Fault-tolerant Sensor Fusion for Marine Navigation

Blanke, Mogens

Published in:
Proceedings of the 7th IFAC Conference on Manoeuvring and Control of Marine Craft

Publication date:
2006

Document Version
Publisher's PDF, also known as Version of record

Citation (APA):
Blanke, M. (2006). Fault-tolerant Sensor Fusion for Marine Navigation. In *Proceedings of the 7th IFAC Conference on Manoeuvring and Control of Marine Craft* (pp. 1385-1390). Elsevier.

FAULT-TOLERANT SENSOR FUSION FOR MARINE NAVIGATION

Mogens Blanke

Automation at Ørsted•DTU, Technical Univ. of Denmark, Kgs. Lyngby, Denmark and CeSOS, Norwegian Univ. of Science and Technology, Trondheim, Norway
e-mail: mb@oersted.dtu.dk

Abstract:
Reliability of navigation data are critical for steering and manoeuvring control, and in particular so at high speed or in critical phases of a mission. Should faults occur, faulty instruments need be autonomously isolated and faulty information discarded. This paper designs a navigation solution where essential navigation information is provided even with multiple faults in instrumentation. The paper proposes a provable correct implementation through auto-generated state-event logics in a supervisory part of the algorithms. Test results from naval vessels shows events where the fault-tolerant sensor fusion provided uninterrupted navigation despite temporal instrument defects.

Keywords: Navigation, Fault-tolerance, Sensor Fusion, Fault diagnosis.

1. INTRODUCTION

Precise and reliable navigation information is crucial for heading, manoeuvring and motion control, thus temporal faults or artifacts in navigation signals may compromise safety. Even when essential navigation instruments are duplicated, present navigation systems require manual selection of a healthy device. Manual selection is time-consuming and prone to errors, and time to reconfigure instruments can itself be critical on vessels at high speed or in critical phases of a mission.

This paper addresses fault-tolerant sensor fusion where the entire suite of navigation instruments are considered, and a reliable estimate of navigation data is obtained. When faults occur, faulty instruments are autonomously isolated and faulty information discarded. Analytic redundancy is employed to diagnose faults and autonomously provide valid navigation data, disregarding any faulty sensor data and use sensor fusion to obtain a best estimate for users. Where the majority of research on fault diagnosis considers only single faults, this paper analyzes cases of multiple simultaneous faults, and designs a navigation computer solution where essential navigation information is provided even with multiple faults in instrumentation. Handling cases of multiple faults give rise to considerable complexity in the software implementation, which is therefore prone to errors. To avoid implementation defects, the paper also proposes how a provable correct implementation can be obtained through auto-generated state-event logics in the supervisory part of the algorithms.

The paper introduces the sensor fusion problem and the requirements to fault tolerance. System structure and redundancy relations for fault diagnosis are analyzed. Multiple faults are considered and automated generation of software code for residual generators are suggested. Finally, test results are shown from naval vessels.

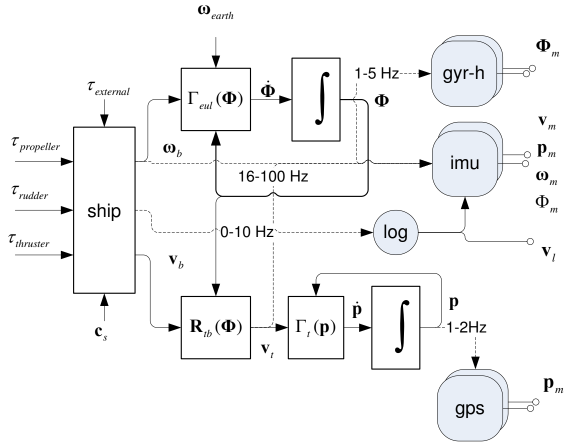

Fig. 1. Ship sensors: imu units, conventional gyro, ships log and GPS receivers.

2. STRUCTURAL MODEL

A structural model Blanke et al. (2006) describes the behavior one could expect between variables in a normal, non faulty system. For a sensor, the normal behavior will be to provide a signal, which gives a correct information about the physical variable being measured. With position denoted p, a GPS position sensor would provide an output signal p_m and the normal behavior would be described by the simple constraint $c_1 : p_m = p$. A fault in the sensor would mean the constraint c_1 was violated. When the normal behavior of an entire system is described by constraints, any violation of one or more constraints would mean one or more faults are present in the system. If the violation is isolated to a particular constraint, this will not inform on the physical reason behind the fault, but indicate that the particular device could not be trusted. The aim is to automatically exclude defect sensors, so reliable fault isolation is needed.

