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Publication date:
1972

Document Version
Publisher's PDF, also known as Version of record

AFDELINGEN FOR

BÆRENDE KONSTRUKTIONER

DANMARKS TEKNISKE HØJSKOLE

STRUCTURAL RESEARCH LABORATORY

TECHNICAL UNIVERSITY OF DENMARK

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WEIGHT OPTIMIZATION OF FRAMED STRUCTURES UNDER EARTHQUAKE LOADS

RAPPORT NR. R 33 1972

WEIGHT OPTIMIZATION OF FRAMED STRUCTURES UNDER EARTHQUAKE LOADS

by Julius Solnes and Ole Holst [a)]

ABSTRACT

The paper deals with minimum weight design and analysis of plane frames under earthquake loads.

The minimum weight design of plane frames for different combinations of vertical and horizontal loads may be solved using mathematical programming techniques. In the case of earthquake loading, however, the corresponding design loads are found to be highly dependent on the stiffness of the frame and hence on the design variables.

In the present study the mathematical programming formulation is presented and a stepwise iterative solution procedure is given, which involves a first order Taylor approximation of the object function and the stress and deflection constraints. Expressions for the various partial derivatives involving the stiffness and mass matrices and the external loads are given, together with a short description of the solution technique.

Finally a numerical example is worked out applying the period dependent loads defined in the Californian SEAOC code.

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INTRODUCTION

Optimization of structural systems using the powerful method of mathematical programming has received considerable attention during the last few years, [1], [2], [3], [4].

The application of such optimization techniques can result in rapid automatic design procedures which choose the "best" values for the design variables while analysing the structure and checking whether all imposed restrictions are observed. All load cases can be run simultaneously and any deflection constraints or other kind of restrictions may be introduced at the same time. At no extra cost, even if it may be of secondary importance, an optimization criterion such as minimum weight governs the selection of the design variables.

For any structural design problem there does not exist a unique solution because the restrictions are in the form of inequalities. For instance, the stresses in any number of critical sections must be less than or equal to the allowable material stresses and so on. An optimization criterion, however, enables one to look for a unique solution, i.e. the solution which yields an extreme value of the function to be optimized.

The more classical design procedures involving random search and check techniques are especially inadequate when regarding complicated structures. Such analysis can be very costly and time consuming. An initial design has to be selected, and the thus defined structure is checked for all static load cases. This will generally result in complete reappraisal of all structural elements, which need to be increased or decreased and a new design is found. Finally, the structure may have to be put through dynamic analysis, which again can prove the design to be inadequate.

The success of the above design procedure is entirely dependent on the design engineer's intuition and experience. Especially, the selection of a good initial design will be fundamental in obtaining reasonable results, and even then, the final design may be very far from the optimum design although all design restrictions are observed.

Structures which have to be checked for earthquakes or other time dependent loads introduce yet another difficulty. The design earthquake loads, f.inst., are strongly period-dependent and each design yields a different load system to be accommodated as the stiffness varies. Design convergence may therefore be very difficult to establish.

Fox and Kapoor [5] have recently discussed structural optimization in the dynamic response regime. They try to evaluate the dynamic response parameters using modal superposition and then run the structure through an optimization analysis in which the response values have to be re-evaluated as the stiffness and hence the frequency characteristics of the structure change.

In the present work, however, a different approach is proposed. Working with stiffness dependent earthquake design loads, an optimization process is studied which yields an optimum structure before the dynamic analysis, thereby ensuring that such analysis is not attempted with a non-feasible design. Moreover, the optimum design thus obtained will be superior within the range of the design loads.

The theory has been formulated for plane structural frames, and a numerical example is given.

PROBLEM FORMULATION

To recapitulate the mathematical programming formulation of any structural frame design problem, consider a structure which is fully determined by the n-dimensional design vector

$$\underset{\sim}{X} = \{x_1, \; \ldots\ldots \; x_n\}^T \qquad (1)$$

Here the design variables $\{X_i\}$ may be taken as a parameter description of each structural element (f.inst. area, moment of inertia, etc.), a geometric description of the structure (f.inst.,column spacing, height etc.) or a description of different features such as the degree of fixation of element ends, etc.

Once all the design variables have been assigned values, there exists one complete structure, and its weight or even the total cost of erection can be expressed as a function (to be minimized),

$$Z(\underset{\sim}{X}) \rightarrow \text{MINIMUM} \tag{2}$$

termed the object function of the problem.

