Railway Substructure System Based on Asphalt

Bose, Tulika

Publication date: 2020

Document Version
Publisher's PDF, also known as Version of record

Citation (APA):
Bose, T. (2020). Railway Substructure System Based on Asphalt. Technical University of Denmark, Department of Civil Engineering.

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.
This study focused on analyzing the mechanical behaviour of a ballastless track-type based on asphalt concrete. A full-scale laboratory prototype was built and tested under different types of vertical loads. Sleepers were sequentially loaded to simulate train passages at different speeds and axle load levels. The test results showed that ballastless asphalt tracks demonstrated time-dependent and nonlinear behaviour. Moreover, measured substructure responses were of very low magnitude, indicating little or no damage under service conditions. Further, a comprehensive 3D FEM model of the prototype was built, which incorporated inertial effects, material nonlinearity, and time-temperature dependency. Separate element tests were carried out to obtain the majority of model parameters. The model predictions showed a good agreement with the experimental measurements. Additionally, an analytical model was developed to investigate track responses during train braking. It was found that for ballastless asphalt tracks, if sliding resistance at a sleeper base was governed solely by frictional mechanism, then a non-standard or heavier-sleeper type was required.

Tulika Bose
Railway Substructure System Based on Asphalt
Railway Substructure System Based on Asphalt

Tulika Bose

PhD Thesis
Department of Civil Engineering
Technical University of Denmark
February 2020
Railway Substructure System Based On Asphalt

Tulika Bose

February 2020

Supervisors:
Associate Professor Varvara Zania,
Department of Civil Engineering, Technical University of Denmark

Associate Professor Eyal Levenberg,
Department of Civil Engineering, Technical University of Denmark

Copyright: Reproduction of this publication in whole or in part must include the customary bibliographic citation, including author attribution, report title, etc.

Cover photo: [Tekst]
Published by: DTU, Department of Civil Engineering,
Brovej, Building 118, 2800 Kgs. Lyngby Denmark
www.byg.dtu.dk

ISSN: [0000-0000] (electronic version)
ISBN: [000-00-0000-000-0] (electronic version)

ISSN: [0000-0000] (printed version)
ISBN: [000-00-0000-000-0] (printed version)
Preface

This thesis is submitted as partial fulfilment of the requirements for the PhD degree at the Technical University of Denmark (DTU). The study has been undertaken at the Department of Civil Engineering, DTU between September 2016 and February 2020 under the joint supervision of Associate Professor Varvara Zania and Associate Professor Eyal Levenberg. This study has been funded by Innovations Fund Denmark (Innovationsfonden) and is part of the project “Roads2Rails: Innovative and cost-effective asphalt based construction system” (Grand Solutions 5156-00006B).

This is a paper-based thesis that consists of two parts: Part A and Part B. Part A introduces the motivation, background and research objectives of the study. It also includes an extended abstract of the papers, along with supplementary information and conclusions from the study. Part B is a collection of five scientific papers (four journal papers and one conference paper) that were written during the PhD study.

Tulika Bose
February, 2020
Acknowledgements

This study was funded by Innovations Fund Denmark (Innovationsfonden) and is part of the project “Roads2Rails: Innovative and cost-effective asphalt based construction system” (Grand Solutions 5156-00006B). This PhD study would not have been achieved without the help and support of a number of people, and I would like to express my gratitude to them.

I am deeply grateful to my supervisors Associate Professor Eyal Levenberg, and Associate professor Varvara Zania. It was a great learning experience for me to work with both of them, and I thank them for their extensive supervision, valuable academic guidance and for always inspiring and motivating me. I sincerely thank them for their active involvement in all stages of this study.

I would like to express my gratitude to all the project members for their valuable contribution and support. Special thanks to Ole Grann Andersen, Bjarne Schmidt, René Xavier Victor Fongemie, Peter Juel Jensen and Russel da Silva. In particular, I would like to thank Jens Borgmann, who was responsible for the construction of the majority of the experimental setup.

I am grateful to several laboratory technicians at the Technical University of Denmark, without whom this research would not have been possible. This is specially extended to Henrik Børglum, Johan Bartholdy, Christian Peter Rasmussen, Troels V. Kristensen, Ian Rasmussen and John C. Troelsen. I would like to thank laboratory managers Kristian Riskær Povlsen and Christian Berggreen for their support and for providing laboratory equipment.

I would like to thank Shafiqur Rahman and Abubeker Ahmed from the Swedish National Road and Transport Research Institute for their valuable contributions in the experimental investigations. It was a very nice experience collaborating with them. I would also like to thank Martin Andersen from VBM Laboratory and Jens Pedersen from SWECO for their contribution in experimental investigations. I would further like to thank Nawrat Wojciech from PCM RAIL.ONE AG for his help.

I would like to thank all my colleagues in Building 119, Department of Civil Engineering for a great work environment and making it a joy to work at DTU. Thank you Tobias Orlander and Ole Grann Andersen for helping me with the Danish abstract. I would like to thank all my friends. Thank you Lin Alexandra Mortensgaard and Benthe Hansen, for being such lovely and understanding people to live with.

Finally, I would like to thank the most important people in my life, my husband Soumya Bandyopadhyay, my parents Kishore Bose and Dalia Bose, my sister Chandrima Bose and my lovely niece Harshita Bhuyan. Thank you for being so patient and supportive, and always motivating me.
Abstract

Over the past few decades, the rail industry has been aiming to provide an infrastructure capable of accommodating faster, heavier and more frequent trains along with lower tolerance for maintenance-related delays and no compromise on travel safety. Under these conditions, conventional ballasted tracks are approaching their performance limits – mainly due to ballast breakage and fouling that undermine track stability and require ever more frequent maintenance. This state of affairs has led to the development of slab stack technology – a ballastless track infrastructure based on Portland cement concrete. The slab track solution was able to increase track stability, reduce maintenance frequency, and essentially provide for all above-mentioned requirements. However, the initial construction costs are very high, and maintenance activities (when necessary) are both expensive and time-consuming. Slab track technology is also known to amplify noise levels.

In this context, the current study focused on another type of ballastless track technology - based on asphalt concrete. While asphalt concrete is widely used for the construction of various transport infrastructures, its use within the rail industry has been very limited with only few reported implementations of ballastless asphalt tracks - mainly in tunnels. Although the solution seems workable and promising, data from the existing tracks are not available (at least not publically) and related scientific literature on the idea is very limited. Therefore, the overall aim of this work was to contribute to the understanding of the behaviour of ballastless asphalt tracks – with emphasis on mechanical responses to train-like loading. This goal was pursued by combining a full-scale experimental investigation with numerical and analytical model developments.

Experimental investigation was carried out by constructing and mechanically interrogating a full-scale mockup. Part of the mockup was built inside a steel container; it was further constructed, instrumented, and tested in a laboratory environment. It consisted of three concrete sleepers resting on asphalt concrete layer underlain by an unbound granular layer, followed by a mat to represent subbase and subgrade. The mechanical interrogation was carried out under different types of vertical loads such as ramp, pulse, and sinusoidal. Moreover, passages of a train with maximum axle loads of 200 kN and speeds of up to 200 km/h were simulated by sequentially loading the three sleepers. Different types of mechanical response were measured in the mockup, such as vertical stresses, horizontal strains, vertical accelerations, and relative displacements between components. It was observed that the ballastless asphalt track exhibited time-dependent and non-linear behaviour. In general, the measured responses within the asphalt track were of very low magnitude, indicating little or no damage under the applied loads.

A three-dimensional finite element model of the mockup was developed, including all major track components and trackbed layers. The non-linearity of the unbound granular layer was considered through stress-dependent elasticity. The time-temperature dependency of asphalt layer was incorporated using linear viscoelasticity theory assuming thermorheological simplicity. The properties of these two layers were obtained from separate laboratory element tests. Implicit dynamic analysis was carried out under simulated train passages, and calculated model responses were validated by the ability to predict response measurements in mockup.
While the experimental investigation and the numerical modelling focused on analyzing responses due to vertical loads, attention was also given in this study to the evaluation of track responses due to longitudinal loads. This was deemed essential - considering that sleepers in ballastless asphalt tracks rest on the top of the pavement, often without any physical anchorage or side support (unlike traditional ballasted tracks where sleepers have horizontal support from crib and shoulder ballast). In this context, a two-dimensional model was developed within an analytical framework to analyze track responses under longitudinal loads induced by train braking. All equations were purposefully developed in closed-form to serve as an easily implementable first-order engineering tool. The model clearly demonstrated that if longitudinal train loads were only intercepted by means of slipper-asphalt friction then heavier sleeper types would be needed to ensure longitudinal track stability under train braking events.

Though this study focused primarily on analyzing mechanical responses in ballastless asphalt tracks, some initial research effort was expended on developing an approach that can be useful for design. Specifically, work was done on a priori estimation of track modulus - a basic engineering parameter needed in new track design, as well as in condition evaluation of existing tracks. The concept of track modulus is closely associated with the idealization, commonly employed for railway tracks, of rails as infinite beams resting on a continuous spring support. A semi-analytical method was suggested for calculating track modulus based on elasticity solutions, and a parametric investigation was carried out to demonstrate the sensitivity to different input parameters. By means of the elastic-viscoelastic correspondence principle this work can be extended to address time-dependent track support. Thus, the track modulus concept can be extended to ballastless asphalt tracks - further promoting industry acceptance of the idea.
Resumé

Gennem de sidste årtier har jernbaneindustrien oplevet væsentligt øgede krav til hastighed, lastkapacitet og frekvens, samt mindre tolerance for forsinkelser og en nul-tolerance med hensyn til sikkerhed. De øgede krav har resulteret i, at konventionel skærveballast til jernbanespor har nået grænserne mht. nedbrydningshastighed, hvilket kompromitterer den samlede sporstabilitet og øger behovet for vedligeholdelse markant. Derfor er udvikling af løsninger med ballastfri jernbanespor introduceret forskellige stedet i verden, hvor hovedparten indtil nu er konstrueret med brug af betonplerader, hvorpå skinnerne fastholdes. Løsninger med betonplader har en positiv effekt med hensyn til øget sporstabilitet og et reduceret behov for vedligehold, men dette skal sættes i forhold til store etableringsomkostninger, samt større omkostninger og tidforbrug i forbindelse med nødvendigt vedligehold.

I denne sammenhæng har fokus i dette studie været på brug af asfalt som en ballastfri løsning til jernbanespor, som et alternativ til beton. Forskellige typer af infrastruktur til transport har en udbredt anvendelse af asfalttopbygning. For jernbaner gælder dette dog nærmest udelukkende for tunneller i den tyske jernbaneindustri, hvor data vedrørende drift etc. er utilgængelig for offentligheden. Ligeledes er antallet af videnskabelige studier omhandlende asfalt som løsning til ballastfri jernbanekonstruktion meget begrænset. Gennem eksperimenter og sammenhørende modelleringer er formålet med dette studie derfor at bidrage til den videnskabelige forståelse af strukturelle egenskaber for ballastfri jernbane sporkonstruktioner med asfalt.


En 3D time-domain finite element model af den anvendte mockup, indeholdende alle hovedelementer af strukturelle lag, blev udviklet, hvor ikke-lineariteten observeret i underballasten blev inkorporeret gennem spændingsaflæggelse elasticitet og tids/temperatur afhængighed af asfaltlaget gennem lineær viskoelastisk teori. Egenskaberne af underballasten samt asfalten blev bestemt i laboratoriet. For simuleringen af det forbi passerende tog blev der foretaget dynamiske analyser og efterfølgende blev model-respons valideret ved sammenlignet med målingerne på den anvendte mockup. Dette blev anset for et nødvendigt første skridt inden videre skridt mod brug på et allerede etableret jernbanespor.

Da de eksperimentelle forsøg og den numeriske modellering har fokuseret på analyser af den mekanisk respons fra lodret belastning, blev et sidestudie etableret til evaluering af respons fra langsgående vandrette belastninger. Dette sidestudie anses for nødvendigt, da konventionel
skræveballast sikrer svellerne mod flytning i flere retninger, hvilke ikke er tilfældet for en ballastfri opbygning, hvor svellerne er placeret direkte på et asfaltlag uden mekanisk fastgørelse. Modstand mod vandret flytning bliver derfor udelukkende modvirket af egenvægten i grænseflade mellem svelle og asfaltlag og dette er måske ikke nødvendigvis tilstrækkeligt til at sikre stabiliteten.

List of scientific publications

This thesis is based on the following publications:


Additional publications that were part of the PhD study but not included in the thesis:


# Table of Contents

Preface .................................................................................................................. iii

Acknowledgements ................................................................................................. iv

Abstract ................................................................................................................... v

Resumé ...................................................................................................................... vii

List of scientific publications .................................................................................. ix

Table of Contents ...................................................................................................... x

Part A: Introduction and Summary ......................................................................... 1

Chapter 1 Introduction ............................................................................................... 2
  1.1. Background and motivation ........................................................................... 2
  1.2. State-of-the-art and research challenges ......................................................... 4
  1.3. Thesis objectives ............................................................................................ 9
  1.4. Thesis outline ................................................................................................ 10

Chapter 2 Synopsis .................................................................................................. 11
  2.1. Overview ....................................................................................................... 11
  2.2. Experimental investigation ........................................................................... 11
  2.3. Numerical modelling and validation ............................................................... 23
  2.4. Longitudinal track responses ........................................................................ 32
  2.5. A priori estimation of track modulus ............................................................. 35

Chapter 3 Summary and conclusions ...................................................................... 39
  3.1. Summary and findings ................................................................................... 39
  3.2. Conclusions .................................................................................................. 41
  3.3. Future work ................................................................................................... 42
  3.4. Thesis contributions ...................................................................................... 43

References ................................................................................................................ 44

Appendix A: Additional test results ....................................................................... 50
  A.1 Ramp load ..................................................................................................... 51
  A.2 Pulse load ..................................................................................................... 53
  A.3 Simulated train passage ................................................................................... 55
    Axle load = 120 kN, Speed = 120 km/h ........................................................... 55
    Axle load = 200 kN, Speed = 120 km/h ........................................................... 57
    Axle load = 120 kN, Speed = 200 km/h ........................................................... 59
    Axle load = 200 kN, Speed = 200 km/h ........................................................... 61
Part B: Appended papers ........................................................................................................... 63


Part A: Introduction and Summary
1.1 Background and motivation

Chapter 1

Introduction

This chapter is based on (includes reuse of text) the following scientific articles:


1.1. Background and motivation

Railway systems are a significant part of the modern transportation network that transports large amounts of goods and commuters between cities, ports, in a relatively short period. The railway track structure can be divided into two parts, superstructure and the substructure. Track superstructure consists of an assembly of steel rails, sleepers (wooden or concrete) and fastening system. Together, these components react to transfer the train load to the track substructure below. Ballasted tracks are the most traditional type of track substructure system consisting of granular ballast, subballast and subgrade. Some of the primary functions of the ballast include (Li et al., 2002):

i. Providing adequate vertical, lateral and longitudinal stability to the sleepers
ii. Maintaining the track alignment and accommodating repair works
iii. Transmitting traffic-induced stresses to the subgrade below at a reduced and acceptable stress level
iv. Contributing to the overall track resilience
v. Providing sufficient permeability to allow for track drainage

Over time, under repeated loading cycles, the ballast accumulates plastic deformations, undergoes breakage and progressively becomes fouled, i.e., voids are contaminated with fine particles generated from ballast breakage, upward subsoil pumping, cargo trains and dust. This results in reduced shear strength of particles, impaired track drainage, differential track settlement, geometry defects; eventually leads to an overall degradation in track stability (Sussmann et al.,
2012; Indraratna et al., 2011; Indraratna et al., 2016; Tennakoon et al., 2012). Maintenance is therefore required to sustain the desired safety level, design speed and passenger comfort. In severe cases, complete replacement of fouled ballast is needed (Indraratna et al., 2016).

Over past decades, the railway industry is required to cope with demands for higher train speeds, increased train frequencies, expansion of rail network and a robust infrastructure to support these demands. However meeting these demands on conventional tracks further exacerbates the problem of ballast breakage and ballast fouling (Sun et al., 2014; Indraratna et al., 2016), leading to frequent and costly maintenance works; disrupting traffic and hindering service. Additionally, ballasted tracks in high-speed networks also suffer from ballast particles flying out and churning at high speeds (Quinn et al., 2010).

Consequently, in past decades, there has been the emergence of ballastless tracks based on Portland cement concrete, commonly known as slab tracks. Slab tracks are known to improve track stability and load distribution, reduce maintenance needs and provide longer service life (Esmaeili et al., 2014; Esveld 2001; Esvel et al., 2003; Gautier 2015; Michas 2012). Moreover, the issues related to flying ballast particles or churning at high speeds are eliminated. However, the initial cost of constructing concrete slab tracks is much higher compared to conventional tracks (Esveld 2003; Gautier 2015; Michas 2012). Additionally, they are reported to generate higher noise levels (Poisson 2015). Further, repair and maintenance operations (when needed), are expensive and requires long track possessions times.

In this context, recent attention has been given to developing another type of ballastless track - based on asphalt concrete (referred to as asphalt in the rest of the thesis). Asphalt is a well-researched material that is extensively used for construction purposes within the transport infrastructure (e.g., roads, airports, bridges) but its application in the railway sector has so far been limited. Asphalt has been primarily used in traditional tracks as a substitute of granular materials in the subballast layer, also commonly known as underlayment system (Teixeira et al., 2006; Teixeira et al., 2010; Di Mino et al., 2012; Rose et al., 2000; Rose et al., 2010; EAPA 2003; Esmaeili et al., 2014). In slab tracks, it has been used as a support layer to the concrete slab and as a waterproofing layer on the surface (Yang et al., 2015; Li et al., 2016; Liu et al., 2018; Liu et al., 2019; Fang et al., 2011). Ballastless tracks built with asphalt as the main load-bearing layer (commonly known as overlayment system) is relatively recent and, very implementations have been identified so far.

The potential advantages of this system compared to ballasted tracks include (EAPA 2003, Rose et al., 2000):

i. Positive contribution to the bearing capacity of the structure  
ii. Improved load distribution and higher tolerances to minor weak spots  
iii. Reduced water infiltration to the substructure layers below; hence, adverse effects associated with moisture retention in subsoil can be reduced  
iv. Elimination of issues related to ballast breakage and generation of fines, ballast flight, and dusty environment  
v. Reduce maintenance needs and support a longer service life.

The potential advantages of this system compared to concrete slab tracks include (EAPA 2003):

i. Lower noise and vibration levels  
ii. Lower construction cost
iii. Construction without need for any joints  
iv. Possibility to accommodate minor repairs and corrections by milling off or paving new layers  
v. Lower track possession time for maintenance works as the track can be operational soon after cooling

Ballastless asphalt tracks present a potential for use in railway track infrastructure. Till date, implementations of these track types are few in number. Alongside there have been limited scientific studies dealing on this topic. Consequently, this study aims to contribute to the understanding of the mechanical behaviour of ballastless asphalt tracks. This is sought by combing full-scale laboratory investigations and with numerical and analytical model developments.

1.2. State-of-the-art and research challenges

1.2.1. Project implementations

The typical structure of a ballastless asphalt track is shown schematically in Figure 1.1 (DS / EN 16432-2: 2017). From top-down it consists of: (i) rails fastened to prefabricated concrete sleepers or slabs, (ii) an asphalt pavement, (iii) aggregate base layer (unbound, hydraulically bound or bituminous bound), (iv) subbase or frost protection layer, and (v) subgrade. This arrangement is structurally quite similar to highway asphalt pavements, the significant difference lies in the load distribution. Unlike in highways, where the vehicular loads are applied directly to the asphalt surface, in railways tracks, an assembly of rail-fastener-sleeper together receives and distributes the train loads to the underlying asphalt pavement.

Till date, very few ballastless asphalt tracks have been built, most of them were identified within the German railway industry, such as ATD, SBV, SATO, Walter, and GETRAC systems (Rose et al., 2010; EAPA 2003; Michas 2012; Lechner 2005; Lechner 2013). In order to prevent transverse movements under horizontal loads, the sleepers were anchored to the underlying pavement using different techniques such as concrete sleepers with grooves, e.g., ATD system, bituminous sealing, e.g., SBV, steel anchor rods, e.g. Walter system, and concrete anchor blocks, e.g., GETRAC (Michas 2012, Lechner 2005; Lechner 2013). Some designs included steel-based sleepers with anchoring systems, e.g., SATO system (Lechner 2005). However, there is very little literature on these asphalt track designs that is publicly available.

The GETRAC is the most recent version of the ballastless asphalt track design (Rose et al., 2000; Rose et al., 2010, Michas 2012; EAPA 2003). It has been implemented mostly in tunnel sections with standard highway asphalt and specially designed concrete sleepers that include a
geotextile at the bottom. Every third sleeper is attached to the asphalt layer through concrete anchor blocks. The GETRAC design has two versions: A1 and A3. The design of the A1 type includes standard sleepers that are 2.6 m long, while, the design of the A3 type consists of slightly smaller sleepers that are 2.4 m long, deeming it suitable for projects with space restrictions, such as a tunnel. However, the sleepers used in A3 system are much wider at the base and heavier. The heavy sleepers and the anchor blocks together accounted for horizontal stability. The basic structure includes a 0.20 m thick asphalt layer, 0.30 m thick base layer and a subbase of 0.50 m. For designs where the base layer was not present, a much thicker asphalt layer of 0.35 m was suggested. While literature exists on the construction details of this track, data is not available publicly on modelling, design or performance of this track under service conditions.

1.2.2. Laboratory and field investigations

In recent years, there has been growing research on ballastless asphalt tracks focusing on asphalt mix design (Lee et al., 2014) as well as laboratory tests on full-scale prototypes (Lee et al., 2016; Lee et al., 2017). A test section was built in Korea based on high-speed railway specifications to evaluate the performance of ballastless asphalt tracks under vertical loads (Lee et al., 2016; Lee et al., 2017). The test section was 4.0 m wide and 4.0 m deep, consisting of 3.6 m subgrade, 0.4 m aggregate base course and three sections with a variable asphalt layer thickness of 0.20 m, 0.29 m, and 0.35 m. In each section, five wide sleepers were used that were 2.4 m long and 0.5 m wide, with a geotextile attached at the underside. A continuous rail was connected to the sleepers. Stationary loads were applied directly to the rails, above the central sleeper, ramping up in increments up to a maximum value of 200 kN, holding at each load level and then unloaded back in increments (Lee et al., 2016). At each hold stage, ‘stable’ values of vertical stresses in the base and subgrade layers, horizontal strains at the bottom of asphalt layers, and vertical displacements on top of the rails, sleepers and base layers were reported. The exact waiting time after which the measurements were reported were not specified. It was shown that measured responses varied non-linearly with an increase in the load level. The stress recordings showed negligible hysteresis between loading-unloading curves, while prominent hysteresis was noted in the strain and displacement measurements. For the entire loading regime, the maximum strains in the asphalt layer were reported to be less than 100 µε. Based on this study, an overall asphalt layer thickness of 0.30 m was proposed for static wheel loads less than 180 kN. The design was based on the maximum allowable stress limit of 133 kPa at the top of the base layers.

The long term behaviour of the chosen asphalt thickness was studied (Lee et al., 2017) on a level and sloped track under moving wheel loads. The latter were approximated as sinusoidal loads applied at a single frequency of 8 Hz and 7 Hz (for the level and sloped track respectively), directly to the rails for 2 to 3 million consecutive cycles, without any rest period. The loading frequency was obtained based on the wheel-to-wheel distance and simulated train speed. However, there was no explanation provided on the methodology used to simulate the moving load in the test facility, i.e. if the load was at a fixed point or multiple points along the rail. At the end of 2 million cycles, for a straight track the settlement measured on top of the rail, sleeper and the asphalt trackbed was 3 mm, 2.6 mm and 0.75 mm respectively. The relative settlement between the rail and sleeper was minor, representing an elastic behaviour of the rail-pads, while, that between the sleeper and the asphalt trackbed was quite large which was attributed to the compression of the geotextile attached at sleeper-bottom. From the measured values, overall track settlement was estimated for 60 million cycles (representing 30 years of rail traffic for straight track) to be 3.4 mm, which was lower than a target value of 4 mm (as per Korean standards).
1.2 State-of-the-art and research challenges

Recently, field studies were conducted by Lee et al. (2019) in a test track of length 207 m that was built outdoors in the live network and monitored for one year to assess the influence of seasonal temperature variations on track responses. The locomotive operating on this line had average speed and axle load of 70 km/hr and 200 kN respectively. The average temperature in the asphalt layer (in the bottom part) was measured during the winter and summer season to be around 0°C and 33°C respectively. The measurements of vertical stresses in the base layers and the horizontal strains in the asphalt layers were influenced with temperature, increasing significantly in the summer season compared to winter. The maximum vertical stresses on top of the base layer were measured to be around 50 kPa in winter and 85 kPa in summer. The horizontal strains at the bottom of the asphalt layers also showed a similar increase with a rise in temperature, with the peak lateral strains changing from 50 µε to 180 µε while the longitudinal (along rail direction) strains changed from 30 µε to 100 µε. However, the study did not mention the loading conditions under which peak responses were obtained, i.e., if the locomotive was stationary, if so, for how long, or was it recorded during passage of locomotive.

The horizontal track forces include both lateral (e.g., lateral wheel force) and longitudinal forces (e.g., change in temperature, train acceleration and braking). One of the fundamental challenge associated with ballastless asphalt tracks is the safe transmission of horizontal forces at superstructure-substructure interface. In slab tracks, the lateral stability is very high as the sleepers are encased within the slabs, or the rails are fixed to the slab directly (Gautier 2015). In ballasted tracks, the sleepers are partially embedded in the ballast which also surrounds it from all sides, in between sleepers (crib ballast) and on the sides (shoulder ballast). All the entities, friction at the sleeper underside, crib and shoulder ballast, contribute to the overall sliding resistance (Le Pen et al., 2011). For ballastless asphalt tracks, the sleepers may simply rest on the pavement- possibly without crib or shoulder ballast or any additional anchoring to the pavement structure. In this situation, horizontal forces at the superstructure-substructure interface are solely mitigated by the sliding resistance available at the sleeper base, which may be inferior to other track forms. In this context, recently, a field study was carried out by Esmaeili et al. (2018) to compare the horizontal resistance of ballastless asphalt tracks and conventional ballasted tracks. A test track section was built that consisted of a ballasted track and an asphalt overlayment track. Identical sleepers were used in both track types; B70 type which are 2.6 m long, 0.3 m wide and weigh around 280 kg. The sleepers did not have any geotextile at the underside, nor any anchoring elements to attach with the underlying asphalt pavement. For the overlayment track, crib ballast was not used, while in the ballasted track it was present. Two types of tests were done, pull out of a single sleeper as well as a track panel of 6 m length consisting of several sleepers. It was reported that the lateral track resistance of ballastless asphalt tracks was on average 41% lower than ballasted tracks. The tests were also done in the presence of shoulder ballast in which case the lateral resistance of ballastless asphalt tracks increased by about 16%.

1.2.3. Track modelling

In a very early study by Huang et al. (1987), a 3D model was developed for designing asphalt tracks (both underlayment and overlayment) using a combined formulation based on finite element method (FEM) and multilayered elasticity theory. The model for ballastless tracks included: rails and sleepers, fasteners and a two-layered elastic track structure composed of asphalt layers and subgrade. Two noded beam elements were used for discretizing the rails and sleepers, and fasteners were modelled as springs between them. Static analysis was performed with multiple concentrated loads applied to the rails. Stresses, strains and displacements in the
layered structure were calculated using multilayered elastic theory. At the bottom of the asphalt layer, maximum tensile strains were evaluated. At the top of the subgrade, maximum compressive stresses were calculated. These were used to estimate the maximum number of load repetitions to prevent failure by fatigue cracking in the asphalt layers or by excessive deformation of the subgrade layers. The calculations were done based on laws that were developed for highway pavements considering damage. The model was used to design the thickness of the asphalt layer, which was estimated to be between 250 mm to 450 mm. This model was validated with field measurements for underlayment systems but not for overlayment tracks.

Very recently, a 3D FEM model of an outdoor test track section was developed (Lee et al., 2019). The model consisted of a pair of rails (modelled with beam elements), fasteners (simulated as springs), sleepers and a multi-layered track structure (modelled with solid 3D elements). The sleepers used in the field study included a geotextile at the underside which were not included in the model. All track layers were assumed to be linear elastic. The asphalt layer was additionally modelled as linear viscoelastic. However, no explanation was provided in this study on how the elastic properties of the different model entities, including the asphalt layer, were obtained. The vehicle loads were simulated in a very approximate manner to be distributed loads acting on the rails. Quasi-static analysis was performed, and a single value of strain and stress below the asphalt layer were compared with field data. The model predictions with viscoelastic analysis were reported to show a poor match with field data (especially the asphalt strains).

### 1.2.4. Design guidelines and standard provisions

Recently, guidelines for ballastless tracks were introduced in the Danish standard DS/EN 16432-2:2017. Some of the provisions of this standard applicable for ballastless asphalt track design and construction (excluding the mix design) include:

i. An overall asphalt layer thickness of 300 mm is suggested
ii. Allowable modulus of deformation \( E_{v2} \) of unbound granular layer is 150 MPa and for frost blanket layer/subbase is 120 MPa
iii. Interface elements such as a geotextile are recommended to be use between prefabricated sleepers and the asphalt pavement
iv. The pavement design criteria is based on limiting the maximum flexural tensile stresses
v. The recommended values of mean flexural fatigue strength and Young’s modulus of asphalt is given as 0.8 MPa and 5000 MPa, respectively.
vi. Allowable contact stress due to traffic loads on top of the asphalt pavement is required to be lower than 0.5 MPa
vii. Allowable stress due to traffic loads on top of the base layer is required to be lower than 50 kPa or can be calculated using fatigue models

### 1.2.5. Summary and research gaps

The state-of-the-art knowledge on ballastless asphalt tracks is summarized as:

i. Laboratory experiments were performed on full-scale prototypes to analyze mechanical responses (stress, strain and displacement) under different intensities of stationary loads (Lee et al., 2016). Based on the findings, an overall thickness of the asphalt layer was...
1.2 State-of-the-art and research challenges

Chapter 1: Introduction

designed to be 0.3 m, by limiting the maximum vertical stresses on top of the granular layer to be lower than 133 kPa.

ii. A full-scale prototype was tested to evaluate long term track settlement under moving loads utilizing consecutive cyclic loads (Lee et al., 2017).

iii. A field test was conducted to measure mechanical responses (stresses and strains) induced by a moving locomotive at different seasons of the year (Lee et al., 2019). The study confirmed the temperature sensitivity of track responses and presented peak values of response due to seasonal temperature variations.

iv. A field study demonstrated the lateral resistance of ballastless asphalt tracks to be inferior compared to traditional ballasted tracks (Esmaeili et al., 2018).

v. Very few implementations of ballastless asphalt tracks have been found, most of them within tunnel sections in the German railway industry. While construction details are available for some recent designs such as the GETRAC system (Rose et al., 2000), the literature on measured system responses under service loads, modelling and design methods is not publicly available.

vi. Modelling approaches were based on finite element method; including pure FEM (Lee et al., 2019) as well as FEM combined with multilayered elasticity theory (Huang et al., 1987). 3D quasi-static linear models were developed which incorporated the time-dependent behaviour of the asphalt layer using linear viscoelasticity theory (Lee et al., 2019).

The major research gaps in the field of ballastless asphalt tracks are summarized as:

i. Laboratory simulation of moving loads (as consecutive cyclic loads at a constant frequency) was not realistic given rest periods were not included between cycles, and that the loading scheme consists of a single unique frequency.

ii. Very limited experimental data exists on track responses measured under service conditions, (i.e., train passage at different speeds and loads). Measurements of different types of responses (histories) during the train passage at different locations in the track are not available.

iii. The influence of different load amplitudes and frequencies have not been investigated.

iv. Experimental data on vibration and noise levels in ballastless asphalt tracks are not available for comparison with other track types.

v. A single field study evaluated track response to horizontal pull-out forces. No further experimental studies have been conducted, and modelling approaches have not been developed to investigate responses to other types of horizontal loads, i.e., both longitudinal loads and lateral loads.

vi. While the geotextile is part of the sleeper design of both German and the Korean solutions, experimental studies have not focussed on its behaviour and impact on the overall track response, and track models have not included it.

vii. Modelling efforts have been limited to quasi-static analysis of linear finite element based models; track dynamics, and non-linearity were not addressed. Moreover, trainloads were simulated in a very simplistic manner to be a sequence of stationary loads or distributed loads acting on the rail. Further, vehicle dynamics were not incorporated, allowing for analysis of wheel-rail interactions issues and near and far-field vibrations.

viii. Models were not adequately validated by comparing calculated responses with experimental measurements. So far, validation efforts have been very limited.
Comparisons were made with field measurements for a single value of stress and strain calculation value for an entire track model.

1.3. Thesis objectives

This study addresses some of the research gaps identified in section 1.2.5. The primary focus of this study is to analyze mechanical response of ballastless asphalt tracks under vertical loads. The goal is to conduct full-scale laboratory investigations, and to develop and validate an advanced computational model. Given the challenges associated with the horizontal stability of ballastless asphalt tracks, a secondary scope is defined within which track responses are studied due to longitudinal loads. A simplified 2D analytical formulation is used to model the problem. Though this study focused primarily on analyzing mechanical responses in ballastless asphalt tracks, some initial research effort was spent on developing an approach that can be useful for design. In this context, attention was paid to a standard approach of simulating railway tracks as infinite beams resting on a continuous spring foundation; the parameter track modulus characterizes the latter (Cai et al., 1994; Li et al., 2002). Subsequently, a semi-analytical method was suggested for calculating track modulus based on elasticity solutions.

The thesis objectives are summarized below:

Objective 1 (O1): Construction, instrumentation and experimental investigation of the mechanical behaviour of a ballastless asphalt track mockup under vertical loads.

As a starting point for experimental investigations a full-scale mockup is constructed, instrumented and tested inside a controlled laboratory environment under different types of vertical loads, including simulation of train passages. The focus is on recording different track responses without destroying the integrity of the system, i.e., design limit states are not in the scope of the study.

Objective 2 (O2): Development and validation of a numerical model of the ballastless track mockup.

An implicit, time domain, 3D, dynamic FEM model of the ballastless asphalt track mockup is developed that includes material non-linearity and time-temperature dependency. The intention is on obtaining the majority of model parameters through standard material tests without using model fitting approach. The scope is limited to evaluation of resilient responses, i.e., fully recoverable deformations. Moreover, vehicle dynamics and wheel-rail interaction are not in the scope of the study. The model is used to investigate the sensitivity of selected responses to some model parameters. Ultimately, the model is validated by comparing calculated results against measured responses in the mockup.

Objective 3 (O3): Development of a mechanical model for analyzing track responses to longitudinal loads induced by train braking.

A 2D analytical model is developed that is capable of analyzing longitudinal track responses to braking loads. The purpose is to provide a mechanical portrayal of how braking loads are handled within a track superstructure. The intention is to present solutions in a closed-form that are easily
implementable and fast to compute. The model is used to demonstrate track responses during braking of a single axle and a full train.

**Objective 4 (O4): Development of a method for a priori determination of track modulus based on elasticity solutions.**

A semi-analytical framework is used to develop a method for calculating track modulus using a 3D linear elastic track model. The method calculates track modulus by establishing a strong connection between the developed 3D model and the existing standard track model of an infinite beam on a continuous spring foundation. The proposed method is illustrated considering a wide set of values for the different model parameters.

The four research objectives (O1 to O4) were addressed in five publications (Paper I to Paper V) that were written during the PhD study, as shown in a flowchart in Figure 1.2.

### 1.4. Thesis outline

This is a paper-based thesis consisting of two parts: **Part A** and **Part B**. Part A includes three chapters and an appendix. Chapter 2 gives a synopsis of the study. It presents an extended abstract of the papers along with supplementary information that helps in presenting a holistic picture. Chapter 3 summarizes the major findings and presents generalized conclusions of the entire study. Appendix A illustrates some additional test results that are not included in the papers. In Part B, five scientific publications that were written during the PhD study are appended.

![Figure 1.2: Workflow in the PhD thesis](image-url)
Chapter 2

Synopsis

This chapter is based on (includes reuse of text and figures) the following scientific articles:


2.1. Overview

This chapter presents a synopsis of the entire PhD study. It consists of four additional sections which present the extended abstract of Paper I to Paper V. The primary scope of this thesis is the experimental and numerical investigation of ballastless asphalt track responses under vertical loads. Section 2.2 presents the experimental investigation, which is based on Paper I. The experimental investigation involved detailed planning in the different stages such as construction, instrumentation and testing. Due to practical reasons, it was not possible to include all the information in Paper I. The author feels that it is necessary to provide some additional information in order to present a holistic picture and explain the logical progression of the work. Consequently, Section 2.2 is not only a summary of Paper I but has been elaborated (as and when required) to include other necessary details. Additionally, some of the test results that were not included in Paper I are also presented separately in Appendix A. In Section 2.3, the numerical modelling has been presented which is based on Paper II and Paper III. The parts dealing with material characterization from laboratory tests has been elaborated in this section. Section 2.4 deals with the analysis of longitudinal track responses; this is presented as an extended abstract of Paper IV. Section 2.5 deals with a method to calculate track modulus; this is an extended abstract of Paper V.