A systematic analysis of normal behaviors is a powerful strategy since the correctness of an analysis of faults does not rely on having a list available of all possible faults, but a far more simple description is needed, namely of the normal behavior of components and subsystems.

Following the notation in Fossen (2002), $\mathbf{v} \equiv \mathbf{v}_o^b$ is the ship's inertial velocity vector in body coordinates, $\mathbf{p} \equiv \mathbf{p}^n$ the position in (North, East) coordinates, λ is latitude, Θ is the attitude vector (Euler angles roll, pitch and yaw), ω the angular velocity and $\mathbf{v}_c^n$ the sea current, $\mathbf{R}_b^n$ the rotation matrix from body to navigation frame. Assuming forces are unknown, constraint c_4 in Eq. (1) describes the principal kinematics.

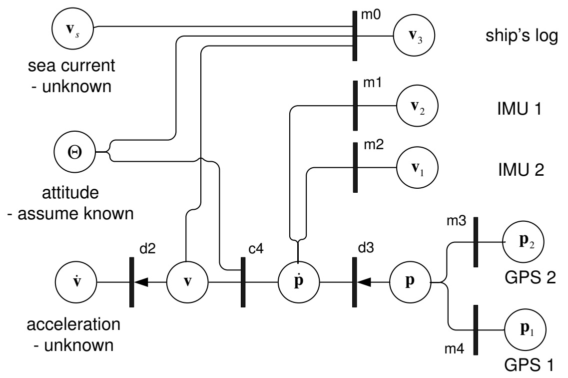

Fig. 2. Structure graph for velocity and position.

$$
\begin{aligned}
c_4 : \quad \dot{\mathbf{p}}^n &= \mathbf{R}_b^n(\Theta, \lambda)\mathbf{v}^b \\
d_2 : \quad \dot{\mathbf{v}}^{\mathbf{b}} &= \frac{d}{dt}\mathbf{v}^b \\
d_3 : \quad \dot{\mathbf{p}}^n &= \frac{d}{dt}\mathbf{p}^n \\
m_1 : \quad \mathbf{v}_1^b &= \mathbf{R}_n^b(\Theta, \lambda)\dot{\mathbf{p}}^n \\
m_2 : \quad \mathbf{v}_2^b &= \mathbf{R}_n^b(\Theta, \lambda)\dot{\mathbf{p}}^n \\
m_3 : \quad \mathbf{p}_1^n &= \mathbf{p}^n \\
m_4 : \quad \mathbf{p}_2^n &= \mathbf{p}^n \\
m_0 : \quad \mathbf{v}_3^b &= \mathbf{v}^b - \mathbf{R}_n^b(\Theta)\mathbf{v}_c^n
\end{aligned}
\qquad (1)
$$

The relations d_2 , d_3 are differential constraints, these serve to tell there is a relation between the time derivative and the variable itself. Measurement constraints $m_1 - m_0$ show we have two GPS receivers available for position monitoring. Two IMU units measure absolute velocity, and velocity through water is measured by a two-axis log. The sea current is $\mathbf{v}_c^n$. The measurement part of Eq. 1 is illustrated in Fig. 1.

It is noted that instrument compensation is implicit in the measurement constraints. With GPS receiver antenna location $\mathbf{l}_g^b$, position at the antenna is $\mathbf{p}_a^n = \mathbf{p} + \mathbf{R}_b^n(\Theta)\mathbf{l}_g^b$. Antenna compensation in the receiver makes the compensation $\mathbf{p}_1^n = \mathbf{p}_a^n - \mathbf{R}_b^n(\Theta)\mathbf{l}_g^b$. With IMU position $\mathbf{l}_{imu}^b$, measurements are compensated for the angular velocity sensitivity, $\mathbf{v}_{imu}^n = \mathbf{v}^n + \mathbf{l}_{imu}^b \times \omega_b^n$. These details are not shown in the constraints since ω and Θ are assumed known, to keep the focus on the principal elements of the example .

