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Abstract.   The primary objective of this work is to demonstrate how the condition for biaxially loaded off-axis tunnelling crack growth can be reproduced in a modified laminate under uniaxial loading. More specifically, laminates with the stacking sequences $[0/\theta/0/-\theta]_s$ and $[0/\alpha/0/-\alpha]_s$ have been considered for the biaxial and uniaxial loadings, respectively. A steady-state crack growth condition is analysed for an isolated off-axis tunnelling crack. The crack tip stress state is expressed in terms of a through thickness averaged mode mixity and energy release rate. The presented framework uses an energy accounting method with the crack opening displacements extracted from a finite element analysis. Results shows that the tunnelling crack tip stress state of the modified laminate under uniaxial loading matches with the biaxially loaded laminate. With a modified off-axis lamina orientation and the additional load amplitude from the biaxially loaded laminate, a simplified uniaxial testing laminate can be designed that yields the same biaxial tunnelling crack tip conditions.

1. Introduction

Over the past few decades, fibre reinforced polymer composites (FRP) have been an excellent choice of material in several industry sectors like aerospace, automotive, marine and wind energy. This is primarily due to their high strength and stiffness properties, combined with the low density. Long aligned FRP composites are laminated materials in which the individual plies are tailored to orient in one single or multiple directions depending on the direction of loads. Laminates consisting of only 0° oriented plies are called unidirectional laminates, otherwise, when stacked up with different ply angles like ±45°, 90° etc. they are called multi-directional laminates. However, these off-axis plies which are inclined at an angle to the primary load direction are always prone to the very first mode of damage during the service [1].

A typical wind turbine blade has load carrying composite laminates in spar caps [2]. Often, these laminated composites are made from non-crimp fabrics which has secondary oriented fibre bundles with respect to the main load carrying fibres, alongside few off-axis plies in between the quasi-unidirectional plies [3]. Cracks in off-axis plies or bundles conventionally called tunnelling cracks, propagate along the direction of off-axis fibres. The multiple tunnelling cracks not only cause the initial stiffness loss but also leads to other failure modes like fibre failures at the crack tips [4] and delaminations [5] at the interface of the neighbouring load carrying unidirectional laminates. Thus, it is necessary to understand and control the formation of the first mode of damage in order to prolong the fatigue life of the composite laminate.

1.1. Background

Generally, the onset of a tunnelling crack occurs either from a flaw like void within the ply or from free edges of the test specimen. Tunnelling cracks with an off-axis angle 90°, called transverse cracks, have been first studied in the late 1970s [6, 7]. From the experimental investigations, it was found that thickness of the transverse ply largely influences the crack spacing [6, 7]. In later years, the effect of multiple cracks and the crack spacing in transverse ply has been studied by finite element models with respect to the changes in the laminate stiffness [8, 9]. In one of the study by Nakamura and Kamath [10], it was shown that the energy release rate of a thin film crack bonded to a rigid substrate and oriented perpendicular to the loading direction reaches a steady state value (remains constant) when the crack length is approximately twice the ply thickness.

For tunnelling cracks propagating in steady state condition, Beuth [11] and Ho and Suo [12] presented an energy accounting method to calculate the energy release rate. Moreover, the cracking condition in transverse plies is pure Mode I [12]. In such conditions, the local energy release along the crack front remains constant and is equal to Mode I fracture energy, $G = G_{Ic}$. Further, the steady-state energy release rate found to be proportional to square of the applied stress and thickness of the lamina [12].

In case of off-axis angles other than 90°, the cracking occurs under mixed mode condition i.e., in both Mode I and Mode II. Besides, the fracture energy is found to depend on the mode mixity [14], defined as the phase angle of the stress intensity factors.

An interesting observation by Nakamura and Kamath [10] was that the far field stress state from the crack tip is found to be independent of the actual crack front shape. Therefore, the energy accounting method [12] which uses the stress state far ahead of the crack tip and crack opening displacements far behind the crack tip will therefore not depend on the actual crack front shape. This approach for calculating energy release rate was further extended to off-axis plies by Quaresimin et al. [15]. Following the studies of Quaresimin et al. [15] and Mikkelsen et al. [16], the present study uses this approach to calculate the average energy release rate.