2.2. Experimental investigation

This section describes the experimental investigation of a ballastless asphalt track mockup under vertical loads. It is presented in the form of a summary of Paper I, supplemented with additional information.
2.2 Experimental investigation

2.2.1. Background and motivation

The mechanical responses of ballast and concrete slab tracks have been extensively investigated through experimental studies. This includes analysis of the behaviour of track components and layers through small-scale tests, (Indraratna et al., 1998; Lackenby et al., 2007; Kaewunruen et al., 2008; Thompson and Verheij 1997; Sol-Sánchez et al., 2014), laboratory mockup tests (Tarifa et al., 2015; Momoya et al., 2005; Cox et al., 2006; Brown et al., 2007; Chen et al., 2013; Chapeleau et al., 2013; Bian et al., 2014; Kennedy et al., 2013; Yu et al., 2019; Al Shaer et al., 2008; Čebašek et al., 2018; Sainz-Aja et al., 2020), and field tests (Liu et al., 2017; Indraratna et al., 2010; Connolly et al., 2014; Connolly et al., 2015; Degrande and Schillemans 2001; Galvín 2010). The latter has the advantage of assessing responses under live loads in as-built conditions including the subsoil. Nonetheless, field studies are expensive to carry out and requires multiple approvals from railway authorities. In this regard, as a first step, before implementing a new material or concept in the railway network and testing it under live loads, laboratory mockup testing can prove to be useful. It is a cost-efficient way of gaining preliminary insight into the component functionalities under different loading conditions while being in a controlled environment. In addition, the ability to reproduce moving loads within a limited test section has been achieved reasonably well (Brown et al., 2007; Bian et al., 2014), which can indicate the expected magnitude and behaviour of track responses under live loads.

Limited experimental investigations have been carried out for ballastless asphalt tracks. A full-scale mockup was built in Korea, instrumented and tested under different intensities of stationary loads (Lee et al., 2016). The study focused on determining the required thickness of the asphalt layer under the loading regime, which was designed to be 0.3m. Subsequently, Lee et al., 2017 estimated long term settlement of this design by applying consecutive cyclic loads at a single frequency. Recently, a field study (Lee et al., 2019) was carried out in which a test track section was built outdoors, and track responses were measured during different seasons of the year.

This study aims to contribute to the understanding of mechanical behaviour of ballastless asphalt tracks by constructing and testing an instrumented full-scale mockup under vertical loads. The focus is on investigating resilient responses without affecting the integrity of the system.

2.2.2. Methodology

2.2.2.1. Initial design and planning

The experimental investigation was aimed at constructing a full-scale mockup, instrumenting the track layers, installing at least three sleepers on top of the asphalt pavement and applying various types of vertical loads to the sleepers directly, including simulation of moving train load with the help of servo-hydraulic actuators supported by a reaction frame. As part of initial planning, the mockup testing was decided to be carried indoors in the Structural Laboratory in Technical University of Denmark (DTU). The reasons were: (i) existing hydraulic and electricity connection in the test hall, (ii) possible damage to the loading actuators and electronic systems in an outdoor environment, and (iii) necessity of elaborate construction work in outdoor location to build an all-around protective shed and foundations to support the columns of the reaction frame.

The test hall did not have a pit inside which the construction of the track layers could be carried out; a large steel container was considered for the purpose. The construction required several heavy types of equipment requiring a large operating area; this was not feasible inside the
test hall. Consequently, the construction was planned in a different outdoor environment inside a steel container after which it was transported to DTU, craned inside the test hall and placed on a thick concrete floor. To allow for portability, only the asphalt layer and the underlying unbound granular layer was built in full-scale. The subbase and subgrade were simulated by utilizing a thin polyurethane mat with equivalent mechanical properties. This was necessary to ensure that the overall weight (self-weight of the container inclusive of the materials inside) of the constructed facility was within the handling capacity of the cranes in the test hall.

An instrumentation scheme was planned to monitor different mechanical responses in the track. The planning process required the knowledge of the following:

i. Locations where peak responses were anticipated?
ii. The expected magnitude of peak responses?
iii. Choice of instrumentation and their precision

In order to answer the above questions, a 3D linear elastic finite element model of the mockup, while still in its preliminary stage (with approximate dimensions) was developed. Static analysis was performed by considering different magnitudes of axle load (maximum of 200 kN) distributed between three sleepers in the ratio of 1:2:1. Parametric studies were conducted by varying the material properties of the track components and layers over a realistic range. The results of this study were used to plan the instrumentation layout. Furthermore, model responses showed very low magnitudes, almost within the precision limit of some sensors. Additionally, the embedded sensors were required to survive the construction loads related to compaction of granular layer and the asphalt layer. Hence, a redundancy rate of 50% was chosen for all sensors to ensure a greater reliance on extracting data from embedded sensors.

2.2.2.2. Construction, instrumentation and loading setup

A full-scale mockup of a ballastless asphalt track was constructed (inside a steel container) with a length of 4.00 m and width of 2.21 m. From bottom-up, the track layers included, (i) a 0.025 m thick polyurethane mat, (ii) a 0.275 m thick unbound granular layer (UGL), and (iii) a 0.280 m thick asphalt layer (Paper I). The mat selection was based on guidelines that specified a minimum $E_{v2}$ value of 120 MPa on top of the subbase layer for a newly constructed track (Paper I). Other considerations behind mat selection were that it should be strong enough to endure the construction work and provide a stable base for building the top layers. The commercial name of the mat used in this study is Regufoam Vibration 990 Plus.

The earth pressure cells (PCs) were the first layer of embedded sensors that were placed at the top of the mat (see Figure 2.1a). These sensors were installed to measure the vertical stress at the bottom of the UGL. For the mock-up, five pressure cells were used with a maximum pressure rating of 1 MPa. The rating was selected based on the maximum anticipated loads that the sensors would have to withstand in order to survive the construction work (failure limit was set as two times the pressure rating). To have a better resolution in the stress range expected during testing, the sensors were calibrated within a narrow range of 0 kPa to 250 kPa.

The PCs were laid flat on the mat at their desired location and then secured in place by screwing onto the mat. The cables were diverted through holes on the sides of the box (see Figure 2.1b) and stacked inside a hollow frame (see Figure 2.1d). The PCs were protected by covering them with generous amounts of unbound granular materials. Large rocks/stones were removed
so that the pressure cells were in immediate contact only with relatively small-sized particles which were compacted manually to form a stable base (see Figure 2.1c).

Figure 2.1: (a) Pressure cells placed at the top of the mat, (b) holes for taking out cables of embedded sensors, (c) protection of the pressure cells with unbound granular materials, and (d) stacking cables inside a hollow frame along sides of the box

The UGL was constructed on top of the mat. For this layer, a gravel base (type SGII) complying with the Danish road standard specifications was chosen (Danish Road Directorate AAB 2012). The aggregates used in the mockup were extracted from Løng Grusgrav (NCC). Before construction, laboratory experiments were carried out (by VBM Laboratoriet A/S) to ensure conformity with SGII requirements. The aggregate properties as determined from the laboratory tests are given in Table 2.1, and the aggregate gradation curve is shown in Figure 2.2a.

Table 2.1: Properties of aggregates used in unbound granular layer

<table>
<thead>
<tr>
<th>Tests</th>
<th>OMC</th>
<th>Max dry density</th>
<th>Max particle size</th>
<th>LL</th>
<th>PL</th>
<th>LA Abrasion</th>
<th>Methylene blue</th>
<th>Sieve analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>6.7</td>
<td>2210</td>
<td>31.5</td>
<td>18</td>
<td>NP</td>
<td>25</td>
<td>1.8</td>
<td>Figure 2.2a</td>
</tr>
</tbody>
</table>

The UGL was constructed in two lifts by compacting a loose mix of unbound granular materials (see Figures 2a and 2b) using multiple passes of a vibratory tandem roller. Using a nuclear moisture density gauge, the dry density was estimated at three different locations to be 98.1 % on average, relative to a vibrating table reference compaction (see Figure 2c). Light Weight Deflectometer (LWD) tests were performed to characterize the stiffness of the UGL inside the box (see Figure 2d), but the test results were not meaningful. Following the completion of on-site tests, the second layer of embedded sensors was placed on the finished surface of the UGL.
These sensors were asphalt strain gauges (ASGs) that were deployed to measure the horizontal strain at the bottom of the asphalt layer. Eleven such sensors were used, each of them was positioned at the desired location and kept flat (see Figure 2.4a) so that they did not rock in place. As before, the cables were taken out through holes on the side of the box and stacked inside the frame (see Figure 2.4b). Generous amounts of hot mix asphalt were placed covering each sensor and compacted manually using wooden blocks (see Figure 2.4c). This aimed maximum protection for the sensors from the heavy equipment involved in the construction of the asphalt layer.

Figure 2.2: Aggregate gradation for (a) unbound granular layer, and (b) asphalt layer

Figure 2.3: (a) Placement of unbound granular materials at the top of the mat, (b) compaction of unbound granular materials, (c) on-site measurement of density and water content, and (d) LWD test at the top of the unbound granular layer
The asphalt layer construction (see Figures 2.5a to 2.5c) consisted of a surface course with a lift thickness of 0.04 m and three underlying identical base course lifts with a total thickness of 0.24 m. The aggregate gradation curve for the surface and base course layers are shown in Figure 2.2b. The asphalt mix design was not in the scope of the PhD study and was carried out by Danish Technological Institute (one of the partners in this project). The final construction was completed in three attempts; in the first two attempts, the surface course did not meet the target requirements of binder content and was milled off and repaved. After the second attempt, a part of the base layer was also milled off to gain access to repair some sensors that were damaged during the construction process. In all the attempts, the quality of construction was assessed by the asphalt contractor by testing asphalt cores from the mockup (98 mm in diameter). The cores were preserved for subsequent laboratory characterization of viscoelastic properties of the asphalt layer.

An FWD (Falling Weight Deflectometer; Model: PRIMAX 3500) test was performed on top of the asphalt layer (see Figure 2.6a). In an FWD test, an impulse load is applied to the pavement surface, and the resulting response (deflection basin) is recorded by a set of geophones. Additionally, an FWD test was also performed directly on the ground supporting the steel box, in an attempt to characterize the subsoil. An initial estimate of the elastic properties of the different layers may be obtained by utilizing test data obtained from inside and outside the box. However, due to time constraints, it was not possible to engage in extensive analysis of FWD test data.
After the construction was completed, the test section was transported (see Figure 2.6b) to DTU, the overall weight was measured to be 15,640 kg. The mockup was craned inside the test hall and placed on a thick concrete floor. To ensure full contact between bottom of the box and the test floor, a wet mix of high strength mortar was spread, and the steel box was lowered on it (see Figure 2.6c). A rigid reaction frame was designed and constructed to house three servo-hydraulic actuators (MTS, Model No: 244.22) with a force rating of 100 kN, and a dynamic stroke of 150 mm. The rigidity of the reaction frame was essential to minimize upward bending of the frame under loads that would otherwise add to the actuator stroke. The actuators were mounted to the reaction frame using steel plates; the spacing between them corresponded to standard sleeper spacing of 0.6 m. The hydraulic system for the actuators was designed and built using an accumulator (top blue part in Figure 2.6e) to be able to generate fast loading and unloading events.

Three wide concrete sleepers of type BBS 3 W60 (used in GETRAC A3 system) was used in the mockup (supplied by Rail.One). Vertical loads were applied to the sleepers directly; each actuator loaded the sleeper directly beneath it. To assist in load transfer, small rail segments
were mounted onto the built-in fastening system (Vossloh System 300). This was done by cutting continuous rail pieces (of type 60E1/60E2) into small segments of length 0.2 m by a hydraulic bandsaw and using a torque wrench to mount the rails (see Figure 2.6d). As there was one actuator for loading each sleeper, a spreader beam was installed to distribute the load equally on top of the two rail segments (see Figure 2.6e). The spreader beams were directly bolted to the swivel base of the actuators. The contact between the spreader beam and the rail segment was established through a circular steel disc of diameter 10 mm glued at the top of the rail-segment at its centre in order to ensure the best possible alignment and symmetry in the loading (Paper I). The constructed facility is shown in Figure 2.6e.

Surface sensors (shown in Figure 2.7) were installed, which included displacement transducers and accelerometers. The displacement transducers were Linear Variable Differential Transformers (LVDTs) that were installed to measure relative displacement between central sleeper and the asphalt surface at the middle (see Figure 2.7a) and at the edges (see Figure 2.7b), and potentiometers that were mounted to measure the rail pad compression (see Figure 2.7c). Subsequently, all sensors were connected to the data acquisition system (MTS FlexDac). The displacement transducers were calibrated, while for the other sensors, the calibration chart provided by the supplier was used. Complete details of the instrumentation layout are given in Tables 1 and 2 in Paper I and positions are illustrated schematically in Figure 3 in Paper I.

2.2.2.3. Test plan

There are two control modes for an actuator, displacement and force, which were separately tuned for the three actuators to obtain the correct control (PID) settings. Through trial tests it was observed that force control mode was unsatisfactory and risk-prone, especially for rapid loading and unloading. On the other hand, the displacement control mode was found to be satisfactory and quite safe to operate. Consequently, all experimental investigations in the mockup were carried out by controlling the displacement history of the individual actuators to generate a target force level and not the force directly.

During trial tests, the functioning and reliability of sensor readings were also assessed. The sensor data showed three asphalt strain gauges to be damaged, and one displacement transducer to be non-functional. All tests were carried out using the MTS Test Suite software; the sampling rate of the tests was set as 1024 Hz, except in very few cases where it was 512 Hz. The following types of tests were carried out in the mockup:
i. Type I: Ramp loading with rapid unloading (see Figure 2.8a)
ii. Type II: Pulse loading (see Figure 2.8b)
iii. Type III: Cyclic loading with variable amplitudes and frequencies (see Figure 2.8c)
iv. Type IV: Moving axle loading (see Figure 2.8d)
v. Type V: Moving train loading (see Figure 2.8e)

Figure 2.8: Different types of vertical loads applied in the mockup. (a) Ramp loading with rapid unloading, (b) pulse load, (c) cyclic load, (d) moving axle load, and (e) moving trainload

2.2.3. Major findings

Findings from tests: Type III, Type IV and Type V have been presented and discussed in detail in Paper I. Additional results from the other test types (Type I, Type II, and including some from Type V) are given in Appendix A. All responses that are illustrated are a manifestation of the externally applied loads without self-weight. The sign convention adopted follows from Paper I, whereby downwards load and displacement is positive and compressive stresses and strains are positive (for coordinate system refer to Figure 3 in Paper I). The average temperature in the test hall varied between 21°C and 22°C.

Type I: Ramp loading with rapid unloading:
The findings from this test are illustrated in Appendix A (see A.1). In this test, only the central sleeper was loaded by Actuator 2; the applied force history is shown in Figure A.1.1a, and the force-displacement curve is shown in Figure A.1.1b. The peak load applied by Actuator 2 was 52 kN, and the loading-unloading event lasted for approximately 1 s. It is seen that the loading and unloading curves form a closed hysteresis loop, the area enclosed within the loop is a measure of the overall energy dissipated in the track during loading and unloading. It should be noted that the actuator’s displacement not only includes displacements from the track but from other components like spreader beams and reaction frame as well.
The time history of vertical displacement measured by two potentiometers (POT 39 and POT 41) are shown in Figure A.1.1c. The two sensors show nearly identical readings and follow the overall shape of the load with a gradual increase of displacement with the load level, followed by rapid unloading to initial conditions without noticeable time effects and no residual displacements, indicating an elastic behaviour of the rail pads. The average peak rail pad compression is measured to be 0.81 mm.

The time history of vertical displacement measured by two LVDTs (LVDT 1 and LVDT 2) are shown in Figure A.1.1d. These two sensors measure the relative vertical displacement between central sleeper and asphalt surface at the edge and middle (for location see Figure 2.7) respectively. The LVDT readings represent a combined effect of geotextile compression and surface displacement at these locations (ignoring sleeper bending). Readings of LVDT 4 were similar to LVDT 1, and that of LVDT 3 was similar to LVDT 2; hence they were not included. The peak value measured by LVDT 1 was higher than LVDT 2, which could be attributed to the difference in surface deflections at these two locations (ignoring bending of the sleeper). These sensors followed the overall shape of the load with a gradual increase of displacement with load level, but in contrast to potentiometers, a prominent time-dependent recovery is observed after unloading. Within the experimental window, the LVDT recovery was still ongoing.

The horizontal strains measured by asphalt strain gauges (ASGs) are shown in Figures A.1.2a and A.1.2b (for strain gauge layout refer to Figure 3 in Paper I). This figure only includes the strain gauge readings which were found to be reliable (in terms of overall shape). Horizontal strains along the direction of the track (along the width of the box) are shown in Figures A.1.2a, and horizontal strains perpendicular to track direction (along the length of the box) are shown in Figures A.1.2b. All strain gauges followed the overall shape of the load, showing a gradual increase in strain with the load level. On unloading, a time-dependent recovery was observed to the initial conditions; residuals strains were not observed. ASG 6 and ASG 7 (situated below the loaded sleeper i.e., Sleeper 2) recorded tensile strains while ASG 3 (situated below Sleeper 3) measured compressive strains. In the perpendicular direction, strains measured below Sleeper 3 (by ASG 2) and that below Sleeper 1 (by ASG 10) were tensile in nature. Due to symmetry ASG 2, and ASG 10 were expected to provide similar readings (which was not the case). The maximum tensile strain was recorded along the track direction to be 38 µε.

Vertical stresses measured by pressure cells (PCs) at the bottom of UGL are shown in Figures A.1.2c (for pressure cell layout refer to Figure 3 in Paper I). PC 16 and PC 14 (located below rail pads on Sleeper 2) recorded nearly identical peak stresses of 20.5 kPa. PC 15 (located below the middle of Sleeper 2) recorded 13 kPa, while PC 13 (below Sleeper 3) and PC 17 (below Sleeper 1) measured nearly the same peak values of 5 kPa. No significant time effects were observed in the readings of pressure cells.

**Type II: Pulse loading:**
The findings from this test are illustrated in Appendix A (see A.2). In this test also, only the central sleeper was loaded by Actuator 2. The applied force history is shown in Figure A.2.1a, and the corresponding force-displacement curve is shown in Figure A.2.1b. The peak load applied was 48 kN which was 7% lower than before, and the entire loading-unloading event was much faster lasting for 0.0625 s. Qualitatively, the discussions presented previously for the different sensor responses in the context of Type I tests was applicable in this case as well. The significant difference was the response magnitudes, all of which were much lower than before (except for rail pad compression).
Type III: Cyclic loading with variable amplitudes and frequencies
In these tests, the central sleeper was subjected to sinusoidal excitations of different amplitudes (varying between 10 kN and 30 kN) and frequencies (0.1 Hz, 1.0 Hz, and 10.0 Hz) for 200 cycles. The steady-state sensor readings were analyzed to understand the effects of different loading amplitude and frequencies.

It was found that the overall load-displacement response of the track was frequency-dependent with a higher complex stiffness norm at a higher frequency. All measured responses displayed frequency-dependence, the rail pad compression to a lesser extent; while the relative vertical displacement between the asphalt surface and the sleeper, the vertical stresses below the UGL and the horizontal strains below asphalt layer, to a major extent. The vertical stresses below the UGL scaled almost linearly with the load amplitude while the rail pad compression and horizontal strains below the asphalt layer showed minor non-linear behaviour. In contrast, the relative displacement between the sleeper and the asphalt surface was strongly non-linear. The findings of Type III tests are illustrated and discussed in detail in Paper I.

Type IV: Moving axle loading
In this test, the passage of a single axle was simulated by sequentially loading the three sleepers with a time delay that was calculated based on sleeper spacing and simulated axle speed \( V \). The magnitude of the axle load \( 2P_{Z,j} \) was chosen as 120 kN, and the simulated speed was 120 km/h (travelling from Sleeper 1 towards Sleeper 3). Here, \( P_{Z,j} \) is the wheel load and \( j \) is an index that denotes the axle number (in this case, \( j = 1 \)). The sleeper loading history was calculated using a simplified track model of an infinite beam on a Winkler foundation. The procedure of simulating a moving load is explained in Section 2.3.2 in Paper I. The findings from this test are illustrated graphically and discussed in Paper I.

Type V: Moving train loading
In these tests, the passage of a full train was simulated from Sleeper 1 towards Sleeper 3. The Danish IC3 train was chosen for this purpose which consists of 8 axles and has a length of 56 m. The train configuration is shown in Figure 4 in Paper I. The sleeper loading history was calculated as before using a simplified track model of an infinite beam on a Winkler foundation. Four tests were carried out: \( V = 120 \text{ km/h}, \text{ and } 200 \text{ km/h} \) \( \{2P_{Z,j} = 120 \text{kN and } 200 \text{kN}\}, j = 1...8 \). Some of the findings from the test involving the train passage at a speed of 120 km/h and axle load of 120 kN are illustrated in Figure 2.9. Figure 2.9a shows the force history of Actuator 2. Similar force histories were applied by Actuator 1 (see Figure A.3.1a) and Actuator 3 (see Figure A.3.1b) with a simulated time delay of 18 ms between the individual actuator signals. In this figure, eight distinct peaks can be observed with an average value of 38 kN, which corresponds to the eight axles of the IC3 train. Further, the chart shows four sets of ‘M’ shapes, each of the ‘M’ signifies two axles located on the same bogie.

For this loading scheme, the time history of rail pad compression measured by two potentiometers is shown in Figure 2.9b. The overall shape of the sensors is very similar to the applied load without any noticeable time effect, and the average rail pad compression is measured to be 0.6 mm. The time history of relative vertical displacement between the asphalt surface and the central sleeper is shown in Figure 2.9c. Contrary to the rail pad compression, this response
illustrates a pronounced time-dependent behaviour in between the bogies passes; minor accumulation was also observed in between successive axle passes.

![Figure 2.9](image)

Figure 2.9: Time history of sensor responses measured during simulation of Danish IC3 train passage (axle load level = 120 kN, speed = 120 km/h). Responses are plotted for: (a) Actuator 2, (b) potentiometers, (c) LVDTs, (d) pressure cells, and (e) - (f) asphalt strain gauges [based on Paper I]

The vertical stresses measured at the bottom of the UGL by two sensors are illustrated in Figure 2.9d. The peak stresses are found to be low (< 25 kPa.) The overall shape is very similar to the applied load without any noticeable time effect. The horizontal strains measured at the bottom of the asphalt layer is shown in Figures 2.9e and 2.9f. The travel of the load along the track direction can be observed by noting the delay in peak strains between ASG 6 (below Sleeper
2) and ASG 3 (below Sleeper 3). It is also seen that strains in ASG 3 reverse sign from compression to tension as the load progresses from Sleeper 1 towards Sleeper 3. The overall behaviour is time-dependent with strain accumulation occurring between successive axle passes. The readings of strain sensors along the longitudinal direction are shown in Figure 2.9f. The strain history is tensile at these locations, and a time-dependent is observed. The peak strains are found to be low (< 25 µε.).

Changing the train speed from 120 km/h to 200 km/h did not have any noticeable effect on the responses of the sensors except the accelerations, which increased with an increase in train speed. In the range of axle loads and speeds simulated in the mockup: (i) maximum vertical displacement due to rail pad compression and relative displacement between sleeper and asphalt surface was less than 1.5 mm, (ii) maximum horizontal tensile strains at the bottom of the asphalt layer was less than 50 µε, (iii) maximum vertical stresses below the UGL was less than 50 kPa and (iv) maximum vertical surface accelerations on the track centre line was less than 2 m/s².

Sensor readings and further discussions are presented in Paper I. Additional results that are not included in Paper I are presented in Appendix A.

2.3. Numerical modelling and validation

This section presents the development, calibration and validation of a numerical model of the ballastless asphalt track mockup. It is presented as a summary of Paper II and Paper III, supplemented with additional information.

2.3.1. Motivation and background

Experimental investigations give insight into the mechanical behaviour of a system under specific test conditions. Additional tests or even the construction of a new testing facility will be needed to determine responses under a different set of conditions, such as changes in load intensity and frequency, layer thickness and material properties. In this context, the development of a predictive model allows evaluating responses to similar conditions. However, the predictive capabilities of a model should be verified by comparing it with experimental measurements (if available).

There is limited literature on modelling of ballastless asphalt tracks. An early study developed a model using FEM (Huang et al., 1987) for the track superstructure and multilayered elasticity for the substructure (composed of an asphalt layer and the subgrade). Static analysis was performed with concentrated loads applied to the rails, neglecting dynamic effects. The model was not validated with field measurements. Recently, a FEM model of a field test section was developed (Lee et al., 2019). The model included all major track components and layers, except for the geotextile (attached at the sleeper bases). Asphalt layer was modelled as both linear elastic as well as linear viscoelastic; all other materials were treated to be linear elastic. However, the calibration of the model parameters was not explained in the paper. Vehicle loads were approximated in a simplistic manner to be distributed loads acting on the rails. Quasi-static analysis was performed to calculate stresses and strains; a single computed value was compared with field measurements that did not match well, especially for the viscoelastic material model.

This study develops a 3D time-domain FEM model of the mockup with a focus on evaluating resilient track responses during train passage. The model incorporates (i) time-temperature dependency of asphalt, (ii) non-linear stress-dependent elasticity of the UGL, and (iii) track dynamic effects, (vehicle dynamics are not in the scope of this study). Particular
attention is given on minimizing calibration operations, i.e. to obtain the governing model parameters by means of standard material testing procedures. In the end, the model's predictive ability is checked by comparing calculated responses against measurements recorded in the mockup.

2.3.2. Methodology

2.3.2.1. General description

A 3D FEM model of the mockup was developed with the commercial code ABAQUS (see Figure 2.10). The model geometry was built as a rectangular cuboid with a length of 4.00 m, a width of 2.21 m, and a thickness of 0.58 m. The thickness was divided into three layers representing (from bottom-up): (i) mat (0.025 m), (ii) UGL (0.275 m), and (iii) the asphalt layer (0.280 m). Three sleepers were modelled based on the geometry of BBS 3W 60. A layer of 0.007 m thickness was included at the sleeper underside to model the geotextile. The rail pads were modelled as square cuboids with a dimension of 0.15 m and thickness of 0.01 m. All layers were assumed to be fully bonded.

The finite domain was discretized with 120,857 eight noded 3D linear brick elements (type C3D8). Boundary conditions similar to the mockup were adopted wherein, the model base was fixed in all directions, and lateral faces along the mat and the UGL were not allowed to move in the direction of their respective normal.

The model did not include the entities from the loading system in the mockup. Instead, uniformly distributed vertical stresses were applied directly to the sleepers on top of the rail pads. Non-linear dynamic analyses were performed using direct time integration with implicit scheme utilizing time increments that were 0.00097 s (Paper II). Sign convention adopted in this section has been changed to be consistent with the convention followed in ABAQUS, tensile stresses and strains are positive and upwards load and displacement is positive.

![Figure 2.10: 3D finite element model of a ballastless asphalt track mockup [taken from Paper II]](image)

2.3.2.2. Calibration of model parameters from laboratory tests

The model parameters for the UGL and the asphalt layer were calibrated from laboratory tests.
2.3 Numerical modelling and validation

Unbound granular layer

The UGL was modelled as a homogenous non-linear elastic isotropic medium with a constant Poisson’s ratio, and a bulk stress-state dependent resilient modulus $M_r$ (Seed et al., 1967):

$$M_r = k_1 P_a \left( \frac{\theta}{P_a} \right)^{k_2}, \quad \theta > 0$$

where $k_1$ and $k_2$ are dimensionless regression constants, $P_a$ is atmospheric pressure (introduced as a normalizing factor) taken as 101 kPa, and $\theta$ is the governing bulk stress (i.e., the first stress invariant) at the location where $M_r$ is evaluated (Paper II). The material model was implemented in ABAQUS through a user-defined subroutine USDFLD; the numerical implementation is presented in Section 3.2 (in Paper II).

The regression constants $k_1$ and $k_2$ were obtained from laboratory tests and not through model fitting approach. For that purpose, the UGL was reconstituted in laboratory into cylindrical specimens of diameter 0.15 m and length of 0.3 m. This was done by compacting a loose mix of unbound granular materials (similar to what was used in constructing the UGL in the mockup) to the target dry density (2168 kg/m$^3$) and moisture content (5.45 %) as measured on-site. The laboratory compaction level achieved was 96.4 % compared to the on-site measurements. These specimens were subjected to repeated load triaxial tests carried out in accordance with the European standard (Dansk Standard DS/EN 13286-7:2004) at ‘constant’ radial stress and ‘low-stress level’. Three such tests were carried out by the Swedish National Road and Transport Research Institute (VTI), and the experimental data was provided to the author. Further data analysis was carried out by the author. Figure 2.11a shows the triaxial cell for testing unbound granular materials at VTI.

In the tests, the specimens were exposed to different stress states; at each state, 100 cycles of load-unload was applied without any rest period in between. The average of the last 10 cycles was used to calculate $M_r$ (at a particular stress state) as the ratio of cyclic deviator stress ($\sigma_d$) to the recoverable part of the axial strain ($\varepsilon_r$). The variation of $M_r$ with $\theta$ as obtained from laboratory tests is shown in Figure 2.12. Subsequently, the regression constants $k_1 = 1345$ and, $k_2 = 0.64$ (see Equation 1) were obtained using a non-linear optimization technique by minimizing an error function between experimentally obtained and calculated values of $M_r$.

Figure 2.11 also shows a comparison of experimentally obtained and calculated values of $M_r$.

Figure 2.11: (a) Triaxial cell for repeated loading test of unbound granular materials, and (b) asphalt specimen subjected to indirect tensile test
2.3 Numerical modelling and validation

Chapter 2: Synopsis

Figure 2.12: Resilient modulus of unbound granular layer – model and experiments [based on Paper II]

Asphalt layer
The asphalt layer was modelled as a homogenous linear viscoelastic (LVE) isotropic solid with a constant Poisson’s ratio (Paper II). LVE properties were characterized from laboratory tests applying indirect tension to cylindrical specimens (IT-CY) in accordance with Annex C of the European standard DS/EN 12697-26:2018. These tests were carried out by the Swedish National Road and Transport Research Institute (VTI), and the experimental data was provided to the author. The author carried out subsequent data analysis and calibration of LVE parameters.

In these tests, cylindrical asphalt specimens that were cored from the base course asphalt layer in the mockup were loaded in compression from opposite ends consisting of 25 sequences of load-unload-rest of about 3 s duration, followed by a rest period of 1800 s (Paper II). The tests were conducted at five different temperatures: -5°C, 10°C, 15°C, 20°C and 30°C. For each test, the applied load history and change in diameter of the specimen was recorded (by two extensometers). Figure 2.11b shows the asphalt specimen subjected to indirect tensile tests at VTI.

The following equation is applicable to an isotropic LVE cylinder with a constant Poisson’s ratio and loaded in diametral compression (Levenberg and Michaeli 2013):

\[
R^{\text{VE}}(t) = F_H^e \int_{\tau=0}^{t} D(t-\tau) dP(\tau) \tag{2}
\]

where, \( R^{\text{VE}} \) is a viscoelastic response of interest, in this case it represents the change in horizontal distance between the ends of the asphalt specimen. \( F_H^e \) [units of length\(^{-1}\)] is the kernel of the elastic problem related to the chosen response and depends on the specimen geometry and Poisson’s ratio. \( D(t) \) is the creep compliance [units of stress\(^{-1}\)], \( \tau \) is a time-like integration variable, \( P \) is the applied load history.

To capture recoverable response, an analytic four-parameter function was chosen to represent LVE solid creep compliance (Smith 1971):

\[
F^e_H = \frac{1}{\eta} \left( 1 - e^{-\eta t} \right)
\]

where, \( \eta \) is the relaxation modulus [units of time\(^{-1}\)].
2.3 Numerical modelling and validation

Chapter 2: Synopsis

Equation 3 appears as a symmetric sigmoid when plotted on a log-log scale. Here, $D_0$ and $D_\infty$ are the instantaneous (short-term) and long-term (equilibrium) values of the creep compliance [units of stress\(^{-1}\)]. $\tau_D$ [units of time] and $n_D$ [unitless] are positive parameters controlling the shape of the curve between the two extreme values.

A linear dashpot model with coefficient $\eta$ [units of time $\times$ stress] was assumed for capturing viscoplastic response, i.e., irrecoverable deformation accumulating in a given sequence (Paper II). The dashpot coefficient was changed between cycles to approximately simulate a hardening type of behaviour. Including a viscoplastic component was necessary in order to isolate the irrecoverable part of the deformation and accurately calibrate the LVE parameters. Out of these parameters, $\tau_D$ and $\eta$ are temperature-dependent while the rest are independent of temperature.

For a Poisson’s ratio of 0.4, and assuming plane stress conditions, for the given specimen geometry, $F_H$ was calculated to be $16.49 \times 10^3$ mm\(^{-1}\). For each test independently, with an assumed set of values for parameters $(D_0, D_\infty, \tau_D, n_D, \eta)$, and with the applied loading history, the change in diameter of the specimen was calculated numerically using Equation 2 (Levenberg and Michaeli 2013). Five separate optimization problems were solved (each corresponding to a test temperature) to minimize an objective function (five separate ones) which was set as the sum of the squares of the residuals. At the end of this stage, five ‘ideal’ solution vectors were obtained for the parameters $(D_0, D_\infty, \tau_D, n_D, \eta)$, one vector for each of the optimization problem. It also returned the minimum value of the objective function at each test temperature.

Next, a constrained non-linear multivariable optimization was carried out based on min-max approach (Osyczka 1984) to obtain a global value of the parameters $(D_0, D_\infty, \tau_D(T), n_D)$ describing the LVE creep compliance across the five temperatures. Here, $\tau_D(T)$ denotes that it is a function of temperature $T$ while the other three parameters are not. The dashpot coefficients obtained from the previous step was used in this step. The entire problem was setup in MATLAB and solved with the help of the function $fmincon$ available in MATLAB’s optimization toolbox. The measured and calculated responses for the five temperatures are presented in Figures 2.13a to 2.13e. The LVE creep compliance parameters are provided in Table 2.2 and the creep compliance is illustrated at a reference temperature of $T_0 = 20$°C in Figure 2.13f.

For asphalt, the assumption of thermo-rheological simplicity allows time-temperature superposition, wherein the response at any temperature level $T$ can be calculated with properties obtained at a reference temperature $T_0$ by replacing physical time $t$ with reduced time $t_r$ through a shift factor $(a_T)$ expressed as:

$$t_r = \frac{t}{a_T}$$

(4)

Here, $a_T$ was calculated as:
2.3 Numerical modelling and validation

Chapter 2: Synopsis

\[ a_T = \frac{\tau_D(T)}{\tau_D(T_0)} \]  \hspace{1cm} (5)

Where \( \tau_D(T) \) and \( \tau_D(T_0) \) denotes the value of \( \tau_D \) at any temperature \( T \) and the reference temperature \( T_0 = 20^\circ C \), respectively.

Further, \( a_T \) was assumed to follow the WLF (Williams et al., 1955) equation expressed as:

\[ \log(a_T) = \frac{-c_1(T - T_0)}{-c_2 + (T - T_0)} \]  \hspace{1cm} (6)

where, \( c_1 \) [unitless] and \( c_2 \) [units of temperature] are positive constants.

Using Equations 5 and 6, \( c_1 \) was obtained as 20.17 and \( c_2 \) as 146.08 \(^\circ\)C.

At this stage, the LVE creep compliance described by Equation 3 is rewritten in a finite length Prony series format as (Park and Schapery 1999):

\[ D(t) = D_0 + \sum_{j=1}^{N} D_j \left( 1 - e^{-t/\tau_j} \right) \]  \hspace{1cm} (7)

where, \( D_j \) represents retardation strengths [units of stress\(^{-1}\)] and \( \tau_j \) represents retardation times [units of time] and are positive constants. The constants were obtained by solving a set of linear algebraic equations.