The design constraints together define a design variable subspace, in which the design variables have to be located. It is defined as follows:

The deflection or deformation of the frame and the maximum stresses in the frame must be within acceptable limits set by design philosophy and material strength. This will result in the following inequalities or constraints.

The deflection or drift limitations

$$\underset{\sim}{Y}(\underset{\sim}{X}) \overset{\leq}{\underset{>}{=}} \underset{\sim}{YD} \tag{3}$$

where $\underset{\sim}{Y}$, and $\underset{\sim}{YD}$ are p-dimensional, $\underset{\sim}{Y}$ being the p critical deflections or deformation values to be checked, and $\underset{\sim}{YD}$ the corresponding vector of the admissible values.

$$\underset{\sim}{S}(\underset{\sim}{X}) \overset{\leq}{\underset{>}{=}} \underset{\sim}{SD} \tag{4}$$

however, refers to the stress constraints where $\underset{\sim}{S}(\underset{\sim}{X})$ is the matrix of critical stresses sought after amongst all the critical sections of the frame and $\underset{\sim}{SD}$ is the corresponding matrix of the allowable stresses, which may or may not include stability effects. Then, there may be restrictions on the variables themselves from purely design considerations (beam height, symmetry, identical elements etc.) and fabricational limits, also, f.inst., the period could be restricted to lie in a certain period band etc. These conditions can be represented symbolically as

$$\underset{\sim}{H}(\underset{\sim}{X}) \overset{\leq}{\underset{>}{=}} \underset{\sim}{HD} \qquad (5)$$

Together the constraints (3) - (5) define the design subspace in the n-dimensional variable space. The boundaries of the design space are the above constraints treated as equalities which correspond to n-dimensional hypersurfaces. A two dimensional geometrical representation shown in figure 1 may further clarify the mathematical programming problem at hand.

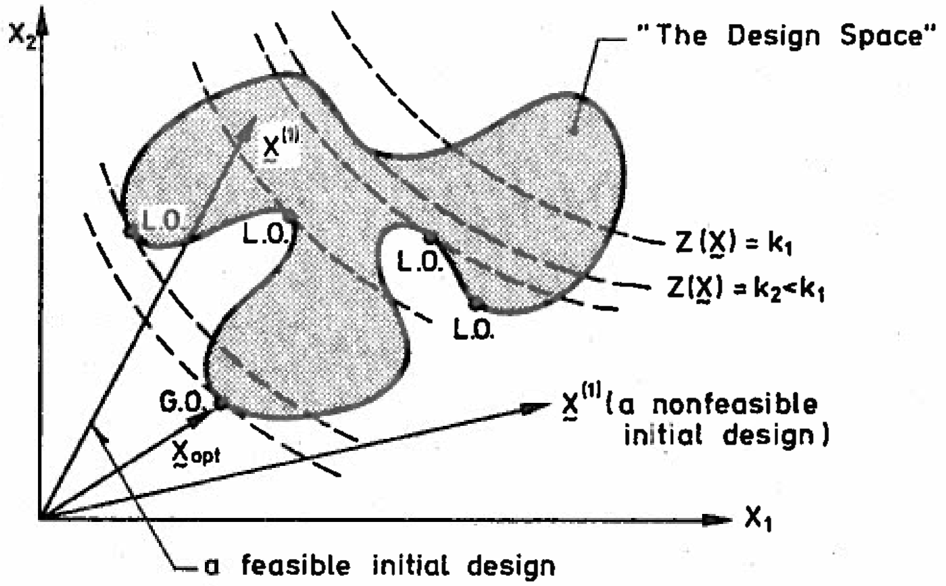

Fig. 1

The constraints (3) to (5) are, of course, due to the effect of all possible load combinations, that the structural frame is to accommodate. Here the design philosophy being studied amounts to determining the optimum values of the design variables such that all load combinations can be withstood through a linear deformation. The design earthquake forces are therefore selected as suitable for elastic behaviour. It is clear, however, that the optimum structure thus determined will afterwards have to be checked for nonlinear behaviour due to more realistic earthquake forces, and will possibly have to be put through a dynamic analysis.

The optimizing process may be briefly described as follows. The numerical procedure is started with an arbitrary initial design ($\underset{\sim}{X}^1$) which may be rather crude (fig. 1). The initial design vector may even be infeasible and it may become infeasible again during the iterative process. In both cases, the numerical procedure will eventually restore its feasibility. The global optimum G.O., which yields the lowest admissible value of the object function within the design space, is then sought through a step by step approach which consists of solving a linear programming problem for each step.

