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ON THE ULTIMATE LOAD OF VERTICAL, KEYED SHEAR JOINTS IN LARGE PANEL BUILDINGS

Den polytekniske Læreanstalt, Danmarks tekniske Højskole

Technical University of Denmark. DK-2800 Lyngby 1975

Preface

The content of this report is a contribution to the International Symposium on Bearing Walls in Warsaw, 8th - 13th September, 1975.

The figures 7,8,and 9 were not in the paper, because the volume of the contributions was limited.

Danish summary

Plastiske beregninger af forskydningsbæreevner er noget nyt. Øvreværdiberegninger synes foreløbigt at give gode resultater, hvilket for bjælker er vist i [4] og [5]. For konsoller og støbeskel er det vist i [8].

Den fortandede vægfuge er et specialtilfælde af støbeskel, og det er heri vist, at bæreevnen af denne også kan findes ved plastiske beregninger. De fundne resultater er i god overensstemmelse med den foreslåede empiriske formel fra [1]. Heri har forskere fra en CIB-arbejdsgruppe behandlet en mængde forsøgsresultater fra Tyskland, Danmark og især Frankrig.

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FORTEGNELSE OVER RAPPORTER, UDGIVET AF
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102	Jensen, Bjarne Chr.	12 forsøg med momentpåvirket bøjlesamling i bjælker, 1973.	udgået
103	Jensen, Bjarne Chr.	Forsøg med stribeformede belastninger på beton, 1973.	
104	Hilbert, Niels-Ole	Undersøgelse af Kelementdæk, 1973.	udgået
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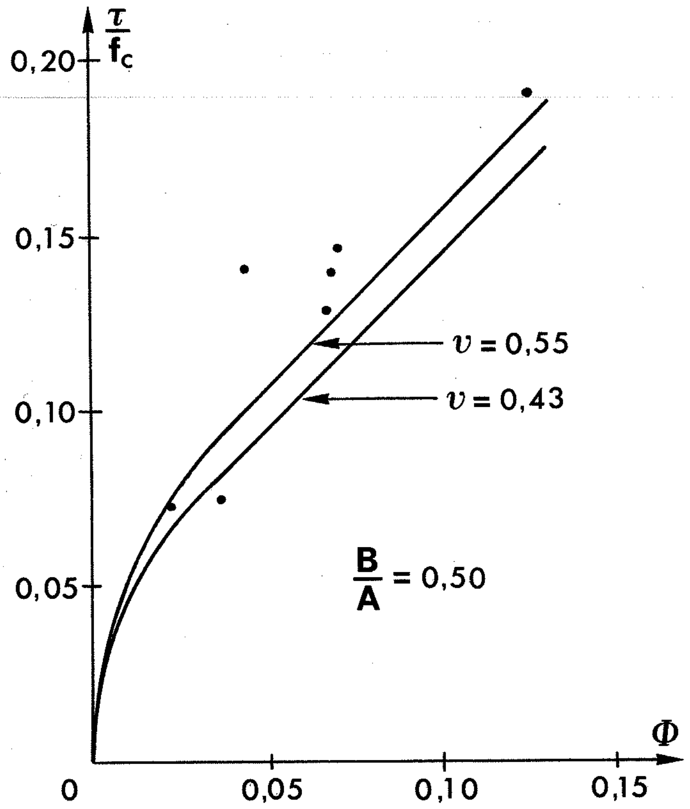

Fig. 7. Ultimate load of vertical keyed shear joints. Tests by Halasz and Tantow, Hansen and Olesen and by Pommeret From [1].