Existing biaxial test methods are classified based on the loading system [17]. Briefly, these methods consists of either applying in-plane loads along the two perpendicular arms of a cruciform specimen or tension-torsion loads to tubular specimens. However, a good testing greatly relies on a good design of biaxial specimen. Additionally, to overcome the limitations associated with the biaxial testing [21], an elegant approach is demonstrated in terms of laminate design such that the same crack tip stress state is mimicked under uniaxial loading.

1.2. Motivation

The stress state in many composite structures is multiaxial. Therefore, a first step moving from a uniaxial fatigue testing is to do a fatigue testing under biaxial loading and there have been a number of testing strategies ([18, 19, 20]). Nevertheless, the fatigue damage initiation for many fibre composites observed to be governed by the evolution of the tunnelling cracks. The tunnelling crack growth condition during a biaxial fatigue testing can be reproduced using a uniaxial fatigue testing on a laminate with a modified layup. Thereby, a complicated biaxial fatigue testing aiming for the characterization of the tunnelling cracks damage can be replaced by a much simpler uniaxial fatigue testing series.

The present study illustrates a novel and scaled down approach of analysing the biaxially loaded off-axis tunnelling cracks under uniaxial loading. For a non-interacting crack under steady-state propagation, the specific mode mixity of biaxially loaded crack tip can be obtained by changing the orientation of the off-axis lamina under uniaxial loading. Furthermore, the uniaxial load applied on the new laminate can be specified to acquire the same energy release at the crack front. A specific biaxial loading scenario is considered on the laminate layup $[0/\theta/0/-\theta]_s$ to determine the crack tip stress state. The aim is to get same crack tip conditions

in the modified laminate $[0/\alpha/0/-\alpha]_s$ subjected to uniaxial loading.

2. Theory of tunnelling cracks

2.1. Cracks in orthotropic materials

Considering the linear elasticity in orthotropic materials, the relationship between Mode I and Mode II energy release rates and the corresponding stress intensity factors is given by

$$\begin{Bmatrix} G_I \\ G_{II} \end{Bmatrix} = \left(\frac{1+\rho}{2E_{11}E_{22}} \right)^{1/2} \begin{Bmatrix} \lambda^{-1/4} K_I^2 \\ \lambda^{1/4} K_{II}^2 \end{Bmatrix} \tag{1}$$

where G_I and G_{II} are the Mode I and Mode II energy release rates, K_I and K_{II} are the Mode I and Mode II stress intensity factors. The two non-dimensional parameters, λ and ρ, are related to the engineering elastic constants ([22, 23]) (see Table 1) as

$$\lambda = \frac{E_{22}}{E_{11}}, \quad \rho = \frac{\sqrt{E_{11}E_{22}}}{2G_{12}} - \sqrt{\nu_{12}\nu_{21}} \tag{2}$$

Using the stress intensity factors at the crack tip for a given loading condition, the linear elastic fracture mechanics mode mixity is defined as ([13])

$$\psi = \arctan\left(\frac{K_{II}}{K_I} \right) \tag{3}$$

In general for off-axis tunnelling cracks, the mode mixity at the crack front varies along the position of the crack front. Furthermore, the fracture energy which is dependent on the mode mixity, $G_c = G_c(\psi)$, also varies along the crack front. However, in the event of steady-state crack propagation, the crack front will realign its shape such that for all points across the front, the local energy release rate is identical to the fracture energy of the associated local mode mixity. Moreover, the energy dissipation at the crack front that of the steady-state crack remains unchanged, even for an unknown crack front shape. As the crack front shape is not known, averaged properties (averaged over lamina thickness) are used to quantify the stress state of the crack front, defined similarly to eqs. (1) and (4). The average mode mixity then becomes

$$\bar{\psi} = \arctan\left(\frac{\bar{K}_{II}}{\bar{K}_I} \right) = \arctan\left(\lambda^{-1/4} \sqrt{\frac{\bar{G}_{II}}{\bar{G}_I}} \right) \tag{4}$$

here $\bar{G}_I$ and $\bar{G}_{II}$ represent average Mode I and Mode II energy release rates respectively, calculated by the formulae presented in the next section.

