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**Svensen, Jan Lorenz; Niemann, Hans Henrik; Falk, A.K.V.; Poulsen, Niels Kjølstad**

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# Model Predictive Control of Overflow in Sewer Networks

**J.L. Svensen\*, H.H. Niemann\*\*, A.K.V. Falk\*\*\*, N.K. Poulsen\***

\* Department of Applied Mathematics and Computer Science, Technical University of Denmark, 2800 Kgs. Lyngby, Denmark , jlsv@dtu.dk, nkpo@dtu.dk
\*\*  Department of Electrical Engineering, Technical University of Denmark, 2800 Kgs. Lyngby, Denmark , hhn@elektro.dtu.dk
\*\*\* DHI Denmark, 2970 Hørsholm, Denmark, akf@dhigroup.com

**Abstract:** In this study, two previously proposed methods for Model Predictive Control (MPC) in sewage systems are compared with regard to prediction of overflow and computation time. The methods considered are a mixed integer quadratic program MPC against a quadratic program MPC with a penalty cost on accumulated overflow volume. The comparison is based on several simulations of the methods applied to a model of the Barcelona sewage system. The numerical simulations show that the quadratic program MPC is more computational efficient and as good as the mixed integer quadratic program MPC at predicting the occurrence and size of the system's overflows.

**Keywords:** Mixed Integer; MPC; Predictive control; Quadratic program; Sewer; Urban Drainage

## Introduction

In sewage systems, optimal control methods such as Model Predictive Control (MPC) have been applied for decades (Marinaki and Papageorgiou, 2005), and generally described in Maciejowski (2002). The design models of the sewer systems utilized by MPC methods are generally simplified models, such as linear reservoir models.  The weir overflows in the sewage system are more complicated though. The overflows depend on whether the volume or flow has exceeded a given threshold, which gives the overflow a binary nature of being zero below the threshold and positive function above it. Previous work (Ocampo-Martinez, 2010) has solved this by utilizing Mixed Integer (MI) MPC, and thereby describing the weir elements using logical variables to switch between models with or without overflows. A different approach to handling the binary nature was suggested by Halvgaard & Falk (2017), were the model is kept linear, and the weir flow is treated as a variable to be determined during the optimization. This allows the MPC to be formulated as a simpler problem, such as a linear or quadratic program (QP). In this study, the two approaches to handling the weir elements of sewage systems are compared on the ability to predict overflow and computation efficiency. For the comparison, the 12-tank model of the Barcelona sewage system presented by Ocampo-Martinez (2010) is utilized for the simulations.

## Methods and Models

The model of the Barcelona sewage system utilized are taken from Ocampo-Martinez (2010), where sections of the system are modelled as virtual tanks, and the interconnections are described with logical expressions. The system model is a discrete model with a sampling time of 5 min. The design model used in the MI method is identical to the system model, but reformulated using the Mixed Logic Dynamic formulation defined by Bemporad & Morari (1999). The cost function of the MI method minimizes the change in control flows (gates) and the deviation of objective flows from references. Both set of objective terms were chosen to be quadratic, which makes the MI method utilized in this work, an MPC of the mixed integer quadratic program (MIQP) type.

The objective flows and their references consist of two sets of objectives; the first being the flows towards the wastewater treatment plants with the maximum pipe capacity as the reference. The second consist of the total flooding (quantified as weir overflows from tanks and pipes), with zero as the reference. In the QP method, the design model consists of the same virtual tanks, but the interconnections are linear expressions, with the weir overflow being handled as an optimization variable. The cost function of the QP method consists of the cost function utilized by the MI method described above, but with an extra linear term, which penalizes the accumulated weir overflow volume at each time in the prediction horizon. The extra term is added to the cost function to force it to only generate overflow, when tanks and pipes have reached their maximum capacity. This enforcement is achieved by given the extra term a weight making the term cost significantly larger than the remaining objective terms. The extra term is a drawback of the QP method, given that the term will be prioritised in comparison to the other terms. This makes the objectives, which are not associated with minimization of overflow less achievable, such objective could be the maximization of the flow to the treatment plant.