<table>
<thead>
<tr>
<th>Table 2.2: Linear viscoelastic creep compliance parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_0 ) [MPa(^{-1})]</td>
</tr>
<tr>
<td>( D_s ) [MPa(^{-1})]</td>
</tr>
<tr>
<td>( n_D ) [-]</td>
</tr>
<tr>
<td>( \tau_D ) [s]</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

The Prony series expansion of the relaxation modulus \( E(t) \) [units of stress] was obtained from the LVE creep compliance through an interconversion procedure (Park and Schapery 1999) invoking the following relationship:

\[ \overline{E}(s) \overline{D}(s) = 1 \]  \hspace{1cm} (8)

Where, \( \overline{E}(s) \) and \( \overline{D}(s) \) are the \( s \) multiplied Laplace transform of \( E(t) \) and \( D(t) \) also known as Carson transform (\( s \) is the transform variable).

The obtained relaxation modulus \( E(t) \) is expressed in a finite-length Prony series format as:

\[ E(t) = E_0 f(t) \]  \hspace{1cm} (9)

where \( E_0 \) is the instantaneous (short-term) relaxation modulus [units of stress], and \( f(t) \) is a dimensionless time function expressed as a finite-length Prony series.
2.3 Numerical modelling and validation

Chapter 2: Synopsis

\[ f(t) = 1 - \sum_{i=1}^{N} m_i \left( 1 - e^{-t/\rho_i} \right) \]  \hspace{1cm} (10)

where, \( m_i \)'s are dimensionless relaxation strengths obtained as \( m_i = E_i / E_0 \). \( \rho_i \)'s are relaxation times [units of time], and \( E_i \)'s are relaxation strengths [units of stress]. The LVE relaxation modulus is illustrated at a reference temperature \( T_0 = 20^\circ\text{C} \) in Figure 2.13.

Figure 2.13: Change in diameter of asphalt specimen in indirect tensile test: calculated and measured values at: (a) -5° C, (b) 10° C, (c) 15° C, (d) 20° C, and (e) 30° C. (f) LVE creep compliance and relaxation modulus
2.3 Numerical modelling and validation

Chapter 2: Synopsis

### 2.3.2.3. Model parameters

In the FEM model, Rayleigh damping was used for all materials except the asphalt layer for which energy dissipation mechanism was part of the basic material model. Inputs parameters (other than those presented in Section 2.3.2.2) for the different layers (from bottom-up) were obtained as: (i) Mat: Young’s modulus, density and damping ratio was obtained from manufacturer’s technical data sheet, (ii) UGL: the at-rest coefficient $K$ was obtained by best matching the mockup measurements and model calculations; the density was obtained from on-site measurements, (iii) Asphalt layer: density was obtained from on-site measurements, (iv) Geotextile: density was provided by the manufacturer; Young’s modulus was obtained by best matching the mockup measurements and model calculations (v) Sleeper: properties of concrete, (vi) Rail pad: Young’s modulus was obtained using direct calculations based on mockup measurements; Poisson’s ratio was obtained by best matching the mockup measurements and model calculations, and (vii) Poisson’s ratio, density and damping coefficient of other materials was taken based on reference ranges suggested in literature. All model parameters are listed in Tables 1 and 2 in Paper II. Further, Section 3.3 in Paper II provides a detailed description of obtaining the different model parameters.

### 2.3.3. Major findings

Before final calibration of all model parameters, assuming a representative set of values, the model was interrogated with a pulse load (see Figure A.2.1a). The sensitivity of selected model responses to temperature variation, and to the at-rest earth pressure coefficient $K$ was assessed. It was found that vertical surface accelerations, horizontal tensile strains at the bottom of the asphalt layer, and vertical compressive stresses below the UGL were all very sensitive to the temperature level. All three responses showed an increase with a rise in temperature level.

Additionally, the accelerations were significantly influenced by the value of $K$; with an increase in $K$, peak accelerations decreased. The horizontal strains at the bottom of the asphalt layer were moderately influenced by $K$, showing a similar trend. On the other hand, vertical stresses below the UGL displayed minimal sensitivity to $K$. Figure 2.14 presents shows sensitivity of horizontal tensile strains at the bottom of the asphalt layer (where the peaks are observed) to temperature variation and to the values of $K$. Further findings from the sensitivity analysis are illustrated and discussed in Paper III.

![Figure 2.14](image.png)

*Figure 2.14: Influence of: (a) temperature (for $K = 1$), and (b) $K$ (for $T=22^\circ C$) on time history of tensile strains at bottom of asphalt layer (at locations of peak strain along Z direction) [based on Paper III]*
After calibration of model parameters, they were fixed to the values given in Tables 1 and 2 in Paper II. Dynamic analysis was performed with a load signal (applied in the mockup) simulating a train passage at a speed of 120 km/h, with an axle load of 120 kN (see Figure 5, Paper II). As model parameters were not changed during this analysis, the computed responses were considered to be forward predictions. The model was validated by comparing predictions and experimental measurements. The model’s predictive ability was quantified using Pearson product-moment correlation coefficient ($r$) and an agreement metric ($\lambda$) highlighted recently by Duveiller et al. (2016).

Predicted stress history at locations of two sensors, PC 15 and PC 16 is shown in Figure 2.15. Predictions and measurements are of the same order of magnitude and agree well in terms of shape as indicated by very high values of ($r > 0.99$) and ($\lambda > 0.96$). In both cases, the model predictions overestimate the extremum values, for PC 15 by 26% (on average), and for PC 16 by 15% (on average).

Figures 2.16a and 2.16b presents the predicted horizontal strains at the bottom of the asphalt layer at locations of ASG 3 (along Z direction) and ASG 10 (along X direction). The model captures the overall shape of the strain measurements (in both directions) as indicated by high values of ($r > 0.96$) and ($\lambda > 0.90$). In Figure 2.16a, the measurements show a strain reversal from compression to tensile at the beginning of every bogie passage. Further, a time-dependent response occurs between bogies and strain accumulation occurs between consecutive axle passes on the same bogie. These features are captured reasonably well in the model predictions. In Figure 2.16b, the overall shape and magnitude is reproduced fairly well, but the model predicts more strain accumulation. In Figure 2.16c and 2.16d, the acceleration time history is presented. It is observed that the accelerations were not predicted with same level of accuracy as indicated by low values of $r = \lambda = 0.3$.

In general, the predictions and measurements showed an overall agreement and responses were of the same order of magnitude. Variations are expected between measurements and predictions due to: (i) uncertainty in laboratory characterization of materials, (ii) uncertainty in responses measured in the mockup, (iii) modelling assumptions related to interface behaviour between layers, (iv) neglecting permanent deformations, (v) assumptions of linearity and homogeneity in the model, and (vi) difference in material responses in tension and compression (Paper II).

![Figure 2.15: Vertical stresses below granular layer for a simulated train passage - measured and predicted time history [taken from Paper II]](image-url)
2.4 Longitudinal track responses

This section presents analysis of track responses to longitudinal loads. It is an extended abstract of Paper IV.

2.4.1. Motivation and background

An essential step in accepting new track concepts, before adaptation within the live network, is the analysis of track responses to anticipated loads. As braking forces are part of such an assessment, this study aims to analyze track responses to longitudinal forces induced by braking. This is considered particularly crucial in the context of ballastless asphalt tracks, in which sleepers may rest on top of the asphalt pavement without any anchorage to the pavement or support from surrounding crib or shoulder ballast. In this situation, longitudinal forces at the sleeper – pavement interface are entirely negated by the sliding resistance available at the sleeper bases; which may be insufficient resulting in slippage. Besides, the analysis is considered necessary in the light of advances in the vehicle design itself, in which stronger brakes, such as ‘track brakes’, have been introduced that enable trains to exert higher braking forces directly onto
the rails. This is different from the traditional way of slowing down or halting the train by clamping the wheels.

The effects of both vertical and longitudinal loads must be considered to evaluate track responses due to braking loads. While there are many studies examining track responses to vertical loads, the literature on longitudinal track response analysis is minimal. Van (1997) numerically studied thermal loads and mechanical loads induced by train braking and accelerating on rails and bridges. The developed model idealized the rails as a single beam resting on longitudinal elasto-plastic springs. However, the study did not analyze responses at the individual sleeper locations; also, the interaction between vertical and longitudinal directions was not accounted for. A FEM model was developed to study longitudinal forces in the fastening system and friction demand at rail-rail pad interface during train acceleration (Zhang et al., 2015). It was reported that longitudinal forces were non-symmetrically distributed between sleepers. Longitudinal force in rail-rail pad interface in front of the wheel was negligible, and five sleepers located behind the wheel mainly carried the loads. However, no explanation was provided on the reason for non-symmetrical load distribution.

The objective of this study is to develop a mechanical model within an analytical framework for analyzing track responses during train braking, considering the effects of both vertical and longitudinal loads. The aim is to present closed-form solutions for computing (i) axial rail stresses, (ii) vertical and longitudinal forces at a sleeper base, and (iii) friction demand at a sleeper base.

2.4.2. Methodology

A 2D response model was developed (see Figure 2.17), focusing on loads that are generated during the braking of a train axle. The model considered one axle side, i.e., a decelerating wheel. The rail was idealized as a homogenous infinite Euler-Bernoulli beam. It was supported by an orthogonal Winkler-type of foundation consisting of a bed of linear springs along the vertical and longitudinal directions, at right angles to each other. These springs represented the collective support offered to the rail by all underlying track components. The spring constants in the vertical and longitudinal direction were $k_Z$ and $k_X$ [units of force/length/length] respectively. The wheel loads were simulated as a collection of two point loads applied to the beam. The vertical load $P_Z$ accounted for half of the axle weight, and the longitudinal load $P_X$ signified the braking effort of a wheel. The weight per unit length of the beam comprised of the rail weight per unit length and the distributed sleeper weight considering one-half of it.

A randomly chosen sleeper within the braking zone was evaluated, with its position coinciding with the origin of the Cartesian coordinate system. Closed-form expressions were derived for calculating different responses at the origin (i.e., the sleeper being evaluated) as a function of the load position $x$ from the coordinate origin. These responses included: (i) vertical beam displacement (ii) vertical force at the base of a sleeper, (iii) longitudinal beam displacement, (iv) longitudinal force at the base of a sleeper, and (v) friction demand at the base of a sleeper. The friction demand represents the required resistance at the sleeper base to ensure no-slip conditions during a braking event.

By means of superposition, the above expressions were extended to analyze the braking event of a full train. In this case, the vertical wheel loads were idealized as a sequence of point loads applied to the beam with a distance corresponding to the axle position. The braking event was simulated as the concurrent application of longitudinal point loads at the same locations.
2.4 Longitudinal track responses

Closed-form expressions were also derived to address modern braking technologies considering distributed loads over a certain length of the rail.

![Figure 2.17: An orthogonal Winkler foundation model for simulating braking of a single wheel [taken from Paper IV]](image)

2.4.3. Major findings

At first, the model was applied for analyzing braking of a single axle. Responses were calculated at the chosen sleeper location for different positions of the load, i.e., values of $x$. This was investigated parametrically by varying the values of $k_z$ and $k_x$, while adopting realistic values for the other model parameters. The dependence of peak friction demand on the spring constants was further investigated for a standard sleeper type as well as a heavier one.

Subsequently, the model was demonstrated for braking event of a full train. The train chosen for this purpose was Thalys HST with a length of 200 m and consisting of 26 axles (Degrande and Schillemans 2001). The vertical axle loads were considered as 170 kN (for the locomotives and central carriages), and 145 kN (for outer carriages). The longitudinal loads were taken to be 25% of the vertical loads. The braking event was simulated as the concurrent application of 26 force pairs (vertical and longitudinal) acting at the individual axle locations. The values of spring constants in vertical and longitudinal direction were taken to be 50 MPa and 15 MPa, respectively. A standard sleeper type was chosen with a mass of 290 kg and a spacing of 0.6 m. All responses were evaluated at the coordinate origin for different values of $x$, taken to be the distance between the rear axle of the train and the sleeper being evaluated (i.e. the coordinate origin).

Figure 2.18 illustrates the outcomes of this investigation. It consists of four curves, the vertical and longitudinal force at a sleeper base is shown in Figures 2.18a and 2.18b, respectively. Figure 2.18c shows the longitudinal beam stress and Figure 2.18d presents the friction demand (ratio between longitudinal and vertical forces) at sleeper base. The dashed lines in each figure refer to the peak values for a braking event of a single axle. From this figure, it can be observed that longitudinal responses (forces and friction demand at sleeper base and beam stresses) exhibit a much wider influence zone compared to vertical responses. In comparison to single axle analysis, the longitudinal responses for a full train were significantly higher, indicating prominent load interaction between axles. On the contrary, the vertical responses for a single axle and full train were quite similar. The values of friction demand at the sleeper base is unrealistically high to be provided by a standard sleeper.

Based on these observations, it can be inferred that the results of a single axle analysis cannot be used to obtain longitudinal track responses for a full train. It has to be dealt on a case by case basis with the specific axle configuration of the train being analysed. Furthermore, for track types such as ballastless asphalt tracks, if sliding resistance of a sleeper is solely derived...
from frictional mechanisms at the base then standard sleeper types are inadequate; heavier or non-standard sleeper types need to be employed.

![Figure 2.18: Model responses for a full train as a function of the distance from rear axle: (a) vertical force at the base of a sleeper, (b) longitudinal force at the base of a sleeper, (c) longitudinal beam stress, and (d) friction demand at the base of a sleeper [taken from Paper IV]](image)

### 2.5. A priori estimation of track modulus

This section introduces a method for a priori estimation of track modulus. It is presented in the form of a summary of Paper V.

#### 2.5.1. Motivation and background

A standard approach to modelling railway tracks involves idealizing the rails as infinite beams supported on a continuous spring foundation. The parameter track modulus characterizes the foundation; it is a measure of the continuous support offered to the rail by all underlying components such as rail pads, sleepers, under sleeper pads, ballast mats and all soil-like materials like ballast, subballast, and subgrade. It is defined as the supporting force per unit rail length per unit vertical rail displacement. Track modulus is used as an input parameter to calculate rail deflections and bending stresses, longitudinal distribution of wheel loads along the rail. It is an accepted indicator of track quality.

Various studies have focused on assessing track modulus from field measurements. There are, however, very few studies that offered a model for calculating track modulus a priori,
2.5 A priori estimation of track modulus

Chapter 2: Synopsis

using the material properties of the components and layers that underlie a rail. This is deemed necessary both in the new design of traditional railways and also for evaluating the performance potential of new track concepts. GEOTRACK (Chang et al., 1980) was an elaborate 3D track model that included all major track components and layers and could be applied for estimating track modulus. This was done by equating maximal vertical rail displacement in GEOTRACK program and in the standard track model of an infinite beam on springs (under identical loading conditions). The issue with this method was the incompatibility of vertical load distribution along the rail at different sleeper locations. Cai et al. (1994) offered a model for calculating track modulus based on analysis of a single sleeper idealized as a finite beam resting on a Winkler foundation. Concentrated loads were applied at rail seats, and track modulus was calculated based on the displacement of the finite beam under the loads after adding the displacement due to the rail pads. The primary limitation of this study was disregarding the effects of adjacent sleepers which leads to overestimation of the values of track modulus.

Consequently, it is the aim of this study to develop a method for a priori determination of track modulus based on elastic solutions. With a focus on the possible acceptance and long-term utility of the work, the emphasis has been given on developing solutions that are analytical and closed-form. The proposed method aims to determine track modulus by establishing a solid connection with the standard track model by considering equal maximal rail displacement while ensuring compatibility of vertical force distribution along the rails.

2.5.2. Methodology

Figure 2.19a shows the standard model of an infinite beam (IB) on springs characterized by a track modulus $k_Z$ and subjected to concentrated forces of intensity $P_Z$. In order to calculate $k_Z$, a 3D linear elastic quasi-static track model was developed (see Figure 2.19b). It included: (i) an array of equidistant sleepers modelled as finite Euler-Bernoulli beams (abbreviated as FB), (ii) two rail pads on each sleeper modelled as a pair of discrete springs (DS), and the substructure representing all soil-like materials, modelled as an isotropic homogenous half-space (HS).

As a starting point in the analysis, in the standard track model, $k_Z$ was assumed, and the reaction forces in the springs were calculated after converting continuous springs to individual ones conforming to sleeper positions in the 3D model. Following this, the discrete springs in the 3D model were each loaded with the respective force intensities $S_Z^{\text{FB}}$ obtained from the standard track model. The overall vertical displacement at the top of the DS $(n = 0)$, i.e., above the central FB was calculated. This was a result of the compression of the DS itself as well as the surface deflection of the half-space (assuming finite beams were incompressible) directly below it, considering the loading on all adjacent FBs. The representative track modulus $k_Z^*$ was obtained iteratively by matching the overall displacement calculated at the top of the DS $(n = 0)$ with the maximal displacement of the infinite beam in the standard model. Thus, track modulus was calculated based on a strong connection between the 3D model and the standard track model accounting for maximal rail displacement while ensuring compatibility in the load distribution along the rail.

As an intermediate step, it was required to resolve the contact stress distribution at the interface of HS-FB to compute the vertical displacement of the HS. For that purpose, two closed-form solutions were established, (i) vertical displacement of a finite beam subjected to two line-loads and a non-uniform support reaction, and (ii) surface deflection of a half-space loaded
uniformly over a rectangular area. The two solutions were combined to analyze the contact stresses at the interface and subsequently calculate the surface deflection of the HS.

![Figure 2.19: A sketch depicting the models considered for track modulus determination (a) standard model of infinite beam on continuous springs, and (b) elastic 3D model [taken from Paper V]](image)

2.5.3. Major findings

The method was illustrated by a parametric study that investigated the sensitivity of track modulus to (i) stiffness of discrete springs (i.e. rail pads) $K_{Z,DS}$, (ii) half-space modulus $E_{HS}$, and (iii) spacing between finite beams (i.e., sleepers) $s$. All other model parameters were fixed to a reference set of values. The findings of the study are illustrated in Figure 2.20. There are two charts in this figure corresponding to two different values of $s$, 0.6 m, and 0.5 m. In both the charts, five different discrete spring stiffnesses are considered varying from 25 MN/m to 250 MN/m. The elastic modulus of the half-space has been varied up to 300 MPa.

From the figures, it is observed that track modulus increased with an increase in half-space elastic modulus but not indefinitely; it had an upper bound controlled by the rail-pad stiffness. This is because the surface deflection of a half-space with a very high Young’s modulus is negligible in relation to rail-pad shortening and therefore the track modulus for these cases is controlled solely by $K_{Z,DS}$, i.e., rail pad stiffness and finite beam spacing. When comparing the track modulus values in Figures 2.20a and 2.20b, it is observed that track modulus increases with a decrease in the finite beam spacing.

It was further investigated that when contact stress was approximated to be uniform at FB-HS interface, then track modulus values were only marginally overestimated. Consequently, an assumption of uniform stresses did not have a significant effect on the calculation of track...
modulus. The obtained results were considered valid as they were similar in magnitude and exhibited similar trends to the variation in the input parameters when compared to results from GEOTRACK.

Figure 2.20: Track modulus determined as a function of half-space modulus for different rail pad stiffnesses considering (a) $s = 0.6$ m, and (b) $s = 0.5$ m [taken from Paper V]
Chapter 3

Summary and conclusions

This chapter is based on (includes reuse of text) the following scientific articles:


3.1. Summary and findings

This study analyzed mechanical responses in ballastless asphalt tracks under vertical loads through laboratory testing of a mockup and subsequent, development and validation of a numerical model of the mockup. To this end, a full-scale mockup (4.00 m long and 2.21 m wide) was constructed inside a large steel container consisting of asphalt layer, underlain by an unbound granular layer, and supported by a thin mat (that simulated subbase and subgrade). The mockup was built in an outdoor location and later transported to an indoor laboratory environment. This allowed full-scale construction equipment to be used, not scaled down.

Embedded sensors (installed during construction) and surface sensors (mounted after-construction) were used to measure different types of mechanical responses. Embedded sensors were installed to measure the vertical stresses below the UGL (by pressure cells) and horizontal strains at the bottom of the asphalt layer (by asphalt strain gauges). The embedded sensors were at risk of being damaged due to exposure to extreme loading conditions during the construction of top layers. In order for them to survive, they were adequately protected during installation, higher pressure ratings were selected relative to the estimated operating range (for pressure cells), and 50% redundancy was considered. The surface sensors were installed to measure rail pad compression, the relative displacement between sleeper and asphalt surface, and vertical accelerations on the surface.

Three sleepers were mounted on the asphalt pavement at a standard spacing of 0.6 m. The sleepers were wider at the base, heavier and included a geotextile on the underside. The sleepers were loaded directly by using servo-hydraulic actuators. The mockup was subjected to different types of vertical loads such as a ramp, pulse, and cyclic (different excitation amplitudes and frequencies). Trains passages with different speeds (maximum 200 km/h) were simulated in
the mockup (maximum axle load 200 kN) by sequentially loading the three sleepers. Post-
processing of various sensor data showed all pressure cells to be functional and 30% of the asphalt
strain gauges to be damaged during construction. Further, the pressure cells exhibited reliable
behaviour across all tests, although the working range was very low (not exceeding 100 kPa)
relative to the operating range (1 MPa). In contrast, the asphalt strain gauges showed somewhat
irregular recordings. Moreover, the measured responses were of very low magnitude, almost
within the precision limits of the gauges. Hence, including redundancy in the mockup (by about
50%) was particularly helpful for the strains.

Next, a detailed 3D FEM model of the mockup, including all major components and
layers, was developed. The model considered non-linear stress-dependent elasticity of the UGL
which was successfully implemented with a user-defined subroutine USDFLD. The time-
temperature dependency of asphalt layer was incorporated using linear viscoelasticity theory
assuming thermorheological simplicity. All other materials were treated as linear elastic. Majority
of model parameters were obtained from laboratory elements tests and independent sources. The
specific intention was to minimize calibration activities, i.e., deriving the parameters through a
model-fitting approach. Implicit dynamic analysis was carried out to consider the effect of inertia
on mechanical responses. The three sleepers in the model were sequentially loaded with a force
history that simulated a train passage. The model was validated by comparing the numerical
predictions with the experimental measurements. Predictions and measurements showed an
overall agreement in terms of shape and behaviour and responses were of the same order of
magnitude. It was possible to predict stress-strain history more accurately compared to the
accelerations.

Further findings from the experimental and numerical investigation are summarized
here. The ballastless asphalt track mockup demonstrated frequency-dependent and non-linear
behaviour under applications of cyclic loads of different frequencies and amplitudes. While the
rail pad compression exhibited moderate frequency dependency; the relative vertical
displacement between the asphalt surface and the sleeper, the vertical stresses below the UGL and
the horizontal strains below asphalt layer showed a higher frequency dependence. Further, the rail
pad compression and horizontal strains below the asphalt layer were non-linear to a minor extent.
The vertical stresses below UGL varied linearly with the load amplitude, while the relative
displacement between the sleeper and the asphalt surface was strongly non-linear.

Time-dependent behaviour was demonstrated by the horizontal strains at the bottom of the
asphalt layer and the relative vertical displacement between the sleeper and asphalt surface.
On the contrary, rail pad compression was nearly elastic, showing instantaneous recovery to initial
conditions (without noticeable time effects) on unloading. Similarly, no significant time effects
were noted in vertical stresses below the UGL.

All track responses were highly sensitive to temperature, namely the horizontal asphalt
strains, vertical surface accelerations, and vertical stresses below the UGL. Further, the horizontal
asphalt strains and vertical surface accelerations exhibited sensitivity to initial horizontal stresses
(induced during construction) within the UGL.

For the simulation of train passages up to speeds of 200 km/h and axle load of 200 kN,
peak horizontal tensile strains at the bottom of the asphalt layer were found to be less than 50 µε,
which is well below the flexural fatigue endurance limit of the material (DS / EN 16432-2: 2017). The
strains along the direction of the moving load showed sign reversal which was not observed
along the other direction. Minor strain accumulation was demonstrated between successive axle
passes. For the same loading scheme, peak vertical stresses below the UGL varied between 15
kPa to 40 kPa, representing approximately 60% of the calculated average stress below a sleeper.
The peak stress magnitudes are low, implying compliance with the standard requirements (DS / EN 16432-2: 2017). Peak vertical accelerations measured on the centre line of the track was less than 2 m/s², which was lower than ballasted tracks where comparable values were reported at a much larger distance, approximately 3 m away from the centre line of the track, (Galvín et al., 2009). Except, for vertical surface accelerations, no noticeable effect was observed on other track responses with an increase in simulated train speed from 120 km/h to 200 km/h.

While the previous study focused on response under vertical loads, a further study was conducted to evaluate response under longitudinal loads caused by train braking. Within an analytical framework, a 2D model was developed idealizing rail as an infinite beam resting on an orthogonal Winkler foundation consisting of a continuous bed of springs in both vertical and longitudinal directions. The formulation was general-purpose; it was applicable for different track types. Braking event was simulated as the simultaneous application of concentrated loads in the vertical and longitudinal direction at locations of the train axles. Closed-form expressing were derived for calculating different track responses. Subsequently, for a range of realistic parameters, the model results were presented for a single axle and then a full train.

It was found that the maximum longitudinal rail stress was independent of the supporting foundation spring constant, but its decay was a function of the latter. Longitudinal track responses were distributed over a much larger zone compared to vertical track responses. Peak friction demand was unrealistically high for a standard sleeper weight and within acceptable limits for a heavier sleeper (such as the one used in the mockup). The full-train analysis showed a significant increase in all responses in the longitudinal direction compared to the single axle case. This result was due to a pronounced load interaction in the longitudinal direction. On the contrary, vertical track responses for a single axle and a full train were nearly similar.

Finally, a limited side study was conducted to suggest a method for a priori estimation of track modulus. A semi-analytical approach was developed using elasticity solutions, capable of addressing different types of tracks. To this end, a 3D linear elastic track model was built that idealized sleepers as finite beams, rail pads as linear springs and all soil-like materials as a homogenous half-space. Track modulus was calculated by achieving equal maximal vertical rail displacement in the two models, while, preserving compatibility of load distribution along the rail. The method was demonstrated to show the sensitivity of track modulus to rail pad stiffness, half-space elastic modulus and sleeper spacing. Other model parameters were not found to have any significant influence on track modulus. Furthermore, it was demonstrated that for a high elastic modulus of half-space, track modulus is solely governed by rail pad stiffness and sleeper spacing.

### 3.2. Conclusions

The findings of the study can be generalized to arrive at the following conclusions:

i. The majority (75 %) of vertical rail displacement in the mockup could be attributed to the rail pad compression. The geotextile compression constituted the remaining part; the displacement resulting from the three mockup layers (asphalt + UGL + mat) was negligible in comparison. Moreover, the geotextile by its compressibility played a role in bridging texture and construction unevenness between sleeper underside and asphalt surface.

ii. The measured mechanical responses (stresses and strains) were of very low magnitude, indicating little or no mechanical damage. This should also be considered for material
characterization of track layers using laboratory standard testing methods which operate at a much higher range.

iii. Ballastless asphalt tracks showed non-linearity and time-temperature dependency, hence; modelling efforts must consider them.

iv. An instrumented mockup with measurements from the surface and embedded sensors provided means for calibrating different model parameters and its subsequent validation. Specifically, measurements of rail pad compression allowed for obtaining realistic stiffness at the applied load amplitude and frequency. Measurements of relative displacement between asphalt surface and sleeper allowed for calibration of elastic properties of the geotextile; the latter can also be characterized from laboratory tests. Initial horizontal stresses induced in the UGL due to construction cannot be obtained from mockup measurements and has to be calibrated using a model.

v. For the simulation of recoverable responses, only the resilient properties of the materials must be used as inputs to the model. The laboratory element tests of the asphalt layer showed significant permanent deformation, especially at higher temperatures. For meaningful characterization of viscoelastic parameters of asphalt, the irrecoverable part of the response must be carefully separated.

vi. In the context of longitudinal loads, responses for a full train cannot be deduced from a single axle analysis; estimates must be made on a case-by-case basis with specific axle configurations, as all resulting peak reactions are anticipated to be higher. For track types in which friction resulting from the sleeper weight is solely responsible for providing sliding resistance at the sleeper base, a heavier sleeper type may be needed.

3.3 Future work

As part of future research, resilient track responses can be studied in the mockup for other types of sleepers with different fasteners and interface layers between the sleeper and asphalt surface (such as the geotextile used in the study). The responses to horizontal loads can be analyzed by employing a different loading and instrumentation scheme. Further experimental investigations are needed towards asphalt mix design explicitly tailored for railway infrastructure. As a natural progression, field conditions can be investigated at a later stage. This will have the benefit of assessing responses in the live network under as-built conditions, including the subsoil. Test data will allow comparison of responses between ballastless asphalt tracks and other traditional track types.

The validated model can be further used to analyze other loading conditions and optimize layer thicknesses and material properties. Future modelling efforts will be aimed at simulating field-like conditions by scaling up the current model size and replacing the mat to include the subsoil domain by utilizing special elements and boundary conditions. Incorporation of a rail will additionally permit to include vehicle dynamics and analyze near and far-field vibrations. Horizontal track responses can be investigated numerically after incorporating interface properties. In this context, standard sleeper geometries can be analyzed, and responses can be contrasted with heavier sleeper types. Moreover, the geotextile plays an important role in the overall track response; laboratory element tests can be carried out to characterize its behaviour and implement it in the FEM model.

The model developed for analyzing longitudinal track responses can be further improved by considering the coupling of responses in the two directions, and including dynamic effects in the formulation. Moreover, it can be employed to study modern braking loads, and
effects of vertical track gradient. While the current study focussed on longitudinal loads due to braking, the model can be developed and extended to study other types of horizontal loads caused by the vehicle, or from temperature effects. Results of such studies will enable us among other things to optimize sleeper dimensions and spacing, interface layer selection, to design anchoring mechanism or other mechanism contributing to horizontal stability.

In future studies, the method proposed for a priori estimation of the track modulus may be extended to evaluate track modulus in the lateral and longitudinal directions. In addition, the modelling approach can be enhanced by the elastic-viscoelastic correspondence principle (e.g. Schapery, 1965) to include time-dependence and thermal sensitivity of components and materials. This will allow a priori identification of track modulus for asphalt solutions for different train speeds and temperatures.

### 3.4. Thesis contributions

The novel contributions of this thesis include:

1. Analysis of diverse mechanical responses in a ballastless asphalt track mockup under service conditions by simulating train passages at a maximum speed of 200 km/h and with a maximum axle load of 200 kN.
2. Investigating effects of different loading amplitudes and frequencies on mechanical responses in ballastless asphalt tracks.
3. Development and validation of an elaborate 3D dynamic finite element model of a ballastless asphalt track mockup including all major components and layers, and considering material non-linearity and time-temperature dependency.
5. Proposing a semi-analytical method for a priori estimation of track modulus using elasticity solutions.
References


Dansk Standard DS/EN 933-1: 2013: Methods for testing the geometrical properties of aggregates - Part 1: Determination of grain size distribution - Sieve analysis


Dansk Standard DS/EN 1097-2: 2010: Methods for testing the mechanical and physical properties of aggregates - Part 2: Methods for determining crush resistance


MATLAB. Version 7.10.0 (R2019a). Natick, Massachusetts: The MathWorks Inc.


http://vibratec.dk/products/vibration-990-plus/


https://www.railone.de/fileadmin/daten/05-presse-mediendownloads/broschueren/de/GETRAC.pdf
Appendix A: Additional test results
A.1 Ramp load

Figure A.1.1: Measured responses for ramp loading. (a) Vertical force applied by Actuator 2, (b) force displacement curve of Actuator 2, (c) potentiometer readings, and (d) LVDT readings
A.1 Ramp load

Figure A.1.2: Measured responses for ramp loading. (a) - (b) asphalt strain gauge readings, and (c) pressure cell readings
A.2 Pulse load

Figure A.2.1: Measured responses for pulse loading. (a) Vertical force applied by Actuator 2, (b) force displacement curve of Actuator 2, (c) potentiometer readings, and (d) LVDT readings.
A.2 Pulse load

Figure A.2.2: Measured responses for pulse loading, (a) - (b) asphalt strain gauge readings, and (c) pressure cell readings
A.3 Simulated train passage

Axle load = 120 kN, Speed = 120 km/h

Figure A.3.1: Measured responses for simulated train passage. (a) Vertical force applied by Actuator 1, (b) Vertical force applied by Actuator 3, (c) potentiometer readings, and (d) LVDT readings
Axle load = 120 kN, Speed = 120 km/h

![Graphs showing asphalt strain gauge readings and pressure cell readings for simulated train passage.](image)

Figure A.3.2: Measured responses for simulated train passage. (a) - (b) asphalt strain gauge readings, and (c) – (d) pressure cell readings
Axle load = 200 kN, Speed = 120 km/h

Figure A.3.3: Measured responses for simulated train passage. (a) Vertical force applied by Actuator 1, (b) Vertical force applied by Actuator 3, (c) potentiometer readings, and (d) LVDT readings
Axle load = 200 kN, Speed = 120 km/h

Figure A.3.4: Measured responses for simulated train passage. (a) - (b) asphalt strain gauge readings, and (c) – (d) pressure cell readings
Axle load = 120 kN, Speed = 200 km/h

Figure A.3.5: Measured responses for simulated train passage. (a) Vertical force applied by Actuator 1, (b) Vertical force applied by Actuator 2, (c) Vertical force applied by Actuator 3, (d) potentiometer readings, and (e) LVDT readings
Axle load = 120 kN, Speed = 200 km/h

Figure A.3.6: Measured responses for simulated train passage. (a) - (b) asphalt strain gauge readings, and (c) – (d) pressure cell readings
Axle load = 200 kN, Speed = 200 km/h

Figure A.3.7: Measured responses for simulated train passage. (a) Vertical force applied by Actuator 1, (b) Vertical force applied by Actuator 2, (c) Vertical force applied by Actuator 3, (d) potentiometer readings, and (e) LVDT readings
Axle load = 200 kN, Speed = 200 km/h

Figure A.3.8: Measured responses for simulated train passage. (a) - (b) asphalt strain gauge readings, and (c) – (d) pressure cell readings
Part B: Appended papers
Experimental Investigation of a Ballastless Track based on Asphalt

Tulika Bose
Department of Civil Engineering
Technical University of Denmark
Nordvej, Building 119
Kgs. Lyngby 2800, Denmark
Email: tulbo@byg.dtu.dk

Varvara Zania (Corresponding author)
Department of Civil Engineering
Technical University of Denmark
Nordvej, Building 119
Kgs. Lyngby 2800, Denmark
Email: vaza@byg.dtu.dk

Eyal Levenberg
Department of Civil Engineering
Technical University of Denmark
Nordvej, Building 119
Kgs. Lyngby 2800, Denmark
Email: eylev@byg.dtu.dk
ABSTRACT

This study presents experimental results from a laboratory investigation into the mechanical behaviour of a ballastless asphalt track under vertical loads. A full-scale test section of the track was constructed inside a steel box, consisting of three wide-base sleepers resting on an asphalt layer that was underlain by an unbound granular layer supported on a rubber mat (representing subbase and subgrade). Cyclic loads were applied to investigate the effects of different excitation amplitudes and frequencies. Also, train passages were simulated with axle loads up to 200 kN and a maximum speed of 200 km/h; this was done by sequentially loading the three sleepers. Resulting responses were recorded, consisting of stresses at the bottom of the unbound granular layer, strains at the bottom of the asphalt layer, relative displacements between various track components, and surface accelerations. From the investigation, it is found that the ballastless asphalt tracks exhibit both time-dependent and non-linear behaviour. Moreover, it is observed that stresses and strains are of very low intensity, suggesting very little mechanical damage under service loads.

Keywords: Asphalt pavement; Ballastless asphalt track; Cyclic loading; Full-scale laboratory testing; Railroad testing; Simulated train passage; Track vibration.

1. Introduction

There are increasing demands for faster and heavier trains and subsequently the expansion in railway line capacity, because of the rise in passenger and freight-railway traffic volume. Traditional ballasted tracks are reaching their performance requirements, owing to ballast breakage and deterioration and, in turn, frequent maintenance activities [1, 2]. Consequently, ballastless tracks have emerged, primarily concrete slab-tracks, which are associated with high initial costs, amplified noise and vibration levels, and difficulties in accommodating maintenance activities even though they provide improved support conditions [1, 3]. Recently, an alternative ballastless track-type based on asphalt concrete (referred to as asphalt in the text) has attracted research and industry attention, as it could combine the advantages of both ballasted and slab tracks [4]. Asphalt is a well-researched material that is traditionally used for construction purposes within the transport infrastructure, but its application in the rail sector has so far been limited.

The current practice in railway construction regarding the use of asphalt in ballasted tracks is primarily as a subballast layer [5-7], while in ballastless tracks the functionality of asphalt is as a secondary support for the concrete slab tracks and as a waterproofing layer [8-11]. Only a few occasions were identified, mainly in the German railway industry, wherein ballastless tracks were constructed with asphalt as the uppermost support layer (overlayment)
These tracks were built mostly in tunnels, with standard highway asphalt and specially designed sleepers, which were wider, heavier, and included at the bottom a geotextile as well as an anchor block.