The set of constraints in Eq. (1) are referred to as the structure S of the problem. The set of variables in a system are separated into the sets X (unknown) and K (known),

$$
\begin{aligned}
C &= \{c_4, d_2, d_3, m_1, m_2, m_3, m_4, m_0\} \\
X &= \{\dot{\mathbf{v}}, \mathbf{v}, \dot{\mathbf{p}}, \mathbf{p}, \mathbf{v}_c\} \\
K &= \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{p}_1, \mathbf{p}_2, \boldsymbol{\Theta}\}
\end{aligned}
\qquad (2)
$$

A graphical view of the structure graph of the translation part of the navigation equations, Eq.

$\diagdown$	*known*					*unknown*				
	v_1	v_2	v_3	p_1	p_2	$\mathbf{p}$	$\dot{\mathbf{p}}$	$\mathbf{v}$	$\dot{\mathbf{v}}$	$\mathbf{v}_c$
c_4							1	1		
d_2								x	1	
d_3						x	1			
m_1	1						1			
m_2		1					1			
m_3				1		1				
m_4					1	1				
m_0			1					1		1

Table 1. Structure as incidence matrix

(2), assuming attitude and angular rate are known, is shown in Fig. 2.

Arrows in Figure 2 indicate direction of calculability. An equivalent representation is the incidence matrix in Table 2 where x indicate direction of calculability: p can not be calculated by integration of $\dot{p}$ unless the initial value of p is known.

2.1 Analysis of structure

The structure of the system is a bipartite graph that shows which variables are used in each constraint. Matching a variable can be interpreted as an advise on the order in which a set of equations can be solved. Fundamental graph-theory results are found in the seminal paper Dulmage and Mendelsohn (1959). An over-constrained graph has excess constraints such that all variables can be matched to constraints, leaving excess constraints as analytical redundancy relations.

In an over-determined part of a system, a variable $x_i \in X$ being matched by constraint c_j is isolated in the graph such that e.g.

$$c_j(x_i, c_{j-1}(c_{j-2}(k_l)), c_{j-3}(k_{l-1})..) = 0,$$

where x_i could be determined using constraint c_j and prior matchings of other variables. Eventually, this trace leads back to known variables $k_l, k_{l-1},.. \in K$. Unmatched constraints express analytical redundancy relations, also referred to as parity relations, are

$$p_k(k_l, k_{l-1}, ...) \equiv c_{j+1}(c_j(c_{j-1}(c_{j-2}(k_l)), ...). \quad (3)$$

Normal behavior is confirmed of all constraints that are used in a parity relation if it equals zero when known (measured) variables are inserted,

$$p_k(k_l, k_{l-1}, ...) \equiv 0.$$

2.2 Parity relations, detectability and isolability

When a matching has been found, backtracking to known variables will provide a set of parity relations that could be used as residual generators. A system with m constraints and n parity relations will give a relation showing which residuals depend on which constraints. One view on these relations is the boolean mapping,

$$\mathcal{F} : r \leftarrow M \otimes (c_i \neq 0) \quad (4)$$

from which structural detectability and isolability can be found.

$\diagdown$	m_1	m_2	m_3	m_4	m_0
p_1	0	1	0	1	0
p_2	1	1	0	0	0
p_3	0	0	1	1	0

Table 2. Dependency matrix

Definition 1. Structural detectability. A fault is structurally detectable $c_j \in C_{det}$ iff it has a nonzero boolean signature in the residual, $C_{det} = \{c_j \,|\exists j : c_j \neq 0 \Rightarrow r_j \neq 0\}$.

Definition 2. Structural isolability A fault is structurally isolable iff it has a unique signature in the residual vector, i.e. column m_i of M is independent of all other columns in M, $c_i \in C_{iso}$ iff $\forall j \neq i : m_i \neq m_j$

A dependency matrix for the reduced navigation system is shown in Table (2), where columns are formed by all constraints that could fail, rows are the parity relations. Constraints that cannot fail are definitions $d2$, $d3$ and c_4.The result of a single matching of the system in normal condition is

The three primary parity relations are

$$\begin{array}{lll} p_1 : & d3(m2(v2), m4(p2)) = 0 & v_2 - \frac{d}{dt}p_2 = 0 \\ p_2 : & m1(v1, m2(v2)) = 0 & v_2 - v_1 = 0 \\ p_3 : & m3(p1, m4(p2)) & p_2 - p_1 = 0 \end{array} \quad (5)$$

The result obtained if each of 19 possible matchings are found, is a set of parity relations, which are all linear combinations of the primary relations listed in Eq. 5. This is a fundamental consequence of the structure graph for systems with pure kinematic relations.