## Results and Discussion

The comparison of the two methods is based on simulations of the system for both methods, driven by idealised rain patterns. These rain patterns consist of an hour of dry weather before the rain and 19 hours of dry weather after the rain. The rain period is varied in intensity and duration, an example is shown in the top graph of figure 1. The intensity is varied from 2 µm/s to 14 µm/s with 2 µm/s increments, while the rain duration is varied from half an hour to five hours with half hour increments. In the simulations, the methods have a predictive horizon of 20 min.
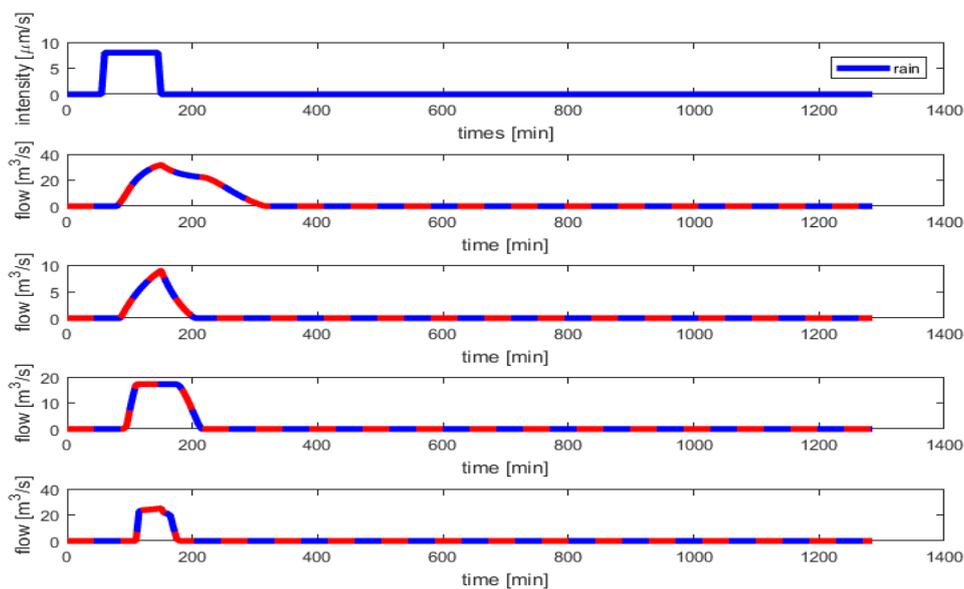


**Figure 1** Idealised rain pattern (top graph) and a selection of weir overflows from a simulation with the QP method (bottom 4 graphs), the actual overflow (red line) vs the predicted overflow (blue line).

Given the model of the system and the design model in MI method are identical, the predicted overflows of the MI method match the actual overflow of the sewage system in the simulations. In the four lower graphs in figure 1, selected overflows from a representative simulation of the system with the QP method is shown. It can be observed that the QP method likewise is capable of predicting the overflows of the

sewage system, which applies for all overflows observed during the simulations. The results show that the QP method is an alternative to MI methods, with regards to handling the binary nature of weir elements in sewage systems.

With both methods capable of predicting the weir overflows occurring in the system, a comparison of the computation time of the two methods is relevant. The maximum computation time of optimization of both methods are shown in figure 2. The computation time of the MI method is clearly dependent on the intensity and duration of the rain, while the QP method's computation time appears to be less sensitive towards the rain characteristics.
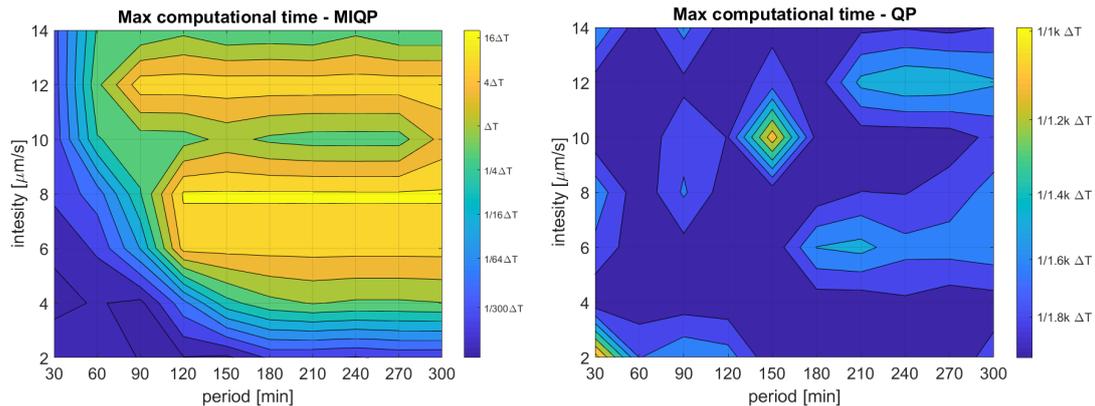