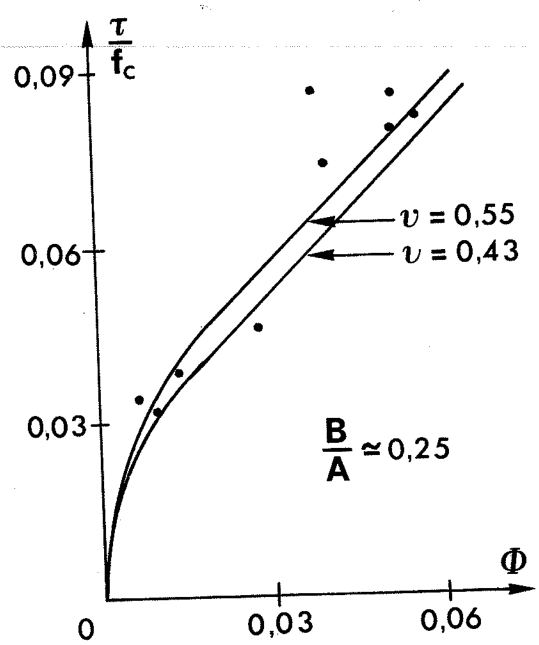

Fig.8. Ultimate load of vertical keyed shear joints. Tests by Pommeret. From [1].

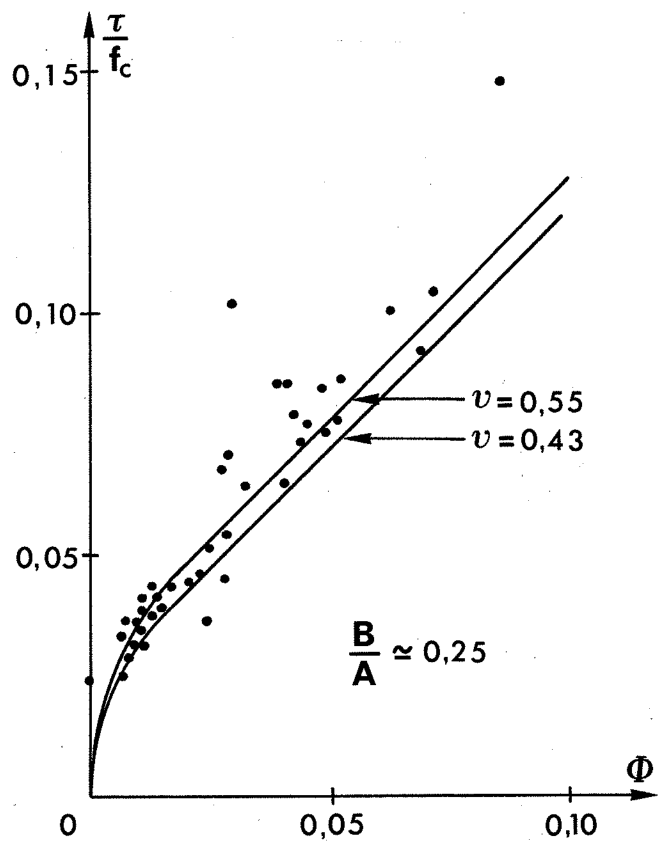

Fig.9. Ultimate load of vertical keyed shear joints. Tests by Pommeret. From [1].

[3] Nielsen, M.P.: Om forskydningsarmering af jernbetonbjælker (On the shear reinforcement in reinforced concrete beams, in Danish) Bygningsstatiske Meddelelser, No.2, 1967.

[4] Nielsen, M.P. and M.W. Bræstrup: Plastic Shear Strength of Reinforced Concrete Beams. Bygningsstatiske Meddelelser, No.3, 1975

[5] Bræstrup, M.W.: Plastic Analysis of Shear in Reinforced Concrete. Magazine of Concrete Research, Vol.26. Dec.1974.

[6] Chen, W.F. and D.C.Drucker: Bearing Capacity of Concrete Blocks and Rock. Journal of the Eng. Mech. Div., ASCE, Vol.95, No.EM4, Aug.1969.

[7] Drucker, D.C. and W.Prager. Soil Mechanics and Plastic Analysis or Limit Design. Quarterly of Applied Mechanics. Vol.10, 1952.

[8] Jensen, B.C.: Plasticitetsteori for Coulomb-materialer.(The Theory of Plasticity of Coulomb-materials, in Danish) Internt notat No.7. Institute of Building Design, Technical University of Denmark, Lyngby 1974.

[9] Jensen, B.C.: On Lines of Discontinuity for Displacements in the Theory of Plasticity of Plain and Reinforced Concrete. Magazine of Concrete Research, in press.