The mechanical response of railway tracks has been extensively investigated and researched for ballasted and slab tracks. Simplistic models were initially developed to analyse the track behaviour under moving loads considering beam on spring foundation or halfspace, either elastic or viscoelastic [15-19]. Thereafter, more elaborate and advanced models with detailed geometry, material behaviour and loading conditions were suggested aiming for a more precise estimation of the dynamic track response and ground borne vibrations [20-23]. This progression was made possible through a better understanding of the track behaviour by conducting small-scale element tests [24-27] and small-scale and full-scale mockup tests [28-36]. Field investigations under live loads have the advantage of evaluating as built track conditions, including the response of the subsoil and allowing for the assessment of ground borne vibrations and noise [37-42]. Nevertheless, testing new materials or design concepts in the live railway network requires extensive certification. In this context, full-scale mockup tests can be particularly useful as they are relatively cost efficient compared to field testing, and provide insight into the functionality of the various components under different loading conditions, while in a controlled environment. In addition, the capability to simulate the effects of a train passage in a limited test section [30, 33, 35] has also been achieved within reasonable precision, allowing the railway industry to obtain estimates on the pattern and magnitude of the expected responses in the live network. Finally, such testing could help to advance modelling efforts and increase their reliability.

One of the early studies to assess the behaviour of asphalt overlayment systems, developed a 3D program (KENTRACK) [43], combining FEM and multilayered elasticity theory. It was used to design the service life of asphalt based tracks. The model included rails and sleepers (beam), fasteners (springs) and a two-layered track structure consisting of asphalt layers and subgrade. Design charts have been developed using the maximum tensile strains at the bottom of asphalt and the maximum compressive stresses on top of subgrade as criteria to estimate the allowable number of load repetitions. The estimations were based on damage laws developed for highway pavements, and it was concluded that the required thickness of the overlayment was governed by fatigue cracking, ranging between 250 mm to 400 mm.

In recent years, full-scale test sections (asphalt overlayment) have been built to evaluate the performance of asphalt based railway tracks [4, 44]. The construction of the test section was as per Korean high speed railway specifications with a focus on designing the overall thickness
of the asphalt layer under static loads (maximum of 200 kN). The experimental facility was constructed with five sleepers that had a wide base and a geotextile at the underside, overlain with a continuous rail-fastener assembly. Measurements of vertical stresses in the base and subgrade layers, transverse strains at the bottom of asphalt layers, and vertical displacements on top of the rails, sleepers and base layers were reported. The measured responses showed a non-linear trend with an increase in the load level, with negligible hysteresis between the loading-unloading curves in the stresses but noticeable hysteresis in the strain and displacement measurements. Based on this study, a design recommendation of 0.30 m asphalt layer thickness was suggested, complying with the allowable stress requirement (133 kPa) at the top of the base layer. The long term settlement of this proposed design was investigated [45] on a level and sloped track under moving wheel loads, approximated as sinusoidal loads, (frequency of 8 Hz and 7 Hz for the level and sloped track respectively) applied to the rails for 2 to 3 million consecutive cycles. The loading frequency was based on the wheel-to-wheel distance and simulated train speed but the methodology used to simulate the passage of a train in the test facility was not explained. The differential settlement between the rail and sleeper was reported to be small, indicating an elastic behaviour of the rail pads, while, that between the sleeper and the asphalt trackbed was quite large, which was attributed to the compression of the geotextile and the track layers.

Field studies were also conducted [46] in a test track of length 207 m that was built outdoors in the live network and monitored for one year. The locomotive operating on this line had an average speed and axle load of 70 km/h and 200 kN respectively. The study reported the average temperature in the bottom of asphalt layer during the winter and summer to be around 0 and 33 degrees Celsius respectively. Vertical stresses in the base layer and horizontal strains in the asphalt layer were influenced by temperature, as was expected, increasing significantly in the summer season compared to winter. However, the loading conditions under which the peak responses were measured were not mentioned. As a part of this study, a 3D FE model was also developed, consisting of a pair of rails (beam elements), fasteners (springs), sleepers and a multi-layered track structure (solid 3D elements). Quasi-static analysis was carried out by assuming wheels loads to be acting as distributed loads on each rail. All the layers were modelled as linear elastic, except for the asphalt layers, which were also modelled as linear viscoelastic. However, it was not clear from the study, whether the material properties used in the model were calibrated separately from independent tests or obtained by fitting with the field data. Selected mechanical responses from the numerical simulations were compared with the measurements, in which the asphalt strains showed a poor match.
This study derives motivation from the limited research available on ballastless asphalt tracks, as the understanding of mechanical behaviour can be strengthened by testing full-scale mockups, and further, developing advanced models by translating the know-how available in the context of the more traditional trackforms.

The main objective of this study is to experimentally investigate the mechanical behaviour of a ballastless asphalt track under vertical loads. As a starting point, a full-scale mockup is constructed and tested indoors. The focus of this study is on investigating the responses of the different track components without impairing the integrity of the system. Service behaviour under nominally pristine conditions is targeted, and thus design limit states are not investigated as in existing studies. First described are the construction and instrumentation of the mockup. Presented next are measured results across different loading conditions. Finally, observed behaviours of general interest are highlighted and discussed.

2. Ballastless asphalt track mockup

2.1 Construction

A full-scale mockup of a ballastless asphalt track was constructed having a length of 4.00 m and width of 2.21 m. From bottom-up, the track layers included, (i) a 0.025 m thick polyurethane mat utilized as a substitute for subgrade and the subbase [47], (ii) a 0.275 m thick unbound granular layer, and (iii) a 0.280 m thick asphalt layer. The different layers were built inside a steel box in an outdoor location after which the box was craned inside a test hall and placed on the thick concrete floor. In order to ensure full contact between the bottom of the box and the floor, a wet mix of cement mortar was spread prior to the box placement. A schematic view of the test facility is shown in Figure 1, and the different stages involved in the construction of the mockup are shown in Figure 2.

The selection of the polyurethane mat was based on guidelines [48] that specified a minimum value of modulus of deformation ($E_{v2}$) of 120 MPa on top of the subbase layer for a newly constructed track. The static elastic modulus of the mat based on the technical specification data provided by the manufacturer was 20 MPa. Light Weight Deflectometer (LWD) tests were performed to characterize the stiffness of the mat inside the box (see Figure 2a), which resulted in dynamic deflection modulus ($E_{vd}$) between 40 MPa and 50 MPa. The variation obtained is attributed to the boundary conditions of the test, as the thickness of the mat does not comply with the homogeneous half space assumption applied for the estimation of $E_{vd}$. The unbound granular layer constructed on top of the mat was categorized as Danish type SGII [49] with a maximum aggregate size of 31.5 mm and fine content of 3.7 %. The base
layer was compacted in two lifts using multiple passes of a vibratory tandem roller (see Figure 2b). The achieved dry density was estimated based on in-situ measurements at three different locations with a nuclear moisture density gauge as 98.1 % on average, relative to a vibrating table reference compaction [50]. The asphalt concrete layer constructed on top of the unbound granular layer (see Figure 2c and 2d) consisted of three identical base course lifts with a total thickness of 0.24 m, and a surface course with a lift thickness of 0.04 m. The base course lifts were coarse graded with a maximum aggregate size of 16 mm (Danish GAB 0 type 16). They included 5.1 % of unmodified binder graded as 70/100. The surface lift was a stone mastic asphalt (Danish SMA 11), characterized by an S shaped aggregate gradation curve with a maximum size of 11 mm. This mix included a styrene based polymer added to 70/100 bitumen (during asphalt production) with binder content of 5.4 %. The void content was estimated for the base course and the surface course as 3.1 % and 2.7 % (average values from 8 samples) respectively, which complies with the EN 16432-2 requirements [48].

Three wide concrete sleepers were placed on top of the asphalt layer with a center to center spacing of 0.6 m (see Figure 2e). The sleepers were type BBS 3 W60 of the GETRAC A3 system; each 2.40 m long and 0.57 m wide at the base, with a mass of 540 kg, and having a thin geotextile attached at the bottom. Small rail segments (type UIC60) of length 0.20 m were mounted onto the sleepers with the built in Vossloh 300 fastening system. According to the manufacturer datasheets, the static stiffness of the rail pads was 23 kN/mm (± 10 %).

2.2 Loading setup and Instrumentation

Vertical loads were applied to the mockup via a rigid reaction frame that was built over the box (see Figure 2e). This frame housed three identical 100 kN servo-hydraulic actuators (MTS Model No. 244.22). The spacing between the actuators corresponded to the sleeper spacing of 0.6 m (see Figure 1a). As there was one actuator for loading each sleeper, a spreader beam was installed to distribute the load equally on top of the two rail segments (see Figure 1b and 2e). The contact between the spreader beam and the rail segment was established through a circular steel disc of diameter 10 mm glued at the top of the rail-segment at its centre in order to ensure the best possible alignment and symmetry in the loading.

The sensors employed for monitoring the different mechanical responses in the mockup are depicted in Figure 3. These included: earth pressure cells (PCs), asphalt strain gauges (ASGs), potentiometers (POTs), linear variable differential transformers (LVDTs), and accelerometers (ACCs). Figures 3a and 3b show a schematic layout of the embedded sensors at two different elevations inside the box when viewed from the top. Figure 3c shows a schematic layout of the surface sensors. These figures also depict a Cartesian coordinate system placed at
the center of the box, which is defined for the location reference of the various sensors. The images of the instrumentation as installed in the setup are shown in Figures 3d-3g. Full details about the embedded and the surface sensors are listed in Table 1 and Table 2 respectively. Additional strain gauges (not listed in Table 1) were mounted along the sides of the steel box and at the bottom flange of the spreader beams to monitor the boundary conditions in the setup.

2.3 Test plan

In general terms the mockup was interrogated in two phases, by two types of tests: (i) cyclic loads applied only to Sleeper 2, and (ii) simulated moving train loads, which consisted of loading all three sleepers sequentially. The experiments were performed by imposing a deformation controlled time history at the respective actuators, and at an average temperature of 22 °C. More details on each of the test types are provided hereafter.

2.3.1 Test Phase 1: Cyclic loads

In this phase of testing, the displacement of Actuator 2 was controlled to apply sinusoidal excitations of different amplitudes and frequencies to the central sleeper for 200 cycles. The aim was to establish steady state conditions and analyse the sensor readings for the effects of different loading frequencies and amplitudes. The tests carried out in this phase are shown in Table 3. The tested load amplitudes ranged between 10 and 30 kN, and three different frequencies: 0.1 Hz, 1.0 Hz, and 10.0 Hz were considered.

2.3.2 Test Phase 2: Simulated moving train loads

In this phase of testing, the loading induced by the Danish IC3 train was simulated, with a load arrangement as shown in Figure 4a. This train has a length of 56 m, consisting of three wagons and eight axles. Let \( j \) be an index denoting the axle number, then \( j = 1 \) represents the rear axle while \( j = 8 \) denotes the front axle. Let the individual axle positions relative to the rear axle be denoted as \( \Delta x_j \) (i.e., \( \Delta x_1 = 0 \)) and the respective axle loads be \( 2P_{z,j} \), wherein \( P_{z,j} \) denotes the wheel load on one side of a sleeper. The simulation assumes that during the IC3 train passage any random sleeper is symmetrically loaded by a dual-force. Let this vertical load history on top of a random sleeper (on one side, excluding the weight of the rail) be denoted as \( S_{z,\text{train}} \). Using the simplified track model of an infinite beam on Winkler foundation (see Figure 4b), and based on equations previously described [51], \( S_{z,\text{train}} \) was calculated as:

\[
S_{z,\text{train}} = \frac{s \beta_z}{2} \sum_{j=1}^{8} P_{z,j} e^{-\beta_z \| r' + \Delta x_j \|} \left( \cos \left( \beta_z \| x + \Delta x_j + V t \| \right) + \sin \left( \beta_z \| x + \Delta x_j + V t \| \right) \right)
\]  

(1)

wherein \( s \) denotes center-to-center sleeper spacing, \( t \) is simulation time, \( V \) denotes train travel speed, and
\[ \beta_Z = \frac{4k_z}{EI} \]  

(2)

in which \( k_z \) is the track modulus, \( EI \) is the bending rigidity of the beam (properties as per rail section UIC 60), and \( x \) is the initial (\( t = 0 \)) distance between the sleeper and the rear axle of the train. Figure 4c shows the calculated load history of a random sleeper (on one side) during the IC3 train passage. The Figure also shows that upward beam bending produces tensile forces on the top of the sleeper when an axle just approaches or leaves the location.

In the mockup, passage of IC3 train was simulated by the principle of sequential loading [30, 33]. The rationale behind this method is explained as follows. When the train moves on the rails, the wheel loads are transferred from the rails to the substructure below via the sleepers. Every sleeper along the railway track direction experiences the same load history (\( S_z^{\text{train}} \)) but with a time delay (\( \Delta t \)) depending on the train speed (\( V \)) and the spacing between the sleepers (\( s \)) where \( \Delta t \) is simply given by \( \Delta t = s/V \). Consequently, the vertical loads induced by a moving train can be approximately reproduced by sequentially loading the individual sleepers with the exact same vertical load history. Figure 4d depicts the target actuator load in the mockup set as \( 2S_z^{\text{train}} \) so that load distributed on each side of the sleeper was \( S_z^{\text{train}} \) (after applying a cut-off to the tensile part of the load history). Tensile loads could not be applied in the set-up as the spreader beams were only resting on the rail segments. The loading scheme was sequential (from Sleeper 1 towards Sleeper 3), with a time delay between Actuator 1 and 2 as \( \Delta t \) and that between Actuator 1 and 3 was \( 2\Delta t \). Based on a separate numerical analysis [52], the value of \( k_z \) was adopted to be 42 MPa for the speed and load range simulated in this study. This value is very close to what would be obtained if the track modulus was calculated based on the stiffness of the rail pads. As in test Phase 1, the target loads were achieved by controlling the displacement of the individual actuators. As the assumptions of the Winkler model do not perfectly comply with the mockup, and also to eliminate the coupling between the three actuators, the displacement functions required some manual fine-tuning in order to reach to the target load. Table 4 lists the tests carried out in this experimental phase. As can be seen, the test in the list (No. 10) included a single axle loading history and not a full train. This single axle passage was simply calculated according to the explanation above with \( j=1 \) in Equation 1.
3. Test results and discussions

3.1 Responses for cyclic loading

In this section, the results of the tests carried out in Phase 1 are presented and discussed. The sign convention adopted is as follows: downwards load and displacement is considered positive and compressive stresses and strains are positive.

3.1.1 Influence of load frequency

First, tests 1, 2, and 3 (see Table 3) are addressed where cyclic loads of amplitude 19 kN (on average) were applied by Actuator 2 at three different frequencies: 0.1 Hz, 1.0 Hz, and 10.0 Hz. The resulting responses are presented in Figure 5 for the last ten cycles during which nearly steady state conditions were achieved. In all the three tests, the maximum increase of the mean load and the load amplitude for the last 100 cycles was observed to be less than 5%. The individual steady state sensor responses from the three tests were zeroed at their respective minimum values to bring them into a common origin (for ease of graphical comparison).

Figure 5a depicts Actuator 2’s force-displacement curve; there are three plots in this figure, each corresponding to a different frequency – each illustrating a closed hysteresis loop. It should be noted, that the actuator’s displacements refer not only to the displacements of the test section but they also include the deformation of the spreader beam and the loading frame. However, the latter are purely elastic, which means that the area enclosed within these loops can be considered as a measure of the overall energy dissipated in the track system over a single load-unload cycle. The dissipated energy is seen to be similar for the different frequencies, indicating that the dissipation is weakly linked to the applied frequency. The overall stiffness of the system is complex, with a norm indicated by the inclination of the hysteresis loop. The norm of the complex stiffness is estimated to be: 35.0 kN/mm @ 10 Hz, 33.4 kN/mm @ 1 Hz, and 32.0 kN/mm @ 0.1 Hz, indicating a minor increase with the applied load frequency.

Figure 5b illustrates the force exerted by Actuator 2 against the vertical displacement recorded by POT 40. Nearly identical readings were obtained by POT 39 and POT 41 (and therefore not shown). These potentiometers measured the vertical compression of the rail pads. A closed hysteresis loop is formed, which is similar in shape for the different applied frequencies, while the area remains similar for frequency of 0.1 Hz and 1 Hz and decreases at 10 Hz. The norm of the complex stiffness of the rail pads is estimated to be: 30.7 kN/mm @ 10 Hz, 29.4 kN/mm @ 1 Hz, and 28.3 kN/mm @ 0.1 Hz. Thus, it is observed that the behaviour of the rail pads is mildly sensitive to the excitation frequency within the considered range.

The force exerted by Actuator 2 against the vertical displacements recorded by LVDTs 1 and 2 are shown in Figure 5c. Both LVDTs measured the differential displacement between
Sleeper 2 and the asphalt surface, i.e., the combined effect of the geotextile compression and the asphalt surface displacement at these two locations. LVDT 1 (as well as LVDT 4) was located at the edge of Sleeper 2 while LVDT 2 (as well as LVDT 3) was located at the middle of Sleeper 2. The readings from LVDTs 3 and 4 are not shown as they contain essentially identical information to the readings of LVDTs 1 and 2. There are six curves in Figure 5c, three of which correspond to the readings of LVDT 1 across the three frequencies, and the other three correspond to LVDT 2. All six curves display a pronounced hysteresis loop for a load-unload cycle. The area of the loops show a gradual decrease with an increase in the frequency. The amplitude of LVDTs 1 and 2 also show a decrease by 30 % and 47 % respectively with increase in loading frequency from 0.1 Hz to 10 Hz. These results depict that the relative displacement between the sleeper and the asphalt surface is very sensitive to the excitation frequency.

Figure 5d presents the force exerted by Actuator 2 against the vertical stress recorded by PC 16 (PC 14 gave similar readings and therefore not shown). The three hysteretic curves in this Figure correspond to the three test frequencies. Due to the location of PC 16, the recorded stress amplitudes are also the maximum that is experienced at the top of the mat (or equivalently at the bottom of the unbound granular layer). The measured stress amplitudes show a 38 % decrease with increase in frequency from 0.1 Hz to 10 Hz. This behaviour is a manifestation of the frequency-dependent behaviour of the overall track structure. The stress and the load curves also forms a closed loop; the area of which is lower at higher frequency.

Figure 5e depicts the force exerted by Actuator 2 against the measurements of all the five pressure cells. The five hysteretic curves were all obtained at the same frequency of 1 Hz. A progressive decrease in the stress amplitude can be observed with increasing distance from Sleeper 2 in both X and Y directions. The peak stress amplitude that occurs in this location are registered by PC 14 (7.3 kPa) and PC 16 (7.1 kPa) which are below the loading positions, while PC 15 which is at the middle of Sleeper 2 measure about 4.7 kPa. PC 13 and PC 17, which are below Sleeper 3 and Sleeper 1 respectively, measure about 1.5 kPa.

The horizontal strain measured by ASG 6 are shown in Figure 5f. The strain amplitude measured by ASG 6 is the maximum expected strain amplitude to be occurring at the bottom of asphalt layer in the X direction (amongst the functioning sensors). The sensor ASG 7 records similar measurements as ASG 6 and is not shown in the Figure here. The trend of the other strain gauges with changing frequencies could not be reliably estimated from these tests, as the recorded strains were quite small and within the precision limits of the sensors. The Figure shows that with increase in frequency from 0.1 Hz to 10 Hz, the strain amplitudes in ASG 6 decrease by about 48 %, indicating a pronounced frequency dependency of the asphalt strains.
3.1.2 Influence of load amplitude

In order to investigate the influence of load amplitude on the track behaviour, cyclic loading was performed at 1 Hz at an approximate mean load level of 31.2 kN, while the load amplitude was varied. An amplitude factor of two was applied between Test 6 and Test 4, and a factor of three was applied between Test 8 and Test 4 (see Table 3). The same procedure was repeated at 10 Hz in between Tests 7 and Test 5, and between Test 9 and Test 5. Only the steady state responses, corresponding to the last 10 cycles, were utilized for subsequent analysis. The sensor amplitude ratios in Test 6/Test 4 (@ 1 Hz) and Test 7/Test 5 (@ 10 Hz) are displayed graphically in Figure 6a and compared to the corresponding load amplitude ratio of approximately 2. The sensor amplitude ratios in Test 8/Test 4 (@ 1 Hz) and Test 9/Test 5 (@ 10 Hz) are shown in Figure 6b and compared with the load amplitude ratio of approximately 3.

From Figure 6a it can be seen that the LVDT amplitude ratio is 30% higher than that of the load amplitude ratio. This indicates that the relative vertical displacement between the asphalt surface and the sleeper varies non-linearly with the load level. The amplitude ratios of the potentiometers and the asphalt strain gauges are seen to be only slightly higher (about 10%) while the pressure cells are almost similar to that of the load. This indicates that the rail pad compression, horizontal strains below the asphalt layers and the vertical stresses below the granular layer scale almost linearly with the load amplitude. The sensor amplitude ratios are observed to be similar at both the frequencies of 1 Hz and 10 Hz. As the load amplitude ratio increases to 3 in Figure 6b, the non-linearity shown by the LVDT readings is pronounced and is higher at a frequency of 1 Hz. The amplitude ratios of the potentiometers and the asphalt strain gauges are seen to be slightly higher than before (about 15%) while, the pressure cells are again similar to that of the load. This indicates minor non-linearity in the rail-pad compression and horizontal asphalt strains.

3.2 Simulated moving loads

In this section, the results from simulating a moving load are presented. First, the findings from simulation of a single moving axle load (Table 4, Test 10) are presented and discussed. Thereafter, the outcomes from simulating the Danish IC3 train (Tests 11 to 14 in Table 4) are discussed. All the responses that are presented graphically in this section are a manifestation of the externally applied load only without the initial conditions.

3.2.1 Response for a single axle passage

The results from the simulation of single axle load of magnitude 120 kN, moving at a speed of 120 km/h from Sleeper 1 towards Sleeper 3 are shown in Figure 7. The time history of the loads applied by the three actuators is illustrated in Figure 7a along with the theoretical values...
calculated from the Winkler model using Equation 1 (2.S_{z}^{\text{train}} shown in dashed lines). The shape of the applied force and the corresponding theoretical target force are similar overall, with some deviations in the beginning of the loading curve and in the unloading region. These differences are due to the fact that the setup was exited in displacement-control mode. The peak force applied by the three actuators are 38.0 kN, 38.6 kN, and 38.4 kN. The time delay between the peak force in Actuator 1 and Actuator 2 is measured as 19.2 ms, while that between Actuator 2 and Actuator 3 is 18.5 ms. Comparing these values with the theoretical values (given in Table 4 for Test 10) shows that the simulation of the moving axle load is achieved reasonably well in the mockup. According to the analysis of an equivalent Winkler model, for k_{z} = 42 MPa, when a wheel load is located on top of any sleeper, the axle load is mainly distributed between five sleepers, i.e., the central (loaded) sleeper plus two on either side; the relative amounts are (%): 10.3, 24.2, 34.0, 24.2, 10.3. For a case where only three sleepers are present, the force in the loaded sleeper (40.9 kN) and the adjacent ones (29.1 kN) amounts to 82.4 % of the axle load. In Figure 7a, it is seen that corresponding to the peak force of 38.6 kN in Actuator 2, the average forces in Actuators 1 and 3 were 22.5 kN amounting to a maximum of 70.5 % of the axle load.

The time history of the vertical displacements recorded by POT 39 to POT 41 are presented in Figure 7b. The readings of the potentiometers follow the overall shape of the applied load, increasing gradually with the force level and then unloading to initial conditions without any residual displacement, indicating an elastic behaviour for the rail pads. As expected, the overall behaviour of the three sensors are quite similar with POT 40 registering slightly higher values. The average peak displacement recorded is 0.626 mm, the corresponding spring stiffness of the rail pads is therefore 30.8 kN/mm, calculated as 0.5×38.6/0.626.

In Figure 7c the time history of the vertical displacements recorded by LVDT 1 to LVDT 4 are displayed. The readings of the four LVDTs also follow the path of the applied load but contrary to the potentiometers, a time-dependent recovery is observed after the unloading, reflecting the time-dependent nature of the mockup, particularly the geotextile and the asphalt concrete. LVDT 1 and LVDT 4 recorded similar peak values of around 0.195 mm, while LVDT 2 and LVDT 3 recorded similar peak values of 0.130 mm. The difference in the peaks of the displacement readings can be attributed to the different deflection at the surface of the asphalt layer below the edges and around the middle of Sleeper 2 (neglecting the bending of the sleeper). Comparing the LVDT and the POT readings it can be concluded that most of the vertical displacement in the track (75 %) is due to the rail pad compression.
The time histories of the vertical stresses recorded by the sensors named PC 13 to PC 17 are illustrated in Figure 7d. All the five sensors follow the overall shape of the applied load without any noticeable time-dependent behaviour on unloading. The time delay between the peak stresses recorded by the pressure cells, depict the movement of the axle from Sleeper 1 to Sleeper 3 as presented before in Figure 7a. PC 17 located below Sleeper 1 registers the first stress peak of about 13 kPa. The next peaks occur simultaneously for PC 14 (18.5 kPa), PC 15 (15.6 kPa) and PC 16 (20.3 kPa), which are all located below Sleeper 2. The last stress peak of 9.5 kPa is recorded by PC 13, which is located below Sleeper 3. As a first order approximation, the stresses below the sleepers can be assumed to be uniform over the entire loaded area, amounting to a calculated average value of 33.85 kPa below Sleeper 2. The corresponding stresses below Sleeper 1 and 3 are approximately 60% of this value. At this loading condition, the peak vertical stress recorded below the granular layer (i.e., on top of the mat) by PC 16 is approximately 60% of the average stress below Sleeper 2. The peak stresses recorded below Sleeper 1 (PC 17) and Sleeper 3 (PC 13) are lower than those recorded under Sleeper 2 (PC 15) because these two sleepers are at the boundaries and the load from adjacent sleepers is missing on one side. PC 17 records peak stresses that are 35% higher than that of PC 13 which is not consistent as these sensors are expected to show similar readings because of their positioning (Figure 3) and symmetry in the applied loading (Figure 7a).

Figures 7e and 7f display the history of the horizontal strains at the bottom of the asphalt layer in the X and Y directions respectively. The axle movement can also be visualized here from the time delay between the peak readings of the different strain gauges. In Figure 7e, the peak strains occur successively from ASG 11 (below Sleeper 1) to ASG 6, ASG 7 (below Sleeper 2) and ASG 3 (below Sleeper 1) while, in Figure 7f the peaks occur consecutively from ASG 10 (below Sleeper 1), ASG 9 (below Sleeper 2) and then ASG 2 (below Sleeper 3). The overall behaviour of the strain gauges reflect the shape of the applied load, with a gradual increase in the strains and then a time-dependent recovery to the initial conditions without any permanent strain accumulation. In some of the strain gauges the recovery is still ongoing within the test window. Among the functioning sensors in X direction, the peak tensile strains are expected under ASG 6 and ASG 7, which record similar values of around 16.7 microstrains. ASG 3 (Sleeper 3) shows a similar peak value with a strain reversal from compressive to tensile as the load moves progressively towards Sleeper 3. ASG 11 (Sleeper 1) records a peak strain that is 50% more (24 microstrains), which is not consistent with the other sensor responses. The peak tensile strains in Y direction as recorded by ASG 9 (Sleeper 2) is around 18 microstrains which is similar to the peak strain in the X direction. The strain readings of ASG
2 and ASG 12 are similar to ASG 9 while the strains measured below Sleeper 1 (ASG 10) are lower by 35%. This is not consistent as these sensors (ASG 2, ASG 10, and ASG 12) are expected to provide similar peak responses considering the nearly symmetric applied loading history (Figure 7a) and their installed positions (Figure 3).

3.2.2 Response for a full train passage at different speeds and loads

In this section, selected results from simulation of the Danish IC3 train at different speeds and axle loads are presented and discussed. The load and displacement curves of only Actuator 2 from Test 11 to Test 14 (see Table 4) are shown in Figure 8. There are four charts in this figure corresponding to simulations involving two different axle loads: 120 kN and 200 kN, and two train speeds: 120 km/h and 200 km/h. The loading part of the four curves appears as a straight line with an average slope of about 28.3 kN/mm; this slope is seen to be similar in all the four cases. The unloading part is curved, and forms a hysteresis loop generating similar overall load-displacement curves at the two test speeds. The corresponding traces of all sensor readings in the mockup (except for the accelerometers), were also very similar for the two speeds. These observations indicate that an increase of train speed from 120 km/h to 200 km/h has little effect on the track responses. Therefore, in the following only results from a simulated train passage at speed of 120 km/h at two different axle loads (120 kN and 200 kN) are presented and addressed (Tests 11 and 12 in Table 4).

Figure 9a shows the force history of Actuator 2 in Test 11 and Test 12 (see Table 4). The forces in Actuators 1 and 3 were similar, but with a simulated time delay of 18 ms as given in Table 4 (and therefore not shown). Eight distinct peaks can be seen in the Figure, each corresponding to one of the eight IC3 train axles (see Figure 4a). For the axle load of 120 kN, the average value of the peak force applied on top of Sleeper 2 was 38.2 kN, while for the axle load of 200 kN it was 64.3 kN, i.e., very close to their respective theoretical peak values (see Table 4).

The vertical displacement history of the three potentiometers (POT 39, POT 40, and POT 41) corresponding to the two axle load levels is shown in Figure 9b as an average. The overall shape of the two curves is similar, consisting of eight peaks corresponding to the eight axles and conforming to the shape of the applied force shown in Figure 9a (both during loading and unloading). The average values of the peak displacements (i.e., rail pad compression) in the two tests are 0.94 mm and 0.62 mm and the average stiffness of the rail pads is calculated to be 30.5 kN/mm and 34.0 kN/mm respectively. This slight difference in rail pad stiffness indicates the minor dependence on loading rate and amplitude. The average peak displacement
observed at axle load level of 120 kN is very similar to the case of single moving axle at the same load level and speed (see Figure 7b).

Figure 9c shows the vertical displacement history of the LVDT readings for the two tests. The readings of the LVDTs measuring vertical displacements at similar locations are separately averaged, i.e., LVDT 1 and LVDT 4, and LVDT 2 and LVDT 3. In each of the four curves, eight distinct peaks corresponding to the train axles can be identified. A time-dependent recovery is observed after passage of sets of closely spaced axles on the same bogie. The displacements show minor accumulation with successive axle passes. The average peak displacements noted at axle load level of 120 kN are again very similar to those in Figure 7c, simulating a single moving axle load. Within the test window, the recovery of the LVDT readings is still ongoing.

The vertical stress history of PC 16 and PC 15 which are located below Sleeper 2 are shown in Figures 9d and 9e respectively. The overall behaviour of each chart resembles the applied load signal shown in Figure 9a. For the two axle loads, the peak stress (average) recorded below the granular layer (i.e., on top of the mat by PC 16) are 35 kPa and 21 kPa, which are approximately 60% of the calculated average stress below Sleeper 2, assuming a uniform stress distribution. The peak stresses measured at this elevation below the unloaded part of Sleeper 2 (i.e., at the middle by PC 15) are 25 kPa and 15.3 kPa, which are about 45% of the calculated surface stress.

Figures 9f and 9g shows the horizontal strain history at the bottom of asphalt layer as recorded by ASG 7 in X direction and ASG 2 in Y direction respectively. ASG 7 is located below Sleeper 2 where less boundary influence is expected on the peak strains in the X direction. Readings of ASG 2 are displayed in Figure 9g instead of ASG 9 (where higher responses are anticipated) as the latter showed unreliable measurements in these tests. The choice of ASG 2 was also based on the reliability of measurements established from Test 10 (see Figure 7f). The peak tensile strains recorded by ASG 7 in X direction at the two load levels are 25 microstrains and 15 microstrains respectively, while that recorded by ASG 2 in Y direction is about 39 and 20 microstrains respectively. The strain gauges depict minor strain accumulation between successive load peaks with ongoing strain recovery at the end of the test, especially in the readings of ASG 2. The peak strains measured by the two sensors at 120 kN axle load are similar to the single moving axle simulation, as shown in Figures 7e and 7f.

The vertical surface acceleration history as recorded by sensor ACC 36 is illustrated in Figure 9h. The passes of the successive train axles can be observed from the recordings.
peak accelerations at the two load levels were measured to be about 0.9 m/s² and 0.7 m/s² respectively.

In order to investigate the effects of speed, some of the results from the other tests are analysed, both in time and frequency domain. The force history of Actuator 2 in Tests 11 and 13 (Table 4) is converted to the normalized amplitude spectra by Fourier transform and illustrated in Figures 10a and 10b respectively. In these two tests, the simulated train speeds are 120 km/h and 200 km/h and the axle load is 120 kN. The corresponding normalized amplitude spectra of the sleeper force history as calculated from the Winkler model are also displayed in Figure 10. This figure demonstrates that the frequency content of the sleeper force history calculated from the Winkler model was adequately reproduced by Actuator 2 in the two tests in the mockup. Similar observations were verified with the forces applied by Actuator 1 and Actuator 3 (therefore not shown). The normalized amplitude spectrum of the actuator forces at axle load of 200 kN at both the speeds were also verified to be similar to the theoretical spectrum. It is noted that the frequencies associated with vehicle dynamics, wheel-rail contact, soil layer etc. are not captured in the Winkler model and consequently not in the mockup as well. In Figure 10a, for the simulated speed (V) of 120 km/h, the dominant frequency in the spectrum corresponds to the bogie passage frequency (f_b) and its higher order harmonics, which is calculated as: \( f_b = \frac{V}{L_b} = 1.9 \text{ Hz} \), where, \( L_b \) is the distance between bogies = 17.7 m (see Figure 4a). The spectrum is modulated around the axle passage frequency (f_a), which is calculated as: \( f_a = \frac{V}{L_a} = 13.3 \text{ Hz} \), where \( L_a \) = distance between the axles = 2.5 m (shown in Figure 4a). In Figure 10b, the frequency spectrum at speed of 200 km/h is presented where the dominant frequency is at \( f_b = 3.1 \text{ Hz} \) and its higher order harmonics with modulations at \( f_a = 22.2 \text{ Hz} \). The effect of speed on the track responses can be visualized in Figure 11a that shows the vertical surface accelerations as recorded by sensor ACC 36 (see Figure 3) in Tests 12 and 14 (see Table 4). These tests simulated an axle load of 200 kN at two different speeds of 120 km/h and 200 km/h. Peak accelerations were recorded on the track center line in the mockup by this sensor and it increased from 0.9 m/s² to 1.8 m/s² with an increase in the simulated train speed. The corresponding frequency domain responses are shown in Figure 11b. The spectrum shows the frequency peaks that are already associated with the load signal (see Figure 10). In addition, higher frequency content is observed in the range beyond 60 Hz, which could be attributed to the free vibrations of the system.
4. Summary of findings and conclusions

This paper focused on the experimental investigation of a ballastless asphalt track under vertical loads. A full-scale mockup of size 4.00 m × 2.21 m was built inside a steel box, consisting of a rubber mat that simulates subbase and subgrade, unbound granular layer, and asphalt layer (see Figures 1 and 2). Three wide sleepers which are part of the GETRAC A3 ballastless track system were placed directly on top of the asphalt layer. Sensors were installed at different locations to measure: (i) vertical stresses below unbound granular layer, (ii) horizontal strains below asphalt layer, (iii) relative vertical displacement between rail and sleeper, and sleeper and asphalt surface, and (iv) vertical surface accelerations (see Figure 3 and Tables 1 and 2). Main findings from the construction and the instrumentation effort are summarized as follows: (i) construction of the mockup in an outdoor location followed by its transportation to an indoor laboratory environment allowed full-scale construction equipment to be used and not scaled-down, (ii) chosen pressure cells exhibited reliable behaviour across all tests, though the actual working range was very small (< 50 kPa) relative to the operating range (1 MPa) that was chosen to be high in order to withstand the construction process, (iii) the asphalt strain gauges showed somewhat erratic recordings - coupled with the fact that the measured strains were very small, almost within the precision limits, and (iv) including redundancy in the mockup (by about 50%) helped to extract reliable measurements especially for the strains.

In the first testing phase, cyclic loads were applied to the central sleeper at different frequencies and amplitudes (see Table 3), and steady-state responses were analyzed. The significant findings from these tests were: (i) the load-displacement response of the track was frequency-dependent with a higher complex stiffness norm at a higher frequency, (ii) all measured responses were frequency-dependent, the rail pad compression to a minor extent, while the rest displayed strong dependence on frequency, (iii) vertical stresses below the granular layer scaled linearly with the load amplitude. In contrast, horizontal strains below the asphalt layer and rail pad compression showed slightly non-linear behaviour. The relative vertical displacement between the sleeper and the asphalt surface (as measured by the LVDTs) varied very non-linearly with the load amplitude.

