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Feasibility of Quantum Communications in Aquatic Scenarios

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Abstract

Exchanging information in the aquatic environment represents a challenging but yet necessary task for example in military and scientific applications. Quantum cryptography is an already available solution able to guarantee information theoretical secure communication in multiple environment, i.e. free-space, fiber and space link. Yet, the implementation of quantum communications in the aquatic scenario is still a relatively new and unexplored field of research. This paper provides a feasibility analysis of various quantum key distribution protocols simulated in the water medium, by performing an evaluation of their secret key rate, external noise sources and wind effect on the surface of the water. Three different links are considered: line-of-sight, non line-of-sight and free-space to underwater channels. The outcomes of this analysis suggest that underwater quantum communication can be implemented by adopting several protocols in all three scenarios.

1. Introduction

In the last twenty years, the research on quantum information applications has shown how the principles of quantum mechanics provide considerable advantages with respect to the classical information resources [1, 2]. In particular, communication using photons (or attenuated pulses) as elementary information units provides information theoretic security thanks to the inherited purely
quantum properties \[3, 4\]. In fact, the implementation of protocols for quantum cryptography or quantum secure direct communication \[5, 6, 7, 8, 9\], and more specifically quantum key distribution (QKD) \[10\], has already produced incredibly interesting results in terms of performance \[11, 12, 13, 14, 15\].

The experimental implementation of practical QKD systems has been limited to communications through fiber \[10\] and free-space in the atmosphere \[16\] and only few studies were focused on transmission in water \[17, 18, 19, 20, 21, 22, 23, 24, 25\]. However, security in underwater communications is a very sensitive topic due to its great interest in numerous scientific, industrial and military applications \[26, 27, 28\]. For example oceanography investigation, offshore oil exploration and sea floor monitoring are activities which require a high level of security. In addition, the increasing number of vehicles travelling on the sea surface or in the water medium itself is also requesting a channel to exchange information which must be considered secure.

In 2012, M. Lanzagorta proposed the idea of bringing the technology of free-space quantum communications in the water \[17\] by performing a feasibility analysis on the BB84 protocol, in order to implement point-to-point communication between a transmitter and receiver. In the last few years, underwater quantum communication focused mainly on different implementations of the BB84 protocol. For example, P. Shi et al. \[18\] proposed a Monte Carlo analysis of a QKD system in the maritime environment and S. C. Zhao et al. analyzed the performance of a line-of-sight link using polarization encoding \[25\]. Indeed, F. Hufnagel et al. \[29\], L. Ji et al. \[19\] and C. Q. Hu et al. \[23\] experimentally proved that the polarization of photons survives when travelling in pure and sea water up to 55 m. Furthermore, recent studies from F. Bouchard et al. \[21\] and Y. Chen et al. \[24\] investigate the effects of space encoded quantum states transmitted through a water link up to 55 m distance link. Although preliminary studies on the BB84 polarization encoding scheme and the continuous variable approach have been reported, an extensive and complete analysis of different QKD protocols in various link scenarios is still missing.

In this paper, we present a comprehensive feasibility analysis for the potential
performances of various QKD protocols (DV and DPR \[30\]) in three different aquatic scenarios. The first type of channel, defined as line-of-sight (LOS)\[31\], is a point-to-point connection between transmitter and receiver. When LOS is not possible, the reflective non line-of-sight (NLOS)\[31\] channel allows to avoid physical obstructions by using the surface of the sea as a mirror. Finally, the air-water downlink\[20\] is a hybrid between an optical wireless free-space channel and a LOS. The parameters describing the optical effects produced by water are modeled and used to obtain the transmissivity for the water link in the three different scenarios described above. The secret key rate is evaluated using the transmission coefficients relative to the three different types of link (LOS, NLOS and air-water) for different QKD protocols\[10, 32, 33, 34\].

2. Water Channel Modelling

In this section we summarized the main equations useful for the modeling of the aquatic channel.