Theorem 1. Parity relations for systems with pure kinematics. Let a system have pure kinematic constraint(s) $d : \frac{d}{dt}p = \dot{p}$ and measurements of the variables p and $\dot{p}$ described by constraints $m_j^p : p_j = p \; (j = 1, n)$ and $m_i^v : v_i = \dot{p} \; (i = 1, k)$. Define

$$\Pi_d = \left\{ \frac{d}{dt}p_j - v_i = 0 \right\} \forall i, j,$$
$$\Pi_p = \{p_i - p_j = 0\} \, \forall i, j \in [1, n], i \neq j,$$
$$\Pi_v = \{v_i - v_j = 0\} \, \forall i, j \in [1, k], i \neq j.$$

Then the set of all possible parity relations are $\Pi \; : \; p \in \Pi_d \cup \Pi_p \cup \Pi_v$.

Proof 1. A complete matching of the kinematics is obtained in one of two ways. One: $\dot{p}$ is matched via any m^v constraint and p by any m^p. Then d is unmatched and a differential parity relation reads $\frac{d}{dt}p_j - v_i = 0$, hence $p \in \Pi_d \neq \emptyset$. Two: $\dot{p}$ is matched via d. Then if $\exists m^v$ *unmatched*, $v_i - v_j = 0$ are parity relations and $p \in \Pi_v \neq \emptyset$. If $\exists m^p$ *unmatched*, $p_i - p_j = 0$ are parity relations and $p \in \Pi_p \neq \emptyset$. In an over-constrained system, at least one parity relation is unmatched, hence $\Pi : p \in \Pi_d \cup \Pi_p \cup \Pi_v$, *qed*,

This result is essential in a fault-tolerant sensor fusion where all possible parity relations need be generated since any sensor is prone to fault at any time, and simultaneous faults may occur.

2.2.1. Diagnosis of normal and impaired system The general result of the analysis of structure is a set of automatically generated parity relations that are used to test each of the measurement signals for faults
$\Pi : \{p \mid p(k_n, k_{n-1}, .., k_1) = 0\}$, where $\{k_i\}$ is the set of available known variables.

If a fault is present in the system, this implies the set of available sensors is reduced. The dimension of the over-constrained part of the system is reduced by one for each violated constraint (fault) in the system, and the number of primary parity relations are reduced by one for each violated constraint in the over-constrained system.

3. FUSION WITH ASYNCHRONOUS DATA

Sensor fusions aims at collecting measurements from the set of available sensors and get a minimum variance estimate. Needles to say, the solution is the Kalman filter. The commonplace formulation of the Kalman filter assumes synchronous data and data sampled at the same rate. In navigation systems, data are not synchronous, sampling rates differ, data arrive asynchronously and sampling may be irregular. Data from some sensors have computational and network delays, which can not be neglected.

3.1 Sensor data fusion.

With asynchronous sensor data, the formulation of the Kalman filter needs be reconsidered and time needs be specific in the filter.

Both issues have been analyzed in the literature, and it turns out as an advantage to use a formulation of the filter which uses updating of the inverse covariance. The inverse covariance of a signal is the information. If covariance is very large, i.e. the information is close to zero, the inverse update algorithm has the advantage of being robust, since zero information in an incoming signal means the information is unchanged. Another advantage is the possibility to implement the inverse covariance updating filter such that one measurement is processed at a time, when measurements are uncorrelated, see Brown and Hwang (1997) for details.