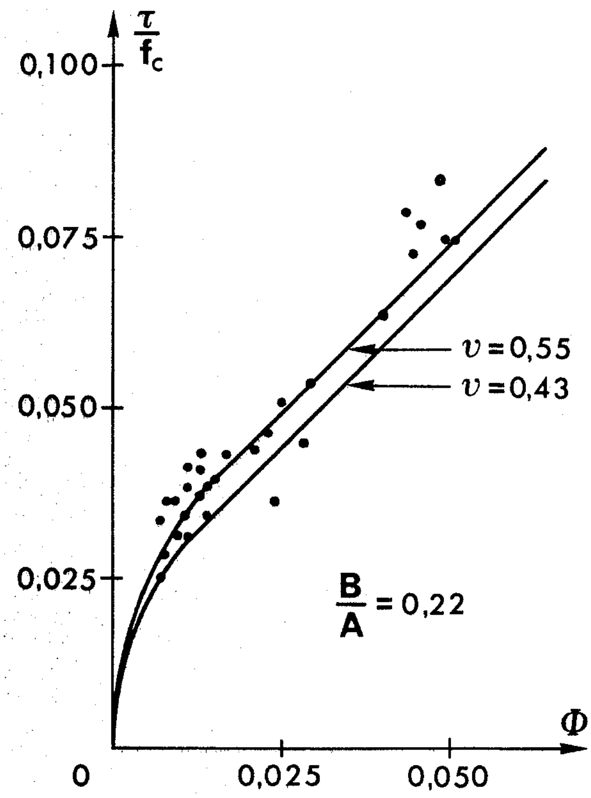

Fig.6. Ultimate Load of Vertical, Keyed Shear Joints. Tests by Pommeret, from [1].

Concluding remarks

The theory described here is a special case of a more general theory, which can be found in [9]. There the author shows some other applications of the theory i.e. construction joints and brackets.

Litteratur

[1] Hansen, Kavyrchine, Melhorn, Olesen, Pume and Schwing: Design of vertical keyed shear joints in large panel buildings. Building Research and Practice, July/August 1974.

[2] Hansen, K and S.Ø.Olesen: Failure Load and Failure Mecanism of Keyed Shear Joints. Test Results II, Report No.69/22, Danmarks Ingeniørakademi, Bygningsafdelingen, Copenhagen, 1969.

Because of that we cannot calculate with the fully contribution from the whole failure surface. We now introduce an efficiency factor $\nu \leq 1$, which must be multiplied with the surface area A to get an equivalent area with fully contribution.

In the vertical keyed shear joints of the failure only occurs in the shear keys, see fig. 5. If the area of the cross-section of the shear keys is B, (14),(15),(16) and (17) are valid if we are not using (10) and (11) but

$$\tau' = \frac{P}{\nu B} \tag{18}$$

$$\Phi' = \frac{F f_y}{\nu B\ f_c} \tag{19}$$

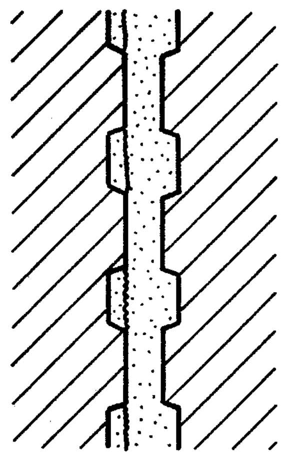

Fig.5. Shearing of shear keys.

The new equations we get, when we replace τ by τ' and Φ by Φ' in (14)-(17), can be expressed by the normal quantities τ and Φ. Hence $\tau' = \frac{A}{\nu B}\tau$ and $\Phi' = \frac{A}{\nu B}\Phi$ we get

$$\frac{\tau}{f_c} = \sqrt{\Phi\left(\frac{\nu B}{A} - \Phi\right)} \tag{20}$$

$$\frac{\tau}{f_c} = \frac{\nu B}{A} \cdot \frac{1-\sin\varphi}{2\cos\varphi} + \Phi\tan\varphi \tag{21}$$

(20) is valid when

$$\Phi \leq \frac{\nu B}{A} \cdot \frac{1-\sin\varphi}{2} \tag{22}$$

and (21) is valid when the sign of inequality in (22) is changed.