2.1. Attenuation Coefficient

The parameter describing the loss of intensity of a light beam travelling through water is called beam attenuation or extinction coefficient. It is defined as \( c(\lambda) = a(\lambda) + b(\lambda) \), where \( a \) and \( b \) represent the absorption and scattering coefficients as a function of the beam wavelength. The absorption coefficient is determined as \[35\]:

\[
a(\lambda) = [a_w(\lambda) + 0.06 a_c(\lambda) C_c^{0.65}] [1 + 0.2 \exp [-0.014 (\lambda - 440)]], \tag{1}
\]

where \( a_w(\lambda) \) is the absorption coefficient in oceanic water, \( C_c \) is the chlorophyll concentration and \( a_c(\lambda) \) is the chlorophyll-specific spectral absorption. The scattering coefficient is obtained from \[35\]:

\[
b(\lambda) = \left(\frac{550}{\lambda}\right) 0.30 C_c^{0.62}. \tag{2}
\]

Thus, the attenuation coefficient depends on the type of water (in the following referred to as Jerlov water types), as reported in Fig. \[30\].
Figure 1: Attenuation coefficient $c(\lambda)$ in dB/km as a function of the wavelength, for the different water types defined in [36].

The minimum values of attenuation occur for wavelengths between 450 and 570 nm, representing the blue-green band of the visible spectrum. It is interesting to compare the results shown in Fig. 1 with the typical attenuation coefficient value of optical fibers, which is 0.2 dB/km for the C-band (1530-1565 nm): the higher loss coefficient of water decreases the propagation distance of light considerably. However, transmission in seawater can still be achieved for channel lengths of the order of a hundred meters, which are relevant for underwater sensors and submarine communications.

2.2. Types of Channel

In this paragraph, three types of channel are proposed and their performances analyzed. The overall link transmissivity depends on the transmittance, geometric factors and the optical efficiencies of the transmitter and receiver, $\eta_T$ and $\eta_R$ respectively.

2.2.1. Line-of-Sight Link.

The first type of channel is defined as line-of-sight link (LOS) and represents an unobstructed, point-to-point connection between a transmitter and
a receiver. In Fig. 2, it is reported an example of this channel where two submarines exchange information through a direct trajectory.

Figure 2: Example of line-of-sight link: $h$ and $x$ are the depths of the transmitter and receiver (TX/RX); $d$ is the perpendicular distance and $\theta$ is the angle with the real point-to-point trajectory.

The transmittance for a LOS link is defined as [31]:

$$t_{\text{los}} = \exp\left(-c \frac{d}{\cos(\theta)}\right),$$  \hspace{1cm} (3)

where $c$ is the attenuation coefficient, $d$ is the perpendicular distance and $\theta$ is the angle, as shown in Fig. 2, between $d$ and the real point-to-point trajectory between the transmitter and receiver.

Moreover, the geometric factor relative in this link is defined as [31]:

$$f_{\text{geom}}^\text{los} = \frac{A_R \cos \theta}{2\pi d^2 [1 - \cos \theta_d]},$$  \hspace{1cm} (4)

Here, $A_R$ is the aperture area of the receiver and $\theta_d$ the laser beam divergence angle, usually few degrees [37]. The beam diameter is inversely related to the beam divergence, thus an optical beam expander will decrease the beam divergence. Intuitively a larger beam diameter will contribute to a higher signal-to-noise ratio at the receiver side, with better performances in terms of quantum bit error rate. However, in the case of large beam diameter, we will be exposed to larger absorption. Thus the advantages of a larger beam diameter are attained.
(a) $T_{los}$ is calculated for different wavelengths in pure sea water (Jerlov water type I).

(b) The wavelength is set to 530 nm and $T_{los}$ is calculated for the five water types.