The original mathematical programming problem ((2) - (5)) is, in general, strongly nonlinear; therefore, for each step the functions (2) - (5) are linearized by a first order Taylor approximation. This involves the numerical computation of the left-hand sides of (3) to (5), which in turn requires the computation of the fundamental period, the force envelope, the mass matrix and the local and the global stiffness matrices. Furthermore, the partial derivatives of all these quantities with respect to all the design variables are needed. Hereafter, it is easy to formulate a linear programming problem which gives the optimum incremental vector $\Delta\underset{\sim}{X}$ that results in maximum decrease of the object function. Care should be taken to avoid the local optima, L.O., at which the process automatically stops.

In the following, the elements of the frame analysis and the optimizing process are described in more detail.

ANALYSIS OF THE STRUCTURAL FRAME

The load combinations may conveniently be split up into lateral and vertical loads. The lateral earthquake loads can be taken as follows:

$$P(T;\ \underset{\sim}{X}) = Q(T) \cdot \underset{\sim}{F}(\underset{\sim}{X}) \tag{6}$$

in which $T(\underset{\sim}{X})$ is the fundamental period, $Q(T)$ is the design seismic shear force at the base and $\underset{\sim}{F}(\underset{\sim}{X})$ is the force distribution envelope. The vertical loads can be written as

$$\underset{\sim}{M}(\underset{\sim}{X})\ g + \underset{\sim}{W} \tag{7}$$

where $\underset{\sim}{M}(\underset{\sim}{X})$ is the mass matrix, dependent on the design variables, g is the acceleration of gravity and $\underset{\sim}{W}$ is the matrix of the portion of the live loads, not to be included with the mass.

Whereas the earthquake design loads (6) can be taken as purely nodal loads, the vertical loads (7) are both nodal loads and distributed loads. It is therefore convenient to define the two load matrices $\underset{\sim}{B}$ and $\underset{\sim}{BF}$ for the combined nodal loads and the equivalent nodal loads respectively; the latter are also referred to as the unbalanced nodal loads.

The areas $\{A_i\}$ of the elements are selected as the design variables of the frame. These variables are assembled in the n-dimensional design vector $\underset{\sim}{X}$.

Denoting the global stiffness matrix of the frame by $\underset{\sim}{KK}$, the following equation relates the nodal loads to the nodal displacements

$$\underset{\sim}{KK} \cdot \underset{\sim}{Y} + \underset{\sim}{BF} = \underset{\sim}{B} \tag{8}$$

Hence the restriction (3) can be written in terms of the external loads and the global stiffness matrix as follows:

$$\underset{\sim}{KK}^{-1}\ (\underset{\sim}{B} - \underset{\sim}{BF}) \lesseqgtr \underset{\sim}{YD} \tag{9}$$

Considering each element in isolation the local end forces and end displacements for the i-th element, $\underset{\sim}{S}_i$ and $\underset{\sim}{V}_i$, are related through the local stiffness matrix $\underset{\sim}{K}_i$, that is

$$\underset{\sim}{S}_i = \underset{\sim}{K}_i \cdot \underset{\sim}{V}_i \tag{10}$$

The local stiffness matrix $\underset{\sim}{K}_i$, which contains all the information on the element stiffness properties and the degree of end fixation, may be found in various textbooks on finite element analysis [6].

Introducing the direction cosines $\underset{\sim}{a}_i$ and the left and right end global displacement sub-matrices $\underset{\sim}{Y}_l$ and $\underset{\sim}{Y}_r$ (see figure 2a), (10) can be written as

$$\underset{\sim}{S}_i = \underset{\sim}{K}_i \cdot \left\{\begin{array}{c|c} \underset{\sim}{a}_i & \underset{\sim}{0} \\ \hline \underset{\sim}{0} & \underset{\sim}{a}_i \end{array}\right\} \cdot \left[\begin{array}{c} \underset{\sim}{Y}_l \\ \hline \underset{\sim}{Y}_r \end{array}\right] \tag{11}$$

where (see fig. 2a)

$$\underset{\sim}{a}_i = \begin{Bmatrix} \cos\theta_{1i} & \cos\theta_{2i} & 0 \\ -\cos\theta_{2i} & \cos\theta_{1i} & 0 \\ 0 & 0 & 1 \end{Bmatrix} \tag{12}$$

The normal force and the moment in the element at a section located ξL_i from the left end can be written as

$$\left.\begin{aligned} n_i(\xi) &= X_i \gamma_i L_i a_{i12} \tfrac{1}{2}(2\xi - 1) - S_{1i} \\ m_i(\xi) &= \frac{L_i}{12}(X_i L_i \gamma_i a_{i11} + p_i)(6\xi(1-\xi)-1) - S_{3i} + \xi L_i S_{2i} \end{aligned}\right\} \tag{13}$$

in which γ_i is the unit volume weight of the element and p_i is the distributed load per unit length acting perpendicular to the element (see figure 2b).