If we have $\varphi = 45^\circ$ and $\nu = 0{,}43$, (21) turn out to be

$$\frac{\tau}{f_c} = 0{,}09\ \frac{B}{A} + \Phi \tag{23}$$

This is exactly what is recommended by Hansen and others [1]. Fig.6 shows results from tests by Pommeret with B/A = 0,22 (from [1]). With $\varphi = 45^\circ$ and $\nu = 0{,}43$ (20) and (21) are drawn in the figur. The slope of the right line is in accordance with tests, but the elected ν is a conservative one. Because the theory shows a circle for small Φ, we can use a bigger ν. As shown in fig. 6, $\nu = 0{,}55$ is a good one. In this case the ultimate load is

$$\frac{\tau}{f_c} = \sqrt{\Phi\ (0{,}55\ \frac{B}{A} - \Phi)} \qquad \Phi \leq 0{,}08\ \frac{B}{A} \tag{24}$$

$$\frac{\tau}{f_c} = 0{,}11\ \frac{B}{A} + \Phi \qquad \Phi \geq 0{,}08\ \frac{B}{A} \tag{25}$$

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ON THE ULTIMATE LOAD OF VERTICAL, KEYED SHEAR JOINTS IN LARGE PANEL BUILDINGS

by Bjarne Chr. Jensen, Akademiingeniør
Institute of Building Design
Technical University of Denmark

Introduction

In this paper it is shown that the ultimate load of vertical, keyed shear joints can be found by using the theory of plasticity.

The ultimate load has been investigated by a lot of researchers. Latest a team of researchers [1] have put plenty of tests together and they recommend an equation by which we can find the ultimate load.

A lower bound solution was found by Hansen and Olesen [2] by supposing a plane stress field in the joint. Because we have plane strain it was not working. The lower bound solution by Hansen and Olesen is valid for the shear carrying capacity of beams. Using the limit theorems in the theory of plasticity the solution is the correct one in fact, because the lower bound solution was found by Nielsen [3] and the upper bound solution by Nielsen and Bræstrup [4], [5].

Here we will use the technique by which we find upper bound solutions, but as it will be seen the solution is in accordance with tests and with the equation from [1].

Discontinuity for displacements

Initially we are considering a plane homogeneous displacement field occurring in a narrow zone with a depth δ between two rigid parts I and II. The ratio between the displacements of part II and part I is V, see fig.1. In the displacement zone we have the strains

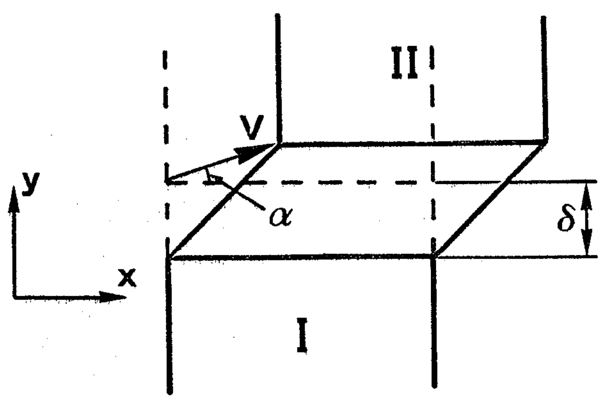

Fig. 1. Displacement zone between two rigid parts

$$\varepsilon_x = 0 \qquad \varepsilon_y = \frac{V}{\delta}\sin\alpha \qquad \gamma_{xy} = \frac{V}{\delta}\cos\alpha \qquad (1)$$

Yield criterion and yield law

Concrete is supposed to obey a modified Coulomb yield criterion, which is shown in fig. 2. The yield criterion consists of two parts. The one is the sliding criterion

$$|\tau| = c - \sigma \tan\varphi \qquad (2)$$

in which τ = the shear stress on the failure section, σ = the normal stress on the failure section (positive in tension), c = the cohesion and φ = the angel of internal friction.