Figure 3: Overall behaviour of $T_{los}$ as a function of the distance. Receiver aperture area $A_R = 0.1 \text{ m}^2$; transmitter optical efficiency $\eta_T = 1$; receiver optical efficiency $\eta_R = 0.9$; angle $\theta = 0^\circ$; laser beam divergence angle $\theta_d = 5^\circ$. 
In our simulation, we consider a feasible and accessible telescope diameter for an underwater transmitter and receiver module. We then empirically change the divergence angle to maximize the transmission distance \cite{37,38}.

By considering the efficiencies of transmitter and receiver (\(\eta_T\) and \(\eta_R\) respectively), the overall transmissivity (\(T_{\text{los}}\)) is defined as:

\[
T_{\text{los}} = \eta_T \eta_R t_{\text{los}} f_{\text{geom}} = \eta_T \eta_R e^{\left(-c \frac{d}{\cos(\theta)}\right)} \frac{\cos \theta A_R}{2\pi d^2[1 - \cos \theta_d]}.
\] (5)

The behaviour of \(T_{\text{los}}\) for an increasing length (from 0 to \(\approx 180 \text{ m}\)) is shown in Fig. 3a for different wavelengths of the blue-green band of the visible spectrum, and in Fig. 3b for the five Jerlov water types \cite{36}.

2.2.2. Reflective Non Line-of-Sight Link.

The second type of link is called non line-of-sight (NLOS) and is illustrated in Fig. 4. NLOS provides a connection between transmitter and receiver whenever point-to-point communication is not possible, for example in case of physical obstructions, e.g. rocks.

![Reflective NLOS](image)

Figure 4: Reflective non line-of-sight link: \(h\) and \(x\) are the depths of the transmitter and receiver (TX/RX); \(d\) is the perpendicular distance and \(\theta\) is the incident angle.

It uses the phenomenon of total internal reflection to reflect the transmitted beam on the sea surface back to the sea bed, towards the receiver. The
transmittance in this case is defined as:

\[ t_{nlos} = \exp \left( -c \frac{(h + x)}{\cos(\theta)} \right), \]  

where \( h \) and \( x \) are the depths of transmitter and receiver, respectively.

The geometric factor for a NLOS link is:

\[ f_{\text{geom}}^{nlos} = \frac{A_R \cos \theta}{2\pi(h + x)^2[\cos \theta_{\text{in}} - \cos \theta_{\text{out}}]}, \]  

where \( \theta_{\text{in}} \) and \( \theta_{\text{out}} \) are the inner and outer angles of the emitted light cone. The values of \( \theta_{\text{in}} \) and \( \theta_{\text{out}} \), as well as the depths of the transmitter and the receiver determine a maximum value of achievable distance. For instance, in the case shown in Fig. 5 the maximum achievable distance is around 75 m due to the link geometry.

In order to compute the overall transmissivity, the optical efficiencies of the transmitter and receiver are introduced, as well as the effects of the sea surface given by the Fresnel reflection coefficient \( r \) and the probability density function \( P(s) \) describing the sea surface slope. Indeed, the expected value of the slope angle of the sea surface is first calculated as \( \theta_{\text{av}} = \int P(s) \cdot s \, ds \) and then subtracted from the transmission angle \( \theta \) to describe the effective angle between the transmitter and the wavy sea surface, \( \theta_{\text{eff}} = \theta - \theta_{\text{av}} \). The final result of \( T_{nlos} \) is given by:

\[ T_{nlos} = \eta_T \eta_R A_R \cos \theta_{\text{eff}} \left( \frac{2\pi(h + x)^2}{(\cos \theta_{\text{in}} - \cos \theta_{\text{out}})^2} \right) \exp \left( -c \frac{(h + x)}{\cos(\theta_{\text{eff}})} \right), \]  

