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Trevisi, Filippo; Gaunaa, Mac; McWilliam, Michael

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# The Influence of Tether Sag on Airborne Wind Energy Generation

Filippo Trevisi, Mac Gaunaa, Michael McWilliam

DTU Wind Energy, Frederiksborgvej 399, 4000 Roskilde, Denmark.

E-mail: [fitr@dtu.dk](mailto:fitr@dtu.dk)

**Abstract.** The aim of this work is to propose engineering models to include the external forces acting on the tether into the power estimation of an Airborne Wind Energy System flying crosswind. The influence of aerodynamic drag, centrifugal force and gravitational force on power production is investigated. These three types of external loading acting on the tether are lumped with the kite and the tether is considered straight. In this way, a set of equations describing the tether shape and its influence on the power production is reduced to just one power equation. The formulation with straight tether and lumped quantities is suitable for preliminary designs and analyses and points out how the tether influence the system. On top of the results concerning the tether modelling, a new analytical model which includes the centrifugal forces into the power estimation for circular paths is presented.

## 1. Introduction

Airborne wind energy (AWE) refers to the harvesting of wind power by means of a flying system tethered to the ground. Typically, a kite flies in crosswind trajectories like circles or figures-of-eight and can harvest power with on-board wind turbines (Fly-Gen) or with a generator placed on the ground (Ground-Gen).

During the design phase, simple analytic models allow a fast estimation of the system performance. As the tether is known to be one of the critical sub systems of AWES, this paper is focusing on its influence on the power generation. During the power generation, the tether is mainly subject to three types of external forces: aerodynamic drag, centrifugal force and gravitation force. Studies have been carried out to investigate, with analytic models, the influence of the tether drag on the power production [1]. The centrifugal forces on the tether are often neglected in the design phase and the effect of the centrifugal force acting on the kite is considered as a loss [1, 2]. In this paper, a new approach to include the centrifugal forces into the power estimation is proposed for circular trajectories. Circular trajectories are chosen because they allow for a closed-form solution, since Coriolis forces are not active. The gravitation forces can be included in the power production estimation with closed-form [3, 4] or with iterative solutions [5]. Detailed effects of the tether physics for AWE applications has been investigated in [6], while in [7] the authors propose an analytical tether model for static kite flight based on catenary curves. In this paper, the authors propose engineering methods to include the contribution of the tether sag due to these three external loadings into the power estimation. This paper is divided into three main sections, corresponding to the three different types of external loading acting on the tether. Each section is divided into four subsections.



The aim of the four subsections is to show that the power estimations considering small tether sags or considering straight tether and lumped forces are equal.

## 2. Methodology

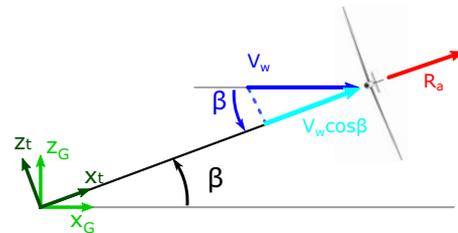
The methodology introduced in this section is used to show the equivalence of the power estimation considering small tether sags and considering straight tether and lumped forces. Power equations with straight tether and lumped forces can be easily analyzed and used, as they do not require the evaluation of the tether shape and its influence on the force balance on the kite. This version of the power equations reduces a set of equations describing the system (tether and kite) to one power equation, which can be used in system design tools. The three following sections are divided into four subsections. The motivations and methods used in each subsection are listed here:

**Tether slope at kite connection:** For a kite to fly, aerodynamic and inertial forces are in equilibrium with the tether force. Since a tether only transfers pure tension, determining the direction of the tether at the kite connection is crucial for solving the force balance and determining the power production. The tether sag and the slope at the kite connection, with respect to the straight shape, due to the three types of forces is derived for small deformations. The kite motion is considered steady, in linear or circular crosswind motion. A visual representation of the reference systems for the linear motion case is shown in Figure 1 and for circular motion in Figure 4. The tether displacement with respect to the straight shape is found by solving the differential equation and boundary conditions [8]:

$$T \frac{\partial^2 \zeta}{\partial \xi^2} d\xi = -dF \quad \zeta(0) = \zeta(l_t) = 0 \quad (1)$$