2.2. Calculation of the energy release rate

To calculate the steady-state energy release rate of an off-axis tunnelling crack, we use the energy accounting method ([15, 16]) as shown in Fig. 1. For an isolated crack in the thick lamina of height $2h$, the average steady-state energy release rate is given by the expression

$$\bar{G}_{ss} = \frac{1}{2h} \int_0^h \sigma_{22}(x_3)\,\delta_n(x_3)\,dz + \frac{1}{2h} \int_0^h \sigma_{12}(x_3)\,\delta_t(x_3)\,dz \tag{5}$$

here x_3 is through-the-thickness coordinate and varies from 0 to the ply thickness h. The stress components σ_{22} and σ_{12} are the normal and shear stresses in the cracked lamina far ahead of the crack tip, whereas δ_n and δ_t are the normal and tangential crack opening displacements far behind the crack tip. In eq. (6) the first integral expression is assumed as Mode I energy release
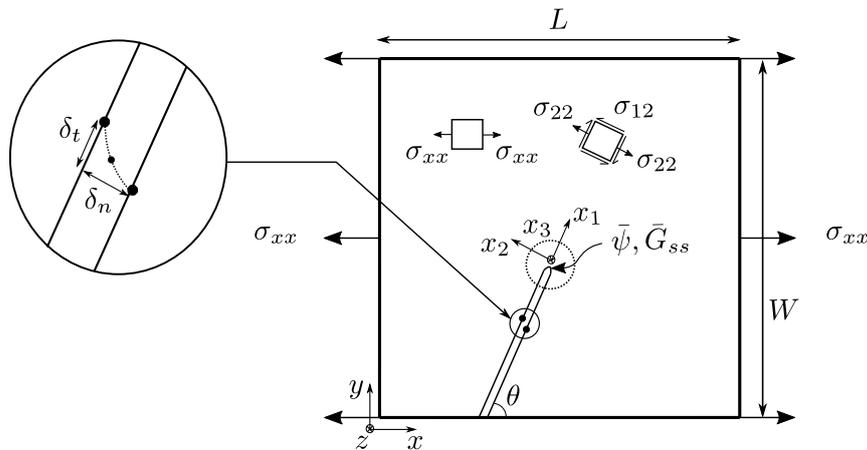
Figure 1: Illustration of the energy accounting method for an off-axis tunnelling crack from the crack normal opening and tangential opening displacements.

rate while the second expression is assumed to be the Mode II energy release rate. Additionally, because of the constant (far field) stress components σ_{22} and σ_{12} along the thickness direction of the off-axis lamina, the eq. (6) can be rewritten as

$$\bar{G}_I = \frac{1}{2}\,\sigma_{22}\,\bar{\delta}_n, \quad \bar{G}_{II} = \frac{1}{2}\,\sigma_{12}\,\bar{\delta}_t \tag{6}$$

where $\bar{\delta}_n$ and $\bar{\delta}_t$ represents average normal and tangential crack opening displacements respectively,

$$\bar{\delta}_n = \frac{1}{h}\int_0^h \delta_n(x_3)\,dz, \quad \bar{\delta}_t = \frac{1}{h}\int_0^h \delta_t(x_3)\,dz \tag{7}$$

therefore, the average total energy release rate is now given by

$$\bar{G}_{ss} = \bar{G}_I + \bar{G}_{II} \tag{8}$$

3. Problem definition and modelling

3.1. Geometry and boundary conditions

Tunnelling crack growth in the present study is analysed in the context of linear elastic fracture mechanics. Symmetric and balanced laminates with the layup a) $[0/\theta/0/-\theta]_s$ for biaxial loading and b) $[0/\alpha/0/-\alpha]_s$ for uniaxial loading has been considered for the steady state crack growth analysis. In case of biaxial loading, the laminate is a square plate with a side length, L_s, while in uniaxial loading the laminate is a rectangular plate with a length, L_r, and a width, W_r, as shown in Fig. 2. Both the plates are assumed to have identical total thickness, $2H$, and individual ply thickness, h. Here, the tunnelling crack is assumed to propagate along the fibre orientation of the thick central lamina (thickness $2h$) in the respective laminates. We operate on two Cartesian coordinate systems, a global coordinate system represented by the axes (x, y, z) along the two edges and thickness of the laminate respectively and a local coordinate system on ply level denoted by the axes (x_1, x_2, x_3), representing fibre direction in the plane, normal to the fibre direction and the out-of-plane thickness direction.