where \( r = 1 \) if \( \theta_{\text{eff}} \geq \theta_c \) and total internal reflection occurs (\( \theta_c \) is the critical angle). The behaviour of the overall transmission coefficient for increasing values of the distance, is shown in Fig. 5a for different values of the wavelength in the green-blue band of the visible spectrum and in Fig. 5b for the Jerlov water types listed in [36]. In the NLOS link, it is important to consider the effect of the wind on the optical beam. The surface of the sea is rarely smooth but, more often, wavy. The studies of C. Cox and W. H. Munk [39] indicate that the surface slope of the ocean shows a nearly-Gaussian probability density function (PDF). In particular, the most accurate analytical model for the sea surface slope is
given by a Gram-Charlier distribution, shown in Fig. 6, which represents a series of approximate Gaussian PDF, expressed as [40]:

\[ P(s) = G(s) \left[ 1 + \left( \frac{c_3}{6} \right) H_3 + \left( \frac{c_4}{24} \right) H_4 \right], \]  

(9)

where \( s \) is the upwind/downwind surface slope, and \( G(s) \) stands for the Gaussian distribution:

\[ G(s) = \frac{1}{\sigma_s \sqrt{2\pi}} \exp \left( -\frac{\eta^2}{2} \right), \]  

(10)

with \( \sigma_s \) the slope standard deviation and \( \eta_s = (s - \langle s \rangle)/\sigma_s \) the normalized slope and \( \langle s \rangle \) represents the mean slope angle. The parameter \( H_n \) represents the \( n \)-th Hermite polynomial in the normalized slope.

The coefficients \( c_n \) vary depending on the wind speed and are shown in Tab. 1 along with \( \sigma_s \); in particular, \( c_3 \) indicates the value of skewness, which is a measure for the absence of symmetry, and \( c_4 \) of kurtosis, which indicates whether the data are heavy-tailed or light-tailed considering the Gaussian distribution.

<table>
<thead>
<tr>
<th>Wind speed [m/s]</th>
<th>( c_3 )</th>
<th>( c_4 )</th>
<th>( \sigma_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.2</td>
<td>-0.283</td>
<td>0.128</td>
<td>11.5</td>
</tr>
<tr>
<td>8.58</td>
<td>-0.165</td>
<td>0.027</td>
<td>8.62</td>
</tr>
<tr>
<td>3.93</td>
<td>0.003</td>
<td>0.129</td>
<td>7.5</td>
</tr>
</tbody>
</table>

Table 1: Coefficients for the Gram-Charlier distributions [40], considering different wind speeds.

In addition, Fig. 7 shows the transmittance according to the variations of the sea surface slope angle. The wind speed is fixed to 3.93 m/s and its effects on the surface slope are described by the Gram-Charlier probability density function. It can be seen that if the angle of the slope is 0°, which means if the surface is smooth, the transmittance is maximized. This gives an indication about how the waves produced by the wind alter the ability of the NLOS to properly reflect the transmitted beam towards the receiver.
(a) $T_{nlos}$ is calculated for different wavelengths in pure sea water (Jerlov water type I).

(b) The wavelength is set to 530 nm and $T_{nlos}$ is calculated for the five water types [36].

Figure 5: Overall behaviour of $T_{nlos}$ as a function of the distance. Receiver aperture area $A_R = 0.1 \text{ m}^2$; inner and outer angle of the cone of light $\theta_{in} = 0^\circ$ and $\theta_{out} = 68^\circ$; transmitter optical efficiency $\eta_T = 1$; receiver optical efficiency $\eta_R = 0.9$; wind speed at 12.5 m height 3.93 m/s; standard deviation of $P \sigma = 7.5$; depth transmitter and receiver $h = x = 15 \text{ m}$. 
Figure 6: Gram-Charlier probability density function for the sea surface slope reproduced from [31]. The values used are shown in Tab. [11].