In the next testing phase, moving loads were simulated by a sequential loading scheme, first, a single axle and then a complete train (see Table 4). The significant findings from the tests were: (i) rail pads behaved elastically without exhibiting any time effects. In contrast, the relative displacement between sleeper and the asphalt surface (as measured by LVDTs) displayed a time-dependent response, (iii) peak vertical stresses at the bottom of the unbound
granular layer ranged between 15 kPa to 40 kPa, amounting to approximately 60% of the calculated average stress below the sleeper. The measured values of the peak stresses are low suggesting compliance with the standard requirements [48], (iv) peak horizontal strains at the bottom of the asphalt layer did not exceed 50 microstrains in tension which is well below the flexural fatigue endurance limit of the material [48], (v) horizontal strains below the asphalt layer in the direction of the moving load exhibited sign reversal which was not observed along the transverse direction, (vi) horizontal strains below the asphalt layer showed minor accumulation with the passes of successive axles, (vii) increasing the simulated train speed from 120 km/h to 200 km/h had no noticeable effect on the track responses except for the vertical surface accelerations, and (viii) the peak vertical acceleration measured on the track centre line in the mockup was less than 2 m/s². Similar values have been reported in ballasted tracks but at a much larger distance from the centre line of the track, approximately 3 m away [53].

Based on the findings from the experimental investigations, some generalized conclusions can be drawn:

(i) majority (75%) of the vertical rail displacement in the track could be attributed to the rail pad compression, (ii) the geotextile compression contributed to the vertical rail displacement and it played a role in bridging texture and construction unevenness between asphalt surface and sleeper underside, (iii) the measured mechanical responses (stresses and strains) were of very low magnitude, suggesting very little mechanical damage, and (iv) modelling for ballastless asphalt tracks should include the effects of time dependence and non-linearity in material behaviour.

As part of future study, resilient track responses can be investigated in the mockup for other sleeper designs with different fasteners and geotextiles. In addition, the response of such a track to horizontal loads can also be studied by using a modified loading and instrumentation scheme. Nonetheless, the findings of the current tests provide an overall intuitive understanding of the mechanics of ballastless asphalt tracks under anticipated train loads and support the development of reliable models for further analysis and design purposes.

Acknowledgement

The support from Innovation Fund Denmark is gratefully acknowledged. This study is part of ‘Roads2Rails: Innovative and cost-effective asphalt based railway construction system’ (Grand Solutions 5156-00006B). The authors would like to thank all the project participants for their valuable contributions. The support from all the laboratory technicians in the Technical University of Denmark is gratefully acknowledged.
References


40. S. Fan, Analysis on experiment of dynamic response in ballastless track subgrade of high speed railway, Southwest Jiaotong University, Chengdu, China (2010).


47. http://vibratec.dk/products/vibration-990-plus/


Figure captions

Figure 1: Schematic view of the test facility showing: (a) cross section and (b) longitudinal section
(All dimensions are in mm)

Figure 2: Major construction stages of the test section. (a) Placement of the mat, (b) compaction of the
unbound granular layer, (c) placement of the asphalt mixture, (d) compaction of the asphalt layer, and
(e) constructed test facility

Figure 3: Instrumentation deployed at different locations in the mockup. Schematic layout (top view)
showing location of: (a) pressure cells on top of the mat, (b) asphalt strain gauges at the bottom of the
asphalt layer, and (c) surface instrumentations. Images of instrumentation from the mockup showing:
(d) pressure cells, (e) asphalt strain gauges, (f) potentiometer (View B), and (g) LVDT and accelerometer
(View A) (All dimensions are in mm)

Figure 4: A sketch showing the process of simulating moving train loads in the mockup. (a)
Configuration of the simulated train, Danish IC3, (b) model of infinite beam on Winkler foundation, (c)
time history of sleeper force (on the top and at one side) during IC3 train passage as calculated from the
above model, and (d) sequential loading of the three actuators in the mockup

(Dimensions are in m)

Figure 5: Comparison of steady state sensor responses at different loading frequencies. Force applied
by Actuator 2 with respect to, (a) displacement of Actuator 2, (b) displacement of POT 40, (c)
displacements of LVDT 1 and LVDT 2, (d) stress in PC 16, (e) stress in all pressure cells at 1 Hz, and
(f) strains in ASG 6

Figure 6: Effect of the load amplitude on the steady state response. The amplitude ratio of the different
sensors readings corresponding to: (a) load amplitude ratio of 2 and (b) load amplitude ratio of 3

Figure 7: Time history of sensor responses measured during simulation of a moving axle load \(2P_{Z,j} = 120 \text{ kN} \) and \( V = 120 \text{ km/h} \). Responses are shown for: (a) actuators, (b) potentiometers (c) LVDTs,
(d) pressure cells, (e) and (f) strain gauges along X and Y direction respectively

Figure 8: Force displacement curve of Actuator 2 from simulation of train passage at two different axle
loads and speeds \( 2P_{Z,j} = 120 \text{ kN} \) and \( 200 \text{ kN} \), \( V = 120 \text{ km/h} \) and \( 200 \text{ km/h} \)

Figure 9: Time history of sensor responses measured during simulation of IC3 train passage at two
different axle load levels \( 2P_{Z,j} = 120 \text{ kN} \) and \( 200 \text{ kN} \), \( V = 120 \text{ km/h} \). Responses are plotted for: (a)
Actuator 2, (b) average reading of potentiometers, (c) average reading of LVDTs, (d) - (e) pressure cell,
(f) strain gauge along X direction, (g) strain gauge along Y direction, and (h) accelerometer

Figure 10: Normalized spectral magnitude of vertical sleeper force history during simulation of train
passage \( 2P_{Z,j} = 120 \text{ kN} \) at two different speeds of: (a) \( V = 120 \text{ km/h} \), and (b) \( V = 200 \text{ km/h} \)

Figure 11: Accelerometer responses during simulation of train passage \( 2P_{Z,j} = 200 \text{ kN} \) at two
different speeds in: (a) time domain and (b) frequency domain
Figure 1: Schematic view of the test facility showing: (a) cross section and (b) longitudinal section  
(All dimensions are in mm)
Figure 2: Major construction stages of the test section. (a) Placement of the mat, (b) compaction of the unbound granular layer, (c) placement of the asphalt mixture, (d) compaction of the asphalt layer, and (e) constructed test facility
Figure 3: Instrumentation deployed at different locations in the mockup. Schematic layout (top view) showing location of: (a) pressure cells (PC) on top of the mat, (b) asphalt strain gauges (ASG) at the bottom of the asphalt layer, and (c) surface instrumentations. Images of instrumentation from the mockup showing: (d) pressure cells, (e) asphalt strain gauges, (f) potentiometer (View B), and (g) LVDT and accelerometer (View A)

(All dimensions are in mm)
Figure 4: A sketch showing the process of simulating moving train loads in the mockup. (a) Configuration of the simulated train, Danish IC3, (b) model of infinite beam on Winkler foundation, (c) time history of sleeper force (on the top and at one side) during IC3 train passage as calculated from the above model, and (d) sequential loading of the three actuators in the mockup. (Dimensions are in m)
Figure 5: Comparison of steady state sensor responses at different loading frequencies. Force applied by Actuator 2 with respect to, (a) displacement of Actuator 2, (b) displacement of POT 40, (c) displacements of LVDT 1 and LVDT 2, (d) stress in PC 16, (e) stress in all pressure cells at 1 Hz, and (f) strains in ASG 6.
Figure 6: Effect of the load amplitude on the steady state response. The amplitude ratio of the different sensors readings corresponding to: (a) load amplitude ratio of 2 and (b) load amplitude ratio of 3
Figure 7: Time history of sensor responses measured during simulation of a moving axle load \( (2P_{Z,j} = 120 \text{ kN and } V = 120 \text{ km/h})\). Responses are shown for: (a) actuators, (b) potentiometers (c) LVDTs, (d) pressure cells, (e) and (f) strain gauges along X and Y direction respectively.
Figure 8: Force displacement curve of Actuator 2 from simulation of train passage at two different axle loads and speeds ($2P_{z,j} = 120$ kN and 200 kN, $V = 120$ km/h and 200 km/h)
Figure 9: Time history of sensor responses measured during simulation of IC3 train passage at two different axle load levels ($2P_{z,j} = 120$ kN and 200 kN, $V = 120$ km/h). Responses are plotted for: (a) Actuator 2, (b) average reading of potentiometers, (c) average reading of LVDTs, (d) - (e) pressure cell, (f) strain gauge along X direction, (g) strain gauge along Y direction, and (h) accelerometer.
Figure 10: Normalized spectral magnitude of vertical sleeper force history during simulation of train passage ($2P_{Z,j} = 120$ kN) at two different speeds of: (a) $V = 120$ km/h, and (b) $V = 200$ km/h
Figure 11: Accelerometer responses during simulation of train passage ($2P_{Z,j} = 200 \text{kN}$) at two different speeds in: (a) time domain and (b) frequency domain.
### Table 1

*Embedded sensors installed in the setup*

<table>
<thead>
<tr>
<th>Sensor name</th>
<th>Location [in m]</th>
<th>Elevation</th>
<th>Model No</th>
<th>Measured entity</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC 13, PC 17</td>
<td>X = ±0.60, Y = 0.00</td>
<td>On top</td>
<td>GEOKON</td>
<td>Vertical stress at interface of unbound granular layer and mat</td>
</tr>
<tr>
<td>PC 14, PC 16</td>
<td>X = 0.00, Y = ±0.75</td>
<td>of the mat</td>
<td>Model No 3500</td>
<td></td>
</tr>
<tr>
<td>PC 15</td>
<td>X = 0.00, Y = 0.00</td>
<td>mat</td>
<td>No 3500</td>
<td></td>
</tr>
<tr>
<td>ASG 2, ASG 10</td>
<td>X = ±0.60, Y = +0.75</td>
<td>At the bottom</td>
<td>CTL Group</td>
<td>Strain at bottom of asphalt layer along</td>
</tr>
<tr>
<td>ASG 4, ASG 12</td>
<td>X = ±0.60, Y = -0.75</td>
<td>of the asphalt</td>
<td>Model No ASG-152</td>
<td>Strain at bottom of asphalt layer along</td>
</tr>
<tr>
<td>ASG 5, ASG 9</td>
<td>X = 0.00, Y = ±0.75</td>
<td>layer</td>
<td>ASG-152</td>
<td>Y direction</td>
</tr>
<tr>
<td>ASG 3, ASG 11</td>
<td>X = ±0.6, Y = 0.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ASG 7</td>
<td>X = 0.00, Y = 0.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ASG 6, ASG 8</td>
<td>X = 0.00, Y = ±0.375</td>
<td></td>
<td></td>
<td>X direction</td>
</tr>
</tbody>
</table>

*ASG 4, 5, 8 were unresponsive in all the tests.*
**Table 2**

*Surface sensors installed in the setup*

<table>
<thead>
<tr>
<th>Name</th>
<th>Location [in m]</th>
<th>Model No</th>
<th>Measured entity</th>
</tr>
</thead>
<tbody>
<tr>
<td>POT 38,</td>
<td>On top of Sleeper 2</td>
<td>Novoteknik</td>
<td>Relative displacement between rail and top of Sleeper 2 at the location of the rail pads.</td>
</tr>
<tr>
<td>POT 39,</td>
<td></td>
<td>Model No</td>
<td></td>
</tr>
<tr>
<td>POT 40, and</td>
<td></td>
<td>TR-0025</td>
<td></td>
</tr>
<tr>
<td>POT 41</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LVDT 1</td>
<td>On top of the asphalt surface</td>
<td>HBM</td>
<td>Relative displacement between Sleeper 2 and the asphalt surface (at the sleeper edges).</td>
</tr>
<tr>
<td>LVDT 4</td>
<td></td>
<td>Model No</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>WA/20 mm-L</td>
<td></td>
</tr>
<tr>
<td>LVDT 2</td>
<td></td>
<td></td>
<td>Relative displacement between Sleeper 2 and the asphalt surface (at the sleeper middle).</td>
</tr>
<tr>
<td>LVDT 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ACC 33, ACC 34</td>
<td>X = ± 0.80, Y = 0.00</td>
<td>Silicon Designs</td>
<td>Vertical surface accelerations</td>
</tr>
<tr>
<td>ACC 35</td>
<td>X = 0.00, Y = -1.30</td>
<td>Inc. Model</td>
<td></td>
</tr>
<tr>
<td>ACC 36, ACC 37</td>
<td>X = ± 0.30, Y = 0.00</td>
<td>No 2210</td>
<td></td>
</tr>
</tbody>
</table>

*POT 38 was unresponsive in all the tests.*
Table 3

Steady state cyclic loads applied by Actuator 2

<table>
<thead>
<tr>
<th>Test No</th>
<th>Min Load [kN]</th>
<th>Max Load [kN]</th>
<th>Mean load [kN]</th>
<th>Load amplitude [kN]</th>
<th>Frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.00</td>
<td>39.70</td>
<td>20.85</td>
<td>19.00</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>2.00</td>
<td>40.90</td>
<td>21.45</td>
<td>19.40</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>2.40</td>
<td>40.00</td>
<td>21.30</td>
<td>18.80</td>
<td>10.0</td>
</tr>
<tr>
<td>4</td>
<td>21.50</td>
<td>40.50</td>
<td>31.00</td>
<td>9.40</td>
<td>1.0</td>
</tr>
<tr>
<td>5</td>
<td>21.50</td>
<td>40.50</td>
<td>31.00</td>
<td>9.70</td>
<td>10.0</td>
</tr>
<tr>
<td>6</td>
<td>12.50</td>
<td>50.90</td>
<td>31.80</td>
<td>19.25</td>
<td>1.0</td>
</tr>
<tr>
<td>7</td>
<td>12.50</td>
<td>50.90</td>
<td>31.80</td>
<td>19.15</td>
<td>10.0</td>
</tr>
<tr>
<td>8</td>
<td>2.80</td>
<td>60.00</td>
<td>31.00</td>
<td>28.45</td>
<td>1.0</td>
</tr>
<tr>
<td>9</td>
<td>2.80</td>
<td>60.00</td>
<td>31.00</td>
<td>28.60</td>
<td>10.0</td>
</tr>
</tbody>
</table>

*Mean load = 0.5 × (Min load + Max load); Load amplitude = 0.5 × (Max load – Min load)
**Table 4**

*Simulated moving loads*

<table>
<thead>
<tr>
<th>Test No</th>
<th>Axle load $2P_{z,j}$ [kN]</th>
<th>Speed $V$ [km/h]</th>
<th>Load type</th>
<th>Peak value of $2S_{z}^{\text{train}}$ [kN]</th>
<th>Time delay $\Delta t$ [ms]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>120</td>
<td>120</td>
<td>Single axle</td>
<td>40.87</td>
<td>18</td>
</tr>
<tr>
<td>11</td>
<td>120</td>
<td></td>
<td></td>
<td>39.3</td>
<td>18</td>
</tr>
<tr>
<td>12</td>
<td>200</td>
<td>120</td>
<td>Danish IC3 train</td>
<td>65.5</td>
<td>18</td>
</tr>
<tr>
<td>13</td>
<td>120</td>
<td></td>
<td>Danish IC3 train</td>
<td>39.3</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>200</td>
<td>200</td>
<td>IC3 train</td>
<td>65.5</td>
<td>10.8</td>
</tr>
</tbody>
</table>
Numerical Modelling of a Ballastless Track Mockup based on Asphalt

Tulika Bose
Department of Civil Engineering
Technical University of Denmark
Nordvej, Building 119
Kgs. Lyngby 2800, Denmark
Email: tulbo@byg.dtu.dk

Eyal Levenberg (Corresponding author)
Department of Civil Engineering
Technical University of Denmark
Nordvej, Building 119
Kgs. Lyngby 2800, Denmark
Email: eylev@byg.dtu.dk

Varvara Zania
Department of Civil Engineering
Technical University of Denmark
Nordvej, Building 119
Kgs. Lyngby 2800, Denmark
Email: vaza@byg.dtu.dk
ABSTRACT
This study focused on the numerical modelling of a ballastless asphalt track mockup. A 3D time-domain finite element model was developed to analyze dynamic responses under simulated train passage. The asphalt layer was modelled as linear viscoelastic, and additionally, the nonlinear stress-dependent elastic modulus of the underlying unbound granular layer was incorporated. Majority of model parameters were obtained through laboratory tests and other independent sources; only a very few were calibrated using experimental data. Implicit dynamic analysis was carried out, and mechanical responses such as stresses, strains, accelerations and displacements were evaluated at different locations. The model was validated by comparing predictions with experimental measurements recorded on the mockup. The measured responses and predictions were in agreement in terms of overall behaviour and magnitude.

Keywords: Ballastless asphalt track; Dynamic analysis; Finite element model; Model validation; Nonlinear elastic; Time-dependent; Viscoelastic.

1. Introduction

Conventional ballasted tracks are still being used in high-speed networks, but they are prone to frequent maintenance due to accelerated ballast breakage and fouling, leading to an overall degradation in track stability [1,2]. As a result, there has been the emergence of ballastless tracks based on concrete (i.e., slab track); these are known to provide improved track stability and reduced maintenance needs but at the same time generating elevated noise levels [1,3,4]. Moreover, compared to the conventional tracks, the initial capital required for a slab track solution is much higher. In this connection, recent attention has been given to developing another type of ballastless track - based on asphalt concrete (referred to as asphalt in the text).

Ballastless asphalt tracks are structurally similar to highway asphalt pavements consisting of an asphalt layer, underlain by granular layers that rest on a subgrade. Unlike ballasted tracks, where sleepers are partially embedded within the ballast, in this case, the sleepers may rest on top of the asphalt pavement (possibly without crib or shoulder ballast).

The use of asphalt in the railway industry is not new, but so far it has been predominantly used in ballasted tracks as a subballast layer (underlayment) [5-7], and in slab tracks as a support layer and as a surface waterproofing layer [8-11]. Ballastless tracks built with asphalt concrete as the main load-bearing layer (overlayment) is fairly recent, with only a few identified implementations [12-14], mostly within the German railway industry. In these tracks, concrete blocks were used to anchor the sleepers to the underlying pavement, and extra reliance was
placed on frictional mechanisms by employing heavier sleeper types with a geotextile at the underside.

The overall goal of this study is to contribute to the basic understanding of the mechanical behaviour of ballastless asphalt tracks under vertical loads. This is sought through the construction, experimental investigation, and numerical modelling of an instrumented full-scale mockup. Doing so is considered a first step prior to wider scale testing and modelling efforts - involving more field-like conditions.

Scientific literature focused on analyzing mechanical responses in ballastless asphalt tracks is limited in number [15-18]. An early study [15] developed a 3D model for designing ballastless asphalt tracks by combining finite element method (FEM) and multilayered elasticity theory. The model consisted of rails and sleepers discretized with beam elements, fasteners simulated as linear springs and the asphalt layer and subgrade as a two-layered linear elastic half-space. Maximum tensile strains at the bottom of asphalt and maximum compressive stresses on top of subgrade were calculated in order to estimate the allowable number of load repetitions to prevent failure by fatigue cracking in the asphalt layers or by excessive deformation of the subgrade layers. The estimations were done based on damage laws developed for highway pavements. The thickness of the asphalt layer was designed using the model and estimated to be between 250 mm to 450 mm. This model, however, was not validated with field measurements from ballastless asphalt tracks.

Recently, laboratory studies were undertaken to investigate the behaviour of ballastless asphalt tracks under different intensities of stationary loads [16] and consecutive cyclic loads [17]. Later, a field study was conducted that analyzed the effects of seasonal temperature variation on track responses [18]. The test track section was 207 m long consisting of 0.3 m asphalt layer, underlain by 0.4 m aggregate base layer that was resting on a subgrade. Heavier concrete sleepers were used that included a geotextile at the underside. As part of the study, a 3D FEM model of the test track was developed, consisting of a pair of rails (as beam elements), fasteners (springs), sleepers and a multilayered track structure (solid 3D elements) with a 3.6 m thick subgrade; the model did not include the geotextile. All the track layers were assumed to be linear elastic; the asphalt layer was also modelled as linear viscoelastic (LVE). However, no explanation was given on how the elastic properties of the various layers were obtained. Train loads were simulated in a very approximated manner as distributed loads acting over the rail. Quasi-static analysis was performed, and peak responses (strain and stress below the asphalt layer) were compared with the field data. The model predictions with viscoelastic
analysis were reported to have shown a poor match with field data (especially the asphalt
strains), compared to the elastic analysis.

Except for the studies mentioned above, there is minimal literature on modelling of
ballastless asphalt tracks. Nonetheless, knowledge is available from a wide range of studies
centered on the modelling and analysis of ballast and slab track responses under vertical loads.
Advanced track models have been developed based on different numerical schemes; in
particular, FEM, to evaluate dynamic track responses, both at the track and the free field. The
vehicle was commonly represented as a sequence of axle loads (i.e., concentrated loads) that
were stationary [19] or travelling at a constant speed on the rails [20-27]. An alternate loading
scheme was also used in which the sleepers were loaded directly and sequentially to simulate
moving loads [22,26]. Such a model was reported to be computationally efficient as continuous
rails (beam elements) were not required to be included [22]. Multibody simulation methods
were also used in which the vehicle model was coupled to the track model through suitable
contact models [28-32]. This was mainly employed to predict vibrations due to train passages
at the track and free field [28-30], and investigate responses from different track defects such
as wheel flats, wheel-rail irregularity [31-32].

In the majority of the models, the track layers are assumed to be linear elastic. However,

it is commonly known that the resilient behaviour of granular soil-like materials, such as
railway ballast, subballast, and subsoil is nonlinear [33-34]. Ballast nonlinearity was modelled
using the well-established $K - \Theta$ model [35,36] and the Universal model [23,33,37]. The
former considers the hardening effect of bulk stress on the resilient modulus, while the latter
also includes the softening effect of shear stress. Advanced elastoplastic constitutive models
were also used to study plastic deformation and degradation response of ballast under
monotonic and cyclic loads [38-39]. The nonlinear behaviour of the subsoil was incorporated
in studies that analyzed train speeds that were comparable to the wave velocity through the
system [37,40]. In such cases, the strains in the subsoil were high, and the assumption of
linearity was incompatible. Conventional track models with asphalt as a supporting layer,
modelled it be linear elastic [15,41] as well as LVE [11,18,42]. The development of a numerical
model, however, advanced it may be, is not enough, as there are several modelling assumptions,
such as uncertainty in material properties, loading scheme, treatment of boundaries, interfaces,
that needs to be verified. This was overcome by validating model responses against available
field data or laboratory measurements [21-23,28-30,43].

Structurally, ballastless asphalt tracks are quite similar to highway pavements; hence,
simulation of material behaviour in the models is of interest. The resilient behaviour of the
unbound granular layers underlying the asphalt pavement was modelled with nonlinear stress-dependent elasticity, using $K-\theta$ model [44], and the Universal model [45-47]; effects of anisotropy was also incorporated [45-47]. The recoverable response of asphalt layer was commonly modelled as viscoelastic, utilizing both linear [45-47] and nonlinear [48-49] models. The recoverable response of asphalt layer was commonly modelled as viscoelastic, utilizing both linear [45-47] and nonlinear [48-49] models.

The objective of this study is to develop a 3D time-domain FEM model of the mockup. The development does not include wheel-rail interaction and only address resilient responses, i.e., fully recoverable deformations. The aim is to consider: (i) time and thermal dependency of asphalt concrete, (ii) nonlinear elasticity of the UGL, and (iii) track dynamic effects. Particular attention will be placed on minimizing calibration activities, i.e., identifying the numerical values of the governing model parameters through standard material testing procedures (and not through fitting). Ultimately, the model’s prediction ability is to be validated by contrasting calculated results against measured responses in the mockup.

The paper is organized as follows: first, the experimental setup and test results are presented. A detailed description of the numerical model is given along with model calibration and input parameters. Finally, the model responses are validated by comparing with mockup measurements, and conclusions are given.

2. Ballastless asphalt track mockup

2.1 General description

A full-scale (length of 4.00 m and a width of 2.21 m) mockup of a ballastless asphalt track was constructed inside a steel box and tested indoors under vertical loads. The schematic view of the mockup is shown in Figure 1a, and the constructed test facility is shown in Figure 1b. From bottom-up, it included: (i) 0.025 m thick polyurethane mat (commercial name Regufoam Vibration 990 Plus) used as a substitute for subgrade and subbase, (ii) 0.275 m thick unbound granular layer (UGL), and (iii) 0.280 m thick asphalt layer composed of 0.240 m of base asphalt mix (paved in 3 lifts), covered by a top (surface) asphalt mix (single lift). The track layers were built inside the steel box in an outdoor location and craned inside a test hall and placed on a thick concrete floor.

The sleepers chosen for the mockup (type BBS 3 W60) were from the GETRAC A3 system, a ballastless asphalt track within the German railway industry. The sleepers were approximately 2.40 m long and 0.57 m wide at the base and included a thin geotextile of thickness 7 mm at the underside over the two ends (each over an approximate area of 1.00 m × 0.57 m). Three sleepers were installed in the mockup on top of the asphalt layer at a spacing of 0.6 m. The sleepers were provided with built-in Vossloh 300 fastening system; rail pads in this
system consists of three plates, the top one, a stiff plastic pad, the intermediate plate made of steel and the bottom one, an elastic rubber pad. The manufacturer’s datasheet reported the static stiffness of the bottom pad to be 23 kN/mm (± 10 %).

Vertical loads were applied to the sleepers directly using three servo-hydraulic actuators (MTS Model No 244.22) that were connected to a rigid loading frame built over the box. Each actuator was responsible for loading the sleeper beneath it (see Figure 1a). Small rail segments (UIC60) were mounted onto the sleepers with the built-in fastening system to aid in load distribution. A spreader beam was connected to the swivel base of the actuator to distribute the loads equally on top of the two rail segments.

Sensors employed to evaluate different mechanical responses included: (i) pressure cells (PCs) to measure vertical stresses at the bottom of the UGL, (ii) asphalt strain gauges (ASGs) to measure horizontal strains at the bottom of the asphalt layer, and (iii) accelerometers (ACCs) to measure vertical surface accelerations. These sensors (names and locations) are shown schematically in Figure 2. Additionally, potentiometers (POTs) were installed to monitor rail pad compression, and linear variable differential transformers (LVDTs) were used to measure the relative vertical displacement between Sleeper 2 (see Figure 1a) and asphalt surface.

2.2 Mockup test results

Results from two tests carried out in the mockup are presented in this study. In both tests, the average temperature in the test hall was 22 °C. The sign convention adopted is as follows: upwards load and displacement are considered positive, and tensile stresses and strains are positive (this is consistent with the sign convention adopted in the numerical model, introduced later). In the first test, a pulse load was applied to Sleeper 2 by Actuator 2, as shown in Figure 3a. This load history is used for calibration of model parameters. For this loading, the time history of rail pad compression and the relative displacement between Sleeper 2 and the asphalt surface (at the edges) is shown in Figures 3b and 3c, respectively. The latter is a net effect of geotextile compression and surface displacement at the sensor locations (neglecting sleeper bending). Further, responses histories measured by selected pressure sensors, asphalt strain gauges and accelerometers are presented in Figure 4.

In the second test, the passage of Danish IC3 train was simulated at a speed \( V \) of 120 km/h and axle load \( 2P_Z \) of 120 kN, \( P_Z \) = wheel load, by the method of sequential loading [50,51]. The principle behind this method is that during train passage every sleeper along the railway track experiences the same loading history, but with a time delay calculated as, \( \Delta t = s/V \), where \( s \) denotes the sleeper spacing. Hence, vertical loads induced by a moving train
can be approximately reproduced by sequentially loading the three sleepers (from Sleeper 1 towards Sleeper 3) with the exact same vertical force history. The latter was calculated from a simplified track model of an infinite beam on a Winkler foundation and equations previously described in a separate study [52]. Figure 5 presents the force history of the three actuators from this test with a simulated delay of 18 ms between them. Visually, each actuator force signal shows four sets of 'M' shapes. Each 'M' includes a pair of extremums with a saddle point in between; the extremums represent two axles on the same bogie. This loading history has been used to validate the model.

For this loading scheme, the measured responses are illustrated in Figures 6 to 8. All sensor readings that are presented graphically are a manifestation of the externally applied load only, without the self-weight. Vertical stress history measured by two PCs are presented in Figure 6. The two charts in this figure follow the shape of the load showing the four sets of 'M', each with a pair of extremum values (may or may not be identical) and a saddle point in between. The stress history showed minor time effects. Due to its location (see Figure 2c), the peak vertical stress at the bottom of the UGL is measured by PC16 to be 21 kPa (see Figure 6b).

The horizontal strains measured by four ASGs are shown in Figures 7. The strains measured along Z direction is shown in Figures 7a and 7b, while, strains along X direction is shown in Figures 7c and 7d. In all the curves shown in Figure 7, the four sets of 'M' shapes are seen, each with two extremum values and a saddle point in between. In contrast to the stresses, the strain measurements show pronounced time dependency. In Figure 7a, the strain history measured below Sleeper 3 (by ASG 3) shows sign reversal as the load progressively moves from Sleeper 1 towards Sleeper 3. This is also recorded below Sleeper 2 in the measurements of ASG 6, but to a minor extent. Further, ASG3 shows strain accumulation between passes of two successive axles on a bogie, the second strain peak being higher on average by 30%. ASG 10, ASG 12 are expected to give similar responses due to symmetry in loading and locations (see Figure 2b). However, test data indicates differences in measured responses with ASG 12 recording values that are higher than ASG 10. Moreover, the measurements of ASG 9 and ASG 2 were not reliable in these tests. Additionally, the measured strains are very low almost within the precision limit of the strain gauges. Hence there is some uncertainty related to the strain measurements especially along X direction. The vertical surface accelerations measured by two ACCs are shown in Figures 8.
3. Finite element model

3.1 General description

A 3D time-domain FEM model of the mockup was developed using the commercial program ABAQUS [53]. The model was built as a rectangular cuboid of dimensions 4.00 m × 2.21 m × 0.58 m. It was partitioned along the depth into three parts to simulate the three layers in the mockup (see Figure 9). The bottom layer represented the mat, the intermediate layer simulated the UGL and the topmost part represented the asphalt layer; layer thicknesses were same as in the mockup. The sleeper geometry was modelled based on dimensions of BBS 3W 60 type, with some simplifications adopted near to the location of the fasteners (see Figure 9). The sleepers consisted of a thin layer at the underside to simulate the geotextile. For the fastening system, only the rail pads were modelled, each as square cuboids with side 150 mm and thickness of 10 mm. This was a simplification given that the rail pads were an assembly of three different plates as explained earlier in Section 2.1.

Entities that were part of the loading system in the mockup, i.e., rails segments, spreader beams, actuators and loading frame were not included in the model. Instead, the sleepers were loaded directly on top of the rail pads as uniformly distributed stresses. The assumption of uniform stress distribution on top of the rail pads was a simplification and it was done considering that the focus of the current study was not on the analysis of contact stress distribution at rail-rail pad interface. Displacement compatibility was enforced in the interfaces between all the successive layers which means that no separation or sliding between nodes was allowed.

The finite domain was discretized with 120,857 eight noded linear 3D solid brick elements of type C3D8 (see Figure 9). The layers simulating asphalt and UGL, each, consisted of five layers of elements along the thickness, while the mat was built with a single layer of elements. An average element size of 50 mm was used for discretizing these three layers. The layers representing the rail pads were discretized into a finer mesh using an average element size of 20 mm with a thickness of 10 mm. The elements modelling the sleeper (multiple element layers) and the geotextile (single element layer) had size ranging between 20 mm and 50 mm.

The boundary conditions were similar to the mockup: (i) the nodes at the base were fixed in all directions, and (ii) the nodes on the lateral faces of the mat and the UGL were restricted from moving in the direction of their respective normal. This was because the bottom surface, the mat and the UGL were fully in contact with the steel plates of the box which were quite rigid.

Nonlinear dynamic analyses were performed in time-domain using direct time integration with implicit scheme. The time increments Δt that were utilized were 0.00097 s.
The sign convention adopted follows from the coordinate system shown in Figure 9 whereby tensile stresses and strains are positive and upwards load and displacement is positive. For model calibration and validation, numerical calculations were compared with experimental measurements. For the purpose of comparing calculations with a given sensor measurements, responses were extracted from all elements covering the area of that particular sensor. The results were then averaged and subsequently used for comparison. The stresses and strains were evaluated at the integration points in an element and the accelerations and displacements were computed at the element nodes.

3.2 Material models

The material models adopted for UGL and asphalt layer and their implementation in the FE model are discussed in subsection 3.2.1 and subsection 3.2.2 respectively. All other materials were modelled as linear elastic.

3.2.1 Unbound granular layer

The UGL was modelled as a homogenous nonlinear elastic isotropic medium characterized by a constant Poisson’s ratio and a stress-state dependent resilient modulus \( M_r \) [35]:

\[
M_r = k_1 P_a \left( \frac{\theta}{P_a} \right)^{k_2} \quad \theta > 0
\]

where \( k_1 \) and \( k_2 \) are dimensionless regression constants, \( P_a \) is atmospheric pressure (introduced as a normalizing factor), and \( \theta \) is the governing bulk stress (i.e., the first stress invariant) at the location where \( M_r \) is evaluated. Any suction effects in the UGL are neglected and therefore total and effective stresses are same.

The bulk stress is seen as being additively composed of two parts, one that originates from the construction phase of the mockup \( (\theta_0) \), representing the compaction effects of the base and asphalt, their self-weight, and also the self-weight of the sleepers (assumed uniformly distributed across the entire area). The other \( (\theta_{ex}) \) represents the effects of external loading. This decomposition can be written as:

\[
\theta(X,Y,Z,t) = \theta_0(Y) + \theta_{ex}(X,Y,Z,t)
\]

where it is shown that \( \theta_0 \) does not vary spatially, but changes within the UGL along the depth coordinate \( Y \), while \( \theta_{ex} \) varies both spatially, depthwise and also temporally. \( \theta_0 \) is expressed as:

\[
\theta_0 = -\sigma_{Y,0}(1 + 2K)
\]
where, $\sigma_{y,0} = \sigma_{y,0}(Y)$ is the initial vertical stress at any point in the UGL and $K$ is the at-rest earth pressure coefficient. $\theta_{ex}$ is a calculated entity that is evaluated for every element constituting the granular layer for applied loads. In effect $\theta_{ex}$ is a function of $M_r$, hence, there is need for an iterative scheme or incremental approach to ensure consistency with the formulation.

With a given set of $k_1$ and $k_2$, the variation of $M_r$ was calculated over a predefined chosen range of $\theta$ values from 0 kPa up to 500 kPa. The results of $M_r$ and corresponding values of $\theta$ were provided to ABAQUS in a tabular form. Subsequently, a user-defined subroutine was written (USDFLD) to implement the remaining formulation. The principle is outlined below as:

for $i = 0$, $t = t_i$ (Base state condition)

External loads ($t = t_i$) = 0, $\theta_{ex}$ ($t = t_i$) = 0

evaluate $\theta_0$ (for all elements within the UGL)

$\theta$ ($t = t_i$) = $\theta_0$ + $\theta_{ex}$ ($t = t_i$)

Table look up for $M_r$ ($t = t_i$) associated with $\theta$ ($t = t_i$) (linear interpolation between data points)

Assign respective $M_r$ to the UGL elements.

Next time instant, ($t = t_{i+1}$), external load increment

Evaluate $\theta_{ex}$ ($t = t_{i+1}$) using $M_r$ ($t = t_i$)

$i = i + 1$

As $M_r$ was updated based on bulk stresses from previous time instant and not the current one, it was verified through utilizing very small time increments ($\Delta t = 0.00097$ s) that there was negligible error on the evolution of $M_r$ with the bulk stress level.

3.2.2 Asphalt layer

The asphalt layer was modelled as a homogenous LVE isotropic solid with a constant Poisson’s ratio governed by a time-dependent shear modulus $G(t)$ and a bulk modulus $K(t)$:

$$G(t) = \frac{E_0f(t)}{2(1+\nu_0)}$$

(4)

$$K(t) = \frac{E_0f(t)}{3(1-2\nu_0)}$$

(5)
where $E_0$ is the instantaneous (short-term) relaxation modulus, $\nu_0$ is the instantaneous Poisson’s ratio, and $f(t)$ is a dimensionless time function expressed as a finite-length Prony series:

$$f(t) = 1 - \sum_{i=1}^{N} m_i \left(1 - e^{-t/\rho_i}\right)$$

(6)

in which $m_i$’s are relaxation strengths (dimensionless) and $\rho_i$’s are relaxation times (units of time).