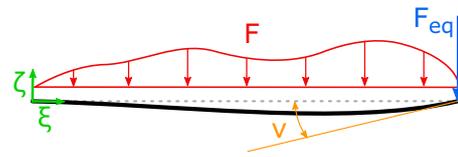
Where  $\zeta(\xi)$  (Figure 2) is the displacement in the direction of the external force,  $\xi$  is the coordinate along the straight shape,  $l_t$  the tether length and the tether tension  $T$  is considered constant. The tether slope at the kite connection is defined as  $\nu = \frac{\partial \zeta}{\partial \xi}(\xi = l_t)$ . The effects of the three forces are analysed individually in 2D. Centrifugal and drag forces act in perpendicular directions to each other for small deformation and for circular paths. The effects of the two can then be superimposed if the tether tension is considered constant. Depending on the kite position in the flight path, the gravitational force is aligned with drag and centrifugal forces, thus no linear superimposition of effects can be used for this force. If a pure linear crosswind motion is assumed (the kite is moving crosswind in the downwind position, as in Figure 1), drag and gravitational forces act in perpendicular directions to each other and thus the effects can be superimposed if the tether tension is considered constant.

**Power equation considering tether slope:** The power equations are derived considering the tether slope at the kite connection. In this step, the tether slope is not assumed to be small, as the linear tether model introduced in Eq. (1) is not needed for the power estimation. The power estimation not assuming small angles can be used only if the sag due to one of the forces is dominant compared to the others.



**Figure 1.** Reference systems for a kite in steady state linear crosswind motion.  $V_w$  is the wind speed,  $\beta$  the elevation angle and  $R_a$  the resultant aerodynamic force.

**Lumped force:** Considering the small tether slope at the kite connection derived with Eq. (1), the distributed force along the tether is lumped with the kite and the lumped force magnitude computed. If the distributed force is lumped, the tether is considered straight. The magnitude of the lumped force is found by matching the torque around the ground station given by the distributed load and by the lumped force (Figure 2). This procedure was first implicitly used by Houska and Diehl [9] for the drag case.



**Figure 2.** Distributed load along the tether  $F$  and equivalent lumped force  $F_{eq}$ .

**Power equation considering straight tether and lumped force:** The power equations are derived considering lumped forces and straight tether. This eases the power estimation. It is shown that the power computed with a small tether slope and with the lumped force and a straight tether are equal. This equivalence could be shown in general way. However, the procedure just presented is chosen to show the peculiarities of each case and to present the power equation with the tether slope at the kite connection not small.

### 3. Drag forces

#### 3.1. Tether slope at kite connection

The drag force acting on the straight tether shape can be approximated by the drag acting on a cylinder. The velocity of a tether section is, as first approximation, proportional to its normalized tether coordinate  $\frac{\xi}{l_t}$  [1]:

$$dF_{dr} = \frac{1}{2} \rho d_t C_{\perp} V_k^2 \left( \frac{\xi}{l_t} \right)^2 d\xi \quad (2)$$

where  $\rho$  is the air density,  $d_t$  the tether diameter,  $C_{\perp}$  the drag coefficient of the tether and  $V_k$  is the kite velocity. The tether slope at the kite connection  $\nu$  due to drag for small deformations can be derived by solving Eq. (1) with the external loading given in Eq. (2):

$$\nu_{dr} = \frac{1}{2} \frac{\rho V_k^2 C_{\perp} d_t l_t}{4T} \quad (3)$$

#### 3.2. Power equation considering tether slope

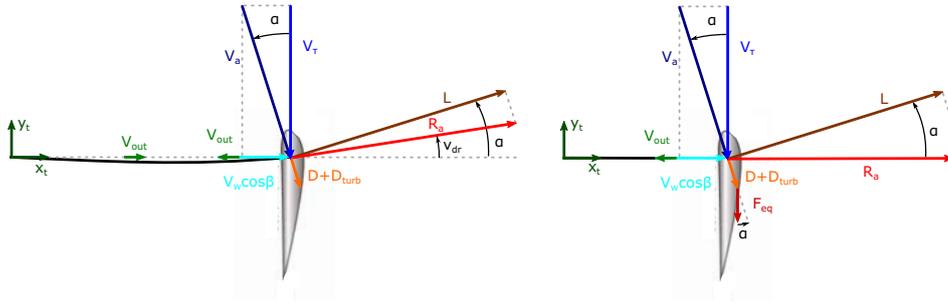
Following the generalization of AWES modelling presented in [3], a system with both on-board and ground generation is initially considered, later the case of Fly-Gen and Ground-Gen are analysed. Considering the system in a steady state linear crosswind motion, the lift force  $L$  is perpendicular to the relative wind velocity  $V_a$  seen by the kite and the resultant aerodynamic force  $R_a$  is along the tether direction (Figure 3(a)).