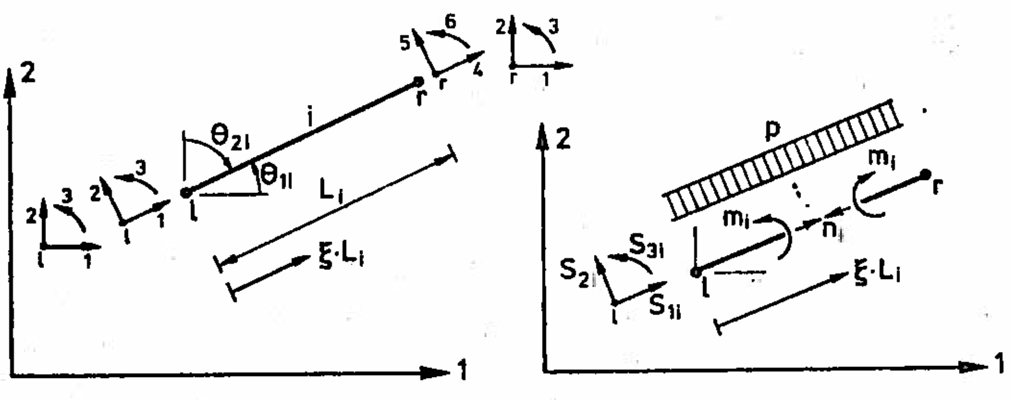

Fig. 2a Fig. 2b

In the RHS of (13), it is the part not involving the $\underset{\sim}{S}_i$ forces that gives rise to the unbalanced nodal force matrix $\underset{\sim}{BF}$.

Now, for any number of sections in each element the member forces m_i (ξ) and n_i (ξ) can be computed and checked against the corresponding allowable forces mo_i and no_i in the member, and the following stress constraints may be formulated,

$$\underset{\sim}{G}(\underset{\sim}{S}, \underset{\sim}{SD}, \xi) = \left|\frac{n_i(\xi)}{no_i}\right| + \left|\frac{m_i(\xi)}{mo_i}\right| \leq 1 \qquad (14)$$

in which

$$no_i = X_i \ \sigma_o^N , \qquad (15)$$

and

$$mo_i = W_i \ \sigma_o^B \qquad (16)$$

where $\{\sigma_o^N, \sigma_o^B\}$ are the allowable stresses, X_i is the unknown cross-section area of the element and W_i is the unknown section modulus. W_i may be expressed in terms of the area as follows:

$$W_i = \alpha X_i^{\beta} \tag{17}$$

and the constants α and β are then universally determined from manufacturers' tables of standard profiles. In this manner, the design variables are still limited to the member cross-section areas.

THE OPTIMIZATION PROCESS

The optimization of the object function (2) subject to the constraints (5), (9) and (15) is in general strongly nonlinear. Various nonlinear optimizing procedures such as gradient methods [7], [8] and penalty function techniques [9] have been proposed. In the following, however, the successive linearization technique, applied by Reinschmidt et al. [10] and Romstadt and Wang [11], is employed. This method is based on a first order Taylor linearization of both the object function and the constraints in each iterative step, followed by a linear programming solution for the optimum incremental vector $\Delta\underset{\sim}{X}$.

Consider a framed structure for which the total weight can be written as

$$Z(\underset{\sim}{X}) = \underset{\sim}{L}^T \underset{\sim}{X} \tag{18}$$

where $\underset{\sim}{L}$ is a vector of all element lengths times unit weight of element.