Furthermore, it is assumed that the model conforms to the assumption of thermo-rheological simplicity wherein the response at any temperature level $T$ can be calculated with properties obtained at a reference temperature $T_0$ by replacing physical time $t$ with reduced time $t_r$ according to: $t_r = t/a_T$ with $a_T$ denoting the time-temperature shift factor. Herein, the WLF equation [54] was chosen for expressing $a_T$:

$$\log a_T = \frac{-c_1(T - T_0)}{c_2 + (T - T_0)}$$

(7)

Where, $c_1$ (unitless) and $c_2$ (units of temperature) are material parameters at reference temperature $T_0$.

3.3 Model calibration and input parameters

Inputs for the different material models were determined using: (i) laboratory element tests for the parameters $m_i$, $\rho_i$, $E_0$, $T_0$, $c_1$, and $c_2$ (see Equations 4 - 7), and $k_1$, $k_2$ (see Equation 1); (ii) manufacturer’s technical data sheet providing the Young’s modulus, density and damping ratio of the mat, and density of the geotextile; (iii) direct calculations based on mockup measurements to provide Young’s modulus of the rail pad; (iv) best matching the mockup measurements to arrive at $K$ (see Equation 3), Young’s modulus of geotextile, and Poisson’s ratio of rail pad. This was done separately for each of the parameters - adjusting their numerical values until a visually good match was attained between calculations and measurements; (v) measurements from mockup providing density of UGL and asphalt layer, and (v) reference ranges available in literature for the Young’s modulus, Poisson’s ratio, density and damping coefficient of all other materials.

The model input parameters (their values and process of deriving) for the different layers are explained below in the sequence in which they appear (from bottom-up).
1. Mat: The elastic modulus and damping ratio of the mat was taken from the manufacturer’s technical data sheet (see Table 1).

2. UGL: Parameters describing the stress-dependent resilient modulus of the UGL ($k_1, k_2$) were obtained from the analysis of cylindrical specimens subjected to repeated load triaxial tests at ‘constant’ radial stress conditions and ‘low stress level’ [55]. For this purpose, the UGL was reconstituted inside a triaxial cell by compacting loose unbound granular materials to the target dry density and moisture content as measured in the mockup. The specimen was exposed to different stress paths, and resilient modulus was evaluated at each stress level (as a ratio of cyclic deviatoric stress and recoverable axial strain). Subsequently, the regression constants: $k_1$ and $k_2$ (Equation 1) were obtained using a nonlinear optimization technique by achieving best match between experimentally obtained and calculated values of $M_r$ for different values of bulk stress, $\theta$ (Figure 10a).

The value of $K$ (Equation 3) was fixed to be 1 after model calibration. This was done by comparing model calculations and experimental measurements for all the sensors that are shown in Figure 4. The choice was guided by a study [56] in which the sensitivity of these model responses to variation of $K$ was separately investigated. From a practical standpoint, values lower than 1 were not considered as residual stresses induced by compaction are not expected to be so low.

3. Asphalt layer: LVE solid parameters describing the asphalt layer, i.e., $\mu$, $\rho$, $E_0$, $T_0$, $c_1$, and $c_2$ ($\nu_0$ assumed), were obtained from analysis of a disk-shaped specimen cored from the base asphalt in the mockup. The specimen was tested in indirect tension mode [57] under five different temperature levels: -5 °C, 10 °C, 15 °C, 20 °C, and 30 °C. In each temperature level the specimen was exposed to 25 load-unload-rest sequences that were 3 s in duration, as well as a relatively long rest period of 1800 s executed after the last sequence; the change in diameter was monitored and recorded throughout the test. The analysis consisted of following steps: (i) assume an analytic LVE solid creep compliance function $D(t)$ governed by four-parameters to capture recoverable response [58]; (ii) assume a linear dashpot model for capturing any irrecoverable deformation accumulating in a given sequence; (iii) apply the load history from the indirect tension tests to a disk model composed of the above assumed properties and calculate change in diameter [59]; (iv) manipulate all model parameters until best match is achieved across all test results. This was done with a multicriterion optimization approach for considering measurements across different temperature levels [60]. As an example, Figure 10b
shows a comparison between measured and calculated responses at 20°C; (v) calculate $c_1$ and $c_2$ with $T_0 = 20°C$; (vi) interconvert $D(t)$ into a LVE relaxation modulus utilizing a Prony series approach [61] to arrive at $\rho_i$’s and $E_i$’s ($i = 0...15$) from which $m_i = E_i / E_0$ ($E_i$’s denote relaxation strengths). Figure 10c shows LVE creep compliance and relaxation modulus at $T_0 = 20°C$.

4. Geotextile: The Young’s modulus of the geotextile was calibrated by visually obtaining a good match between calculations and measurements of relative vertical displacement between central sleeper and asphalt surface. The response history, after calibration, is shown in Figure 3c (Poisson’s ratio was chosen as 0.15). The responses agree well in the loading phase in terms of shape and peak magnitude, while variations are noted in the unloading phase. The computed responses show a rapid recovery while the LVDTs in the experiments show a time-dependent recovery after unloading. This could be attributed to the simplified material model adopted for the geotextile, wherein, the effects of time dependency or nonlinearity (if present) is not captured. Advanced material modelling of the geotextile is not included in the scope of the present study. The calibration was thus limited to mostly matching the peak response.

5. Sleeper: Elastic properties and density of concrete were used for sleepers.

6. Rail pads: The static spring stiffness of rail pad ($k_{rp}$) was provided by the manufacturer to be 23 kN/mm (± 10 %). The Young’s modulus of the rail pad ($E_{rp}$) can be calculated from spring stiffness as:

$$E_{rp} = \frac{k_{rp} h_{rp}}{A_{rp}}$$  \hspace{1cm} (8)

where, $h_{rp}$ and $A_{rp}$ are the initial thickness and area of the rail pads

The dynamic stiffness of the rail pad in the mockup was calculated as 33 kN/mm (for the applied load in Figure 3a and measured rail pad compression in Figure 3b). Inserting this value in Equation 6, $E_{rp}$ was obtained as 14.7 MPa. Using this value as an input to the model, the Poisson’s ratio of rail pad was calibrated to be 0.05. The predicted and measured response histories (rail pad compression) after calibration are illustrated in Figure 3b.

Damping

Rayleigh damping was used in this analysis; the damping ratio of this model is frequency dependent. The input parameters ($\alpha_R, \beta_R$) required to define the model were calculated using Equations 9 and 10:
\[
\alpha_R = \frac{2\xi \omega_1 \omega_2}{\omega_1 + \omega_2}
\]
\[
\beta_R = \frac{2\xi}{\omega_1 + \omega_2}
\]

(9)

(10)

where, \( \xi \) is the damping ratio at two frequencies \( \omega_1 = 25 \text{ rad/s} \) and \( \omega_2 = 400 \text{ rad/s} \). The target was to achieve an approximately damping ratio of \( \xi \) between \( \omega_1 \) and \( \omega_2 \), which represent frequency range of interest. As the asphalt layer had energy dissipation mechanism as part of the basic material model, additional damping was not included.

All the above model parameters are summarized in Tables 1 and 2. Selected model responses calculated for the load history shown in Figure 3a, using these input parameters, are illustrated in Figure 4. In general, it illustrates the capability of the model in reproducing the results. The differences in the calculations and measurements will be examined in detail during validation.

4. Model validation

At this stage, the model parameters were fixed to the values given in Tables 1 and 2. Dynamic analysis was performed with a loading history that was applied in the mockup (as shown in Figure 5). As model parameters were not changed during this analysis, the calculated responses can be considered as forward predictions. These predictions were compared to the measurements recorded in the mockup in order to validate the model. The loading scheme (as shown in Figure 5) chosen for validation is representative of service conditions, i.e., train passage. The loading was implemented in the model as uniformly distributed stress on top of the respective rail pads (one half on either side). The goodness of model predictions is quantified using Pearson product-moment correlation coefficient \( (r) \), and a modern agreement metric \( (\lambda) \) [62].

Predicted stress history for PC 15 and PC 16 (for location, see Figure 2c) is shown in Figures 6a and 6b. In Figure 6a, the average value of the extremums is overestimated by 26 %, while in Figure 6b, it is overestimated by 15 %. With reference to Figure 6a, the predicted value of the saddle point in the first and fourth 'M' shapes is lower than the average value of the extremums (in the first and fourth 'M') by 23.5 % (in measurements it is 21 %). The predicted value of second and third saddle point is lower than the respective extremum pair by 32 % (in measurements it is 29 %). Similarly, in Figure 6b, the predicted saddle point in the first and
fourth' M' shapes is lower than the average value of the extremums by 20.5 % (in measurements it is 18.5 %). The predicted value of the second and third saddle point is lower than the extremum pair by 30 % (in measurements it is 28 %). Further, in Figure 6b, both predictions and measurements show a minor time-dependent behaviour between the ' M ' shapes and the values of the eight extremums are of nearly the same magnitude. Comparing Figures 6a and 6b, the predicted value of the extremum (the average) of PC 15 is lower than that of PC 16 by 18 % (in measurements it is 25 %). Based on the above qualitative assessments and from the high values of $r$ (> 0.99) and $\lambda$ (> 0.96) it can be said that the model predictions and measurements agreed well.

The predicted horizontal strains at the bottom of the asphalt layer for sensors ASG 3, ASG 6, ASG 10 and ASG 12 are shown in Figure 7. Figures 7a and 7b compare strains along the Z direction (for coordinate system see Figure 9), while, Figures 7c and 7d compare strains along the X-direction. From Figures 7a and 7b, it is seen that the model captures the overall shape of the measurements but underestimates the extremum values. In Figure 7a, the predicted strains at the beginning of every ' M ' shape, show a sign reversal and in-between the ' M ' shapes, and at the end of the last ' M ', shows a time-dependent response. Further, within every ' M ' shape, the second extremum value is on average higher than the first one by 30 %. All of these features are observed in the experimental measurements of ASG 3.

In Figure 7b, it is observed that the model underestimates the average values of the extremums by 22 %. The predicted value of the saddle point in the first and last ' M ' shapes is lower than the average value of the extremums by 50 % (in measurements it is 54 %). The predicted value of the second and third saddle point is lower than the average value of the extremum pairs by 57 % (in measurements it is 59 %). Moreover, predicted strains show a change in the sign at the beginning and end of an ' M ' shape; in-between the ' M ' a time-dependent response is observed. Also, the eight extremum values are predicted to be of similar magnitude with minor strain accumulation. All these features are present in the recordings measured by ASG 6. The recordings of ASG 11 were unreliable in this test and hence not used for model validation. In Figures 7c and 7d, the model captures the overall shape and magnitude of the strain history along the X-direction. In both figures, the predictions show a higher strain accumulation between the extremum pairs. Time-dependent behaviour observed between the ' M ' shapes is reproduced. The above observations and high values of $r$ (> 0.96) and $\lambda$ (> 0.85) indicate good prediction abilities of horizontal asphalt strains.

The computed and measured vertical acceleration histories at the surface of the asphalt layer are shown in Figures 8a – 8b. For visual clarity, a part of the acceleration time history
from $t = 26.2$ s to $26.5$ s is zoomed on and separately presented in Figures 8c and 8d. The measurements and predictions corresponding to sensor ACC 37 were similar to ACC 36, hence not shown here. The figures show that in general, the predictions and measurements have a similar overall shape and magnitudes are of the same order. In contrast to stress and strains predictions, much lower values of $f_r$ (> 0.25) and $\lambda$ (> 0.24) were obtained. This is because the accelerations are a more chaotic response compared to stress and strains and hence, it is difficult to predict them with the same level of accuracy.

In reality, a perfect match between model predictions and measurements cannot be produced and differences will arise from: (i) uncertainty in laboratory characterization of materials, (ii) uncertainty in responses measured in the mockup, (iii) modeling assumptions related to interface behaviour between layers, (iv) neglecting permanent deformations, (v) assumptions of linearity and homogeneity in the model, (vi) difference in material responses in tension and compression.

5. Contour plots

Figure 11a presents a contour of the vertical stress distribution in the X-Y plane through the middle of Sleeper 2. The time instant shown (at $t = 25.219$ s) coincides to a load position when the first train axle is on Sleeper 2. The stresses shown here are only due to external applied loads, without initial self-weight. For visual clarity, only the track layers are displayed, downwards from the asphalt surface (geotextile, sleepers are rail pads are not included). In general, nearly uniform compressive stresses occur on the asphalt surface just below the geotextiles (loaded areas); with a magnitude of around 34 kPa. At the middle (in-between the geotextiles), the surface is essentially unloaded without any stresses. The stresses gradually decrease with depth, the peak value at the bottom of the UGL occurs below the loaded areas and the stress distribution in-between is fairly uniform. Figure 11b presents a contour of the stress distribution in the Y-Z plane through the center of the rail pads for the same time instant.

The vertical displacement contours for the same conditions are presented in Figure 12. The peak surface displacement resulting from the substructure layers, i.e., asphalt, UGL and the mat, in combination, is very low, only about 46 microns occurring below the loaded areas. The displacement is fairly uniform in this region. In the unloaded part it is slightly lower, around 35 microns. The total displacement at this time instant calculated on top of the rail pad is 0.75 mm. This overall displacement is mostly contributed by compression of rail pad and the remaining due to geotextile compression. The displacement from the substructure is negligible in comparison.
For visual understanding, Figure 13 presents contour plots along the X-Z plane showing horizontal strain distribution at the bottom of the asphalt layer. Figures 13a, 13c and 13e illustrate strains along X direction, while Figures 13b, 13d and 13f show the strains along Z direction. Three time instants are presented, which tracks the movement of the first train axle from Sleeper 1 (in Figures 13e and 13f, for \( t = 25.198 \) s) to Sleeper 2 (in Figures 11c and 11d, for \( t = 25.219 \) s) and finally to Sleeper 3 (in Figures 13a and 13b, for \( t = 25.240 \) s). In Figures 11a, 11c and 11e, for the three load positions, the strains just below the loaded areas are tensile in nature, while those in the middle and near to the edges are compressive. The magnitude of the peak tensile strains are higher than the peak compressive strains. The change in horizontal strain distribution along the Z direction with the progressive movement of the load can be visualized from Figures 11b, 11d and 11f. The strain show sign reversal from compression to tension as the load progressively moves from Sleeper 1 towards Sleeper 3. The peak tensile strain magnitudes are much higher than peak compressive strains. The magnitude of peak tensile strains along X and Z direction are comparable.

### 6. Conclusions

This paper focused on developing and validating a numerical model of a ballastless asphalt track mockup. For that purpose, a 3D time-domain dynamic FEM model was developed that considered nonlinear stress dependent modulus of unbound granular layer and time-temperature dependent behaviour of asphalt layer. Majority of the input parameters were obtained from laboratory tests and other sources and only a few model parameters were calibrated using mockup data. Model responses (such as stresses, strains, accelerations) were evaluated at different locations and validated for a vertical load history that simulated a moving train.

Some generalized conclusions from this study are summarized: (i) an instrumented mockup with measurements from embedded sensors, provided means for model calibration and validation, under realistic loading conditions; (ii) not all model inputs could be quantified a priori, some needed to be characterized from as-built conditions, such as asphalt as granular materials; (iii) specifically, measuring the rail pad compression allowed for obtaining a realistic stiffness, values provided by datasheet values may not provide a complete picture in terms of stiffness at the applied load amplitude and frequency; (iv) measuring the relative displacement between asphalt surface and sleeper allowed for calibration of elastic properties of the geotextile, this, however can also be done from separate laboratory tests; (v) it was possible to implement a nonlinear stress-dependent elastic model for the unbound granular layer with user
defined subroutine USDFLD; (vi) measured and calculated responses were very small, while, current laboratory material characterization standards operate in a much higher range, (vii) laboratory results on element tests for the asphalt layer displayed permanent deformation, especially at higher temperatures. For modeling resilient responses, only resilient properties must be used as input to the model. This means careful characterization of the viscoelastic parameters by separating out the viscoplastic response, and (viii) in general, there was good agreement between the mockup measurements and the model predictions as revealed by values of $r$ and $\lambda$. It was possible to predict stresses and strains more accurately compared to accelerations.

The validated model can be further employed to investigate other loading conditions in terms of intensities, speeds, loading durations and temperature levels. Moreover, layer thicknesses and material properties can be optimized using model responses. As future developments of this study, modelling efforts will aim at simulating field like conditions. To address this, the present model can be scaled up and the infinite domain of the subsoil can be integrated using special elements. Inclusion of a rail will further allow to include vehicle dynamics and analyze near and far-field vibrations in ballastless asphalt tracks. By including frictional properties of the sleeper- asphalt interface, mechanical responses under horizontal loads can be studied. In this context, standard sleeper geometries can also be used and responses can be compared with heavier sleeper types (such as the one used in the current study). Moreover, laboratory tests can be carried out to characterize the geotextile and implement more realistic models.

Acknowledgement

The support from Innovation Fund Denmark is gratefully acknowledged. This study is part of ‘Roads2Rails: Innovative and cost-effective asphalt based railway construction system’ (Grand Solutions 5156-00006B). The authors would like to thank Mr Shafiqur Rahman and Mr Abubeker Ahmed from the Swedish National Road and Transport Research Institute for their valuable contribution in the experimental investigations. The support provided by all the project participants and the laboratory technicians in the Technical University of Denmark is gratefully acknowledged.
References


Figure 1: A full-scale mockup of a ballastless asphalt track. (a) Schematic illustration of the mockup cross section, and (b) final constructed test facility.
Figure 2: Schematic illustration (top view) of the sensors (names and locations) installed at different elevations in the mockup. (a) Accelerometers (ACC) on the surface of the asphalt layer, (b) asphalt strain gauges (ASG) at the bottom of the asphalt layer, and (c) pressure cells (PC) below the unbound granular layer.

(All dimensions in mm)
Figure 3: (a) Load history applied by Actuator 2 on top of middle sleeper. Calculated and measured time history of: (b) rail pad compression, and (c) relative vertical displacement between Sleeper 2 and asphalt surface.
Figure 4: Calculated and measured time history of: (a) – (b) vertical stresses below UGL, (c) – (d) horizontal strains below asphalt layer, and (e) – (f) vertical surface accelerations.
Figure 5: Simulated train passage by sequential loading of three sleepers in the mockup (axle load = 120 kN, and speed = 120 km/h)
Figure 6: Vertical stresses below unbound granular layer for a simulated train passage - measured and predicted time history.
Figure 7: Horizontal strains below asphalt layer for a simulated train passage - measured and predicted time history along: (a) – (b) along Z direction, and (c) – (d) along X direction.
Figure 8: Vertical surface accelerations for a simulated train passage - measured and predicted time history
Figure 9: 3D finite element model of a ballastless asphalt track mockup
Figure 10: (a) Resilient modulus of unbound granular layer – model and experiments, (b) computed and measured values of horizontal deformation of an asphalt specimen in indirect tensile test, and (c) viscoelastic creep compliance and relaxation modulus of the asphalt layer.
Figure 11: Vertical stress distribution through depth. (a) Contour plot in X-Y plane through model center, and (b) contour plots in Y-Z plane through center of rail pads (for $t = 25.219$ s).

(Rail pads, sleepers and geotextile are not included)
Figure 12: Vertical displacement through depth. (a) Contour plot in X-Y plane through model center, and (b) contour plots in Y-Z plane through center of rail pads (for $t = 25.219$ s).

(Rail pads, sleepers and geotextile are not included)
Figure 13: Contour plots in X-Z plane showing evolution of horizontal strains at bottom of asphalt layer during simulated train passage: (a) – (b) $t = 25.240$ s, (c) – (d) $t = 25.219$ s and (e) - (f) $t = 25.198$ s
### List of Tables

#### Table 1

*Model input parameters simulating following entities*

<table>
<thead>
<tr>
<th>Entities</th>
<th>Young’s Modulus $E$ [MPa]</th>
<th>Poisson’s ratio $\nu$ [-]</th>
<th>Density $\rho$ [kg/m$^3$]</th>
<th>Damping ratio $\zeta$ [-]</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail pad</td>
<td>14.7</td>
<td>0.05</td>
<td>800</td>
<td>0.055</td>
<td>-</td>
</tr>
<tr>
<td>Sleeper</td>
<td>32,000</td>
<td>0.20</td>
<td>2500</td>
<td>0.03</td>
<td>-</td>
</tr>
<tr>
<td>Geotextile</td>
<td>1.6</td>
<td>0.15</td>
<td>900</td>
<td>0.07</td>
<td>-</td>
</tr>
<tr>
<td>UGL</td>
<td>-</td>
<td>0.35</td>
<td>2286</td>
<td>0.055</td>
<td>$K = 1$, $k_1 = 1345$, $k_2 = 0.64$</td>
</tr>
<tr>
<td>Asphalt</td>
<td>-</td>
<td>0.40</td>
<td>2438</td>
<td>N/A</td>
<td>see Table 2</td>
</tr>
<tr>
<td>Mat</td>
<td>46</td>
<td>0.48</td>
<td>990</td>
<td>0.055</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 2

Properties of the asphalt layer ($T_0=20^\circ C$)

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\rho_i$ [s]</th>
<th>$m_i$</th>
<th>$i$</th>
<th>$\rho_i$ [s]</th>
<th>$m_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0E-08</td>
<td>0.0082</td>
<td>9</td>
<td>1.0E+00</td>
<td>0.0298</td>
</tr>
<tr>
<td>2</td>
<td>1.0E-07</td>
<td>0.0111</td>
<td>10</td>
<td>1.0E+01</td>
<td>0.0112</td>
</tr>
<tr>
<td>3</td>
<td>1.0E-06</td>
<td>0.0410</td>
<td>11</td>
<td>1.0E+02</td>
<td>0.0044</td>
</tr>
<tr>
<td>4</td>
<td>1.0E-05</td>
<td>0.0967</td>
<td>12</td>
<td>1.0E+03</td>
<td>0.0017</td>
</tr>
<tr>
<td>5</td>
<td>1.0E-04</td>
<td>0.2245</td>
<td>13</td>
<td>1.0E+04</td>
<td>0.0007</td>
</tr>
<tr>
<td>6</td>
<td>1.0E-03</td>
<td>0.2982</td>
<td>14</td>
<td>1.0E+05</td>
<td>0.0002</td>
</tr>
<tr>
<td>7</td>
<td>1.0E-02</td>
<td>0.1862</td>
<td>15</td>
<td>1.0E+06</td>
<td>0.0001</td>
</tr>
<tr>
<td>8</td>
<td>1.0E-01</td>
<td>0.0784</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$c_1$ [-] $20.17$ $E_0$ [MPa] $26.493$

$c_2$ [\textdegree C] $146.08$
Numerical analysis of ballastless asphalt tracks subjected to dynamic loads

T. Bose, V. Zania* and E. Levenberg

Department of Civil Engineering, Technical University of Denmark
Nordvej, Building 119, Kgs. Lyngby 2800, (Denmark)
vaza@byg.dtu.dk*

Abstract This study presented the results of numerical analyses for a ballastless asphalt track. The asphalt layer was modelled as a linear viscoelastic (LVE) solid that was resting on an unbound granular layer (UGL) simulated as a non-linear (stress-state dependent) elastic medium. The mechanical properties of these layers were calibrated from laboratory element tests. For a given structural arrangement, the model was interrogated under a vertical impulse load to assess the sensitivity of selected responses to temperature and to the initial compaction-induced horizontal stresses in the UGL. It was found that horizontal tensile strains at the bottom of the asphalt layer, vertical stresses below the UGL, and vertical surface accelerations were all very sensitive to temperature level. The vertical surface accelerations were also found to be sensitive to the level of compaction-induced stresses in the UGL. In contrast, the other two responses exhibited a moderate dependency on the latter stresses. The results from this numerical study provide a better overall understanding on the mechanical behaviour of ballastless asphalt tracks.

Keywords: Ballastless asphalt track, Compaction induced stress, Dynamic analysis, Finite element model, Temperature.

1 Introduction

Over the years, the traffic in the railway network has increased in terms of speed, axle loads and frequency of the trains. In order to adapt to this, the design of the railway tracks has evolved accordingly. Instead of conventional ballasted tracks, high speed networks are increasingly being built with ballastless tracks, mainly concrete slab tracks to date. Lately, attention is being paid to another type of ballastless track made of asphalt concrete. The use of asphalt in railway tracks is not recent and it has been used as a secondary support for the main load bearing layers, both ballast and concrete slab tracks (Rose and Souleyrette, 2014; Yang et al. 2015). Nevertheless, ballastless tracks based on asphalt are still quite rare, with only a few known field applications, e.g., some tunnel sections in Germany (Rose and Souleyrette, 2014). The mechanical behavior of railway tracks has been extensively investigated by means of numerical modelling. The early
modelling approaches idealized rails as infinite beams and tracks as a Winkler foundation, either elastic (Auersch, 1996) or viscoelastic (Vostroukhov, 2003). Later, the subgrade was integrated with the traditional models as an elastic or viscoelastic half-space (Knothe and Wu, 1998). Subsequently, a variety of problems were investigated, i.e., track vibrations, stresses and displacements within the track, and modal analysis of different track components employing several numerical methods, of which the finite element method (FEM) was widely applied (Galvín et al. 2010; Powrie et al. 2001; Poveda et al. 2015).

Similar studies on ballastless asphalt tracks are quite limited. Huang et al. 1987 developed a FEM based code to analyze asphalt tracks wherein the substructure was modeled as a multilayered elastic system and the rails and sleepers as finite beams connected by linear springs (representing rail-pads). Design charts for asphalt railway tracks were suggested based on empirical equations reflecting failure criteria commonly adopted in highway pavements. In recent years, full-scale test sections were built to evaluate the performance of ballastless asphalt tracks under different intensities of stationary loads (Lee et al. 2016). Later, Lee et al. 2019 conducted field studies in which a test track was built outdoors and effects of seasonal temperature variation on track responses were assessed. As part of this study, a 3-D FEM model was developed in which the asphalt layers were modeled as linear viscoelastic (LVE) solids. Only quasi-static analysis was performed considering distributed loads over the rails. The calculated responses showed a poor match when compared to field measurements.

Subsequently, a numerical model was developed as part of a research project to investigate the mechanical behaviour of ballastless asphalt tracks. The purpose was to mimic a full-scale, limited size test section of an asphalt track which was built inside a steel box and tested indoors. The numerical model and the sensitivity of selected model responses on temperature and compaction stresses are presented herein.

2 Finite element model

A 3-D FE model (Fig. 1) of the asphalt track mockup (4.0 m long and 2.2 m wide) was developed with the commercial program ABAQUS. From bottom up, it includes: (a) 0.025 m thick mat that simulates the cumulative effects of subgrade and subballast, (b) 0.275 m thick unbound granular layer (UGL), (c) 0.280 m thick asphalt layer, and (d) three wide concrete sleepers (used in the GETRAC A3 system) at 0.60 m spacing, including a 7 mm thick geotextile attached at the bottom. The sleepers are 2.40 m long and 0.57 m wide at the base and the geotextile is attached at the two ends of the sleeper in contact with the asphalt top surface over an area of 1.00 m × 0.57 m. The boundary conditions applied in the model were similar to the mockup whereby: (i) the bottom was completely constrained, (ii) the sides along the mat and the UGL were restricted from moving in the direction of their respective normal, and (iii) the sides along the asphalt layer were free. Displacement compatibility was enforced in the interfaces between the successive layers. The finite domain was discretized using eight noded linear brick elements with an average element size of 50 mm. A finer mesh was adopted close to the
loading areas. Table 1 lists the input material parameters considered for the modelling. The material properties of the asphalt layer and the UGL were calibrated from laboratory element tests.

Fig. 1 3-D Finite element model of a ballastless asphalt track mockup (For boundary conditions, see text)

<table>
<thead>
<tr>
<th>Model entity</th>
<th>Young’s modulus (MPa)</th>
<th>Poisson’s ratio (−)</th>
<th>Density (kg/m³)</th>
<th>Damping ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail pad</td>
<td>13</td>
<td>0.40</td>
<td>800</td>
<td>5</td>
</tr>
<tr>
<td>Sleeper</td>
<td>32,000</td>
<td>0.20</td>
<td>2500</td>
<td>3</td>
</tr>
<tr>
<td>Geotextile</td>
<td>1.2</td>
<td>0.15</td>
<td>900</td>
<td>7</td>
</tr>
<tr>
<td>UGL</td>
<td>Eq. (1)</td>
<td>0.35</td>
<td>2286</td>
<td>5</td>
</tr>
<tr>
<td>Asphalt</td>
<td>Fig. 2a</td>
<td>0.40</td>
<td>2438</td>
<td>-</td>
</tr>
<tr>
<td>Mat</td>
<td>46</td>
<td>0.48</td>
<td>990</td>
<td>5</td>
</tr>
</tbody>
</table>

The Young’s modulus of the mat was taken as in the technical specifications, while the rail-pad and the geotextile were obtained from separate tests. The asphalt layer was modelled as a LVE solid with a Prony series expansion of the relaxation modulus (Fig. 2a), obtained from laboratory tests. The UGL was modelled as nonlinear elastic with a bulk stress dependent resilient modulus \( M_r \) as proposed by Seed et al. 1967:

\[
M_r = k_1 P_a \left( \frac{-\sigma_{v,0}[1+2K]+P}{P_a} \right)^{k_2}
\]  

where \( k_1 \) and \( k_2 \) are regression constants obtained from laboratory tests to be 1345 and 0.64 respectively, and \( P_a \) is the atmospheric pressure. \( \sigma_{v,0} \) is the initial vertical stress at any point in the UGL resulting from the combined weight of the sleepers (approximated as a uniform pressure over the area), asphalt layer, and the UGL itself. The initial horizontal stresses \( \sigma_{x,0}, \sigma_{z,0} \) are the residual stresses induced during UGL compaction, which are unknown and are expressed as a factor of the vertical stress in terms of the
parameter $K$ ($\sigma_{x,0} =\sigma_{z,0} =K\sigma_{y,0}$) which is the at-rest earth pressure coefficient. $p$ is the bulk stress at any point in the UGL induced only by external loads:

$$p = - (\sigma_x + \sigma_y + \sigma_z)$$  \hspace{1cm} (2)

where, $\sigma_x$, $\sigma_y$, and $\sigma_z$ are normal stresses along X, Y, and Z directions (see Fig. 1). Incorporation of the material model described in Eq. (1) into ABAQUS was done through a user-defined subroutine USDFLD. Rayleigh damping was assigned to approximate an almost constant value of damping ratio within the frequency range of interest. The damping ratio was chosen based on ranges available in literature. As the asphalt layer has dissipation sources as part of the material model (LVE), additional damping was not considered. The sign convention adopted is as follows: tensile stresses and strains are positive and upwards load and displacement is positive.

### 3 Sensitivity analyses

In the numerical model, vertical loads were applied as uniformly distributed stress on top of the two rail-pads of the middle sleeper. The load function is as shown in Fig. 2b, split into half on each of the two rail-pads. Nonlinear implicit dynamic analyses were performed utilizing small time increments, verified to ensure that the evolution of $M_r$ with the stress level follows Eq. (1).

![Fig. 2](image)

(a) Relaxation modulus of the asphalt concrete (b) Vertical load history applied to the middle sleeper

The first set of sensitivity analyses involved three temperature levels: 0 °C, 22 °C, and 35 °C; each assumed uniform for the whole model. The temperature of 22 °C was chosen because it corresponds to the neutral track temperature in Denmark. The lower and upper bounds were chosen as representative of winter and summer conditions. The second set of sensitivity analyses involved three values for the parameter $K$: 1, 4, and 8. The upper bound on $K$ was based on keeping it lower than the passive earth pressure coefficient, assuming an internal friction angle of 51°.
4 Results and discussions

Presented in Fig. 3 is a contour plot of the vertical stress distribution in the X-Y plane through the model center for $T = 22^\circ C$ and $K=1$. The stresses shown are here are excluding the self-weight and the time instant being presented corresponds to the peak load in Fig. 2b. For visual clarity only the track substructure is displayed, extending downwards from the asphalt surface. Compressive stresses are observed on the asphalt surface occurring just below the geotextile locations (loaded areas); these are fairly uniform with a magnitude of around 40 kPa. At the middle (in-between the geotextiles), the surface is essentially unloaded. The stresses gradually decrease with depth approaching a peak value at the bottom of the UGL (top of mat) of approximately 45% of the peak surface stress. Presented in Figs. 4a and 4b are contour plots of horizontal strain distribution at the bottom of the asphalt layer in the X-Z plane for $T = 22^\circ C$ and $K=1$. The time instant being presented corresponds to when the peak strains occur, which is at a delay from the occurrence of the peak load owing to the VE behaviour of the asphalt layer. Along the X direction, the bending of the asphalt layer results in tensile strains below the loaded areas and compressive strains at the middle. Along the Z direction, tensile strains occur below the loaded sleeper and compressive strains below the adjacent ones. The peak tensile strain in the Z direction is about 100% higher than that in the X direction.

The sensitivity of selected responses to the temperature variation has been presented in Figs. 5 and 6 (assuming $K=1$). Fig. 5a illustrates the vertical stress history below the UGL (i.e., on top of the mat) at a location where the peak compressive stress occurs. The three curves in the image follow the overall shape of the applied load (see Fig. 2b) and the peak stresses show an increase with rise in temperature. This can be attributed to the asphalt layer which has a lower stiffness at higher temperatures resulting in increased stresses within the track. Fig. 5b shows the vertical acceleration history of a location that is along the track center line and at the middle of the sleepers (on the asphalt surface). The peak accelerations (upwards) increase from 0.47 m/s$^2$ to 0.8 m/s$^2$ as the temperature increases. This is associated with the higher vertical displacements occurring with rising temperatures. It is observed that after unloading (at around 20.15 s), the system vibrates at its fundamental frequency, which is calculated to be 73Hz. The subsequent acceleration peaks in this vibration mode show a gradual decay owing to the associated overall model damping. Fig. 6 illustrates the horizontal tensile strain history at the bottom of the asphalt layer at location where the peak strains occur. The overall shape of the curves follow the load with a pronounced time dependent strain recovery.
occurring at higher temperatures. The peak strain magnitudes are very sensitive to the change in temperature. From a reference state of 22 °C, lowering of temperature to 0°C, decreases the strains by 70 %, while, increase of temperature to 35 °C, increases the strains by 110 % (both along X and Z directions).

Fig. 4 Contour plots showing horizontal strain distribution at bottom of asphalt layer along (a) X direction and, (b) Z direction \([K=1, T=22°C]\).

Fig. 5 Influence of temperature on time history of: (a) vertical stress on mat top (at location of the peak) and, (b) vertical surface acceleration along track center line in between the sleepers \([K=1]\)

Fig. 6 (a) Influence of temperature on time history of tensile strains at bottom of asphalt layer (at locations of peak strains) along: (a) Z direction, (b) X direction \([K=1]\)

The sensitivity of the same responses to the variation of the parameter \(K\) is presented in Figs. 7 and 8 (assuming, \(T=22°C\)). Fig. 7a illustrates a mild influence of \(K\) on the peak vertical compressive stresses occurring below the UGL, (i.e., on top of the mat). The
vertical acceleration histories shown in Fig. 7b depicts high sensitivity to \( K \), decreasing by around 60% with an increase in \( K \) from 1 to 8. This would be attributed to lower vertical displacements caused by a stiffer UGL with increasing \( K \). The natural frequency of vibration increases to about 90 Hz when \( K=8 \). The influence of \( K \) on the horizontal tensile strains at the bottom of the asphalt layer are shown in Fig. 8. The strains are moderately sensitive to \( K \); decreasing by about 25% with an increase of \( K \) from 1 to 8.

![Fig. 7 Influence of initial stresses in the UGL on time history of: (a) vertical stress on mat top (at location of the peak) and, (b) surface acceleration along track center line in between the sleepers \( [T=22^\circ \text{C}] \)](image)

![Fig. 8 Influence of initial stresses in the UGL on time history of tensile strains at bottom of the asphalt layer (at locations of strain peaks) along: (a) Z direction, (b) X direction \( [T=22^\circ \text{C}] \)](image)

### 5 Conclusions

A 3-D FE model of asphalt-based track was developed in this study, and dynamic analyses were performed for a vertical pulse load. A parametric study was conducted to investigate the sensitivity of selected model responses to the change in temperature and the initial compaction induced horizontal stresses in the UGL. The findings can be summarized as: (a) the horizontal tensile strains at the bottom of the asphalt layer are highly sensitive to the temperature and moderately sensitive to the initial modulus of the UGL; (b) the vertical stresses at the bottom of the UGL are also quite sensitive to temperature.
but less influenced by the initial modulus in the UGL; and (c) the vertical surface accelerations are found to be sensitive to both the change in temperature and the initial modulus of the UGL.

Acknowledgements The activity presented in the paper is part of the research grant from Innovations fund Denmark, ‘Roads2Rails: Innovative and cost-effective asphalt based railway construction system’ (Grand Solutions 5156-00006B).