2.2.3. Water-Air Link.

The last type of scenario studied is an air-water downlink, as depicted in Fig. 8. The transmission coefficient in the atmosphere can be defined as [41]:

\[
t_{air} = t_0 \left[ 1 - \exp \left( -\frac{2 r_R^2}{w_{LT}^2} \right) \right],
\]

where \( t_0 \) is the link efficiency taking into account the atmospheric losses, \( r_R \) is the receiver aperture radius and \( w_{LT} \) is the long-term beam width. The parameter \( w_{LT} \) can be calculated as [16]:

\[
w_{LT}^2 = w_0^2 \left( 1 + \frac{l^2}{Z_0^2} \right) + 2 \left( \frac{4l}{kF_0} \right)^2,
\]

where \( Z_0 \) is the Rayleigh parameter of the beam and is equal to \( \frac{w_0^2}{\lambda} \), \( l \) is the altitude of the receiver, \( w_0 \) the beam waist, \( k \) the wave number and \( F_0 \) is the Fried parameter, and \( C_n^2 \) is called refractive index structure constant [16].

The second stage of the link happens in water and reflects the behaviour of a LOS link with transmittance as in Sec. 2.2.1. When the light beam reaches the sea surface it is affected by the effects of the wind, thus the variations of the sea surface slope must be considered. The overall transmissivity for this type
of link can be described as:

\[ T_{aw} = (1 - r) \eta_f \eta_{R} t_{\text{los}} f_{\text{geom}}^{\text{com}}(t_{\text{air}}), \]  

(13)

where \( r \) is the reflection coefficient. Similar to the previous paragraph, the effect of the wavy sea surface are included by using the expected value \( \theta_{av} \).

3. Quantum Communications

The quantum key distribution protocols analyzed in the LOS, NLOS and air-water downlink channels can be divided into two families: discrete-variable (DV) and distributed-phase-reference (DPR) [10, 34]. The main difference between them consists in the technique chosen to encode the bits [30]. The analysis of the security parameters of the protocols is performed for collective attacks [10], where an eavesdropper Eve uses the same strategy to attack every quantum states. If we consider the discrete-variable protocols, the security bound for collective attacks actually equals the one for coherent attacks, which represents the most general attack Eve can perform [10]. A final consideration is that the uncalibrated-device scenario [10] is assumed in the following discussion on the protocols. This assumption gives as a consequence that all the losses and errors
introduced by the devices and the transmission through the channel are counted as errors produced by Eve.

3.1. Secret Key Rate

The parameter used to estimate the performances of QKD protocols is called secret key rate and is given by the product of the raw-key $R$ rate and the secret fraction $r$:

$$K = Rr.$$  

(14)

The coefficient $R$ takes into account the losses and efficiencies of the transmitter and receiver. It depends on the repetition rate of the transmitter and the probability that the receiver achieves a successful measurement. The secret fraction $r$ is extracted from the raw key rate by using classical information post processing algorithms: the error correction and privacy amplification procedures.

Indeed:

$$r = I(A : B) - \min(I_{EA}, I_{EB}).$$  

(15)

If the choice of sending a 0 or a 1 is equally likely, the mutual information becomes $I(A : B) = 1 - h(Q)$, where $h(\cdot)$ is the binary entropy and $Q$ the quantum bit error rate (QBER). Finally, the information available to the eavesdropper
from the raw key of Alice (Bob) is $I_{EA}$ ($I_{EB}$): it must be removed to achieve security.

3.2. Noise Analysis

The total noise in a QKD system is given by multiple factors. The following analysis is focused on the noise produced by dark counts, which are the registered counts without any incident light, and by background light. Therefore, the noise probability is given by:

$$p_{tot} = p_{dark} + p_{back}.$$ (16)

The main contributor of background noise in a maritime environment is given by solar radiation and its effect limits the propagation of a transmitted beam in sea water. Nevertheless, solar radiation is naturally more intense close to the water surface and becomes negligible, or even not present, for deeper waters.