Theorem 2. Inverse covariance updating Kalman Filter (Brown and Hwang (1997)). The optimal estimator for the linear, time-invariant system

$$\begin{aligned} \mathbf{x}(t_2) &= \mathbf{\Phi}(t_2 - t_1)\mathbf{x}(t_1) + \mathbf{w}(t_2) \\ \mathbf{y}(t_2) &= \mathbf{C}(t_2)\mathbf{x}(t_2) + \mathbf{v}(t_2) \\ E\{\mathbf{w}\mathbf{w}^T\} &= \mathbf{Q} \\ E\{\mathbf{v}\mathbf{v}^T\} &= \mathbf{R} \end{aligned}$$

is given by the following algorithm,

1. *Predict in time*
$$\Delta t = t_k - t_{k-1}$$
$$\hat{\mathbf{x}}_k^- = \mathbf{\Phi}_k(\Delta t)\hat{\mathbf{x}}_{k-1}$$
$$\left(\mathbf{P}_k^{-1}\right)^- = \mathbf{\Phi}_k(\Delta t)\mathbf{P}_{k-1}\mathbf{\Phi}_k^T(\Delta t) + \mathbf{Q}_{k-1}$$
2. *Update inverse covariance:* (6)
$$\mathbf{P}_k^{-1} = \left(\mathbf{P}_k^{-1}\right)^- + \mathbf{C}_k^T\mathbf{R}_k^{-1}\mathbf{C}_k$$
$$\mathbf{P}_k = \left(\mathbf{P}_k^{-1}\right)^{-1}$$
$$\mathbf{K}_k = \mathbf{P}_k\mathbf{C}_k^T\mathbf{R}_k^{-1}$$
3. *Update estimate*
$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}_k\left(\mathbf{y}_k - \mathbf{C}_k\hat{\mathbf{x}}_k^-\right)$$

Proof 2. See Brown and Hwang (1997) chapter 9 for a proof of this updating algorithm.

It is noted that with Eq.(6), both mean and covariance can be propagated hindcasted in time. A measurement with a computational delay will arrive at time $t_j > t_{k-1}$but due to the delay, $t_k < t$ hence $\Delta t < 0$. It is thus essential to use a timestamp t_k that informs on when a measurement was generated, not when it was received. Another salient feature of this algorithm is that, if measurements are uncorrelated, the inverse covariance update of Eq. (6) reduces to individual measurement updates,

$$\begin{aligned} \mathbf{P}_k^{-1} &= (\mathbf{P}_k)^{-1} \\ &+ \left[\mathbf{C}_1^T, \mathbf{C}_2^T, \ldots\right] \begin{bmatrix} \mathbf{R}_1^{-1} & 0 & 0 \\ 0 & \mathbf{R}_2^{-1} & 0 \\ 0 & 0 & \ddots \end{bmatrix} \begin{bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \\ \vdots \end{bmatrix} \\ &= (\mathbf{P}_k)^{-1} + \mathbf{C}_1^T\mathbf{R}_1^{-1}\mathbf{C}_1 + \mathbf{C}_2^T\mathbf{R}_2^{-1}\mathbf{C}_2 + ... \end{aligned} \qquad (7)$$

where each of the measurements are treated independently. There is a significant computational saving associated with measurement covariance reducing to a scalar or a small dimension when the alternative was to invert the entire measurement covariance matrix. Inversion need only be done of an $n \times n$ matrix where n is state dimension.

3.2 Fault-tolerant sensor fusion

Fault-tolerant sensor fusion in brief serves the purpose to prevent any faulty measurement from entering into the sensor fusion filter, Eq.(6). Data can be invalid when: an artefact is present is a signal; a measurement is timed out; a fault is present is a sensor. Once validated, measurements update the sensor fusion filter. Should one measurement be absent, due to a fault, the remaining set of available measurements will be used to update the sensor fusion filter. When an instrument becomes available, it can re-enter the calculations without causing jumps in state or covariance estimates.

Fault-tolerant sensor fusion consists of three steps

Algorithm 1. Fault-tolerant sensor fusion:
1. Check sanity of input signals for signal age (timestamp) and outliers
2. Diagnose signals as normal or not normal
3. Make an optimal sensor fusion using the Kalman filter of Equations (6, 7).

4. AUTOMATON FOR RESIDUAL GENERATION

With asynchronously arriving data, residual generation is more difficult than when data are synchronous. The parity relations that can be generated at any instant of time depends on the set of valid and available signals. The state event diagram for residual generation from three measurements is shown in Fig. 3. States indicate which measurements have arrived. Each state is associated with an action table that determines which computational actions shall be taken, i.e. which parity relations shall be calculated. The automaton must also consider non-valid data events, shown as $\tilde{~}v$ in the Figure.