References


Analyzing Track Responses to Train Braking

Tulika Bose
Department of Civil Engineering, Technical University of Denmark
Nordvej, Building 119
Kgs. Lyngby 2800, Denmark
Email: tulbo@byg.dtu.dk

Eyal Levenberg (corresponding author)
Department of Civil Engineering, Technical University of Denmark
Nordvej, Building 119
Kgs. Lyngby 2800, Denmark
Tel: +45 4525 1907; Email: eylev@byg.dtu.dk

Varvara Zania
Department of Civil Engineering, Technical University of Denmark
Nordvej, Building 119
Kgs. Lyngby 2800, Denmark
Email: vaza@byg.dtu.dk
ABSTRACT
The objective of this study was to suggest a response-analysis framework for railway tracks subjected to braking. An analytical formulation was developed, in which the rail-track system was modeled as an infinite beam supported by an orthogonal Winkler foundation consisting of linear springs in perpendicular directions. The spring constants were varied over a wide range in order to represent different track types. Braking loads were simulated as representative sets of vertical and longitudinal forces, either concentrated or distributed. Considering a realistic set of model parameters, the approach was demonstrated by evaluating track responses for a single axle and for a full train. The computations included determination of axial rail stresses, forces at the base of a sleeper, and the associated friction demand required to resist longitudinal slippage. Based on these analyses, it is concluded that longitudinal track responses have a much longer influence zone compared to vertical track responses. This implies that calculations involving a full train must be done on a case-by-case basis, i.e., they cannot be deduced from a single axle analysis. It is also found that high values of friction demand may develop at the sleeper bases - indicating possible slippage. Overall, the proposed formulation provides a highly adaptable and easily implementable first-order mechanistic tool for analysis of track responses to decelerating vehicular loads.

Keywords: Railway, Train Braking, Longitudinal loads, Winkler foundation, Friction demand.

Introduction
Over the years, the rail industry has been facing constant public and economic demands to expand service, especially in accommodating faster and heavier trains, and increased line capacities.\textsuperscript{1} This motivated improvements in vehicles, e.g., powering individual railcars for increased acceleration and top speed,\textsuperscript{2} and developing newer/stronger braking systems.\textsuperscript{3,4} The demands also triggered advancements to the physical infrastructure, both within the realm of traditional ballasted tracks, e.g., incorporating new sleeper designs\textsuperscript{5,6} and upgrading vibration absorbing components,\textsuperscript{7-9} as well as by exploring non-traditional track types, e.g., reinforcing ballast layers,\textsuperscript{10} replacing subballast with asphalt concrete, i.e., asphalt underlayment solution,\textsuperscript{11-13}
and switching to ballastless track based on Portland cement concrete, i.e., slab track solution.\textsuperscript{14} Another ballastless track-type that has been gaining recent attention is asphalt overlayment.\textsuperscript{15-17}

An essential step in accepting new track concepts or new vehicles (or both), before full-scale adaptation within the live network, is evaluating mechanical infrastructure responses (i.e., forces, stresses, displacements, strains) to anticipated train loads. As an integral part of such evaluation, the current work focuses on analyzing responses to loads exerted during a braking event. Doing so is deemed a priority especially for asphalt overlayment solutions wherein precast sleepers are placed on an asphalt pavement surface - possibly without crib or shoulder ballast. Therefore, at the superstructure-substructure interface, longitudinal loads are solely counteracted by the sliding resistance available at the sleeper bases. This resistance may be inferior compared to a ballasted track where crib and shoulder ballast also contribute to the sliding resistance.\textsuperscript{18,19} Analyzing track responses due to braking is also warranted in view of advances in braking systems. The traditional method of slowing or completely stopping trains involves clamping of the wheels. In this case, the braking effort is limited by the wheel-rail friction conditions, which depend on factors such as cleanliness level, and moisture conditions;\textsuperscript{20} it is also influenced by railhead roughness.\textsuperscript{21} For this reason, so-called track-brakes were devised, in which the braking force is applied to the top surface of the rails between the axles over a certain length. These operate either through friction with the rail or by generating eddy currents in the rails, creating forces acting in the direction opposite to train movement.\textsuperscript{3}

Analysis of track responses to train braking requires modeling the effects of both vertical and longitudinal vehicular loads. While modeling the former category is fairly established in railway design, only a handful of studies were identified to address the latter. Van\textsuperscript{22} set forth to numerically study thermal loads on rails and bridges as well as mechanical loads due to braking or accelerating trains. In the proposed model, the two rails were jointly treated as a single beam connected to longitudinal elasto-plastic springs with an assumed (fixed) yield limit. The beam’s axial stresses and displacements were then evaluated under varying temperature conditions and also under the influence of an applied longitudinal load. The latter was uniformly distributed, representing the effect of a braking (or accelerating) train, averaged over its entire length. Some shortcomings of the study are that the effects at individual sleepers or due to distinct axles were not included; also, interaction between longitudinal and vertical loading directions was not accounted for.
The work of Rhodes et al.\textsuperscript{23} was motivated by a number of cases where track structures experienced sleeper movements relative to rails and ballast. These resulted from high traction forces induced by heavy freight trains. The study included field measurements of axial rail stresses and offered some design guidelines. However, no mechanistic model was offered, capable of providing quantitative explanation to the observed field responses. Zhang et al.\textsuperscript{24} carried out finite element analysis to investigate the longitudinal forces within the rail fastening system and the friction demand at the rail rail-pad interfaces during train acceleration. All frictional contact interfaces within the track components were modeled as Coulomb-type; in particular, a value of 0.70 was taken for the sleeper-substructure interface. A wheel was accelerated in the model with an assumed wheel-rail friction coefficient of 0.5. Their analysis found that longitudinal forces at the rail rail-pad interfaces were almost zero in front of the wheel. It was also reported that only about five sleepers behind the accelerating wheel participated in carrying the longitudinal load. The paper did not provide a physical explanation of this non-symmetric distribution of responses. Additionally, relative slippage between components and development of residual stresses in the system were not considered.

Subsequently, the objective of this study is to develop a model suited for analyzing the response of railway tracks to braking events, accommodating different braking systems. The purpose is to provide a mechanistic portrayal of how decelerating vehicular loads are handled within a track superstructure. Specifically, the model should allow: (i) resolving the axial stresses along the rail; (ii) identifying the number of participating sleepers and their relative contribution in transferring the braking-related loads to the substructure; and (iii) describing the friction demand at the sleeper-substructure level (or equivalently at the rail-railpad interfaces). An analytic framework is sought for the development in order to provide closed-form solutions that are easily implementable, fast to compute, and therefore highly adaptable as a first-order engineering tool.

The paper commences by introducing the basic model equations and assumptions; this is followed by demonstrating calculations and interpreting results for the case of a single axle and for a full-length train. This is done assuming a traditional braking system while employing a range of model parameters representing different track-types.
Response modeling

In this section, a new two-dimensional response-model for railway tracks is proposed, with the focus on vehicular loads generated during braking. Consider, as shown in Figure 1, a single train axle decelerating over a rail, supported on sleepers that are resting on a substructure. The sleepers are all identical, sequentially numbered and spaced at an interval of $s$. A Cartesian coordinate system is shown with its origin placed at the rail surface directly above the $i^{th}$ sleeper (i.e., a random sleeper being evaluated within the braking zone). The $Z$ axis points downward and the $X$ axis points along the travel direction. The axle exerts a vertical load with magnitude $2P_Z$ representing its total weight and concurrently a longitudinal load of magnitude $2P_X$ signifying its braking effort. The resulting vertical and longitudinal forces at the base of the $i^{th}$ sleeper are denoted as $S_Z$ and $S_X$ respectively. Equilibrium considerations dictate that the same forces must also operate at the rail-railpad interface (i.e., fastening assembly).

The model of the physical problem described in Figure 1 is presented in Figure 2 by referring to a single axle side, i.e., a decelerating single wheel. The rail is considered as an infinite homogenous Euler-Bernoulli beam with cross-sectional area $A$, Young’s modulus $E$, and moment of inertia $I$. The weight per unit length of the beam is taken as: $\gamma = \gamma_R + W/s$ wherein $\gamma_R$ is the rail weight per unit length, and $W/s$ is the distributed weight considering one-half of a sleeper - implying $2W$ denotes a full sleeper weight. The neutral axis of the beam, passing through the geometric centroid of the cross section, is denoted by the dashed horizontal line.

A Cartesian coordinate system is positioned at the point of evaluation corresponding to the location of the $i^{th}$ sleeper, in analogy to Figure 1; the origin coincides with the neutral axis. Loads exerted by the wheel, $P_Z$ and $P_X$, are applied to the neutral axis at a distance $x$ from the coordinate origin. The beam is supported by an orthogonal Winkler type foundation, which includes both vertical and longitudinal linear springs at right angles, connected to the beam along the neutral axis. The springs are shown at discrete locations, only for illustration purposes, in effect they are continuous. The corresponding spring constants are $k_Z$ and $k_X$, both with units of force per length squared; they represent the cumulative support offered to the rail by all underlying track components, with magnitudes depending on the track-type.
The vertical beam displacement at the coordinate origin \( (u_z) \) is expressed following Hetenyi\(^{25} \) as:

\[
u_z = \frac{P_z \beta_z e^{\beta_z |z|}}{2k_z} (\cos \beta_z x + \sin \beta_z |x|)
\]

(1)

wherein:

\[
\beta_z = \sqrt{\frac{k_z}{4EI}}
\]

(2)

Accordingly, the vertical force at the base of the \( i^{th} \) sleeper including self-weight \( (S_z) \) can be approximated as:

\[
S_z = s(u_z k_z + \gamma) = \frac{sP_z \beta_z e^{\beta_z |z|}}{2} (\cos \beta_z x + \sin \beta_z |x|) + s\gamma
\]

(3)

In this expression, the continuous support model is employed for evaluating the response at a discrete location, i.e., a sleeper. The error involved in such approximation has been verified to be negligibly small by comparison against an exact analytic solution for a periodically supported beam.\(^{26} \)

For determining responses in the longitudinal direction, Figure 3 presents a magnified view of an infinitesimal beam element of length \( dx \) located near, and to the left of, the coordinate origin (refer to Figure 2). This element experiences axial stress along its neutral axis, having magnitude \( \sigma_N \) at the origin and \( \sigma_N + d\sigma_N \) at a distance \( x + dx \) from \( P_x \). The chosen sign convention is such that positive stress or strain indicates compression. Also, the element is displaced by \( u_x \) in the direction of \( P_x \); this activates the longitudinal springs and induces an opposing axial stress along the neutral axis with magnitude \( k_x u_x \).

Under the abovementioned stress conditions, longitudinal force-equilibrium dictates:

\[
u_x = \frac{A}{k_x} \frac{d\sigma_N}{dx}
\]

(4)

This expression provides a link between the displacement \( u_x \) and the change in \( \sigma_N \) as \( x \) changes. Alternatively, the displacement \( u_x \) can be viewed as the summation of all deformation increments.
(at the origin) as the force $P_x$ ‘moves’ from infinity towards $x$. This situation is expressed by the integral:

$$u_x = \int_x^\infty \frac{\sigma_N}{E} \, dx$$

(5)

Combining equations (4) and (5) gives the governing integro-differential equation:

$$\frac{AE}{k_x} \frac{d\sigma_N}{dx} = \int_x^\infty \sigma_N \, dx$$

(6)

This equation is solved with a boundary condition stating that for $x=0^+$ the axial stress at the coordinate origin is $\sigma_N = -0.5P_x / A$ (tensile) and for $x=0^-$ the axial stress at the coordinate origin is $\sigma_N = 0.5P_x / A$ (compressive). The solution is therefore:

$$\sigma_N = -\frac{P_x \, \text{sgn}(x)}{2A} e^{-\beta_x |x|}$$

(7)

wherein $\text{sgn}(\bullet)$ is the signum function, and:

$$\beta_x = \sqrt{\frac{k_x}{EA}}$$

(8)

Utilizing either equation (4) or equation (5), an expression representing the longitudinal beam displacement at the coordinate origin is:

$$u_x = \frac{\beta_x P_x}{2k_x} e^{-\beta_x |x|}$$

(9)

from which the longitudinal force at the base of the $i^{th}$ sleeper ($S_x$) can be approximated as:

$$S_x = su_x k_x = \frac{sP_x \beta_x}{2} e^{-\beta_x |x|}$$

(10)

Similar to the vertical case, the suitability of employing a continuous horizontal support model for analysis at a discrete location, was verified by means of a separate finite element analysis. This longitudinal force is transferred from the rail via the fastener-assembly to the sleepers, with the analysis focusing on the interface with the substructure. Overall, the spring forces were derived
by neglecting any second order effects arising from coupling of responses in vertical and longitudinal directions.

Based on equations (3) and (10), an expression describing the friction demand $\mu_D$ at the base of the $i^{th}$ sleeper is:

$$\mu_D = \frac{S_X}{S_Z} = \frac{P_X \beta_X e^{-\beta_X|x|}}{P_Z \beta_Z e^{-\beta_Z|x|}(\cos \beta_Z x + \sin \beta_Z |x|) + 2\gamma}$$

(11)

At its core, $\mu_D$ represents the required resistance at the sleeper base to ensure no-slip conditions during a braking event. As can be seen $\mu_D$ is a function of both the position and intensity of loading; it also depends on all model parameters including sleeper spacing (via the expression for $\gamma$). A closed form solution for the peak value of the friction demand $\mu_D^{\text{max}}$ or the corresponding load position cannot be presented; however, for a restricted domain of model parameters $\mu_D^{\text{max}}$ may be approximated by assuming it develops where the vertical force $S_Z$ is minimal, i.e., for $x = \pi / \beta_Z$.

If the friction demand cannot be realistically matched, then sliding takes place at the sleeper bases. This sliding gives rise to redistribution of longitudinal forces and development of residual stresses within the superstructure. Such inelastic behavior violates the basic assumptions of the proposed framework, rendering it inapplicable.

As means of addressing modern braking technologies, there is a need to provide a solution for a distributed loading both in the vertical and longitudinal directions. This is because the braking effort may not rely solely on a wheel clamping mechanism but apply over a certain rail length. Therefore, the model is hereafter extended to consider the case of vertical and longitudinal loads with magnitudes $P_Z^{\text{mb}}$ and $P_X^{\text{mb}}$ respectively, each acting over a length $x_0$ (superscript mb is introduced to indicate ‘modern braking’). The responses at the origin become:

$$u_Z^{\text{mb}} = \frac{P_Z^{\text{mb}}}{2k_Z x_0} \begin{cases} 
\left( e^{\beta_Z(x+0.5x_0)} \cos(\beta_Z(x+0.5x_0)) - e^{\beta_Z(x-0.5x_0)} \cos(\beta_Z(x-0.5x_0)) \right) \\
+ H(x+0.5x_0) \left( 2 - \cos(\beta_Z(x+0.5x_0)) \left( e^{-\beta_Z(x+0.5x_0)} + e^{\beta_Z(x+0.5x_0)} \right) \right) \\
- H(x-0.5x_0) \left( 2 - \cos(\beta_Z(x-0.5x_0)) \left( e^{-\beta_Z(x-0.5x_0)} + e^{\beta_Z(x-0.5x_0)} \right) \right) 
\end{cases}$$

(12)
\[
\sigma_{N}^{mb} = -\frac{P_{x}^{mb}}{2A\beta_x x_0} \left( e^{\beta_x (x+0.5x_0)} - e^{-\beta_x (x-0.5x_0)} - H(x-0.5x_0)\left(2 - e^{-\beta_x (x-0.5x_0)} - e^{\beta_x (x-0.5x_0)}\right) \right) 
\]
\[
\mu_{N}^{mb} = \frac{P_{x}^{mb}}{2k_x x_0} \left( e^{\beta_x (x+0.5x_0)} - e^{-\beta_x (x-0.5x_0)} - H(x-0.5x_0)\left(2 - e^{-\beta_x (x-0.5x_0)} - e^{\beta_x (x-0.5x_0)}\right) \right) 
\]

wherein the coordinate \( x \) refers to the (lateral) distance between the evaluation point and the center of the load distribution, \( H(\cdot) \) is the Heaviside step function, and all other symbols as previously defined. It is noted that at the limit, as \( x_0 \to 0 \), the above expressions reduce to the point load expressions.

From the displacement expressions, equations (12) and (14), the friction demand in the case of a modern braking system can be evaluated (at the base of a sleeper) following a similar approach as described above, i.e., calculating the forces \( S_{x} \) and \( S_{z} \), and then their ratio (see equation (11)). The basic analysis in this case deals with a single wagon, given that there is a contribution to \( S_{x} \) and \( S_{z} \) from the axles as well as from the distributed braking unit. Thus, all the above expressions, jointly, can be instrumental in quantifying superstructure responses under braking events, for both traditional and modern braking systems.

**Analysis of track response**

The purpose of this section is to demonstrate model capabilities for evaluating the braking event of a single axle and then of a full train. Only the point force solutions were considered representing a traditional braking system. A loading ratio of \( P_{x} / P_{z} = 0.25 \) was chosen given that it represents an accepted and realistic wheel-rail friction level for dry conditions.\(^{27-29}\) The properties of the beam were chosen as per UIC 60 rail section. A standard concrete sleeper was selected with weight \( 2W = 2845 \text{ N} \) (i.e., mass of 290 kg); and spacing as \( s = 600 \text{ mm} \) (therefore, \( \gamma = 2.96 \text{ N/mm} \)).

To represent different support conditions, a range of spring constants were chosen in vertical and longitudinal directions. The former were considered, from a relatively low value of 20 MPa to a high value of 100 MPa.\(^{30,31}\) Similarly, the longitudinal spring constants were selected over a wide range of 5 MPa to 30 MPa. This choice was deemed reasonable based on measured
force-displacement behavior during sleeper pullout tests\textsuperscript{18} from which longitudinal spring constants in the range of 6 MPa to 13 MPa were extracted. Additionally, in connection with track resistance to longitudinal displacement, similar values are mentioned in the UIC Code dealing with track-bridge interactions.\textsuperscript{28}

**Braking of a single axle**

Figure 4 presents model responses for a single axle with $2P_z = 170\text{kN}$ and $2P_x = 42.5\text{kN}$. Four charts are shown, each depicting a different response parameter as a function of the load position ($x$). For graphical presentation purposes, the abscissas in the charts span different ranges of $x$. Figures 4(a) and 4(b) illustrate forces in vertical ($S_z$) and longitudinal ($S_x$) directions, induced at the base of the $i^{th}$ sleeper, as per equations (3) and (10) respectively. It can be seen that the average peak vertical and longitudinal forces differ by almost two orders of magnitude: $32\text{kN}$ vs. $0.6\text{kN}$. The displacements corresponding to these average peak force levels are 1.0 mm and 0.07 mm respectively.

In addition, forces at the base of a sleeper depend on the spring constant values; smaller spring constants correspond to lower peak forces and a wider region of load influence. Conversely, larger spring constants imply higher peak forces and narrower zone of load influence. Vertical forces are observed to decay much faster (from their peak level) with respect to $x$ as compared to longitudinal forces. The impact of vertical load becomes almost negligible as $|x| > 2\text{m}$, i.e., about three sleeper spacings, while the longitudinal load influences a range of $|x| > 18\text{m}$, i.e., almost 30 sleeper spacings.

Figure 4(c) presents the axial stresses along the beam’s neutral axis ($\sigma_x$) at the coordinate origin as per equation (7). It can be observed that the stress reverses sign with respect to the load position; $\sigma_x$ is compressive for negative values of $x$ and is tensile for positive $x$’s. The reversal occurs at $x = 0$ and the peak longitudinal stresses are attained when the load is closest to the origin (just before and just after). The peak stress is independent of the support condition, but the rate of decay with $x$ is influenced by $k_x$. It is noted that the peak longitudinal stress magnitude is relatively small and constitutes less than 5% of the maximal bending stress caused by $P_z$. 
Finally, Figure 4(d) presents the friction demand at the base of a sleeper ($\mu_D$), as per equation (11). It can be seen, that $\mu_D$ is symmetric about the loading location, consistent with the responses of both vertical and longitudinal forces. As the loading approaches a sleeper from a negative $x$, the friction demand increases until $\mu_D^{\text{max}}$ is attained. Then after $\mu_D$ begins to decrease in value until the load is directly above the sleeper. With further increase in $x$, the above described behavior is mirrored, i.e., the friction demand rises to $\mu_D^{\text{max}}$ and then decreases towards zero as $x \to \infty$. With reference to Figure 4(a) it can be seen that the $x$ corresponding to $\mu_D^{\text{max}}$ is nearly the $x$ corresponding to minimal vertical force (i.e., locations where upward beam bending counteracts self-weight). The results presented in Figure 4 indicate a general trend of increasing $\mu_D^{\text{max}}$ with increasing spring constants. It is noted that $\mu_D^{\text{max}}$ values larger than unity may be too high to be provided by a purely frictional mechanism at the sleeper-substructure interface.\textsuperscript{32,33}

Figure 5 further investigates the dependence of $\mu_D^{\text{max}}$ on the spring constants for a single axle. The considered loads for this case remained: $2P_Z = 170\text{kN}$ and $2P_X = 42.5\text{kN}$. Two charts are presented, depicting the variation of $\mu_D^{\text{max}}$ as a function of the ratio $k_Z/k_X$ within the range of 1.5 to 4.5; this was done for $k_X$ in the range of 5 MPa to 30 MPa. In Figure 5(a) this information is shown for a standard sleeper with mass of 290 kg while in Figure 5(b) a heavier sleeper was considered with a mass of 590 kg. Within this analysis domain, $\mu_D^{\text{max}}$ may be approximated to a relative error of 0.5% by inserting $x = \pi / \beta_z$ in equation (11).

Referring to Figure 5(a), it can be seen that $\mu_D^{\text{max}}$ exhibits a positive relationship with both $k_X$ and $k_Z$, and is governed by the ratio of $k_Z/k_X$. The sensitivity of $\mu_D^{\text{max}}$ to increase in $k_Z/k_X$ is dependent on the value of $k_X$. At low $k_X$ values $\mu_D^{\text{max}}$ is mildly sensitive to increase in $k_Z$; conversely, at higher values of $k_X$ the influence of increase in $k_Z/k_X$ becomes more pronounced. Low values of $k_X$ imply small horizontal forces at the base of a sleeper. For the chosen range of $k_Z/k_X$, these small forces mostly govern the maximal friction demand. High values of $k_X$ are associated with larger longitudinal forces at the base of a sleeper. At the same time, within the considered $k_Z/k_X$ range, larger $k_Z$ values imply lower vertical force (due to upward beam bending
counteracting self-weight). Hence, for these positions, it is the adverse combination of high longitudinal force and low vertical force, which governs the peak friction demand.

Referring to Figure 5(b), it can be seen that $\mu^\text{max}_D$ also exhibits a positive relationship with both $k_X$ and $k_Z$, and is governed by the ratio of $k_Z/k_X$. However, with respect to Figure 5(a), the behavior appears different; $\mu^\text{max}_D$ is mildly sensitive to $k_Z/k_X$ across the entire range of considered $k_X$ values. It is important to note that the $\mu^\text{max}_D$ levels in Figure 5(b) are much smaller than those in Figure 5(a), and are well within acceptable levels of interface friction. The above features are a direct result of employing a heavier sleeper.

Braking of a full train

The track response model is hereafter demonstrated by analyzing a braking event involving a full train. Thalys HST train was selected for this purpose; it includes two locomotives and eight carriages, having total of 26 axles spanning a length of approximately 200 m. For the different train units, Table 1 denotes each axle by an index $j$ such that $j = 1$ and $j = 26$ identify the rear and front axles of the train respectively. The Table also lists the individual axle positions relative to the rearmost axle ($\Delta x_j$) and the axle loads ($2P_{Z,j}$). In the analysis, longitudinal axle loads were taken as 25% of the vertical loads, i.e., $P_{X,j} = 0.25P_{Z,j}$; the spring constants were $k_Z = 50\text{MPa}$ and $k_X = 15\text{MPa}$.

The track responses were evaluated at the origin of the coordinate system by considering a collection of 26 force-pairs applied at locations corresponding to the axles. With respect to the origin, at a particular instant of time, the $j^{\text{th}}$ axle was located at a distance of $x + \Delta x_j$. This implies that $x$ denotes the distance between the rear axle (for which $j = 1$, $\Delta x_j = 0$) and the evaluation point. The derived expressions were:

$$S^\text{train}_Z = s' + \frac{s\beta_Z}{2} \sum_{j=1}^{26} P_{Z,j} e^{-\beta_Z[k_Z/k_X]} \left( \cos(\beta_Z(x + \Delta x_j)) + \sin(\beta_Z|x + \Delta x_j|) \right)$$

$$S^\text{train}_X = \frac{s\beta_X}{2} \sum_{j=1}^{26} P_{X,j} e^{-\beta_X[k_X/k_Z]}$$

(15)
\[ \sigma^\text{train}_N = -\frac{1}{2A} \sum_{j=1}^{26} P_{x_j} e^{-\beta_k |y_j + \Delta v_j|} \text{sgn}(x + \Delta x_j) \]  

(17)

wherein \( S^\text{train}_Z \) and \( S^\text{train}_X \) are the vertical and longitudinal forces at the base of the \( i^{th} \) sleeper, and \( \sigma^\text{train}_N \) is the axial stress at the beam’s neutral axis.

**Table 1**

Thalys HST train configuration and axle loads.\(^{34}\)

<table>
<thead>
<tr>
<th>Train units</th>
<th>Axle indices</th>
<th>Axle positions with reference to rearmost axle ( \Delta x_j ) [m]</th>
<th>Axle weights [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 × Locomotives</td>
<td>( j = 1...4 ); ( j = 23...26 )</td>
<td>0.0, 3.0, 14.0, 17.0; 176.0, 179.1, 190.1, 193.1</td>
<td>( 2P_{Z,j} = 170 )</td>
</tr>
<tr>
<td>2 × Outer carriages</td>
<td>( j = 5...7 ); ( j = 20...22 )</td>
<td>20.3, 23.3, 39.0; 154.2, 169.9, 172.9</td>
<td>( 2P_{Z,j} = 145 )</td>
</tr>
<tr>
<td>6 × Central carriages</td>
<td>( j = 8...19 )</td>
<td>42.0, 57.7, 60.7, 76.4, 79.4, 95.1, 98.1, 113.8, 116.8, 132.5, 135.5, 151.2</td>
<td>( 2P_{Z,j} = 170 )</td>
</tr>
</tbody>
</table>

Figure 6 depicts calculated model responses based on equations (15) – (17) with \( x \) varying from \(-250 \text{m}\) to \(+50 \text{m}\). Four charts are included, illustrating \( S^\text{train}_Z \), \( S^\text{train}_X \), \( \sigma^\text{train}_N \), and the ratio \( \mu^\text{train}_D = S^\text{train}_X / S^\text{train}_Z \), i.e., the friction demand at the base of a sleeper. The horizontal dashed lines appearing in the charts are the corresponding peak responses computed with the same parameters but for a single axle with \( 2P_{Z} = 170 \text{kN} \) and \( 2P_{X} = 42.5 \text{kN} \) (see Figure 4).

Figure 6(a) shows the vertical force \( S^\text{train}_Z \) at the base of the \( i^{th} \) sleeper as per equation (15). There are 26 distinct peaks in this chart, each corresponding to an axle; this indicates that load interaction across axles is not very pronounced. The relative difference in the peak force magnitudes for a full train and a single axle is a direct measure of the load interaction. In the case shown, the peak forces for the single axle and the full train are 32 kN (dashed line) and 31 kN respectively; amounting to 3\% load interaction. Based on equation (3), this peak force translates to a vertical beam displacement of about 1.0 mm.

Figure 6(b) shows longitudinal force \( S^\text{train}_X \) at the base of the \( i^{th} \) sleeper as per equation (16). Here, there are no sharp peaks corresponding to the individual axles as in Figure 6(a). This
indicates a pronounced interaction across axles. The effect can be visualized in the Figure by observing that the force rises or decays over long intervals outside the length of the train (i.e., \(-250 < x < -200\) and \(0 < x < 50\)). This wide-ranging interaction is also the reason why forces are highest when the train ends are closest to the sleeper. For these train locations, the axles are more densely spaced as compared to the rest of the train. Contrasting the peak longitudinal force (2.2 kN) with the single axle case (0.6 kN, see dashed line), provides another clear manifestation of the load interaction; this amounts to an interaction level of 266%. Based on equation (10), this peak force translates to a longitudinal beam displacement of about 0.25 mm.

Figure 6(c) presents the longitudinal stresses in the beam’s neutral axis at the origin. In this case, it is noted that the beam exhibits several stress reversals during the train passage. There are 39 stress reversals whereas the number of axles crossing it are 26; indicating the prominence of the interaction effects. The magnitude of the peak stress is noted to be 3.3 MPa, which is more than two times higher than the single axle case (magnitude of about 1.4 MPa, indicated by dashed lines).

Finally, Figure 6(d) depicts the friction demand at a base of the \(i^{th}\) sleeper. It shows an erratic and fluctuating demand reflecting both the vertical and longitudinal force behaviors from Figures 6(a) and 6(b). The demand consists of only 22 peaks which do not directly correspond to axle locations. The maximal friction demand occurs twice, when either of the train end units are close to the sleeper; specifically when \(x = -181.75\) m (i.e., sleeper is between axles 24 and 25) and \(x = -11.35\) m (i.e., sleeper is between axles 2 and 3). These maxima represent the most adverse combination of high longitudinal force and low vertical force. It should be noted that they are influenced by the choice of the spring constants, the ratio of \(P_{x,j} / P_{Z,j}\), and the unit weight of the beam. Compared to the single axle case (dashed line at a value of 1.0), the peak friction demand due to the entire train is seen to be 3.5 times larger, further highlighting the interaction effects encountered for a full train. The above analysis was also repeated with a heavy sleeper, in which case \(S_{\text{Z}}^{\text{train}}\) retained the same shape while the peak shifted upwards by 1.47 kN representing the added self-weight of the system. The \(S_{\text{X}}^{\text{train}}\) and \(\sigma_{N}^{\text{train}}\) responses remained unaltered, while the maximal friction demand dropped to 0.9.
Conclusions
This paper focused on modeling railway track responses to braking loads. An analytical framework was proposed based on an infinite Euler-Bernoulli beam supported by an orthogonal Winkler foundation, subjected to concentrated or distributed forces (or as a combination), in both vertical and longitudinal directions. Such modeling applies to different track types, and can address both traditional and modern braking systems. Closed-form expressions were provided for calculating diverse set of responses such as: (i) longitudinal rail stresses and displacements, (ii) vertical and longitudinal displacements or forces at a sleeper base (or fastener), and (iii) friction demand at a sleeper base (or fastener) corresponding to zero slippage condition.

Subsequently, for a range of realistic parameters, the model was first demonstrated by application to a single axle. The specific findings from this demonstration were: (i) peak longitudinal rail stress was independent of the support condition but its decay was a function of the latter, (ii) longitudinal track responses were spread over a much longer zone in comparison to vertical track responses, (iii) peak friction demand exhibited a positive relationship with the spring constants, and (iv) peak friction demand was unrealistically high for a standard sleeper weight and acceptably low for a heavier sleeper. The model was demonstrated next for a full train considering a traditional braking system. This analysis showed that responses in the longitudinal direction, as well as peak friction demand, increased considerably compared to the single axle case. This outcome was due to the pronounced load interaction in the longitudinal direction.

These findings can be generalized to arrive at the following conclusions for a braking event: (i) for a full train, all track responses involving longitudinal loads cannot be deduced from a single axle analysis; calculations must be done on a case-by-case basis as all related peak responses are expected to be larger, (ii) for track types in which sliding resistance of the sleepers is solely supplied by a frictional mechanism at the base, a heavy sleeper type may be needed, (iii) given that forces in the sleepers and fasteners are interlinked, when sleepers have higher sliding resistance, greater demand will be placed on fasteners to resist rail pullout.

In future studies, the proposed framework may be employed to address additional cases involving, e.g.: (i) modern braking technologies, (ii) uneven braking conditions between two rails or along a single rail, and (iii) effects of vertical track gradient, i.e., uphill or downhill. Moreover, the framework capabilities may be enhanced further by: (i) considering second-order effects from coupling of responses in the two directions; (ii) utilizing a Pasternak foundation for the vertical track responses, and (iii) including dynamic (inertia) effects in the formulation. Nonetheless, even
in its current form, the proposed approach is deemed flexible, robust, and well suited for analyzing track responses to decelerating vehicular loads.

Acknowledgement
The support from Innovation Fund Denmark is gratefully acknowledged. This study is part of ‘Roads2Rails: Innovative and cost-effective asphalt based railway construction system’ (Grand Solutions 5156-00006B). The authors would like to thank all the project participants for their valuable contributions.

References


23. Rhodes D., Maree J.S., and Barthram P. Track design for extreme traction and braking forces. In 8th International Heavy Haul Association Conference (IHHA), Rio de Janeiro, RJ, Brazil. 2005.


34. Degrande G. and Schillemans L. Free field vibrations during the passage of a Thalys high-speed train at variable speed. *Journal of Sound and Vibration* 2001; 247(1): 131-144.
Figure captions

Figure 1. A sketch depicting a single train axle decelerating over a rail supported on sleepers.

Figure 2. An orthogonal Winkler foundation model for simulating braking of a single wheel.

Figure 3. Magnified view of induced axial stresses in an infinitesimal beam element.

Figure 4. Model responses for a single axle as a function of the load positions: (a) vertical force at the base of a sleeper and (b) longitudinal force at the base of a sleeper, and (c) longitudinal stress at the beam’s neutral axis and (d) friction demand at the base of a sleeper.

Figure 5. Peak friction demand as a function of spring constant ratios for different values of longitudinal spring constants: (a) standard sleeper and (b) heavy sleeper.

Figure 6. Model responses for a full train as a function of the distance from rear axle: (a) vertical force at the base of a sleeper and (b) longitudinal force at the base of a sleeper, and (c) longitudinal stress at beam’s neutral axis and (d) friction demand at the base of a sleeper.
List of Figures

Figure 1. A sketch depicting a single train axle decelerating over a rail supported on sleepers.

Figure 2. An orthogonal Winkler foundation model for simulating braking of a single wheel.

Figure 3. Magnified view of induced axial stresses in an infinitesimal beam element.
Figure 4. Model responses for a single axle as a function of the load positions: (a) vertical force at the base of a sleeper and (b) longitudinal force at the base of a sleeper, and (c) longitudinal stress at the beam’s neutral axis and (d) friction demand at the base of a sleeper.
Figure 5. Peak friction demand as a function of spring constant ratios for different values of longitudinal spring constants: (a) standard sleeper and (b) heavy sleeper.
Figure 6. Model responses for a full train as a function of the distance from rear axle: (a) vertical force at the base of a sleeper and (b) longitudinal force at the base of a sleeper, and (c) longitudinal stress at beam’s neutral axis and (d) friction demand at the base of a sleeper.
A Priori Determination of Track Modulus Based On Elastic Solutions

Tulika Bose

PhD student

Department of Civil Engineering, Technical University of Denmark

Nordvej, Building 119

Kgs. Lyngby 2800, Denmark

Email: tulbo@byg.dtu.dk

Eyal Levenberg [Corresponding author]

Associate Professor

Department of Civil Engineering, Technical University of Denmark

Nordvej, Building 119

Kgs. Lyngby 2800, Denmark

Tel: +45 4525 1907

Email: eylev@byg.dtu.dk
Abstract

The standard approach for modeling railway tracks idealizes the rails as two infinite beams, each supported by a continuous spring foundation. The foundation is characterized by a track modulus that embodies all components and materials underlying each rail as well as any cross-rail interaction. Track modulus is considered a basic parameter governing the field performance of tracks. Therefore, a priori determination of track modulus is needed in design of traditional railways, as well as in evaluating the performance-potential of non-traditional track solutions. In this study, a new method was suggested for a priori track modulus determination based on elastic solutions. Specifically sought were closed-form analytical formulations that could be representative and tractable. In this connection, a 3-D track model was developed, wherein: rail-pads were considered as linear springs, sleepers as finite beams, and all underlying soil-like materials as a homogenous half-space. Ultimately, track modulus was determined by linking calculations in the 3-D model and the standard model. This was done by requiring equal maximal displacement as well as identical load distribution along the rail under the weight of a single railcar axle. The method was illustrated considering a wide set of values for the different model parameters. The calculated results are comparable in magnitude and exhibit similar sensitivities to the input parameters as reported in field studies or as derived from elaborate numerical schemes.