The angle between tangential  $V_{\tau}$  and relative velocity  $V_a$  is:

$$\arctan \left( \frac{V_w (\cos \beta - \gamma_{out})}{V_{\tau}} \right) = \arctan \left( \frac{D + D_{turb}}{L} \right) + \nu_{dr} \quad (4)$$

$\gamma_{out}$  is the ratio between reel-out speed and wind speed:  $\frac{V_{out}}{V_w}$ . The lift force is  $L = \frac{1}{2} \rho A C_L V_a^2$ , where  $A$  is the wing area and  $C_L$  the lift coefficient. The drag force is  $D = \frac{1}{2} \rho A C_{d,k} V_a^2$ , where  $C_{d,k}$  is the kite drag coefficient. The productive drag  $D_{turb} = \frac{1}{2} \rho A C_{d,t} V_a^2$ , where  $C_{d,t}$  is the productive drag coefficient. For high  $\frac{C_L}{C_{d,k} + C_{d,t}}$ , Eq. (4) becomes:

$$\frac{V_w (\cos \beta - \gamma_{out})}{V_{\tau}} = \frac{\frac{C_{d,k} + C_{d,t}}{C_L} + \tan \nu_{dr}}{1 - \frac{C_{d,k} + C_{d,t}}{C_L} \cdot \tan \nu_{dr}} \approx \frac{C_{d,k} + C_{d,t}}{C_L} + \tan \nu_{dr} \quad (5)$$



**Figure 3.** Velocity triangle and force balance of a kite with on board and ground generation flying crosswind for the case with tether sag (a) and the case with straight tether and lumped force (b).

The relative wind velocity  $V_a$ , for  $\left(\frac{C_L}{C_{d,k} + C_{d,t} + C_L \tan \nu_{dr}}\right)^2 \gg 1$ , is:

$$V_a \approx V_\tau \approx \frac{C_L}{C_{d,k} + C_{d,t} + C_L \tan \nu_{dr}} \cdot V_w (\cos \beta - \gamma_{out}) \quad (6)$$

For a Fly-Gen system, the ideal power is  $D_{turb} \cdot V_a$ :

$$P_{FG} = \frac{1}{2} \rho A C_{d,t} V_w^3 \cos^3 \beta \left( \frac{C_L}{C_{d,k} + C_{d,t} + C_L \tan \nu_{dr}} \right)^3 \quad (7)$$

For a Ground-Gen system, the power is  $T \cdot V_{out} \approx L \cdot V_{out}$ :

$$P_{GG} = \frac{1}{2} \rho A C_L \gamma_{out} V_w^3 (\cos \beta - \gamma_{out})^2 \left( \frac{C_L}{C_{d,k} + C_L \tan \nu_{dr}} \right)^2 \cos \nu_{dr} \quad (8)$$

### 3.3. Lumped force

The equivalent lumped drag force  $F_{eq}^{dr}$  is found by computing the torque around the ground station  $T_g$  due to drag force along the tether:

$$T_g = \int_0^{l_t} \xi \cdot \frac{1}{2} \rho C_\perp d_t \left( V_k \frac{\xi}{l_t} \right)^2 dx = \frac{1}{2} \rho C_\perp V_k^2 d_t l_t^2 = l_t \cdot F_{eq}^{dr} \quad (9)$$

The equivalent lumped drag force is [9]:

$$F_{eq}^{dr} = \frac{\frac{1}{2} \rho C_\perp V_k^2 d_t l_t}{4} \quad (10)$$

This force represents one fourth of the drag the tether would have if moving at the kite speed.

### 3.4. Power equation considering straight tether and lumped force

Considering Figure 3(b), the angle  $\alpha$  between tangential  $V_\tau$  and relative velocity  $V_a$  is:

$$\alpha = \arctan \left( \frac{V_w \cos \beta - V_{out}}{V_\tau} \right) = \arctan \left( \frac{D + D_{turb} + F_{eq}^{dr} \cos \alpha}{L - F_{eq}^{dr} \sin \alpha} \right) \approx \arctan \left( \frac{A C_{d,k} + A C_{d,t} + \frac{C_\perp d_t l_t}{4}}{A C_L - \frac{C_\perp d_t l_t}{4} \alpha} \right) \quad (11)$$

Where it is assumed  $\alpha$  to be small. Assuming  $\frac{C_{\perp}}{C_L} \frac{d_t l_t}{4A} \alpha \ll 1$  (equivalent of assuming  $\nu_{dr}$  small), the relative wind velocity  $V_a$  is:

$$V_a \approx V_{\tau} \approx \frac{C_L}{C_{d,k} + C_{d,t} + C_{\perp} \frac{d_t l_t}{4A}} \cdot V_w (\cos \beta - \gamma_{out}) \quad (12)$$