Now, a Taylor linearization of (18) together with the constraints, yields the following linear programming system.

$$\Delta\underset{\sim}{Z}^k = \underset{\sim}{L}^T \, \Delta\underset{\sim}{X}^k \rightarrow \text{MINIMUM} \tag{19}$$

subject to

$$\frac{\delta \underset{\sim}{Y} (\underset{\sim}{X}^k)}{\delta \underset{\sim}{X}} \Delta\underset{\sim}{X}^k \lesseqgtr \underset{\sim}{YD} (\underset{\sim}{X}^k) - \underset{\sim}{Y}(\underset{\sim}{X}^k)$$

$$\frac{\delta \underset{\sim}{G} (\underset{\sim}{X}^k)}{\delta \underset{\sim}{X}} \Delta\underset{\sim}{X}^k \leqq 1 - \underset{\sim}{G}(\underset{\sim}{X}^k) \ , \tag{20}$$

$$\frac{\delta \underset{\sim}{H} (\underset{\sim}{X}^k)}{\delta \underset{\sim}{X}} \Delta\underset{\sim}{X}^k \lesseqgtr \underset{\sim}{HD} (\underset{\sim}{X}^k) - \underset{\sim}{H} (\underset{\sim}{X}^k) \ .$$

where $\underset{\sim}{X}^k$ is the value of the design vector in the k-th iteration.

In order to keep the linear approximations within a reasonable range of errors, the inequalities (20) are further supplemented by move limits set for the incremental vector $\Delta\underset{\sim}{X}^k$ or

$$\underset{\sim}{LB}^k \leqq \Delta\underset{\sim}{X}^k \leqq \underset{\sim}{UB}^k \tag{21}$$

in which the "lower bound" and "upper bound" vectors are usually selected as a certain percentage of the design vector $\underset{\sim}{X}^k$. Move limits can also ensure that the design variables are kept within the limits set by the range of acceptable profiles.

The linear programming problem (19), (20) and (21) may now be solved using the dual formulation of the standard Simplex method, to yield an optimum incremental vector $\Delta\underset{\sim}{X}^k$. This implies that all the partial derivatives in (20) need to be evaluated, either analytically or numerically. Starting with the deflection, the following relation is obtained

$$\frac{\delta\underset{\sim}{Y}}{\delta\underset{\sim}{X}} = \underset{\sim}{KK}^{-1} \frac{\delta(\underset{\sim}{BF}-\underset{\sim}{B})}{\delta\underset{\sim}{X}} - \underset{\sim}{KK}^{-1} \frac{\delta\underset{\sim}{KK}}{\delta\underset{\sim}{X}} \underset{\sim}{KK}^{-1} (\underset{\sim}{BF}-\underset{\sim}{B}) \tag{22}$$

The earthquake forces which are a part of $\underset{\sim}{B}$ have to undergo a special treatment due to the period-dependency. Formally it is possible to write

$$\frac{\delta \underset{\sim}{P}}{\delta \underset{\sim}{X}} = \frac{dQ}{dT} \frac{\delta T}{\delta \underset{\sim}{X}} \underset{\sim}{F}(\underset{\sim}{X}) + Q(T) \frac{\delta \underset{\sim}{F}(\underset{\sim}{X})}{\delta \underset{\sim}{X}} \tag{23}$$

whereby it becomes necessary to compute the period $T(\underset{\sim}{X})$ and its derivatives at each step. The period, then, is computed as follows:

First, from (8) all the horizontal deflection components $\underset{\sim}{YH}$ due to unit horizontal forces applied at all the nodes are successively obtained as

$$\underset{\sim}{KK} \cdot \underset{\sim}{YH} = \underset{\sim}{BU} \tag{24}$$

where $\underset{\sim}{BU}$ is a matrix of column vectors that have zero elements except for the loaded node, which has a unit element. Then, taking the corresponding submatrix $\underset{\sim}{MH}$ from the global mass matrix, the period is computed by the followins scheme,

$$[(\underset{\sim}{MH}^{\frac{1}{2}})^T \cdot \underset{\sim}{YH} \cdot \underset{\sim}{MH}^{\frac{1}{2}} - \frac{1}{4\pi^2} T^2 \underset{\sim}{I}] \cdot \underset{\sim}{EY} = \underset{\sim}{O} \tag{25}$$

where $\underset{\sim}{I}$ is the unit matrix and the transformation indicated has ensured a positive definite, symmetric matrix. $\underset{\sim}{EY}$ is the modified eigenvector, which will not be evaluated.

The derivatives $\delta T/\delta \underset{\sim}{X}$ may formally be obtained by differentiation of (25). However, such a procedure is unnecessarily complicated and a numerical evaluation by a repeated application of (25) is preferable.