Keywords: Track Modulus, Railway, Elasticity, Track stiffness, Rail track modeling.
1. Introduction

Winkler’s (1867) hypothesis of subgrade reaction is widely practiced for design and analysis of soil-coupled constructions such as: (i) pile and raft foundations (Hemsley 2000), (ii) concrete pavements (Westergaard, 1948; Ioannides, 2006), (iii) buried pipes (Rajani et al., 1996; Klar et al., 2005), and (iv) tunnel linings (Wood, 1975; Lee et al., 2001; Mair, 2008). Essentially, this hypothesis is a radical mathematical simplification of actual soil behavior; it does not directly represent any basic material property. Because of this, problem-specific methods or guidelines are needed to suitably determine subgrade reaction values for subsequent structural evaluation. In an early contribution, Biot (1937) offered such guidelines for the case of an infinite beam resting on an elastic half-space and loaded by a concentrated force. The development was founded on theoretical arguments, and was based on requiring equivalency of maximal beam moments. Vesić (1961) extended Biot’s work to include beams of finite length. The proposed guidelines were based on theoretical considerations reinforced by experimental evidence. They also utilized maximal moment as basis for equivalency. Determination methods of subgrade reaction for foundation design were proposed by Terzaghi (1955) after combining full-scale field experience and theoretical considerations. Similarly, full-scale experiments and theoretical considerations were employed to guide practitioners on determining subgrade reaction for slab-on-grade constructions (Vesic and Saxena, 1969; Khazanovich et al., 2001; Setiadji and Fwa, 2009; Daloglu and Vallabhan, 2000).

A mathematically equivalent concept to subgrade reaction - called ‘track modulus’ is commonly utilized within the field of railway engineering (Selig and Li 1994). In words, track modulus expresses the supporting force per unit length of an infinite rail per unit vertical rail displacement. The concept serves as a basic input parameter for: (i) calculating rail bending stresses and deflections under the weight of railcar axles (Sadeghi and Barati, 2010; Hay, 1982; AREMA Manual 2006; Kerr, 2003; Selig and Waters, 1994), (ii) analysis of vibrations caused by vehicle dynamics (Newton and Clark, 1979), and (iii) assessing overall in-service track quality (Ebersöhn et al., 1993; Read et al., 1994; Roghani and Hendry, 2017). Unlike soil-coupled constructions, track modulus embodies the continuous support offered to a rail by discrete track components, e.g., rail-pads, sleepers, under-sleeper pads, and ballast mats, as well as by soil-like materials,
e.g., ballast, sub-ballast, and subgrade. This means that track modulus is governed by a large set of attributes, i.e., material properties, component dimensions, interconnectivity, etc. Perhaps because of this intricacy the vast majority of existing methods for determining track modulus rely on field measurements (Kerr, 2000; Norman et al., 2004; Lu et al., 2008; Zakeri and Abbasi, 2012; Nafari et al., 2017; Narayanan et al., 2004).

The idea of track stiffness, taken to mean the point-load required to produce a unit deflection of the rail (at the location where the load is applied), is often employed for evaluating the quality of existing tracks (Tzanakakis, 2013). In effect, whenever assuming a beam on Winkler model for the rail, track stiffness and track modulus are directly linked. However, while track stiffness encapsulates all track components to provide some global stiffness, track modulus is only focused on the support offered to the rail. The advantage of so doing is related to the fact that rail flexural properties are engineered and well defined whereas the support offered to the rail may vary widely; thus, track modulus is closely linked to the sources governing the track quality.

A priori determination of track modulus can be attained with realistic and representative track models that are able to accept basic material properties as input. Such determination is needed in new-design and rehabilitation-design of railways, e.g., for guiding the selection of components and materials; it is equally important when evaluating the performance-potential of non-traditional track solutions for which limited field experience exists (if any). Nonetheless, only a handful of studies were specifically dedicated to this task.

The GEOTRACK program (Adegoke et al., 1979; Chang et al., 1980), successor of the MULTA code (Prause and Kennedy, 1977), represents one of the first efforts to develop an elaborate and realistic track model. In the program, the rail and sleepers were considered as Euler-Bernoulli beams of finite length, rail-pads were represented by discrete linear springs, and all soil-like layers (ballast, sub-ballast, and subgrade) were represented by a stratified elastic half-space. The total model size was limited to eleven sleepers supported on the surface of the layered half-space, jointly carrying two rails. As means of simulating the weight action of a railcar axle, vertical point-loads were applied to the rails, just above the
central sleeper. Stress distributions at the sleeper bottoms were approximated according to Barden’s (1962) formulation. In turn, the corresponding stress distributions at the surface of the half-space were approximated as circular loads (Burmister, 1945). Ultimately, by means of a stiffness matrix formulation, the displacements at the interaction points between all model elements were matched. The GEOTRACK program was not explicitly designed for performing a priori determination of track modulus. Nonetheless, it was applied for this purpose (Stewart, 1985), with reported reasonable agreement between calculated and field-measured track modulus values. To achieve this the elastic properties of all model components were assumed, and track modulus was determined solely based on a criterion of maximal rail displacement, i.e., by equating maximal rail displacement in GEOTRACK and in the infinite beam on a continuous spring model. One drawback of this approach is the incompatibility in the distribution of vertical forces along the track in the sleeper locations - resulting in a mismatch of bending stress distribution in the rails. Regardless, the GEOTRACK program is neither accessible nor handy at this time; it is incompatible with modern computing platforms and non-upgradable - available only in compiled form (Mishra et al., 2016).

A priori determination of track modulus was specifically targeted by Cai et al. (1994) The work commenced by analyzing a finite length beam representing a single sleeper, supported on a Winkler foundation (representing the entire track structure below the sleeper), and loaded by two equal forces representing loading at the rail seats. The sought track modulus was calculated based on the displacement of the finite beam under one of the forces, taking into account the added flexibility due to rail-pad and sleeper compressibility (jointly represented as an additional discrete spring in series). The subgrade reaction offered to the finite beam was obtained from an approximate formula (Galin, 1943). This formula expresses the interaction between a finite beam and a linear elastic (homogenous) half-space based on the assumption that the beam’s width tends to zero. Steinbrenner’s (1936) formula was also suggested in this study for calculating a single ‘effective’ Young’s modulus from a layered half-space (representing ballast, sub-ballast, subgrade, and deeper soil layers). The main limitation of the work comes from the analysis of a single sleeper. Doing so disregards the effect of adjacent sleepers, and therefore results in overestimation of the track modulus. Other limitations include representing the sleeper as a zero-width finite beam, and
utilizing Steinbrenner’s formula, which is essentially applicable for layered systems wherein the modulus increases (rather than decreases) with depth (Poulos and Davis, 1991).

Consequently, it is the aim of the current study to contribute a new method for a priori determination of track modulus based on elastic solutions. Three development attributes are sought as means of enhancing any possible acceptance and long-term usefulness of the work: (i) the formulation should be founded on closed-form analytic solutions, offering a step-by-step reproducible approach; (ii) the method should accept as input all pertinent material properties, component dimensions, and connectivity to offer user flexibility in evaluating a wide range of scenarios, and (iii) to better strengthen the connection with the track modulus concept, the method should also consider the correct interaction across sleepers, and not rely solely on equivalency of maximal rail displacement. The paper commences with an overview of the method followed by a detailed presentation of two elastic solutions. It then explains how these solutions are to be linked for track modulus determination. Next, the method is illustrated with a reference set of representative values, along with a limited parametric investigation. The paper ends with a short summary and recommendations for further development.

2. Method Overview (a main section)

The standard approach for modeling railway tracks idealizes the rails as two weightless separate infinite beams (IBs), each individually supported on a continuous spring foundation; this model is graphically shown in Fig. 1(a). The figure also includes a Cartesian coordinate system where the X-axis is oriented along the IBs, the Y-axis is transverse to the IBs, and the Z-axis points vertically downward. As can be seen, the two IBs are simultaneously loaded by point forces with intensity \( P_z \) representing (jointly) a single railcar axle. The foundation is characterized by a track modulus with spring constant \( k_z \) that embodies all components and materials underlying each rail as well as any cross-rail interaction.

To facilitate a priori determination of \( k_z \), a 3-D elastic quasi-static track model is developed, comprising all components and materials below the rail that govern the track modulus. It consists of an
equidistant array of sleepers (spacing denoted by $s$) modeled as Euler-Bernoulli finite beams (FBs) with uniform cross-section. Each FB is identified by an index $n$, going from positive to negative, with the zeroth FB located at the center of the 3-D model. The FBs are supported by an underlying elastic half-space (HS) that is linear, homogenous and isotropic. The HS represents all soil-like materials, which for a traditional track includes: ballast (excluding crib and shoulder ballast), sub-ballast, and deeper soil layers; for a non-traditional track the soil-like materials may also include: asphalt concrete, Portland cement concrete, stabilized ballast, etc. The exclusion of materials from in-between and along the sides of the finite beams is justified given their marginal effect (if any) on vertical track responses. Fig. 1(b) shows a part of this model consisting of seven FBs, with the model sides and bottom truncated for visual clarity. The two dark shaded patches seen at the top of each FB represent rail-pad locations; each modeled as a linear discrete spring (DS).

Fig. 1(b) shows that the DSs in the 3-D model are directly loaded by forces with intensities $S^u_z$'s. These forces are spring reactions calculated in the standard model with an assumed $k_z$, after converting the ‘continuous’ springs into individual ones (also with spacing $s$). For this purpose, the two models are aligned such that the position of $P_z$ in Fig. 1(a) corresponds to the center of the 3-D model in Fig. 1(b). The relative size of the arrows schematically indicate the difference in force intensities applied on top of individual DSs. Considering that all 3-D model parameters are known, the displacement at the top of the zeroth DS (on either side) is calculated quasi-statically due to the simultaneous application of all $S^u_z$'s. This displacement is composed of the shortening of the zeroth DS and the surface deflection of the HS directly below it. In this calculation, it is assumed that the FBs are incompressible, and that they remain in frictionless contact with the HS. Therefore, as part of the calculation, it is necessary to resolve the contact stresses at the interface of the FBs and underlying HS. The frictionless contact assumption means that FBs are fully bonded to the HS on which they are supported – allowing for full transfer of stresses in the vertical direction, without allowing shear stresses to develop at the bonding interface. This
neglect of interface shear stresses is justified given their second order effect, and given the focus on track modulus.

Ultimately, a representative track modulus is determined by manipulating the assumed $k_z$ until a match is attained between the calculated displacement at the top of the zeroth DS and the maximal IB displacement from the standard track model. Thus, an intimate and close connection is established between the 3-D model and the standard model accounting not only for maximum displacement equivalence but also for equivalence in the longitudinal load spreadability, i.e., identical moment distribution along the rail. More details on the method and underlying calculations are provided in what follows.

3. Elastic Solutions (a main section)
Two closed-form independent elastic solutions are presented in this section. The first analyzes a FB subjected to two line-loads of equal intensity (i.e., a straight track) and a non-uniform support reaction. The line-loads represent rail-seat forces distributed over the length of rail-pads, and the non-uniform support reaction represents an interaction with a HS. The second elastic solution deals with the surface deflection of a HS that is loaded over a rectangular stress patch. Then after, both solutions are combined to analyze the interaction at the FB-HS interface with the aim of resolving the contact stress distribution between a sleeper and its supporting medium. Ultimately, these solutions are applied to determine the surface deflection of the HS under the action of several $S_z$’s acting simultaneously (refer to Fig. 1(b)).

3.1 Solution1: Analysis of a FB
A FB of length $L$ with free ends is shown in Fig. 2; a 3-D Cartesian coordinate system is also included in the Figure with similar orientation as in Fig. 1. Here, the origin is located at the leftmost end of the FB, with the $Z$-axis pointing vertically downward, the $Y$-axis pointing along the length of the beam, and the $X$-axis oriented perpendicular to the $YZ$ plane. The beam’s elastic modulus is $E_{FB}$, and moment of inertia about the $X$-axis is $I_{X,FB}$. The beam is symmetrically loaded by two line-loads of equal magnitude $w_z$ (units of
force/length), acting over a length $L_2$ at a distance of $L_4$ from the beam ends. It is also loaded at the bottom by a segmented array of opposing line-loads representing a support reaction. There are 12 such segments of equal length ($L_4/12$), numbered from left to right; each with uniform magnitude. The overall shape of the support reaction is variable, governed by the values of five independent unitless coefficients denoted as $a_i$’s. These unitless coefficients represent a reaction intensity relative to unity, i.e., if all five $a_i$’s equal unity then the support reaction becomes uniform across the entire length of the beam. In the interest of clarity, conditions of symmetry have not been invoked in the FB analysis.

The governing differential equation of the FB is:

$$\frac{d^4 u_{Z,FB}(y)}{dy^4} = \frac{p_Z(y) - r_Z(y)}{E_FB I_{X,FB}}$$

(1)

wherein, $u_{Z,FB}(y)$ denotes the displacement of the FB along Z-axis direction, $p_Z(y)$ represents the loading that acts from the top, $r_Z(y)$ is the segmented bottom support reaction, and $y$ identifies the calculation location along the FB, i.e., $0 \leq y \leq L$. Utilizing the Heaviside function $H(\cdot)$, and enforcing equilibrium considerations, these are expressed as follows:

$$p_Z(y) = w_Z \left( H(2y-2L_4+L_2) - H(2y-2L_4-L_2) + H(2y-2L+2L_4+L_2) - H(2y-2L+2L_4-L_2) \right)$$

(2)

$$r_Z(y) = \frac{24w_ZL_2}{La_T} \sum_{k=1}^{4} (a_k - a_{k+1}) \left( H(y-kL/12) - H(y-L+kL/12) \right) + (a_5-1) \left( H(y-5L/12) - H(y-7L/12) \right) - a_i$$

(3)

wherein $a_r = 2(a_1 + a_2 + a_3 + a_4 + a_5 + 1)$.

Eqs. (2) and (3) are inserted into Eq. (1), and a solution for $u_{Z,FB}(y)$ is obtained after applying the boundary conditions of zero shear force and zero bending moment at beam ends as well as zero slope at the beam center. The result is:

$$u_{Z,FB}(y) = \frac{w_Z}{E_FB I_{X,FB}} \left( D_1 + \frac{L_2D_2}{La_T} \right) - yD_3 + D_4$$

(4)
where,

\[
D_1 = \left\{ \begin{array}{l}
(y-L+L_1+L_2/2)^4H(y-L+L_1+L_2/2) - (y-L+L_1-L_2/2)^4H(y-L+L_1-L_2/2) \\
+ (y-L_1+L_2/2)^4H(y-L_1+L_2/2) - (y-L_1-L_2/2)^4H(y-L_1-L_2/2)
\end{array} \right.
\]

\( (\text{5}) \)

\[
D_2 = \sum_{k=1}^{4} (a_k - a_{k+1}) \left\{ (y-kL/12)^4 H(y-kL/12) - (y-L+kL/12)^4 H(y-L+kL/12) \right\} + (a_4 - 1) \left\{ (y-5L/12)^4 H(y-5L/12) - (y-7L/12)^4 H(y-7L/12) \right\} - a_t y^4
\]

\( (\text{6}) \)

\[
D_3 = \frac{9w_Z}{432E_{FB}I_{X_{FB}}} \left( \frac{L-2L_1+L_2}{2} \left( H(L-2L_1+L_2) + H(2L_1-L_2-L) \right) - (L-2L_1-L_2)^3 \left( H(L-2L_1-L_2) + H(2L_1+L_2-L) \right) \right)
\]

\[
- \left( \frac{L^2L_z}{9a_t} (9a_1 + 61a_2 + 37a_3 + 19a_4 + 7a_5 + 1) \right)
\]

\( (\text{7}) \)

and \( D_4 \) is a free constant accounting for rigid body displacement.

### 3.2 Solution 2: HS Loaded over a Rectangular Stress Patch

A linear elastic half-space, characterized by Young’s modulus \( E_{HS} \) and Poisson’s ratio \( \nu_{HS} \), is loaded at the surface by a uniform vertical stress with intensity \( q_Z \) acting over a rectangular area of dimensions \( b \times a \).

This situation is shown in Fig. 3, which also includes a 3-D Cartesian coordinate system placed at the center of the patch; the \( Z \)-axis is pointing downward, and the \( X \) and \( Y \) axes are oriented parallel to the patch sides. The surface deflection of any point along the \( Y \)-axis due to the stress patch is denoted by \( u_{Z,HS}^p (y) \). By integration of Boussinesq’s point-load solution, the related expression is:

\[
u_{Z,HS}^p (y) = \frac{1-\nu_{HS}^2}{2\pi E_{HS}} q_Z f(y)
\]

\( (\text{8}) \)

wherein,

\[
f(y) = (a + 2|y|) \ln \left( \frac{f^+ + b}{f^- + b} \right) + 2b \ln \left( \frac{f^- + (a - 2|y|)}{f^+ - (a + 2|y|)} \right)
\]

\( (\text{9}) \)

and,
The two separate elastic solutions are hereafter combined to resolve the contact stress distribution below a single FB having width \( b \) and supported by a HS. This means determination of the five \( a_i \)’s as well as the rigid body displacement \( D_4 \). To achieve this, the displacement of the FB and the surface deflection of the HS are matched in the least-squares sense. Such matching represents a bonded yet frictionless contact area of dimension \( L \times b \) between the two elastic bodies.

Based on the coordinate system defined in Fig. 2, and for a given FB loaded by an arbitrary \( w_z \), Eq. (4) is utilized to calculate \( u_{z,FB}(y) \) at \( y = y_j = jL/J \) ( \( j = 0, \ldots, J \) ), i.e., at \( J+1 \) equidistant points within \( L \). Concurrently, by means of superposition, Eqs. (8-10) are used to compute \( u_{z,HS}(y) \), i.e., the HS surface deflection at the same \( y_j \)’s:

\[
u_{z,HS}(y) = \frac{(1-v_{HS}^2)}{2\pi E_{HS}} \sum_{m=1}^{12} \frac{1}{b} \varphi_z \left( \frac{mL}{12} - L \right) f\left( y - \left( \frac{mL}{12} - \frac{L}{24} \right) \right)
\]

This equation represents the contribution of the 12 adjacent patches indexed \( m = 1, \ldots, 12 \) with \( m = 1 \) identifying the patch closest to the coordinate origin. With reference to Eq. 8, Expression I represents \( q_z \) and Expression II represents \( f(y) \). Expression I gives the stress intensity at center of a \( m^{th} \) patch calculated using Eq. 3; the division by \( b \) implies that the stress distribution is taken as uniform across the width of the FB. Expression II is computed according to Eq. 9, with \( a = L/12 \) and \( b \).

The five sought \( a_i \)’s as well as \( D_4 \) are determined by minimizing the sum of the square of the differences between the HS and FB displacements across all considered \( y_j \)’s. The formulation is:

\[
(a_i^*, D_4^*) = \arg\min_{a_i, D_4} \sum_{j=0}^{J} \left( u_{z,HS}(y_j) - u_{z,FB}(y_j) \right)^2 \quad (i = 1, \ldots, 5)
\]
where the optimal/final argument values are denoted with an asterisk as superscript, i.e., $a_i^*$'s and $D_i^*$. Note that due to assumption of linearity the $a_i^*$'s are essentially unaffected by choice of $w_z$.

4. Track Modulus Determination (a main section)

All above-described formulations are ultimately interlinked hereafter to comprise a method for a priori track modulus determination. The calculations require all parameters to be known and fixed, with only assumption being a trial value for $k_z$. The first step is to solve Eq. 12 and obtain $a_i^*$'s that represent the interaction between a single FB supported on a HS. Next, the load applied on top of the $n^{th}$ DS, i.e., $S_n''$ (refer to Fig. 1) is calculated from the standard model:

$$S_n'' = \frac{sP_2 \beta_2 e^{-\beta_2|n|}}{2} \left( \cos \beta_2 |ns| + \sin \beta_2 |ns| \right)$$

where $\beta_2 = \sqrt{k_z / (4E_{IB}I_{Y,IB})}$ in which $E_{IB}$ and $I_{Y,IB}$ are the Young’s modulus and moment of inertia about the $Y$ axis of the IB, respectively. This expression is obtained by multiplying $sk_z$ with $u_{Z,IB}^0$, i.e., the displacement of the IB at location corresponding to the $n^{th}$ FB (Bose et al. 2018). Here $n$ varies from $-N$ to $+N$ with $2N + 1$ denoting the total number of FBs considered in the analysis.

Then, the line-load applied on top of the $n^{th}$ FB is:

$$w_z^n = S_n'' / L_2$$

where $L_2$ signifies length of rail-pads (refer to Fig. 2). Also, the maximum IB displacement is given by:

$$u_{Z,IB}^0 = S_Z^0 / sk_Z$$

Further, the shortening of the zeroth DS is:

$$\Delta u_{Z,DS}^0 = S_Z^0 / K_{Z,DS}$$

where $K_{Z,DS}$ is the stiffness of DSs.
Next, Eq. 3 is employed with $a_i$’s and $w_Z^n$ replacing $a_i$’s and $w_Z$ (respectively) to express $r_Z^n(y)$, i.e., the support reaction distribution below the $n^{th}$ FB:

$$r_Z^n(y) = \frac{24w_Z^n L_z}{L a_T^*} \left( \sum_{k=1}^{4} (a_k^* - a_{k+1}^*) \left( H(y - kL/12) - H(y - L + kL/12) \right) \right) + (a_5^* - 1)(H(y - 5L/12) - H(y - 7L/12)) - a_1^*$$

(17)

wherein $a_T^* = 2(a_1^* + a_2^* + a_3^* + a_4^* + a_5^* + 1)$. Note that in doing so any cross-interaction between the adjacent FBs is neglected (with respect to the foundation support stress distribution), i.e., one interaction analysis between a FB and HS is utilized for all FBs.

Subsequently, the deflection of the HS for a point located directly underneath the zeroth DS is sought - denoted by $u_{Z,HS}^T$. For clarifying the calculation rational, Fig. 4 offers a plan view of the HS loaded by several FBs. The point of deflection calculation is depicted in the figure at $y = L_1$ for $n = 0$ (as a triangular marker). Obtaining $u_{Z,HS}^T$ requires calculating $u_{Z,HS}^0(L_1)$, i.e., evaluating Eq. 11 with $r_z = r_z^0(y)$ at $y = L_1$. It additionally requires computing the deflection contributed by all adjacent FBs, i.e., considering the effect from all $r_Z^n(y)$’s excluding $n = 0$. To quantify this latter contribution, it is sufficient to consider Boussinesq’s point-load solution when superposing the effects from all stress patches:

$$u_{Z,HS}^C = \frac{2(1 - v_H^2)}{\pi E_H} \sum_{n=1}^{N} \sum_{m=1}^{12} \frac{P_m^n}{R_m^n}$$

(18)

where,

$$P_m^n = \frac{L}{12} \cdot r_Z^n \left( \frac{mL}{12} - \frac{L}{24} \right)$$

(19)

$$R_m^n = \sqrt{(ns)^2 + \left( L_1 - \frac{mL}{12} - \frac{L}{24} \right)^2}$$

(20)

Eq. 18 represents the deflection contribution of 12 adjacent patches indexed $m = 1, \ldots, 12$ from all considered FBs excluding $n = 0$. Symmetry about the model center permits considering twice the contribution of the
FBs indexed \( n = 1, \ldots, N \) (hence the multiplication by 2 in Eq. 18). Both \( P_m^n \) and \( R_m^n \) are graphically presented in Fig. 4. As can be seen, \( P_m^n \) is the force intensity at the center of the \( m^{th} \) patch for the \( n^{th} \) FB, and \( R_m^n \) is the distance between the calculation point and the center of the \( m^{th} \) patch of the \( n^{th} \) FB. Thus, the sought HS deflection is given by the summation:

\[
U_T^Z = U_Z^C + U_Z^0 (L_1)
\]  

(21)

Ultimately, by matching \( U_T^Z \) plus the shortening of the zeroth DS \( \Delta U_{Z,DS}^0 \) against the maximal displacement in the standard model \( U_{Z,IB}^0 \) the representative track modulus \( k_Z^* \) (associated with all 3-D model attributes) is determined:

\[
U_T^Z + \Delta U_{Z,DS}^0 = U_{Z,IB}^0 \quad \text{for} \quad k_Z = k_Z^*
\]

(22)

Fig. 5 presents a flowchart that summarizes the entire method of track modulus determination. As can be seen the process commences by fixing all model parameters and an assumed/trial value for the track modulus. Then after, calculations and exchange of information take place in the standard track model and in the 3-D track model. The specific equations needed for each step are indicated in the flowchart. The final step (after which the process ends) requires solving Eq. 22, i.e., finding \( k_Z^* \). This is done by improving the initial assumption for \( k_Z \) and iterating until satisfactory convergence. Thus, a track modulus is determined, associated with the entire parameter set, and satisfying both maximal rail displacement and moment distribution along the rail.

5. Method Illustration (a main section)

The purpose here is to illustrate, by means of application, the proposed track modulus determination method. Some intermediate results will also be shown to further facilitate clarity of the underlying calculations. A reference set of input parameters was chosen for this purpose – shown in Table 1. As can be seen, the IB (rail) properties are taken as per UIC60, the stiffness of the DS (rail-pads) is 100 MN/m, the
FBs (sleepers) are of concrete, each 2.5 m long with a rectangular cross-section (base width 0.25 m), spaced apart by 0.6 m; the elastic modulus of the supporting HS is 200 MPa.

All calculations were done considering $P_Z = 80$ kN (a choice that does not influence the final result) and 37 FBs, i.e., a central FB plus 18 FBs on each side (see Fig. 1). This latter choice was taken to guarantee that the load distribution between the sleepers was fully captured across the entire parameter range. With respect to the flowchart in Fig. 5, the FB-HS interaction was solved first (Eq. 12), giving: $a_1^* = 1.94$, $a_2^* = 1.69$, $a_3^* = 1.87$, $a_4^* = 1.56$, and $a_5^* = 1.21$. Subsequently, the shortening of the zeroth DS (Eq. 16) was: $\Delta u_{Z,DS}^0 = 0.308$ mm, corresponding to $S_Z^0 = 30.8$ KN. The HS deflection directly underneath the zeroth DS (Eq. 21) was: $u_{Z,HS}^T = 0.440$ mm of which about 35% was from $u_{Z,HS}^C$ (Eq. 18). Finally, the sought track modulus was obtained by matching the maximal IB displacement $u_{Z,IB}^0$ (Eq. 15) with $u_{Z,HS}^T + \Delta u_{Z,DS}^0 = 0.748$ mm. The result was: $k_Z^* = 68.7$ MPa. It is noted that the $a_i^*$’s above indicate a maximum contact stress to be occurring near the edges of the FB; this is in contrast to the accepted viewpoint that the peak stresses occur below the rails (e.g., Zakeri and Sadeghi, 2007). This is the consequence of the assumption of a FB in contact with an elastic continuum, which mandates high stresses at the edges to fulfill deformation compatibility.

Additionally, a parametric investigation was conducted to study the behavior of $k_Z^*$. For this purpose the following parameter values were varied: FB spacings, half-space modulus $E_{HS}$ and discrete spring stiffness $K_{Z,DS}$. The chosen range for these parameters is also shown in Table 1. All other parameters (within a realistic range) were noticed to have minor influence on track modulus and therefore remained fixed at their reference value.

The results of this investigation are presented in Fig. 6 which depicts track modulus vs. half-space modulus for five different rail-pad stiffnesses and two different sleeper spacings. Fig. 6(a) and Fig. 6(b) refer to $s = 0.6$ m and $s = 0.5$ m respectively. In both figures each line represents the variation of $k_Z^*$ with
change in $E_{HS}$ for a given value of $K_{Z,DS}$. From these figures it can be observed that for any $K_{Z,DS}$, $k^*_Z$ increases when the $E_{HS}$ increases. The increase is not indefinite as each curve is bounded by a horizontal asymptote defined by $k^*_Z = K_{Z,DS}/s$. This can be explained by the fact that at the limit, as $E_{HS}$ approaches infinity, the term $u^T_{Z,HS}$ becomes zero and the track flexibility is solely governed by the rail-pads ($\Delta u^0_{Z,DS}$ remain finite). By comparing Fig. 6(a) against Fig. 6(b) it can be observed that track modulus increases with a decrease in sleeper spacing.

As a final note, the parametric study was repeated with all $\alpha_i$’s presumed unity (see Fig. 2). This is a simplification wherein the stress distribution below the FBs is assumed uniform. Thus, the calculation step in Eq. 12, which solves the FB-HS interaction, is evaded. After doing so it was found that $k^*_Z$ was only slightly overestimated; the error compared with the curves in Fig. 6 was mostly perceptible for higher values of $E_{HS}$ and $K_{Z,DS}$, and reached a maximum of only 8% (for $E_{HS} = 300$ MPa and $K_{Z,DS} = 250$ MN/m).

This finding also means that for the purpose of track modulus evaluation, it is not necessary to conduct detailed analysis (e.g., further refined segmentation) of the interaction between FBs and underlying HS.

6. Summary, Conclusions, and Comments

This paper offered a new method for a priori determination of track modulus based on elastic solutions. Within an analytical framework, a 3-D model was developed (Fig. 1(b)), capable of representing a wide range of track-types, in terms of components and materials that underlay the rails. Closed form equations were presented for calculating displacements of a finite beam subjected to two line loads and non-uniform support reaction, and calculating surface deflection of a half-space loaded over a rectangular stress patch.

These equations were subsequently utilized to determine track modulus by linking the 3-D model and the standard track model of infinite beam on springs (Fig. 1(a)). The linkage was based on simultaneous equivalency of both maximal displacement as well as force distribution along the rail. A flowchart was
introduced to summarize the main calculation steps (Fig. 5). Overall, the method is deemed flexible and robust, while at the same time computationally ‘lightweight’ and straightforward to reproduce.

Track modulus determination was demonstrated via application of the method to a realistic set of input values. To better clarify the approach, intermediate calculation steps as well as final results were shown. The demonstration included a parametric investigation, in which the sensitivity of track modulus to the elastic modulus of the half-space, stiffness of rail-pads, and sleeper spacing was studied (Fig. 6). Results are deemed realistic and valid overall, given that they are similar in magnitude and exhibit similar sensitivity to varying input parameters when compared to GEOTRACK (Selig and Li 1994). Specifically, it was possible to closely reproduce the track modulus results shown in Figure 2 in Selig and Li for the entire range of subgrade moduli, fastener stiffnesses, tie spacings and track types (assuming ballast and subballast moduli were same as subgrade to accommodate homogenous half-space assumption). Also, the results are within the range of track modulus values reported by some field studies (Norman et al. 2004; Zakeri and Abbasi 2012; Narayanan et al. 2004).

The development included simplifying assumptions, some of which can be partially relieved to further enhance the applicability of the method. The treatment of all soil-like materials and track components as linear is not realistic, e.g., loading magnitude is known to affect track modulus determined in the field. Nonetheless, the linear treatment can be utilized to approximate nonlinear behavior by choosing elastic constants such that they reflect the anticipated load level. The treatment of the sleepers as beams with zero compressibility and constant geometry can also be made more realistic. Whenever the vertical compression of the sleepers (FBs) is not negligible, e.g., in the case of wooden sleepers, an additional linear spring can be assumed connected (in series) to the discrete springs representing the rail-pads. Thus, the entire calculation procedure remains unchanged except that the new shortening/flexibility is added to the left-hand-side of Eq. 22. Finally, the issue of sleepers with variable cross-section can be addressed by converting the actual dimensions to equivalent constant dimensions. In this connection, the FB length \( L \) can be taken as the actual length of the sleeper, and the FB width \( b \) can be obtained from dividing the
actual sleeper base-area by $L$. Regarding the inertia moment, the following formula - based on equivalency of bending strain energy - is suggested for calculating $I_{x,FB}$:

$$I_{x,FB} = \frac{\int_0^L M^2(y)\,dy}{\int_0^L \frac{M^2(y)}{I_{x,SL}(y)}\,dy}$$

(23)

where $M(y)$ is the bending moment at a distance $y$ for a FB as shown in Fig. 2, considering an arbitrary $w_z$ and all $a_i$’s taken as unity; and $I_{x,SL}(y)$ denotes the actual moment of inertia, which varies along the sleeper length.

In future studies, the method proposed herein can be extended to determine track modulus in the longitudinal and lateral directions. Moreover, by means of the elastic-viscoelastic correspondence principle (e.g., Schapery, 1965) the modeling approach can be enhanced to include time-dependence and thermal sensitivity of components and materials. This will facilitate, for example, a priori determination of track modulus for asphalt overlayment solutions across different train speeds and environmental conditions.

**Acknowledgement**

The support from Innovation Fund Denmark is gratefully acknowledged. This study is part of ‘Roads2Rails: Innovative and cost-effective asphalt based railway construction system’ (Grand Solutions 5156-00006B).
References


<table>
<thead>
<tr>
<th>Component</th>
<th>Parameter/Role</th>
<th>Symbol</th>
<th>Units</th>
<th>Reference Value</th>
<th>Range</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Infinite Beam (IB)</td>
<td>Moment of inertia about Y axis</td>
<td>$I_{Y,IB}$</td>
<td>m$^4$</td>
<td>3.04x10$^{-5}$</td>
<td>-</td>
<td>UIC 60 rail section</td>
</tr>
<tr>
<td></td>
<td>Young’s modulus</td>
<td>$E_{IB}$</td>
<td>GPa</td>
<td>208</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Discrete Spring (DS)</td>
<td>Stiffness</td>
<td>$K_{Z,RP}$</td>
<td>MN/m</td>
<td>100</td>
<td>25 to 250</td>
<td>Rail-pad stiffness</td>
</tr>
<tr>
<td></td>
<td>Moment of inertia about X axis</td>
<td>$I_{X,FB}$</td>
<td>m$^4$</td>
<td>1.22x10$^{-4}$</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Young’s modulus</td>
<td>$E_{FB}$</td>
<td>GPa</td>
<td>30</td>
<td>-</td>
<td>Concrete Sleeper</td>
</tr>
<tr>
<td>Finite Beam (FB)</td>
<td>Length</td>
<td>$L$</td>
<td>m</td>
<td>2.50</td>
<td>-</td>
<td>Sleeper length</td>
</tr>
<tr>
<td></td>
<td>Base width</td>
<td>$b$</td>
<td>m</td>
<td>0.25</td>
<td>-</td>
<td>Sleeper base width</td>
</tr>
<tr>
<td></td>
<td>Total number on one side</td>
<td>$N$</td>
<td>-</td>
<td>18</td>
<td>-</td>
<td>See Eq. 18</td>
</tr>
<tr>
<td></td>
<td>Spacing</td>
<td>$s$</td>
<td>m</td>
<td>0.60</td>
<td>0.50 or 0.60</td>
<td>See Fig. 1</td>
</tr>
<tr>
<td></td>
<td>Position of $w_z$</td>
<td>$L_1$</td>
<td>m</td>
<td>0.50</td>
<td>-</td>
<td>See Fig. 2</td>
</tr>
<tr>
<td></td>
<td>Length over which $w_z$ operates</td>
<td>$L_2$</td>
<td>m</td>
<td>0.17</td>
<td>-</td>
<td>Rail-pad length</td>
</tr>
<tr>
<td></td>
<td>No. of matching FB-HS points</td>
<td>$J$</td>
<td>-</td>
<td>50</td>
<td>-</td>
<td>Eq. 12</td>
</tr>
<tr>
<td>Half-space (HS)</td>
<td>Young’s modulus</td>
<td>$E_{HS}$</td>
<td>MPa</td>
<td>200</td>
<td>20 to 300</td>
<td>All soil-like materials</td>
</tr>
<tr>
<td></td>
<td>Poisson’s ratio</td>
<td>$\nu_{HS}$</td>
<td>-</td>
<td>0.30</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>
List of Figures

Figure 1. A sketch depicting the models considered for track modulus determination (a) standard model (b) elastic 3-D model.
Figure 2. A finite beam subjected to two line-loads and a non-uniform support reaction.
Figure 3. An elastic half-space loaded over a rectangular stress patch: (a) top view, and (b) cross-sectional view.
Figure 4. Top view of an elastic half-space loaded by stress patches and point forces representing the support-interaction of several finite beams.
Figure 5. Flowchart describing the proposed track modulus determination method.
Figure 6. Track modulus determined as a function of half-space modulus for different rail-pad stiffnesses and sleeper spacings based on Table 1 parameters: (a) $s = 0.6$ m, and (b) $s = 0.5$ m.
This study focused on analyzing the mechanical behaviour of a ballastless track-type based on asphalt concrete. A full-scale laboratory prototype was built and tested under different types of vertical loads. Sleepers were sequentially loaded to simulate train passages at different speeds and axle load levels. The test results showed that ballastless asphalt tracks demonstrated time-dependent and nonlinear behaviour. Moreover, measured substructure responses were of very low magnitude, indicating little or no damage under service conditions. Further, a comprehensive 3D FEM model of the prototype was built, which incorporated inertial effects, material nonlinearity, and time-temperature dependency. Separate element tests were carried out to obtain the majority of model parameters. The model predictions showed a good agreement with the experimental measurements. Additionally, an analytical model was developed to investigate track responses during train braking. It was found that for ballastless asphalt tracks, if sliding resistance at a sleeper base was governed solely by frictional mechanism, then a non-standard or heavier-sleeper type was required.