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# Modelling of Wave-structure Interaction for Cylindrical Structures using a Spectral Element Multigrid Method


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## Highlights

- Time-domain modeling wave-structure interaction of surface-piercing bottom-mounted and truncated cylindrical structures.
- Fully Nonlinear Potential Flow (FNPF) time domain solver based on a high-order Spectral Element Method that has support for geometric flexibility using adaptive meshes and high-order convergence rates.
- Efficient and scalable $\mathcal{O}(n)$ solution effort using a geometric $p$-multigrid method that enable practical calculation times for engineering analysis.

## Introduction

The need for advanced time-domain simulators for improved offshore engineering analysis are growing with the continued improvement in computational ressources. In line with this trend, we consider a fully nonlinear potential flow (FNPF) model discretised with a stabilised Galerkin Spectral Element Method [5] addressing the stability problems and lack of progress made for this type of modelling approach since the work of Robertson & Sherwin (1999) [11]. A recent review of the Spectral Element Method is given in [12]. In a recent study a SEM-based fully nonlinear potential flow model referred to as the FNPF-SEM model [4] was developed and then validated in a blindtest experiment [10] against experimental measurements for focusing waves interacting with a fixed FPSO structure. In this work, we consider a standard benchmark problem for cylindrical structures due to McCamy & Fuchs (1954) [9], with an objective to evaluate a new extension of this solver with a $p$-multigrid method with the aim to enable scalable $\mathcal{O}(n)$ complexity in work effort. Recently, the first proof of an efficient geometric $p$-multigrid method was demonstrated in 2D and in 3D[8], and in this work we provide some additional results based on refined developments of the algorithms with the aim of demonstrating the practical feasibility of using this new SEM-based solver for 3D analysis. Our main objective is to handle the wave propagation problem as well as the curvilinear features of offshore structures such as cylindrical structures within a single FNPF solver. In particular, the new FNPF-SEM $p$-multigrid solver makes it possible to address both wave propagation and wave-structure problems within a single solver. The $p$-multigrid method is designed to exploit the $p$-type convergence property in solving the Laplace problem, and by avoiding $h$-type convergence property it is possible to handle the representation of structural bodies with curvilinear features without refining the underlying mesh-topology. In this sense, this work contributes to demonstrating that the spectral element method is a technology that is useful for engineering analysis. It comes with the ability to represent offshore structures that can also be handled with methods such as HOBEM [7], however, SEM comes with sparse matrices after global assembly in the discrete problem, and therefore provides more efficient and better scalability than boundary element methods that comes with dense operators and much higher costs to achieve asymptotic scalability. By using a FNPF formulation it is possible to predict the horizontal and vertical hydrodynamic forces for marine structures placed offshore, and account for the nonlinear effects that are significant when standard frequency domain analysis falls short and time domain solutions may be used instead.

# Governing equations

A brief description of the modelling equations are presented in line with the derivation given in [2]. We consider the Eulerian formulation based on fully nonlinear potential flow theory. A Cartesian coordinate system is adopted with the origin on the still water $xy$-plane and the $z$-axis pointing vertically upwards. $t$ is time. The fluid domain is bounded by the sea bottom at $z = -h(\mathbf{x})$, the free-surface at $z = \eta(\mathbf{x}, t)$, and an enclosing boundary that defined the dimensions of a Numerical Wave Tank (NWT) as well as structural boundaries of fixed bottom-mounted and fixed floating structures that are surface-piercing. The free surface boundary conditions are expressed in terms of 'free-surface only' variables

$$\frac{\partial}{\partial t}\eta = -\nabla\eta \cdot \nabla\tilde{\phi} + \tilde{w}(1 + \nabla\eta \cdot \nabla\eta), \tag{1a}$$

$$\frac{\partial}{\partial t}\tilde{\phi} = -g\,\eta - \tfrac{1}{2}\nabla\tilde{\phi} \cdot \nabla\tilde{\phi} + \tfrac{1}{2}\tilde{w}^2(1 + \nabla\eta \cdot \nabla\eta). \tag{1b}$$

Here $\nabla = (\partial/\partial x, \partial/\partial y)$ is the horizontal gradient operator, $g$ the gravitational acceleration. These equations describes the temporal evolution of the free surface elevation $\eta$ and free surface velocity potential $\tilde{\phi}$. Subject to numerical discretization these equations defines an initial value problem that can be advanced in time using a classical explicit fourth-order Runge-Kutta method. To obtain closure, the vertical velocity at the free surface $\tilde{w}$ needs to be determined from the known state of the fluid surface defined by $\eta$ and $\tilde{\phi}$, that requires satisfying a continuity equation in the form of the Laplace equation and solid boundary conditions.

$$\nabla^2\phi + \phi_{zz} = 0, \qquad -h(x, y) \le z \le \eta(x, y, t) \tag{2a}$$

$$\phi_z + \nabla h \cdot \nabla\phi = 0, \qquad z = -h(x, y). \tag{2b}$$

$$\mathbf{n} \cdot (\nabla\phi, \phi_z) = 0 \qquad \text{on } \Gamma_b, \tag{2c}$$

where $\boldsymbol{n}$ is representing outward pointing normal vectors at structural surfaces $\Gamma_b$.