Now, turning to the stress function derivatives, these are evaluated as follows:

$$\frac{\delta \underset{\sim}{G}}{\delta \underset{\sim}{X}} = \left\{\frac{\delta G_i}{\delta X_j}\right\} = \left\{\frac{\text{sign}(n_i)}{no_i}\left(\frac{\delta n_i}{\delta X_j} - \frac{n_i}{no_i}\frac{\delta no_i}{\delta X_j}\right) + \frac{\text{sign}(m_i)}{mo_i}\left(\frac{\delta m_i}{\delta X_j} - \frac{m_i}{mo_i}\frac{\delta mo_i}{\delta X_j}\right)\right\} \tag{26}$$

Lastly, the evaluation of the partial derivatives ($\delta \underset{\sim}{H}/\delta \underset{\sim}{X}$) is straightforward.

Thus the linear programming scheme (19), (20), (21) is ready for solution and the result is the incremental vector $\Delta \underset{\sim}{X}$. The new design vector

$$\underset{\sim}{X}^{k+1} = \underset{\sim}{X}^{k} + \Delta \underset{\sim}{X}^{k} \qquad (26)$$

is the basis of the next iteration step, and the process is stopped when the relative reduction of the object function is less than a certain percentage, say 1%, and all design restrictions are observed. As mentioned earlier, this only means that the process has possibly been stopped at any of the local optima. There is no way to direct the process automatically to the global optimum. The only method possible is to rerun the problem with a different initial design vector and, after possibly several attempts, the lowest optimum is taken as the global optimum, or at least a sufficiently close local optimum has been found. The numerical procedures and their implementation, therefore, depend greatly on intuition and sound judgement as to selection of the starting values.

Finally, it may be mentioned that the final solution is based on continuous variation of all the variables. The available sections are of course discontinuous, and the final design has to be selected from the section values closest to the values found. This, of course, will possibly shift the optimum solution considerably. Several methods have been proposed to solve this problem, which involve random search techniques and discrete programming techniques [12], [13]. However, it appears that these methods are very time-consuming and the further weight reduction which is obtained is rather insignificant considering the extra computational effort.

NUMERICAL EXAMPLES

A numerical procedure, written in the PL/I language for the IBM 370/165 computer system at the Technical University of Denmark, has been worked out in order to test the above theory.

Fig. 3 shows two simple frames which have been analysed. In both cases, design constraints were introduced so that only two design variables were considered, by keeping all beams and all columns alike. In this way, much time was saved and the results are more easily displayed.