The power for a Fly-Gen system is:

$$P_{FG} = \frac{1}{2} \rho A C_{d,t} V_w^3 \cos^3 \beta \left( \frac{C_L}{C_{d,k} + C_{d,t} + C_{\perp} \frac{d_t l_t}{4A}} \right)^3 \quad (13)$$

For a Ground-Gen system:

$$P_{GG} = \frac{1}{2} \rho A C_L \gamma_{out} V_w^3 (\cos \beta - \gamma_{out})^2 \left( \frac{C_L}{C_{d,k} + C_{\perp} \frac{d_t l_t}{4A}} \right)^2 \quad (14)$$

Substituting  $\nu_{dr}$  from Eq. (3) (considering  $T \approx L$ ) in Eq. (7) and (8), the power computed considering the tether slope at the kite connection and the lumped forces are equal. Therefore, this derivation brings to the well know formulation of including the tether as an additional drag coefficient applied to the kite [1]. The effective drag coefficient is  $C_d = C_{d,k} + C_{\perp} \frac{d_t l_t}{4A}$  and the so called effective glide ratio is  $G_e = \frac{C_L}{C_{d,k} + C_{\perp} \frac{d_t l_t}{4A}}$ . It should be noted that Eq. (7) and (8) do not assume a small tether slope: these formulations could then be used if a non-linear tether model, which can model large deformations, is used and the contribution to the sag given by the other forces acting on the tether is negligible. Eq. (13) and (14) assume instead small tether slope.

#### 4. Centrifugal forces

To study the influence of the centrifugal forces, a circular path is considered: it represents the simplest case and can be analysed with a closed-form solution.

##### 4.1. Tether slope at kite connection

The component of the centrifugal force perpendicular to the straight shape is:

$$dF_c = m_t \dot{\alpha}^2 \xi \sin \phi \cos \phi d\xi \quad (15)$$

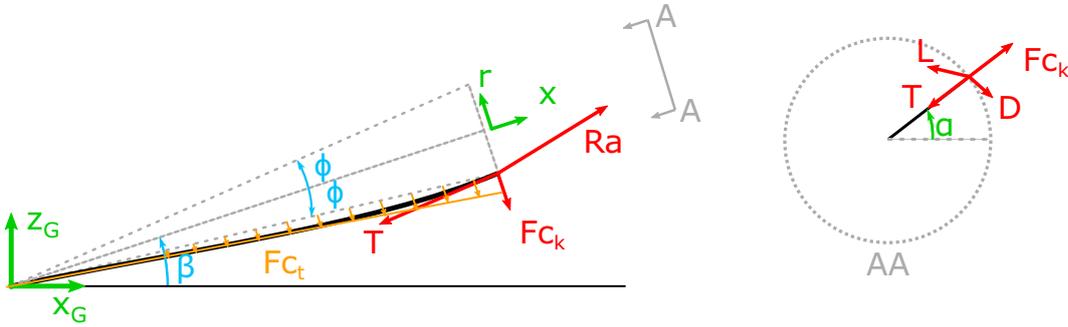
Where  $m_t$  is the tether linear mass,  $\dot{\alpha}$  the rotational speed of the kite in the circular trajectory in rad/s and  $\phi$  the opening angle of the tether with respect to direction defined by the mean elevation angle and the downwind direction (see Figure 4 for clarification). The slope at the connection with the kite ( $\xi = l_t$ ) due to centrifugal forces is found by solving Eq. (1) with the external loading given in Eq. (15):

$$\nu_{ce} = \frac{M_t \dot{\alpha}^2 l_t \cos \phi \sin \phi}{3T} \quad (16)$$

Where  $M_t$  is the total tether mass.

##### 4.2. Power equation considering tether slope

A cylindrical reference frame  $(x, r, \alpha)$  (Figure 4) is considered: the  $x$ -axis is along the direction defined by the mean elevation angle  $\beta$  and the downwind direction, the radial coordinate  $r$  and the angular coordinate  $\alpha$  identify the kite position in the plane perpendicular to the  $x$ -axis. The direction identified by  $x = 0$ ,  $r = 1$  and  $\alpha = 0$  is parallel to the ground ( $Y_G$  in Figure 4).



**Figure 4.** Coordinate system and force vectors for the centrifugal forces case.  $F_{c_t}$  and  $F_{c_k}$  represent the centrifugal force acting on the tether and on the kite respectively,  $R_a$  is the resultant aerodynamic force (summation of lift  $L$  and drag  $D$ ).