Due to symmetry, only lower half of the laminates (H) are modelled. Boundary conditions include the symmetric conditions applied on the mid-plane of the laminates along the z-direction i.e., all the nodes positioned at $z = H$ are constrained in the z-direction, $u_3^s = u_3^r = 0$. For biaxial
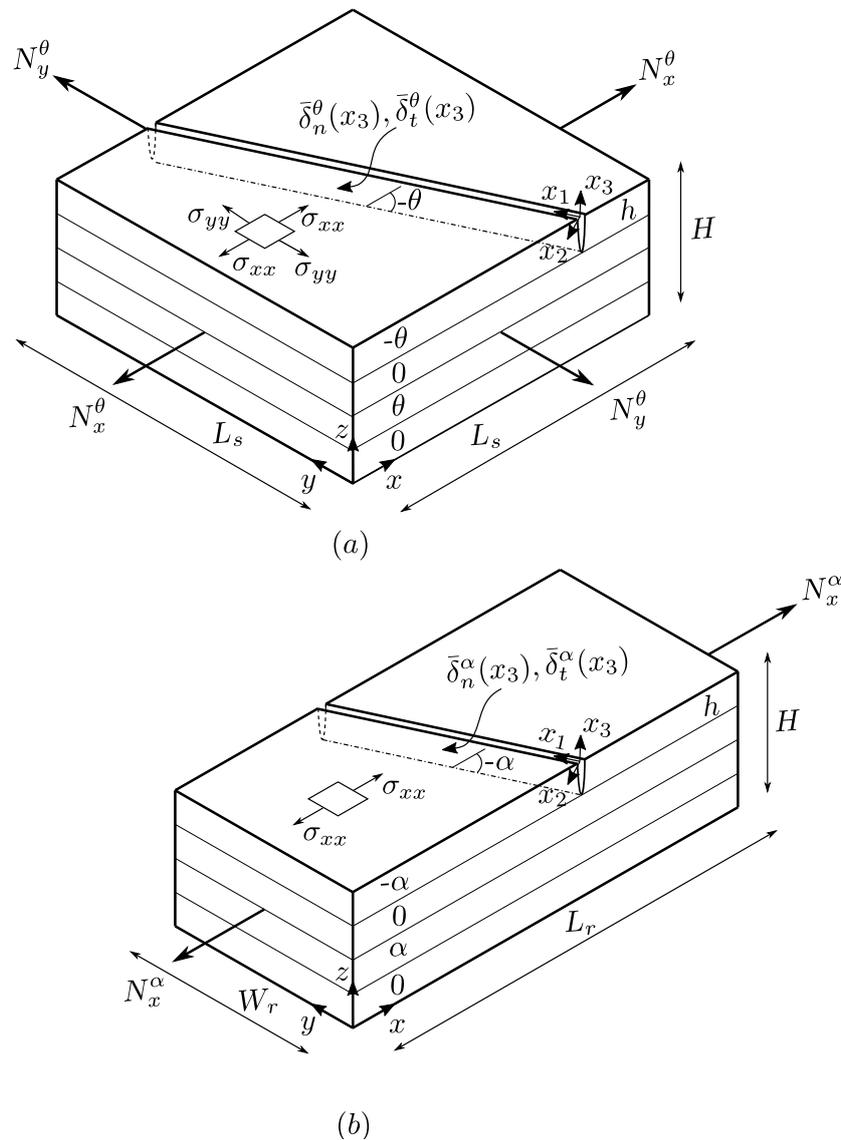
(a)

(b)

Figure 2: Geometries and boundary conditions of the symmetric laminate models along with the full-width cracks shown for a) biaxial loading of $[0/\theta/0/-\theta]_s$ and b) uniaxial loading of $[0/\alpha/0/-\alpha]_s$. The average crack opening displacements are extracted at the centre of the full-width crack for the respective model.

loading, the displacements were applied to the edges of the plate, along x and y-directions. This means, the nodes located at $x = 0$ and $x = L_s$ are subjected to prescribed displacements, $u_1 = \mp u_x^b$ with u_2 unconstrained, likewise, the nodes at $y = 0$ and $y = L_s$ are subjected to displacements in the $y-$direction, $u_2 = \mp u_y^b$ with u_1 left free. While for uniaxial loading, the displacements are applied to the nodes located $x = 0$ and $x = L_r$ as $u_1 = \mp u_x^u$ with u_2 left free.