**Figure 2** Maximum computation time observed of the MI method (left) and the QP method (right).

It can further be observed that the computation time of the QP method is in the worst-case approximately 1/1000 of the sampling time ΔT (corresponding to one third of a second), while the MI methods worst-case computation times are on the scale of a dozen sampling times or roughly in the area of 0.5-1.5 hour. In the MI method the largest computation time occurs, when the logic parts of the system are at the switching point, becoming sensitive to the change in control flows. The largest computation times of the MI method occurs under the more extreme and unlikely weather events, it is also more depending on the size of the prediction horizon in comparison to the QP method. In figure 3, the maximum computation time of both methods are shown as the prediction horizon increases for a rain pattern of a half hour with an intensity of 2 µm/s.
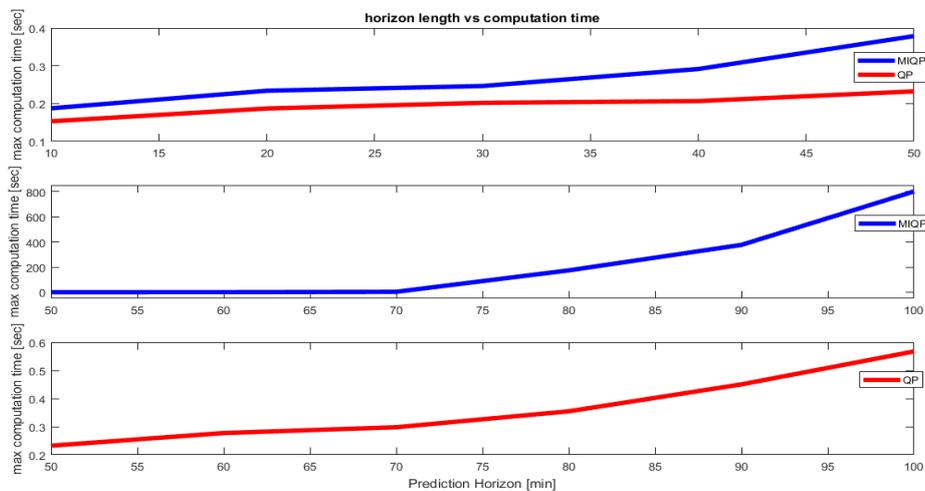


**Figure 3** Maximum Computation time vs the length of the prediction horizon for both the MI method (blue) and the QP method (red). The graph is split in three at 50 min for details being visible.

The computation time of the MI method can be seen to rapidly increase with the horizon length in comparison to the QP method. The MI method uses 800 seconds against the QP method's 0.6 seconds for a 100 min horizon. In sewage system, it is usually preferred to predict at least a few hours ahead, and not a few tens of minutes; therefore, the QP method shows better prospective, given the computation time allows longer prediction horizons, and therefore can compute a long-term operation plan.
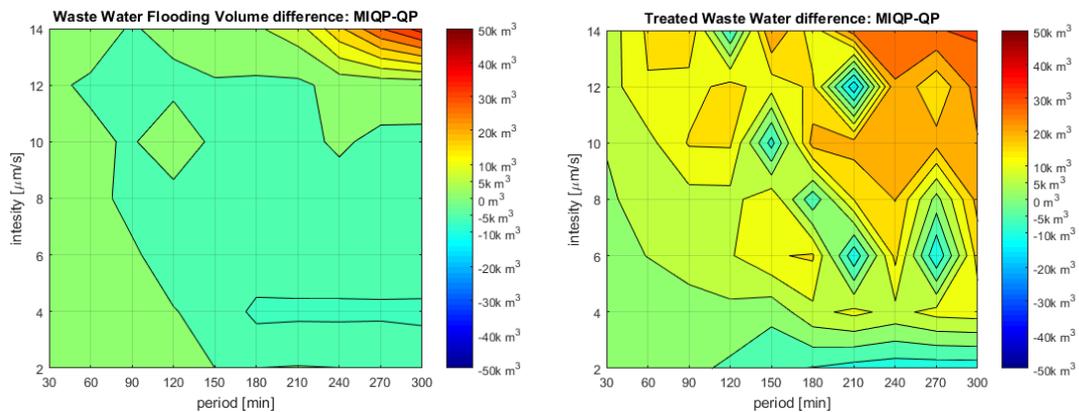


**Figure 4** the difference in flooding volume (left) and the treated waste water at the treatment plants (right) between the MI method and the QP method.

On the operational aspect, figure 4 shows the difference in flooding volume and treated wastewater in $m^3$. It can be seen that the amount of flooding wastewater is relatively similar between the two methods. In the case of the difference in amount of wastewater, being sent to the treatment plants, the MI method generally perform better as the rainfall increases, resulting in more treated water, but for smaller and more likely rain falls, the performance is quite similar.

## Conclusions

In this work, we have compared two methods for handling weir elements with regards to MPC in sewage systems. The methods are a Mixed Integer MPC versus a Quadratic Program MPC with an accumulate weir overflow volume cost term. The simulations with the methods show a clear benefit towards utilizing the QP method: both methods accurately predict the overflows of the system, but the QP method has much lower computation times than the MI method. This allows the QP method to work with longer prediction horizons. For the MI method, the computation time becomes impractical for more extreme weather or longer prediction horizons.

## Acknowledgement

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