The solar background noise power $P_{solar}$ can be defined as [17]:

$$P_{solar} = \frac{(2\Omega)^2 \pi A_R \Delta \lambda L_{solar}}{16},$$ (17)

where $A_R$ is the receiver aperture area, $\Omega$ the field-of-view (FOV) of the detector (the solid angle which defines its sensitivity), $\Delta \lambda$ the bandwidth of the optical filter and $L_{solar}$ is the solar radiance, given by [17]:

$$L_{solar} = \frac{r_{irr} I_{\lambda} \gamma \exp(-cz)}{\pi}.$$ (18)

The factor $r_{irr}$ is the reflectance of downwelling irradiance, $I_{\lambda}$ the spectral radiant intensity, $\gamma$ the radiance factor describing the directional dependence of the radiance, $c$ the attenuation coefficient and $z$ is the depth. In general, the effects of background solar noise can be reduced by introducing a very narrow-band optical filter [21].

The photon number $n_{photons}$ that constitutes background solar noise is evaluated from the noise power $P_{solar}$. It is shown in Fig. 9 for an increasing depth in water, given the parameters $c = 0.042$, $r_{irr} = 0.0125$, $\Delta \lambda = 450 \text{ nm}$, $I_{\lambda} = 1.5 \text{ W m}^{-2} \text{ sr}^{-1} \text{ m}^{-1}$ for daytime, $1.5 \times 10^{-3}$ for full moon night time and $1.5 \times 10^{-5}$ for moonless night time [42]. $\gamma = 333$ valid for an angle of 0°. The solar light
error probability is finally calculated as $p_{\text{diss}} = n_{\text{photons}} \times \tau$, where $\tau$ is the effective sampling period of the detector.

4. Results

The results are reported for the three different types of link: LOS, reflective NLOS and air-water downlink. The protocols simulated are single-photon \cite{10}, weak-coherent pulses (WCP) \cite{10}, decoy states \cite{32} version of BB84 and single photon with high-dimensional encoding \cite{43} for DV schemes and differential-phase-shift (DPS) \cite{33}, coherent-one-way (COW) \cite{33} and differential phase-time shifting (DPTS) \cite{34} for the DPR family. Some assumptions were made in the analysis: the effects of fluorescence and bioluminescence are considered negligible. The wavelength chosen for the simulations is 530 nm, in the green band of the visible spectrum, a good tradeoff in terms of level of attenuation ($c = 0.0733 \, m^{-1}$) and detector efficiency ($\eta = 0.6$) \cite{22}. The probability of dark counts considered is $p_{\text{dark}} = 2 \times 10^{-9}$, in line with commercial detectors. In general, we consider Jerlov water type I, except in the case of the decoy-state discrete variable protocol, where we compare the performance also with Jerlov water types IA, IB, II and III. The parameters used for the simulations are listed in Tab. \cite{2}.

4.1. Line-of-Sight

The behaviour of the secret key rate $K/\nu$ (normalized on the repetition rate $\nu$) in a LOS link is shown in Fig. \cite{10} in the scenario of a moonless clear night, full moon clear night and daytime for the protocols from the DV family. The same results are shown for the DPR protocols in Fig. \cite{11}. Note that the effects of background light can be considerably reduced if the transmitter and receiver are located deeper in the sea (see Fig. \cite{9}).