Implementation of state-event automata associated with sensor events is seen to be fairly complex and is hence prone to implementation defects. Automatic generation of the automaton $\mathcal{S}$ and the action tables are thus suggested as follows,

$$
\begin{aligned}
&Let : \check{k}_i \equiv \begin{cases} 1 & k_i = valid \\ 0 & k_i \neq valid \end{cases} \quad \forall i \qquad (8)\\
&s_i = \{\check{k}_m, \check{k}_{m-1}, \cdots, \check{k}_1\}\\
&\mathcal{S} : s_k \leftarrow S(s_{k-1}, e_k)\\
&\mathcal{A} : \mathbf{r} = \mathbf{p}(k_i) \;\; given \; k_i \text{ in } s_k
\end{aligned}
$$

Since the state-event transition matrix S is a map of all possible combination of states and events.

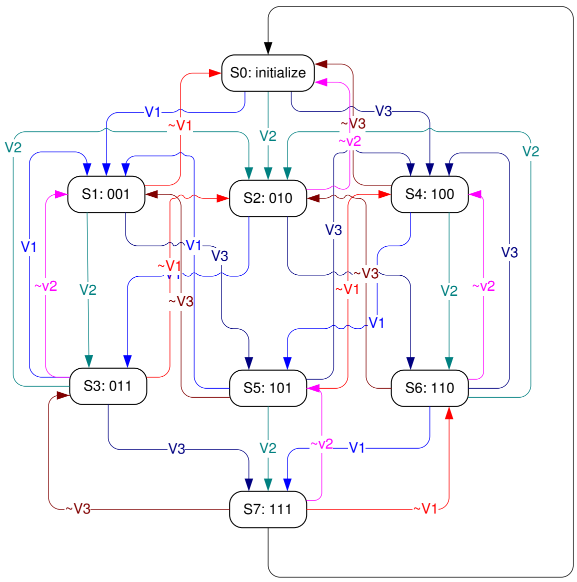

Fig. 3. State-evant automaton for three sensor signals. v1 denotes valid event, ˜v1 denotes not-valid signal event.

The event vector is $e = [v_1, v_2, v_3, \neg v_1, \neg v_2, \neg v_3]$. The state transition matrix for a problem of size N can be generated by

$$
\begin{aligned}
&s = [1, 2, \cdots, 2^N] - 1\\
&for \quad j = 1 : N\\
&\quad S(:, j) = s \vee bitset(0, j, 1)\\
&\quad S(:, j) = s \wedge bitset(2^N - 1, j, 0)\\
&end\\
&S(N, :) = S(1, :);
\end{aligned} \qquad (9)
$$

The parity vector is then generated from the list of parity relations Π obtained automatically from the graph-matching and backtracking. The procedure is hence provable correct for residual generation.

4.1 Change evaluation

Change detection and hypothesis testing in a purely stochastic setting is often based on the log likelihood ratio and a decision function $g(t)$. Looking for faults that are strongly detectable in residuals, a cumulative sum (CUSUM) test function could be chosen using parameters σ_i for standard deviation of residual component r_i and μ_{0i}, μ_{1i} for the mean value of the residual before and after an unwanted change (fault) has a log likelihood test quantity, see Basseville and Nikiforov (1993). For a scalar residual

$$s_i(t_k) = \frac{\mu_{1i} - \mu_{0i}}{\sigma_i^2}\left(r_i(t_k) - \frac{\mu_{1i} + \mu_{0i}}{2}\right) \quad (10)$$

and for a vector valued residual

$$s(t_k) = (\mu_1 - \mu_0)' \mathbf{Q}^{-1} (\mathbf{r}(t_k) - \frac{1}{2}(\mu_1 + \mu_0)) \quad (11)$$

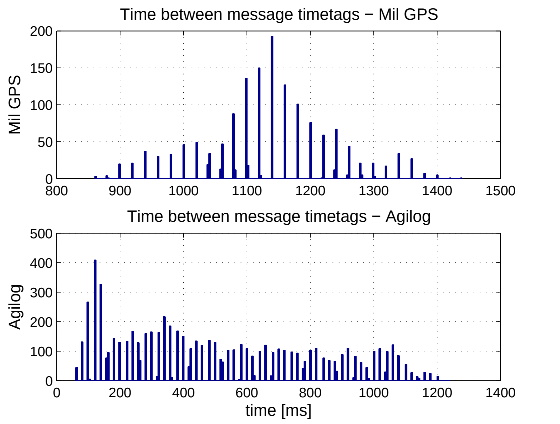

Fig. 4. Histogram of timetags on Military GPS and ship's log signals

where $\mathbf{Q}$ is the covariance matrix for the residual vector. The decision function is here given a slight twist compared to the usual recursive CUSUM,