The kite attitude is described through the unit vector  $\vec{s}$  in the aforementioned coordinate system.  $\vec{s}$  represents the span-wise direction.

$$\vec{s} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} \quad |\vec{s}|^2 = u^2 + v^2 + w^2 = 1 \quad (17)$$

A system with on-board and ground generation is initially considered, later the case of Fly-Gen and Ground-Gen are analysed. The relative wind velocity seen from the kite is:

$$\vec{V}_r = \begin{pmatrix} V_x \\ V_r \\ V_\alpha \end{pmatrix} = \begin{pmatrix} V_w(\cos \beta - \gamma_{out}) \\ V_w \sin \beta \sin \alpha \\ V_w \sin \beta \cos \alpha - \dot{\alpha}R \end{pmatrix} \quad (18)$$

Where  $V_w \gamma_{out}$  is the reel-out velocity along the  $x$ -axis. It is assumed that the span direction  $\vec{s}$  is always perpendicular to the relative wind velocity. This is to avoid side-slip velocity, not contributing to aerodynamic forces:

$$\vec{s} \cdot \vec{V}_r = uV_x + vV_r + wV_\alpha = 0 \quad (19)$$

The lift and drag force can be expressed as:

$$\vec{L} = \frac{1}{2} \rho A C_L |\vec{V}_r| \vec{V}_r \times \vec{s} = \begin{pmatrix} L_x \\ L_r \\ L_\alpha \end{pmatrix} = \frac{1}{2} \rho A C_L |\vec{V}_r| \begin{pmatrix} V_r w - V_\alpha v \\ V_\alpha u - V_x w \\ V_x v - V_r u \end{pmatrix} \quad (20)$$

$$\vec{D} = \frac{1}{2} \rho A C_d (1 + \gamma_t) |\vec{V}_r| \vec{V}_r = \begin{pmatrix} D_x \\ D_r \\ D_\alpha \end{pmatrix} = \frac{1}{2} \rho A C_d (1 + \gamma_t) |\vec{V}_r| \begin{pmatrix} V_x \\ V_r \\ V_\alpha \end{pmatrix} \quad (21)$$

Where  $C_L$  and  $C_d$  are the effective lift and drag coefficient and  $\gamma_t$  is the ratio between productive and effective drag:  $\frac{|\vec{D}_{turb}|}{|\vec{D}|}$ .  $C_L$  and  $C_d$  are considered constant: the kite pitch angle (rotation around the span direction  $\vec{s}$ ), that influences the angle of attack, is not included in the model and it is assumed to be settle to give the prescribed  $C_L$ . The force balance along the three axes is considered. The tether at the kite connection is inclined of an angle  $(\phi - \nu_{ce})$  with respect to the  $x$ -axis. The force balance along the  $x$ -axis is:

$$L_x + D_x + T \cos(\phi - \nu_{ce}) = 0 \quad (22)$$

Where  $T$  is the tether force magnitude. Along the radial direction, the force balance is:

$$L_r + D_r + T \sin(\phi - \nu_{ce}) + M_k \dot{\alpha}^2 R = 0 \quad (23)$$

Where the last term models the centrifugal forces acting on the kite:  $M_k$  is the kite mass and  $R$  is the circular path radius. The kite can accelerate only along the tangential direction, thus the force balance along this axis is:

$$L_\alpha + D_\alpha = M_k \ddot{\alpha} R \quad (24)$$

The problem has 5 time-dependent unknowns ( $u, v, w, \alpha, T$ ) and 5 equations (Eq. (17), (19), (22), (23), (24)). A system with no elevation  $\beta = 0$ , constant wind speed as function of altitude and thus no angular acceleration  $\ddot{\alpha}$  is considered here, in a future work the solution for a generic elevation angle will be shown. Eq. (19) can be reformulated as:

$$u = \frac{\dot{\alpha} R}{V_x} w \quad (25)$$

Considering Eq. (24) with  $\ddot{\alpha} = 0$  and defining for readability  $G_{e,t} = \frac{C_L}{C_d(1+\gamma_t)}$ :

$$v = \frac{\dot{\alpha} R}{V_x G_{e,t}} \quad (26)$$

Combining Eq (17) with the two equations just derived (Eq. (25) and (26)) and considering that for an Airborne Wind Energy System it is expected  $(\dot{\alpha} R)^2 \gg V_w^2$ :

$$w = \frac{\sqrt{(G_{e,t} V_x)^2 - (\dot{\alpha} R)^2}}{G_{e,t} \dot{\alpha} R} \quad (27)$$