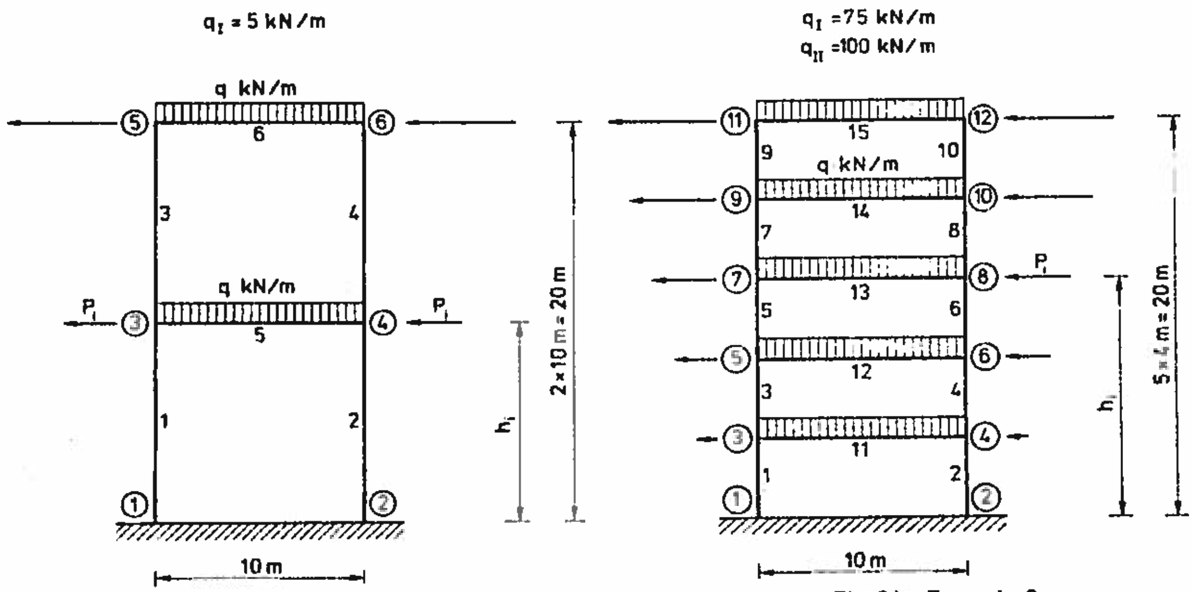

Fig. 3a. Example 1

Fig. 3b. Example 2

The load combinations which the two structures have to accommodate are the following. Firstly, the combined distributed vertical loads (self-weight of all members together with applied loads q_I on all beams) and the horizontal nodal earthquake forces p_i constitute one load case. Then, in example 2 only, the vertical loads q_{II} on all beams and self-weight of all members are treated as a second load case.

The earthquake design loads p_i are as specified by the SEAOC code,

$$p_i = \frac{0.0335}{3\sqrt{T}} \quad \Sigma\, w_i \; \frac{w_i h_i}{\Sigma w_i h_i}$$

where T is the fundamental period of the frame and W_i is the total lumped mass concentrated at the i - th node.

For the sake of simplicity, standard HE-A profiles (Euronorm 53 - 62) were chosen for all the beam and column elements. Therefore, analytical expressions were derived relating the section modulus W and the moment of intertia I to the area for the full range of HE-A profiles available. These are shown in figure 4, together with general information on material properties and strengths used in the analysis.

The computational process is carried out as already outlined in the text. However, it was found that a numerical evaluation of all the gradients using a forward differences technique was much more advantageous than the analytical method. In this way, considerable ease in programming and greater versatility of the procedure was obtained.

The two examples were now run under two different conditions. In the first case, only the stress constraints were considered whereby the stresses are checked for the left end, the right end and

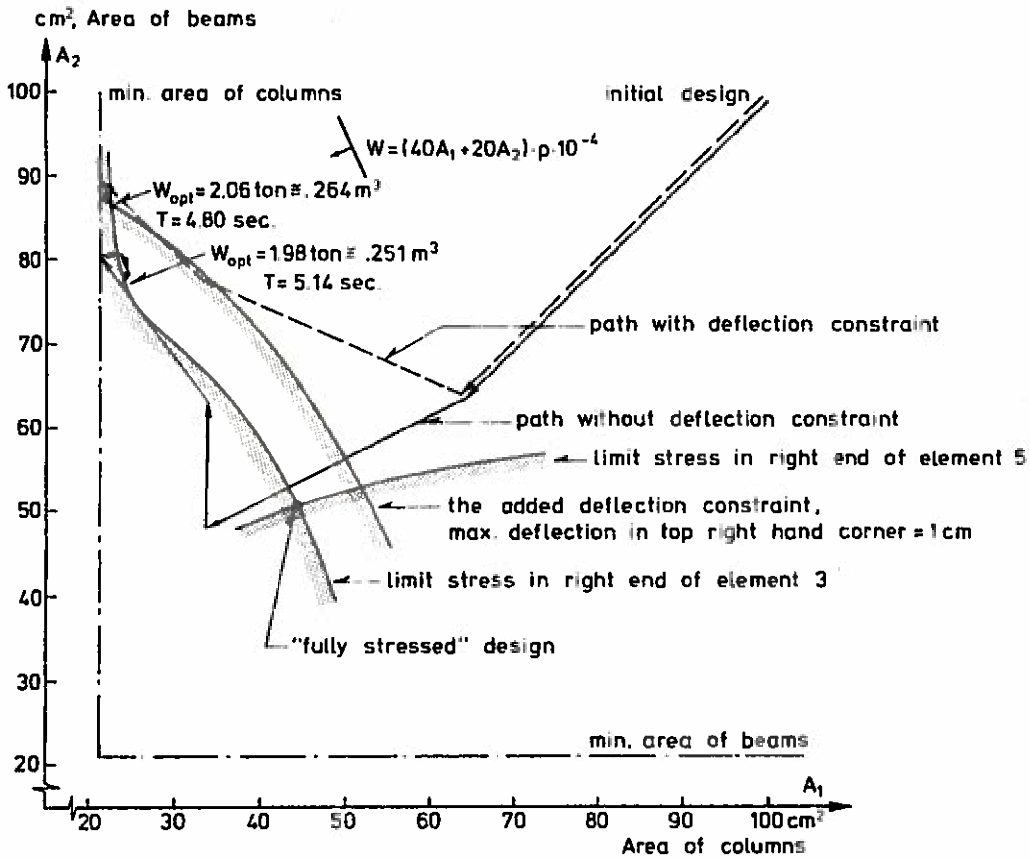

Fig. 5a. Example 1, stress & deflection constraints and iterative paths.