The other part is the separation criterion

$$\sigma_1 = f_t \qquad (3)$$

in which σ_1 = the largest principal stress and f_t = the tensile strength.

Determination of the yield criterion demand 3 parameters, say, the compressive strength f_c, the tensile strength f_t and the angel of friction φ.

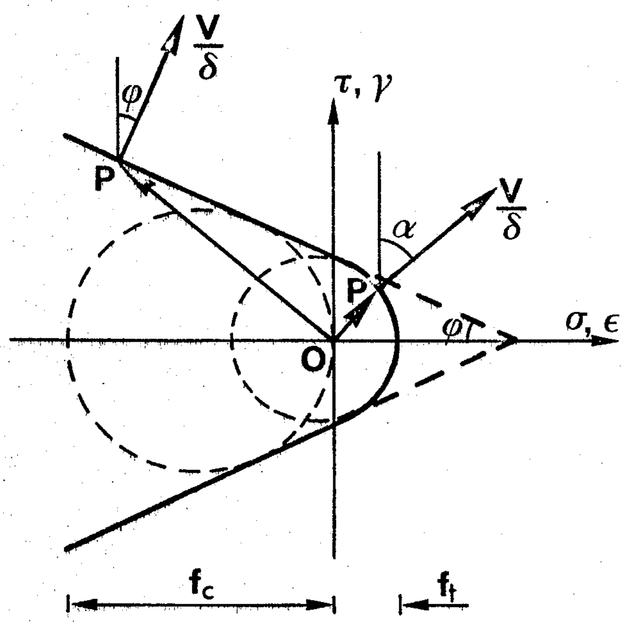

Fig.2. Coulomb's modified yield criterion with displacement vectors.

When a rigid-plastic material is yielding, the strains satisfy the normality-condition. (In the theory of plasticity we normally use the strain rate, but having a rigid-plastic material the strain itself is sufficient). The normality-condition requires that the displacement vector given by (1) is perpendicular to the yield criterion, when a ε,γ -coordinate system is inserted in fig.2 congruent with the σ,τ -coordinate system. Fig.2 shows that $\alpha = \varphi$ along the straight lines in the yield criterion. Along the circular cut-off $\alpha \geq \varphi$.

Internal plastic work

The stress field belonging to the given deformation field is the point or points on the yield criterion at which the normality-condition is satisfied.

Thus, at a point P, the internal plastic work per unit length in the displacement zone is found from

$$W_\ell = \delta \, \vec{OP} \, \frac{\vec{V}}{\delta} = \vec{OP} \, \vec{V} \tag{4}$$

Note that (4) is independent of δ, whereby the lines of discontinuity can be introduced. In this $\delta \to 0$, but V is retained and (4) is still valid.

From the geometry in fig.2 it can be found that along the straight lines (4) will be

$$W_\ell = V c \cos\varphi = V \frac{1 - \sin\varphi}{2} f_c \qquad \alpha = \varphi \tag{5}$$

Along the circular cut-off (4) turn out to be

$$W_\ell = V \left(\frac{1 - \sin\alpha}{2} f_c + \frac{\sin\alpha - \sin\varphi}{1 - \sin\varphi} f_t \right) \qquad \alpha \geq \varphi \tag{6}$$

Chen and Drucker [6] have found (5) and (6) in the manner described here, but earlier the corresponding expressions for $f_t = 0$ was found by Drucker and Prager [7]. Using the yield criterion on principal stress form the author has formulated the same expressions [8].

Ultimate shear load

Now we are looking at the shear problem in fig. 3. The failure mechanism consists of a line of discontinuity between the loads P, and the right-hand part is displaced V in relation to the left-hand part. In the line of discontinuity we have plane strain and then we know $\alpha \geq \varphi$.

Perpendicular to the line of discontinuity, there is reinforcement with a total cross-section of F and with a yield strength of f_y. The cross-section of the concrete is A and concrete is assumed to have the tensile strength $f_t = 0$.

We are using the work equation to get the carrying capacity and we begin with the case $\alpha > \varphi$.