Considering Eq. (22) and (26), with the approximation:  $|\vec{V}_r| \approx \dot{\alpha} R$ , the tether force is:

$$T = -\frac{\frac{1}{2} \rho A C_L (\dot{\alpha} R)^3}{G_{e,t} V_x \cos(\phi - \nu_{ce})} \quad (28)$$

Eq. (23) is used to find the tangential velocity of the kite:

$$\dot{\alpha} R = G_{e,t} V_x \cos(\phi - \nu_{ce}) \left( \hat{M}_k \frac{\sin(\phi - \nu_{ce})}{\sin(\phi)} + \sqrt{1 - \hat{M}_k^2 \frac{\cos(\phi - \nu_{ce})^2}{\sin^2 \phi}} \right) \quad \hat{M}_k = \frac{M_k}{\frac{1}{2} \rho A C_L l_t} \quad (29)$$

The ideal power of a Fly-Gen system, considering centrifugal forces, is  $\vec{D}_{turb} \cdot \vec{V}_r$ :

$$P_{FG}^\circ = \frac{1}{2} \rho A \gamma_t C_d (\dot{\alpha} R)^2 \cdot (\dot{\alpha} R) \quad (30)$$

The ideal power equation for a Fly-Gen, neglecting centrifugal forces, is  $P_{FG} = \frac{1}{2} \rho A \gamma_t C_d V_w^3 \cos^3 \beta G_{e,t}^3$ . The ratio between the power computed considering centrifugal forces and neglecting them, considering  $V_x = V_w$ , is:

$$\eta^{ce} = \cos(\phi - \nu_{ce})^3 \left( \hat{M}_k \frac{\sin(\phi - \nu_{ce})}{\sin(\phi)} + \sqrt{1 - \hat{M}_k^2 \frac{\cos(\phi - \nu_{ce})^2}{\sin^2 \phi}} \right)^3 \quad (31)$$

Considering the Ground-Gen case, the kite is moving along the  $x$ -axis. The power can be computed as:  $T \cos(\phi - \nu_{ce}) \cdot V_w \gamma_{out}$ :

$$P_{GG}^\circ = \frac{\frac{1}{2} \rho A C_L (\dot{\alpha} R)^3}{G_{e,t} V_x \cos(\phi - \nu_{ce})} \cos(\phi - \nu_{ce}) \cdot V_w \gamma_{out} \quad (32)$$

The ideal power equation for a Ground-Gen, neglecting centrifugal forces, is  $P_{GG} = \frac{1}{2} \rho A C_L G_e^2 V_w^3 \gamma_{out} (\cos \beta - \gamma_{out})^2$ . The ratio between the power computed considering centrifugal forces and the classical result is given in Eq. (31).

#### 4.3. Lumped force

The equivalent tether centrifugal force  $F_{eq}^{ce}$  is found by computing the torque around the ground station  $T_g$  due to centrifugal force along the tether:

$$T_g = \int_0^{l_t} \xi \cos \phi \cdot m_t \dot{\alpha}^2 \xi \sin \phi d\xi = \frac{l_t^3}{3} \cos \phi \sin \phi m_t \dot{\alpha}^2 = l_t \cos \phi \cdot F_{eq}^{gr} \quad (33)$$

The equivalent centrifugal force  $F_{eq}^{ce}$  is:

$$F_{eq}^{ce} = l_t \sin \phi \frac{M_t \dot{\alpha}^2}{3} = R \frac{M_t \dot{\alpha}^2}{3} \quad (34)$$

This force represents one third of the centrifugal force the tether would have if moving at the kite speed and radius.

#### 4.4. Power equation considering straight tether and lumped force

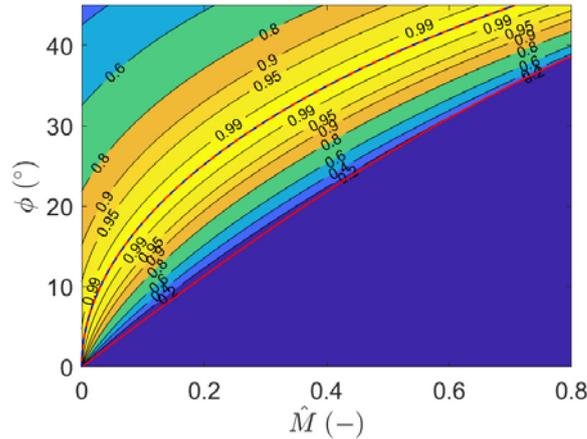
Considering the lumped mass and a straight tether, Eq. (23) is modified to:

$$L_r + D_r + T \sin \phi + \left( M_k + \frac{M_t}{3} \right) \dot{\alpha}^2 R = 0 \quad (35)$$

