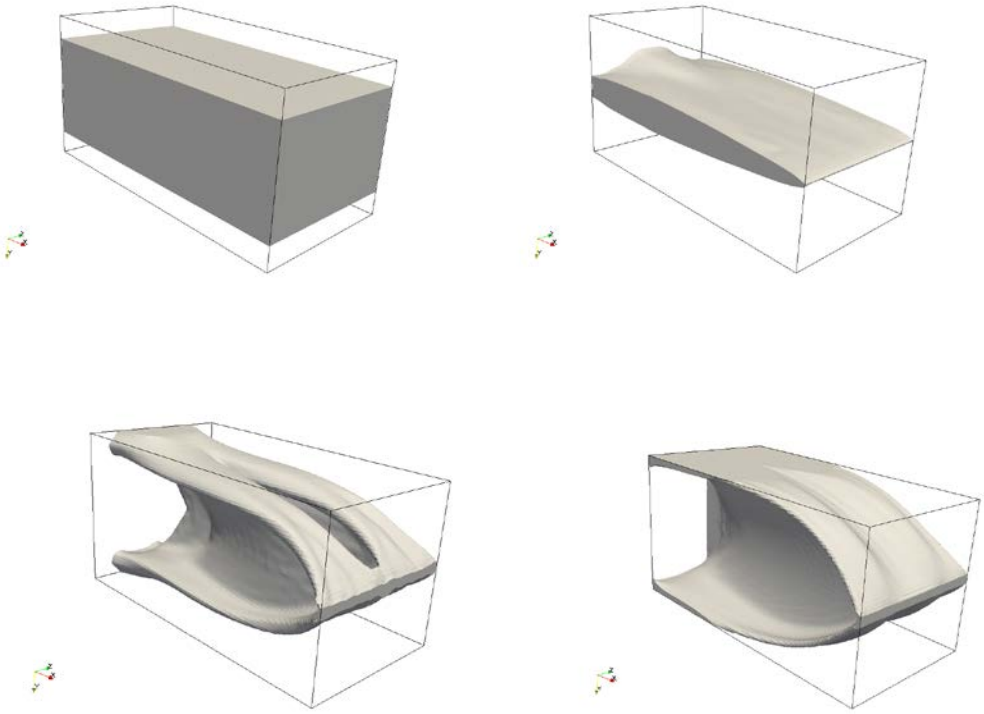

Figure 1. Minimum compliance result and selected design history (iteration 0, 42, 96 and 400, respectively) for a 3D cantilever design problem using 12% volume.

Therefore, to utilize the true strength of the presented CutFEM optimization approach, examples in which interface conditions are paramount will be presented. Focus will be on vibroacoustics, in which the acoustic pressure exerts a pressure load on the structure and the structure provides an acceleration to the fluid. We note that these coupling conditions are particularly easy to incorporate into the CutFEM solver since both couplings arise through Neumann conditions. The method is then applied to design problems using both frequency domain and time dependent problems formulations, and the results are compared to designs obtained using a density method based on the mixed formulation [5]. The study shows that the CutFEM optimized design generally are of a higher quality than the density designs.

## References


[1] Sigmund, O., & Maute, K. (2013). Topology optimization approaches. *Structural and Multidisciplinary Optimization, 48*(6), 1031–1055.

[2] Wang, F., Lazarov, B. S., & Sigmund, O. (2011). On projection methods, convergence and robust formulations in topology optimization. *Structural and Multidisciplinary Optimization*, *43*, 767–784.

[3] Burman, E., Claus, S., Hansbo, P., Larson, M. G., & Massing, A. (2014). CutFEM: Discretizing geometry and partial differential equations. *International Journal for Numerical Methods in Engineering*, 472–501.

[4] Aage, N., Andreassen, E., & Stefanov Lazarov, B. (2015). Topology optimization using PETSc: An easy-to-use, fully parallel, open source topology optimization framework. *Structural and Multidisciplinary Optimization*, *51*, 565–572.

[5] Yoon, G. H., Jensen, J. S., & Sigmund, O. (2007). Topology optimization of acoustic–structure interaction problems using a mixed finite element formulation. *International Journal for Numerical Methods in Engineering*, *70*(9), 1049–1075.