The external work is

$$W_E = P V \cos\alpha \tag{7}$$

By means of (6) the contribution to the internal work from concrete is

$$W_{IC} = V \frac{1 - \sin\alpha}{2} f_c A \tag{8}$$

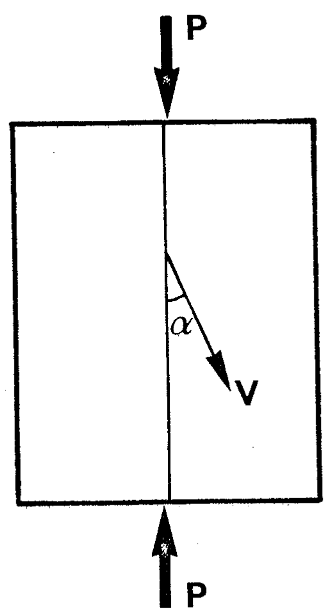

Fig.3. Shear failure in a diaphragm

From the reinforcement we get

$$W_{IR} = F f_y V \sin\alpha \tag{9}$$

The shear stress τ and the ratio of reinforcement Φ are introduced as

$$\tau = \frac{P}{A} \tag{10}$$

$$\Phi = \frac{F f_y}{A f_c} \tag{11}$$

By putting $W_E = W_{IC} + W_{IR}$, we get

$$\frac{\tau}{f_c} = \frac{1 - \sin\alpha}{2\cos\alpha} + \Phi \tan\alpha \tag{12}$$

This is an upper bound solution. The minimum is found for

$$\sin\alpha = 1 - 2\Phi \tag{13}$$

The minimum is found to be

$$\frac{\tau}{f_c} = \sqrt{\Phi\,(1-\Phi)} \tag{14}$$

From $\alpha > \varphi$ and (13) it is evident that (14) is valid when

$$\Phi \leq \frac{1 - \sin\varphi}{2} \tag{15}$$

For $\alpha = \varphi$ we find from (7),(8) and (9) that

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which is valid, when

$$\Phi \geq \frac{1 - \sin\varphi}{2} \tag{17}$$

In a Φ, $\frac{\tau}{f_c}$ -coordinate system (14) is a circle and (16) is a straight line that is tangential to (14), see fig.4.

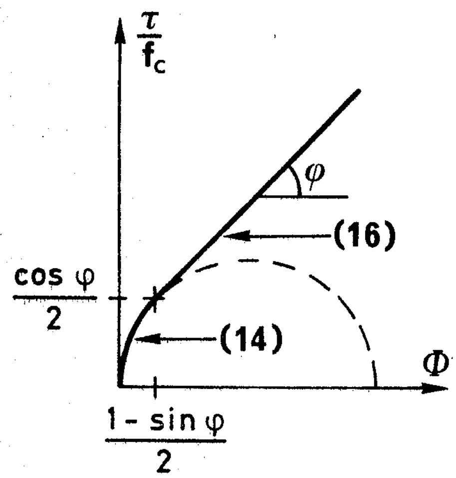

Fig.4. Shear strength

Modification to concrete and vertical keyed shear joints

From the stress-strain curve in simple compression tests we know that concrete is not a rigid-plastic material. After reaching maximum the stress is decreasing until failure, often about $\varepsilon = 3 - 5$ ‰.

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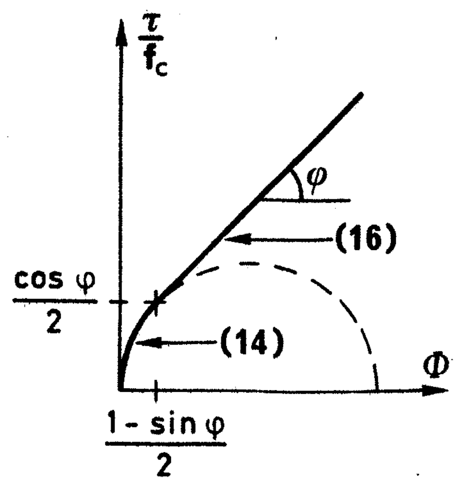

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