Same procedure can be applied to this case, showing that Eq. (31) becomes:

$$\eta^{ce} = \cos \phi^3 \left( \hat{M} + \sqrt{1 - \frac{\hat{M}^2}{\tan^2 \phi}} \right)^3 \quad \hat{M} = \frac{M_k + \frac{M_t}{3}}{\frac{1}{2} \rho A C_L l_t} \quad (36)$$

It can be shown that Eq. (31), considering Eq. (16), and Eq. (36) give the same results. The power efficiency due to centrifugal forces is function of just two parameters: the opening angle  $\phi$  and the non dimensional mass parameter  $\hat{M}$  (Figure 5). For any given  $\hat{M}$  it exists one optimal



**Figure 5.** Efficiency  $\eta^{ce}$  due to centrifugal forces as function of  $\hat{M}$  and the opening angle  $\phi$ .

value of the opening angle  $\phi$  that maximises the efficiency. In particular the equation of the optimal  $\hat{M}$  and  $\phi$  are:

$$\hat{M}^{opt} = \tan \phi \sin \phi \quad \phi^{opt} = \arccos \left( -\frac{\hat{M}}{2} + \frac{\sqrt{\hat{M}^2 + 4}}{2} \right) \quad (37)$$

For low efficiencies, the assumption  $(\dot{\alpha}R)^2 \gg V_w^2$  does not hold and Eq. (36) is expected to predict the performances poorly. According to this model, for values of  $\phi < \arctan(\hat{M})$  a kite cannot fly. For kites with negligible mass Eq. (36) reduces to the well know formulation  $\eta^{ce} = \cos \phi^3$  [1]. If the optimal values (Eq. (37)) are considered,  $u = w = 0$  and  $v = 1$ , meaning that the kite span is aligned with the radial direction and the kite bank angle is equal to the opening angle. In a future work, the same procedure will be applied to a generic case with elevation. Dynamic simulations are needed to investigate more in details this phenomenon, to understand if any benefit to the flight stability and to the power production can be attained. It should be noted that Eq. (31) does not assume a small tether slope, while Eq. (36) does.

## 5. Gravitational forces

### 5.1. Tether slope at kite connection

The gravitational force is constant along the tether. Its component perpendicular to the undeformed shape is:

$$dF_{gr} = m_t g \cos \beta d\xi \quad (38)$$

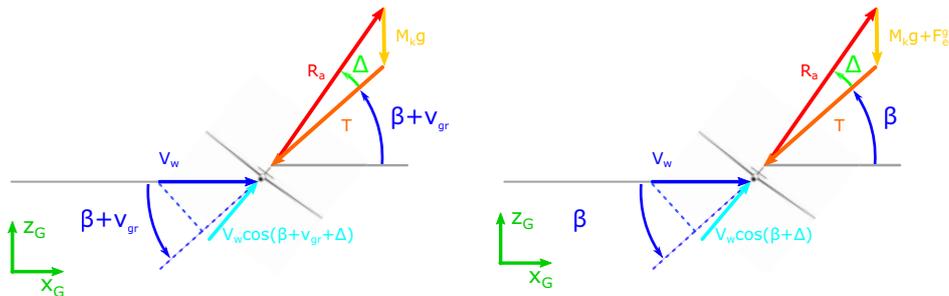
where  $g$  is the gravitation acceleration. If  $m_t$  is constant along the tether, the tether sag is a quadratic function. The slope at the connection with the kite ( $x = l_t$ ) due to gravitational forces is:

$$\nu_{gr} = \frac{M_t g \cos(\beta)}{2T} \quad (39)$$

By removing the assumption of constant tether tension, the tether sag is better modelled as a catenary curve [7].

### 5.2. Power equation considering tether slope

The power efficiencies considering gravitational forces presented in this paper are derived following the procedure presented in [3], taking into account the tether sag.



**Figure 6.** Forces and and velocities for a kite moving along  $y_G$  axis considering tether sag (a) and considering lumped mass and straight tether (b).