3.2. Finite element modelling
Following Mikkelsen et al. [16], a non-crack tip model denoted as a full-width crack model is used in the present study as shown in Fig. 2. Each of the laminate model has only one full-width crack to analyse isolated and non-interacting crack growth condition.

For the two loading scenarios, the average normal and tangential crack opening displacements

are extracted at the center of the respective laminate model and thereby the average mode mixities and the average energy release rates are calculated. The dimensions of the finite element model used for biaxial loading are $L_s \times L_s = 180h \times 180h$, whereas for uniaxial loading, $L_r \times W_r = 140h \times 40h$. Both the models use 8 noded brick elements (C3D8R) with a refined mesh size of $0.04h \times 0.04h \times 0.01h$ focussed at the center of the full-width crack, similar to what is used in Mikkelsen et al. [16]. The biaxial model consists of 6.5 million elements while the uniaxial model has 3.5 million elements. Since the relative difference between the stress, σ_{xx} (σ_{22} and σ_{12}), far away from both the edges and the crack of the finite element and the stress calculated from classical laminate theory is less than 1%, stresses from the latter are used in the energy accounting eq. (7). Linear analysis on the finite element models were performed using a commercial finite element code (Abaqus standard, 2017).

4. Results
4.1. Material properties
Glass fibre composite made up of unidirectional E-glass and epoxy resin is analysed with the elastic properties given in Table 1. The laminate is modelled as a homogeneous orthotropic material with each ply possessing the given elastic properties.

Table 1: Elastic constants of unidirectional E-glass/epoxy composite

Property	Value
Longitudinal modulus, E_{11}	40.8 (GPa)
Transverse modulus, E_{22}	12.25 (GPa)
In-plane Poisson's ratio, ν_{12}	0.27 (-)
In-plane shear modulus, G_{12}	4.62 (GPa)
Out-of-plane Poisson's ratio, ν_{23}	0.40 (-)

[a] Assumption of transverse isotropy:
  $E_{22} = E_{33}$, $\nu_{12} = \nu_{13}$, $G_{12} = G_{13}$ and
  $G_{23} = E_{22}/(2(1 + \nu_{23}))$.
[b] $\nu_{21} = (E_{22}/E_{11})\nu_{12}$.
[c] $\lambda = 0.30$, $\rho = 2.25$.

Following Mikkelsen et al. [16], the finite element solutions of the biaxial and uniaxial loadings are presented in non-dimensional form. The results include the crack opening displacements, δ_n, δ_n, the local stress components σ_{22}, σ_{12}, and the energy release rates, G_I and G_{II}. Parameters used for normalization are cracked ply thickness, $2h$, global stress, σ_{xx}, and global strain, ε_{xx}, in the $x-$ direction for each loading case. defined as follows:

$$\bar{\delta}_n^* = \frac{\bar{\delta}_n}{\varepsilon_{xx}\,2h}; \quad \bar{\delta}_t^* = \frac{\bar{\delta}_t}{\varepsilon_{xx}\,2h}; \quad \sigma_{22}^* = \frac{\sigma_{22}}{\sigma_{xx}}; \quad \sigma_{12}^* = \frac{\sigma_{12}}{\sigma_{xx}}; \quad \bar{G}^* = \frac{\bar{G}}{\sigma_{xx}\,\varepsilon_{xx}\,2h} \tag{9}$$

4.2. Biaxial loading of $[0/\theta/0/-\theta]_s$
With the aim to analyse the effect of biaxial loads on the steady-state crack propagation, the average mode mixity and the average energy release rate was calculated in $-\theta$ layer of the layup, $[0/\theta/0/-\theta]_s$, under investigation. For this, we define load biaxiality ratio as transverse load to horizontal load, N_y^θ/N_x^θ, acting on the laminate. Additionally, positive (tensile) loads are investigated with the condition: $N_y^\theta < N_x^\theta$. In the present study, an example of biaxial loading case with specific N_y^θ/N_x^θ ratio is simulated on the laminate as:
Biaxially loaded condition: $\theta = 30°$ in $[0/\theta/0/\theta]_s$ and $N_y^{30°}/N_x^{30°} = 0.12$