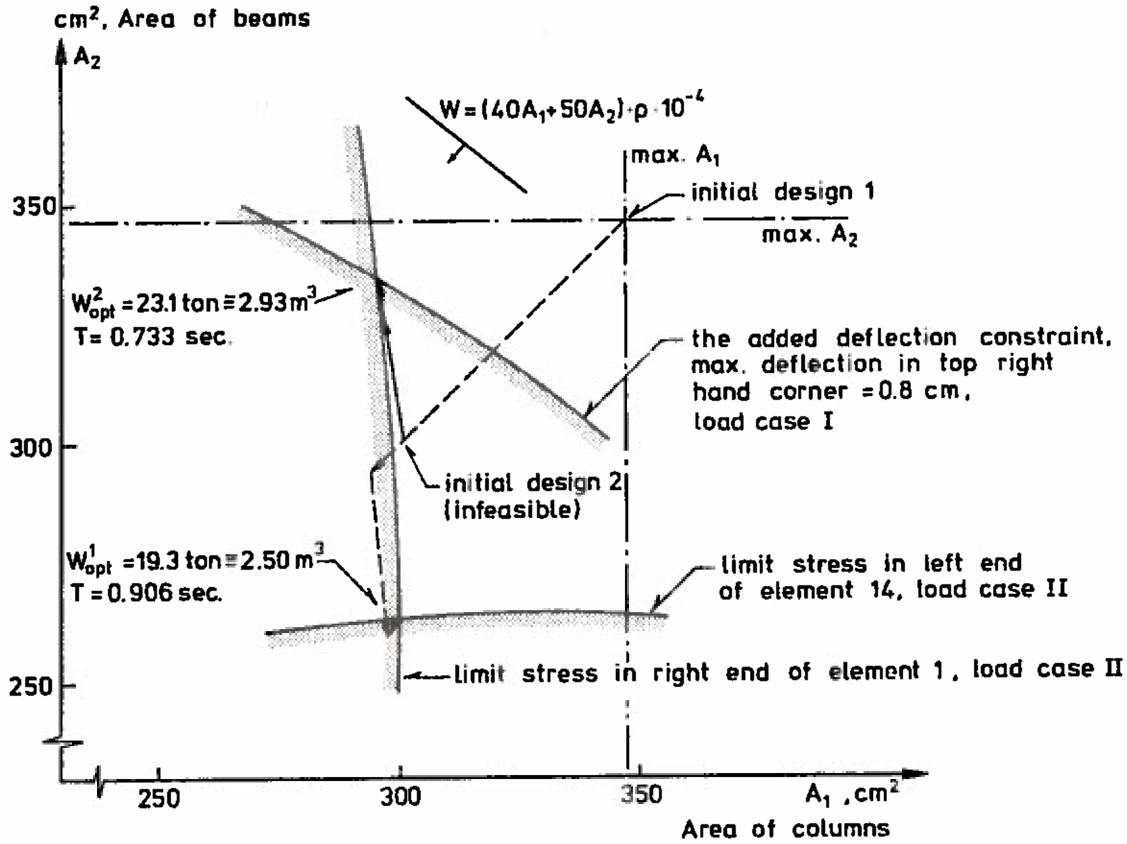

Fig. 5 b. Example 2, stress & deflection constraints and iterative paths.

CONCLUDING REMARKS

The theory which has been outlined in the present work appears to offer interesting possibilities regarding automated earthquake resistant design of framed structures. It has been shown that the optimizing techniques which are becoming an increasingly important aspect of structural design, can successfully be applied in design for stiffness-dependent forces such as earthquakes and wind forces.

In earthquake design analysis, it is especially important to have a good initial design before the more sophisticated methods of dynamic analysis are attempted. The optimum design method proposed here may offer a solution to this problem.

The numerical evaluation of the theory presented above is only in its initial stage. Due to the limited time available, only the two simple examples could by analysed. However, the numerical results obtained so far are encouraging, and as shown by the two examples, design convergence is rather easily obtained. Therefore, the numerical procedure is still being improved and it is hoped that new results for more complicated frames will soon be available for publication.

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