For a Fly-Gen, the power considering gravitational forces is [3]:

$$P_{FG}^{\Delta} = \frac{1}{2} \rho A C_L G_e^2 V_w^3 \gamma_t \left( \frac{\cos(\beta + \Delta + \nu_{gr})}{1 + \gamma_t} \right)^3 \quad (40)$$

Where the angle  $\Delta$ , assumed small, is needed to compensate gravitational forces:

$$\Delta = \frac{M_k g (1 + \gamma_t)^2}{\frac{1}{2} \rho A C_L V_w^2 G_e^2 \cos(\beta + \nu_{gr})} \quad (41)$$

Considering  $\nu_{gr}$  small, the efficiency due to gravity for a Fly-Gen is [3]:

$$\eta_{FG}^{gr} = 1 - 3 \sin \beta \cdot \frac{M_k g}{T} - 3 \nu_{gr} \tan \beta \quad (42)$$

For a Ground-Gen system the power equation considering the gravitational losses is [3]:

$$P_{GG}^{\Delta} = \frac{1}{2} \rho A C_L G_e^2 V_w^3 \gamma_{out} (1 - \Delta \tan(\beta + \nu_{gr})) (\cos(\beta + \Delta + \nu_{gr}) - \gamma_{out})^2 \quad (43)$$

Where  $\Delta$ , assumed small, for a Ground-Gen is:

$$\Delta = \frac{M_k g \cos(\beta + \nu_{gr})}{\frac{1}{2} \rho A C_L V_w^2 G_e^2 (\cos(\beta + \nu_{gr}) - \gamma_{out})^2} \quad (44)$$

Considering  $\nu_{gr}$  small, the efficiency due to gravity for a Ground-Gen is [3]:

$$\eta_{GG}^{gr} = 1 - \frac{3 \cos \beta - \gamma_{out}}{\cos \beta - \gamma_{out}} \sin(\beta) \frac{M_k g}{T} - 2 \nu_{gr} \frac{\sin \beta}{\cos \beta - \gamma_{out}} \quad (45)$$

### 5.3. Lumped force

The equivalent gravitational force  $F_{eq}^{gr}$ , placed at the end of the tether, is found by computing the torque around the ground station  $T_g$  due to gravitational force:

$$T_g = \int_0^{l_t} \xi \cos \beta \cdot m_t g \cos \beta d\xi = l_t \cos \beta \frac{M_t g}{2} = l_t \cos \beta \cdot F_{eq}^{gr} \quad (46)$$

The equivalent gravitational force is  $F_{eq}^{gr} = \frac{M_t g}{2}$ . This force represents one half of the gravitational force the tether has, in accord with Houska and Diehl [9].

### 5.4. Power equation considering straight tether and lumped force

By applying the lumped force (i.e. lumped mass), Eq. (42) can be re-written for an augmented mass and a straight tether:

$$\eta_{FG}^{gr} = 1 - 3 \sin \beta \cdot \frac{(M_k + \frac{M_t}{2})g}{T} \quad (47)$$

It can be shown that Eq. (42) and (47) are identical. For a Ground-Gen, the efficiency considering the lumped mass is:

$$\eta_{GG}^{gr} = 1 - \frac{3 \cos \beta - \gamma_{out}}{\cos \beta - \gamma_{out}} \sin(\beta) \frac{(M_k + \frac{M_t}{2})g}{T} \quad (48)$$

If the definition of the tether slope is substituted into Eq. (45), a difference of  $\frac{M_t g \sin \beta}{2T}$  is found. This is because, in the lumped mass case, half of the tether mass is included into the power estimation, while in the tether slope case, no tether mass is included. If the tension loss along the tether due to gravity is considered, then the results are identical. It should be noted that Eq. (40) and (43) do not assume a small tether slope, while Eq. (42), (45), (47) and (48) do.

## 6. Conclusions

Within this work, the main analytic equations that relate tether sag to power production are shown. The individual effect of the sag due to drag, centrifugal and gravitational forces on the power estimation is presented. This does not require the assumption of small tether slope at kite connection. Assuming small angles, tether drag and tether mass are lumped with the kite and the tether is considered straight. With this approach the power estimation results easier and quicker. By applying on the kite one fourth of the drag force that the full tether would have if moving at the kite speed and location, the power can be computed considering a straight tether and lumped drag. If a circular trajectory is assumed, one third of the tether mass can be lumped with the kite to estimate the power losses due to centrifugal forces. If a linear motion is considered, half of tether mass can be applied to the kite to estimate the power losses due to gravitational forces. Considering a circular trajectory, the effects of tether sag due to drag and to centrifugal forces can be superimposed. For a linear motion, the effects of tether sag due to drag and to gravity can be superimposed. The power equations given in the last subsection of the three main sections represent a tool to evaluate the influence on the power of a tether for a given system. Within the results presented in this work, the power efficiency equation due to centrifugal force and the optimum value of the opening angle, as function of a non-dimensional mass (Eq. (36) and (37)) are new important findings. A future work will investigate the validity and the implications of these equations for any elevation. Validation of these analytic equations with a numerical dynamic model including gravitational, inertia and aerodynamic forces is envisaged.

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