4.2. Reflective Non Line-of-Sight

The results for the reflective NLOS link are shown in Fig. \cite{12} and \cite{13}. First, it is important to notice that the achievable distance is limited by the outer
Figure 9: Number of photons given by the solar background light as a function of the sea depth for different field-of-view values. The parameters are: $c = 0.042$, $r_{irr} = 0.0125$, $\Delta \lambda = 450$ nm, $I_\lambda = 1.5 \text{ W m}^{-2} \text{ sr}^{-1} \mu \text{m}^{-1}$ for daytime, $1.5 \times 10^{-3}$ for full moon night time, $1.5 \times 10^{-5}$ moonless night time, and $\gamma = 333$ valid for an angle of $0^\circ$.
angle of the cone of emitted light and the depths of transmitter and receiver. In a scenario, for example, where the transmitter and receiver are at 15 m of depth and the inner and outer angles of the cone of light are 0° and 68°, respectively, the maximum achievable distance is around 75 meters. The different behaviour of this type of link is due to the fact that, until the angle of transmittance reaches the critical angle for total internal reflection, a portion of the total beam is transmitted outside water and the secret key rate is therefore reduced. The value of $K/\nu$ increases as long as the angle of transmittance approaches $\theta_c$, reaching its maximum at the critical angle. For values above $\theta_c$, the secret key rate starts to decrease with the distance, as in the LOS link.

In this second type of link, the performances are worse compared to the LOS
Figure 10: LOS link: secret key rate as a function of the distance for the DV protocols for water type I, at $\lambda = 530$ nm. Parameters from Tab. 2. The results are shown for a moonless clear night, full moon clear night and daytime.

link: important losses occur as a result of the reflection on the sea surface and the effects of the wind on it. Nevertheless, a few tens of meters can still be reached with a positive secret key rate for most of the protocols of both the DV and DPR families, in a moonless clear night. In a full moon night or in clear daytime, however, extracting a secret key becomes more challenging and limited to few meters, if not feasible at all.

4.3. Air-Water Downlink

The last scenario taken into account is the hybrid between an optical free-space wireless downlink and the line-of-sight link. In this case, the distance travelled in the atmosphere varies depending on the angle $\theta$ between the satellite and the zenith, shown in Fig. 8. In addition, the transmitted angle of the beam entering the sea depends on $\theta$ following Snell’s law. The secret key rate for this link is depicted in Fig. 14 for the single photon protocol, as a function of the total distance travelled in the atmosphere and in the sea. The distance of 11474 km corresponds to the angle $\theta = \pm 85^\circ$, of 2000 km to $\pm 60^\circ$, of 1155 km to $\pm 30^\circ$ and of 1000 km to 0$^\circ$. The parameters used are shown in Tab. 2.
Figure 11: LOS link: secret key rate as a function of the distance for the DPR protocols for water type I, at $\lambda = 530 \text{ nm}$. Parameters from Tab. 2: transmitter and receiver depths $h = 20$ and $x = 25 \text{ m}$. The results are shown for a moonless clear night, full moon clear night and daytime.

4.4. Water types

After a direct comparison between DV and DPR protocols performances, it is interesting to examine the performance in different water types. As an example, we consider the decoy-state protocol in different water types for a moonless clear night LOS scenario. Indeed, the decoy-state solution represents the most attractive one in terms of implementation and security. In Fig. 15, five different values of the attenuation coefficients were used, representing types I, IA, IB, II and III in Jerlov’s definition. The increasing values of attenuation, peculiar of each the water types, decrease considerably the performances of the simulated scheme.

5. Discussion

The LOS link provides the longest reachable distance and the best performances. The outcome of the simulations appears relatively flexible to changes of the attenuation coefficient and, therefore, of the oceanic possible water types. The reflective NLOS, which was analyzed here for the first time for QC in a
Figure 12: NLOS link: secret key rate as a function of the distance for the DV protocols for water type I, at $\lambda = 530$ nm; transmitter and receiver depths $h = 15$ and $x = 15$ m. Parameters from Tab. 2. The results are shown for a moonless clear night.

moonless clear night, brings an interesting solution in case of impossible point-to-point connection between transmitter and receiver. Introducing the effects of the wind on the slope PDF of the sea surface was a necessary requirement to create a more accurate description of a real maritime scenario. In the proposed realistic scenario of light wind ($\approx 4$ m/s), a positive secret key rate can be obtained for several meters in a moonless night.