To solve this Laplace problem we employ a Galerkin Spectral Element discretization where the fluid volume (domain) is first decomposed into a set of non-overlapping elements. These elements are in this work taken to be prism elements in two layers that are generated using the open source mesh generation `Gmsh` [6]. The element in the lowest layer are kept fixed and the elements in the upper layer are adjusted to the free surface using curvilinear elements that are stretched with the position of the free surface at a given instant of time. Across the domain, solution quantities are represented using globally piece-wise continuous polynomial basis functions of arbitrary polynomial order. The discretization leads to a linear system of equations [4] that defines the discrete Laplace problem

$$\boldsymbol{L}\boldsymbol{\phi} = \boldsymbol{b}. \tag{3}$$

This problem is a computational bottleneck problem in our procedure and we solve it efficiently using a geometric $p$-multigrid method tailored to the MarineSEM library and is exploited specifically as an iterative solver in the FNPF-SEM model.

# Geometric $p$-multigrid method

[Wojciech: Describe briefly here the p-multigrid method you employ. If not enough room, just describe in text and give the references to the techniques you are using]

# Numerical experiments

The main purpose of the numerical experiments is to validate the numerical model and highlight the high accuracy and flexibility of the model for nonlinear wave propagation and wave-structure interactions.

## Linear diffraction

We consider the scattering of regular waves on a single cylinder in an open sea with flat bottom. We compare to the exact solutions due to MacCamy & Fuchs (1954) valid for a cylinder positioned in finite depth. This MacCamy & Fuchs solution for small amplitude waves generalises the solution of Havelock (1940) for a cylinder in deep water. The validity of this study on diffraction effects lies in the assumption that the inertial forces dominates the drag forces in interaction of the fluid with the cylinder. The case highlight the ability of the SEM to accurately account for the circular boundary representation of a realistic shape of an object such as a mono-pile foundation. Analytical solutions exist also for multiple cylinders, cf. Twersky (1952) and for a range of different arrangement of two cylinders due to Spring & Monkmeyer (1974). We consider a case for prediction the maximum wave run-up assuming small amplitude wave propagation at relative depth $kh = 1$ and relative size of cylinder $kR = \pi/2$. Excellent agreement between analytical results of MacCamy & Fuchs compared to the high-order SEM solution is given in Figure 2. In the simulation we have used curvilinear iso-parametric elements to represent the cylinder boundary as in the high-order Boussinesq model [1, 3] based on a nodal Discontinuous Galerkin spectral element method. The minor discrepancy in the wave run-up at the front side is due to minor reflections between the wave generation zone and the cylinder. The result demonstrate the high accuracy that can be achieved using the SEM model.

At the workshop we plan to demonstrate results also for nonlinear diffraction cases describing the interaction between nonlinear waves and cylindrical structures.

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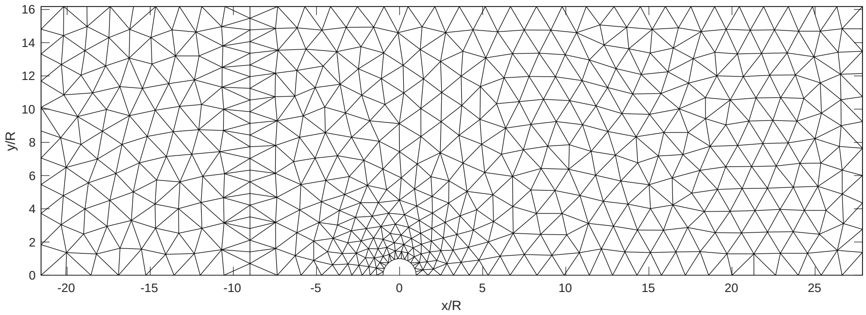


Figure 1: Illustration of mesh for the representation of the free surface plane. The mesh conforms with the interface of the wave generation zone (left part).

(a) Wave envelope (b) Wave runup

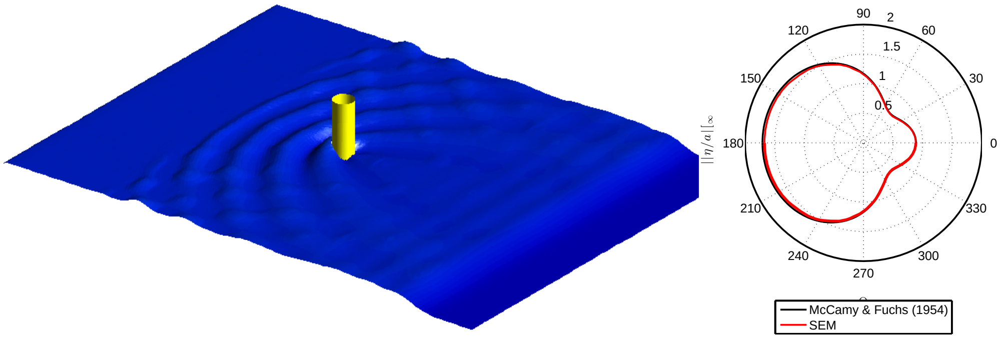


Figure 2: (a) Wave envelope with some visible reflections near boundaries away from the proximity of the cylinder. (b) Comparison of maximum wave run-up on cylinder in an open sea for exact solution and computed results shows excellent agreement. $kh = 1$. $kR = 1$. Maximum wave run-up is predicted to be up to 1.67 times larger on front side relative to the incident wave amplitude as determined from the wave crease envelope. Exact integrations were used in liner model. $(N, N_s) = (5, 6)$. $\Delta t = 0.02$ s.

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