The normalized average crack opening displacements and the stress components are found to be

$$\delta_n^* = 0.4980, \quad \delta_t^* = 0.6985$$
$$\sigma_{22}^* = 0.1522, \quad \sigma_{12}^* = 0.1673 \tag{10}$$

now substituting above values in eq. (7) gives an estimate for the average Mode I and Mode II energy release rates as

$$\bar{G}_I^* = \frac{1}{2}\,\sigma_{22}^*\,\bar{\delta}_n^* = \frac{1}{2}\cdot 0.1522 \cdot 0.4980 = 0.0379 \tag{11}$$

$$\bar{G}_{II}^* = \frac{1}{2}\,\sigma_{12}^*\,\bar{\delta}_t^* = \frac{1}{2}\cdot 0.1673 \cdot 0.6985 = 0.0584 \tag{12}$$

when substituted the energy values into eqs. (5) and (9) gives the average mode mixity and the average total energy release rate for as

$$\bar{\psi}^\theta = 59.2° \text{ and } \bar{G}_{ss}^\theta = 0.21 \text{ N/mm (absolute)} \tag{13}$$

4.3. Uniaxial loading of $[0/\alpha/0/-\alpha]_s$
To match the average mode mixity of approx. 59° at the crack front of an off-axis tunnelling crack under uniaxial loading, lamina orientation α in $[0/\alpha/0/-\alpha]_s$ was altered [24]. Further, the uniaxial load N_x^θ applied to the laminate is varied in order to attain the same average energy release rate at the steady-state crack front [24].
Uniaxially loaded condition: $\alpha = 50°$ in $[0/\alpha/0/-\alpha]_s$ and $N_x^{50°}/N_x^{30°} = 0.57$

For this case, the average crack opening displacements and the stress components and in normalized form are

$$\delta_n^* = 0.6471, \quad \delta_t^* = 0.9530$$
$$\sigma_{22}^* = 0.4425, \quad \sigma_{12}^* = 0.4706 \tag{14}$$

using eq. (7) again, the Mode I and Mode II energy release rates are calculated as

$$\bar{G}_I^* = \frac{1}{2}\,\sigma_{22}^*\,\bar{\delta}_n^* = \frac{1}{2}\cdot 0.4425 \cdot 0.6471 = 0.1431 \tag{15}$$

$$\bar{G}_{II}^* = \frac{1}{2}\,\sigma_{12}^*\,\bar{\delta}_t^* = \frac{1}{2}\cdot 0.4706 \cdot 0.9530 = 0.2242 \tag{16}$$

and thus from eqs. (5) and (9), the average mode mixity and the average energy release rate are estimated as

$$\bar{\psi}^\alpha = 59.4° \text{ and } \bar{G}_{ss}^\alpha = 0.21 \text{ N/mm (absolute)} \tag{17}$$

Thus, it can be seen that the average mode mixity and the average energy release rate of a steady-state tunnelling from the biaxial loading case can be recreated in the uniaxial loading, $\bar{\psi}^\theta = \bar{\psi}^\alpha$ (with less than 1% discrepancy) and $\bar{G}_{ss}^\theta = \bar{G}_{ss}^\alpha$.

5. Summary
Using three dimensional finite element analyses, steady-state crack growth condition of an off-axis tunnelling crack is analysed. The energy accounting method is applied to the isolated cracks in two different laminate layups, $[0/\theta/0/-\theta]_s$ and $[0/\alpha/0/-\alpha]_s$, subjected to the biaxial and uniaxial loading respectively. It is shown that the average mode mixity and the average energy release rate of the biaxially loaded tunnelling crack tip can be attained on the modified laminate in the uniaxial loading. By altering the orientation angle of the off-axis lamina and

the uniaxial load amplitude, mode mixities and the energy release rates at the crack tip can be varied accordingly.

The approach of performing uniaxial testing to understand tunnelling cracks under biaxial loads may be seen as an alternative to the complicated biaxial testing.

Acknowledgements

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