$$g_i(t_k) = \min\left(\max\left(g_i(t_{k-1}) + s_i(t_k), -\beta_a a_i\right), \beta_h h_i\right)$$

where a_i and h_i are threshold values for detection of normal and faulty behavior, respectively and $\beta_a, \beta_h > 1$ are used to control the transition away from a confirmed hypothesis,

$$\begin{array}{ll} H_o: \; g_i \leq -a_i & \text{normal confirmed} \;\; (h_i = 0) \\ H_1: \;\; g_i \geq h_i & \text{fault confirmed} \;\;\; (a_i = 0) \end{array} \tag{12}$$

We use two levels of h for detection/isolation (Nikiforov (2000)) but hypothesis testing is made in parallel for normal and faulty in our approach to avoid restarting of algorithms from scratch to return to normal after temporal faults.

5. EXPERIENCE

A full implementation of the fault-tolerant sensor fusion was implemented as a navigation computer solution on a number of naval vessels. This section highlights experiences and results.

Asynchronous sensor events. The timing issue is illustrated by real data as shown in Fig. 4 where timestamps on arriving signals are plotted as histograms. The nominal transmission frequency of 1Hz from the particular GPS receiver is seen to vary with a 200 ms interval (1 sigma). The plot is the result of statistics made over one hour. The ship's speed log does not supply data at constant rate but after a certain increment in distance.

Sensor faults. If a measurement should be absent due to a fault, the remaining set of available measurements are used to update the sensor fusion algorithms. When the instrument becomes available again, it can re-enter the calculations without causing jumps in state or covariance estimates. Fig. 5 shows a not planned GPS dropout for about 30 seconds before it re-enters (triangles in the plot). The dotted line is the predicted value. The dropout caused a position error of 8 cm which is negligible here but this drift has later been literally removed by adding INS bias estimation.

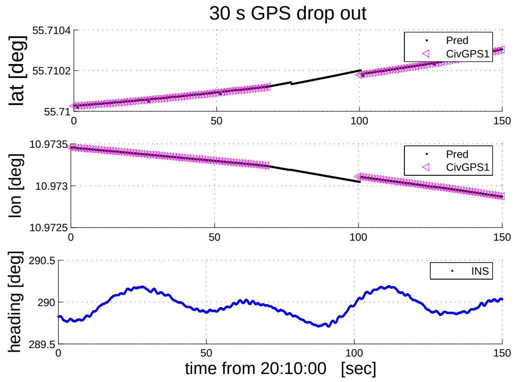

Fig. 5. GPS signal dropout for 30 s during track control with one GPS receiver. Black line is the position estimate.

6. CONCLUSIONS

This paper employed structural analysis to generate parity relations for fault diagnosis and used these in conjunction with an inverse covariance updated Kalman filter to provide a fault-tolerant fusion of navigation sensor data. Fundamental issues on generation of residuals for systems with pure kinematic relations were shown and a provable correct implementation was presented. Experience from implementing a fault-tolerant navigation solution on-board naval ships demonstrated the method and showed how the system could cope smoothly with temporal loss of data, without affecting closed loop control.

REFERENCES

M. Basseville and I. V. Nikiforov. *Detection of Abrupt Changes: Theory and Application.* Information and System Science. Prentice Hall, New York, 1993.

M. Blanke, M.Kinnaert, J.Lunze, and M.Staroswiecki. *Diagnosis and Fault-tolerant Control.* Springer, 2nd edition, 2006.

R. G. Brown and P. Y. C. Hwang. *Introduction to random signals and applied Kalman filtering.* John Wiley & Sons., 3rd edition, 1997.

A. L. Dulmage and N. S. Mendelsohn. A structure theory of bi-partite graphs. *Transactions of Royal Society of Canada. Sec. 3.*, 53:1–13, 1959.

T. I. Fossen. *Marine Control Systems.* Marine Cybernetics, 2002.

I. Nikiforov. A simple recursive algorithm for diagnosis of abrupt changes in random signals. *IEEE Transact. Information Theory*, 46 (7):2740–2746, Nov 2000.