Furthermore, as already introduced in [19], transmission in sea water creates some effects of depolarization, which, however, can be significantly reduced using some spatial filtering. As an example, in the NLOS link, when the light reaches the sea surface, it incurs in some additional depolarization effects, which might destabilize the encoding on the quantum states. This phenomena will affect only polarization based protocols and will less impact on other degrees of freedom e.g., time-bin, path-encoding, frequency-encoding. In addition to that, DPR encoding, relying on the coherence of the phases between consecutive pulses, might remain stable during the underwater transmission as simulated in [46, 47, 48]. To have a better understanding of the behaviour of these protocols in the aquatic scenario, we tested the DPR protocols with a lower visibility
value. In particular, if we consider a visibility of 90% the maximum achievable
distance for the DPS protocol for the LOS link will be decreased to 90 meters,
10 meters less compared to Figure 11. At the same way, all the other protocols
will be proportionally influenced. Furthermore, in the case of DPR protocols
and NLOS links, 90% visibility will not enable a positive key generation rate.
Therefore, it might be particularly interesting to test the stability of this type
of encoding experimentally.

An important consideration needs to be made on the reflective NLOS link.
Total internal reflection is achieved only when the angle of the transmitted beam
is larger than the critical angle $\theta_c$. Indeed, if $\theta < \theta_c$ a fraction of the transmitted
power is refracted outside the water. In this case, there might be some security
risks given by the fact that the eavesdropper Eve could steal that fraction of
transmitted key. However, these losses can be considered as information ob-
tained by Eve in a beam-splitter attack, since they can be predicted easily from
the well known fraction of transmitted beam refracted out of the sea.

To conclude, it is important to notice that the implemented system repre-
sents an approximation of the real aquatic scenario. Indeed, this model aims
to give and initial overview of the behaviour of QKD protocols in a nearly
unexplored environment - the maritime one. For example it is worth noting
that turbulence effects have not be considered in our analysis. Although mul-
tiple works have proposed various models for turbulence characterization in
underwater free-space communication, a definitive model has not been identi-
fied yet. Finally, for future work, a more complete model for such
a complex medium can be provided only with a continuous interaction between
computer simulations and experiments in water.

6. Conclusion

We propose a feasibility analysis of different QKD protocols in three different
types of link in seawater.

A model for the channel in the three links, LOS, NLOS and air-water down-
link, was created and used to evaluate the channel transmissivities in each case. In addition, in order to make the model more faithful to the real oceanic environment, the effects of a light wind were added to the losses caused by the attenuation. The performances of the protocols in terms of their secret key rate for a LOS, NLOS and air-water link were obtained. In particular, the reflective NLOS was introduced and analyzed here for the first time in underwater quantum communications. The results are obtained for a water type I, which represents the clearest oceanic water and for a transmitted beam at 530 nm. The outcome of the simulations indicates that a secret key rate can be extracted for most of the protocols in the three links. Further investigations are required to design a better model for the transmission of the quantum states through an underwater link considering for example turbulence effects.

In conclusion, the outcome of this analysis is quite promising and suggests that a quantum communication system can be indeed implemented even in an environment so hostile to the transmission of light. The different types of link proposed can be suitable for most of the scenarios where a secure communication must be guaranteed.

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Figure 13: NLOS link: secret key rate as a function of the distance for the DPR protocols for water type I, at $\lambda = 530 \text{ nm}$; transmitter and receiver depths $h = 15$ and $x = 15 \text{ m}$. Parameters from Tab. 2. The results are shown for a moonless clear night.

Figure 14: Air-water downlink: secret key rate as a function of the distance in air (depending on the angle between the satellite and the zenith) and as a function of the distance in water. The results are shown for water type I, at $\lambda = 530 \text{ nm}$ and for the single photon protocol. Parameters from Tab. 2.
Figure 15: Secret key rate for the decoy-state protocol in a LOS link as a function of the distance for water types I to III in a moonless clear night scenario.
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Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☒ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

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