DTU Library

A comparison of the tolerance analysis methods in the openloop assembly

Kosec, P.; Škec, Stanko; Miler, D.

[Link back to DTU Orbit](#)

A comparison of the tolerance analysis methods in the open-loop assembly

Kosec, P.[a], Škec, S.[a,b,*], Miler, D.[a]

[a]University of Zagreb, Faculty of Mechanical Engineering and Naval Architecture, Zagreb, Croatia
[b]Technical University of Denmark, Kongens Lyngby, Denmark

ABSTRACT

Dimensional and geometric tolerances affect both the cost and the functionality of a given product. Finding the acceptable trade-off between the two is among the common engineering tasks. Thus, many tolerance analysis methods are developed to help engineers and assist in the decision-making process. In this article, the authors have assessed four tolerance analysis methods by applying them to the open-loop assembly. The results obtained by the tolerance chart (worst-case) method, Monte-Carlo simulation, vector-loop analysis, and the Unified Jacobian-torsor model were analysed and compared. Additionally, the overview and application guidelines are included for each of the methods, aiming to help both researchers and practitioners. The results have confirmed that there are significant variations in the outputs across the observed methods, implying the need for informed method selection.

1. Introduction

During the design phase, tolerances are assigned to nominal dimensions, ensuring successful assembly while retaining the manufacturing costs at an acceptable level. As the complexity of mechanical design increases, keeping track of the tolerances becomes harder. To mitigate the problem, tolerance analysis methods of various complexity are available. The methods range from simple, 1D tolerance chart analysis, to advanced procedures requiring the use of advanced mathematical models. Examples include vector loop, Unified Jacobian-torsor, T-maps, and Skin model Shapes. Furthermore, tolerance analysis methods can be divided by several criteria: an approach to the analysis, identification process, and the calculation procedure of dependent dimensions [1].

Before analysing, tolerances are assigned to assembly features and are organised into stacks, easing the variation analysis. Stacks are then used to analyse the assembly by reading the drawings or by assigning tolerances on computer-aided drawing (CAD). The tolerances are then stacked into loops using points, surfaces, vectors, or joints, among others – depending on the method [1]. Manual charting is frequently used when solving simple problems consisting of few dimensions. As the number of dimensions increases, its reliability decreases – it is error-prone and tiresome. Additionally, manual analysis is hard to perform in 2D and 3D tolerancing problems.

The tolerancing problem complexity further increases when the geometric tolerances are necessary [2]. Geometric tolerances are defined by 3D tolerance zones, rendering most of the simpler methods unusable. Thus, computer-aided tools (CAT) were developed, increasing the capabilities in terms of the number of available approaches and mathematical models. Many such tools are developed and successfully applied (VisVSA, 3DCS, CETOL, OpTol) [3] in the industrial environment. Unfortunately, various proprietary CAT tools use different mathematical models to define and analyse tolerances, meaning that the obtained results may differ [4].

State-of-the-art CAT tools allow users to model assembly stacks with point-to-point features. The contributing tolerances are identified and arranged into suitable stacks or loops [5] as each method is compatible with a specific stacking procedure to build the stacking equation. In recent papers, many researchers have studied differences and similarities of tolerance analysis methods. Studies considered the contributing tolerances from multiple directions [6], the angular deviation of the adjustable element, or a critical assembly feature (functional requirement) [6]. Also, the form [1] and interaction of the multiple tolerances in the 3D context is defined by the geometric drawing and tolerancing (GD&T) standards [1, 10]. Due to frequent changes in GD&T standards [7] such as ISO 8015 [2] and ASME Y14.5 [8, 9], continuous support of the tolerance analysis methods is needed.

Various assembly applications are described as a system of open-loop or closed-loop that must be solved together. The open-loop describes a dimension stack terminated with a gap or a critical assembly feature. The closed-loop defines a closure constraint for the assembly, implying that adjustable elements are in the assembly. Thus, the critical difference between the open-loop and closed-loop assemblies is the existence of gap; in the open-loop assemblies, we anticipate that gap dimension must be properly toleranced to allow us to form an engineering fit with another part (for the schema of the open-loop assembly, (please see Fig. 3). Those elements, gap or functional requirement, are the result of part tolerance accumulation. If there are no adjustable components, there is no need for closed-loops – the assembly model is composed only of open-loops [2].

In recent studies, methods for tolerance analysis were compared using the closed-loop examples. The aim was to determine the advantages and shortcomings of each method, along with the differences in output (e.g. [10-15]). To the best of our knowledge, mentioned research studies have not considered the open-loop assemblies. Hence, the contribution of the article at hand is the evaluation of the tolerance analysis methods on open-loop problems. Furthermore, besides the scientific contribution, this article aims to provide the practitioners with a simple review and guidelines for the application of each method. To achieve this, we have compared four different methods: tolerance chart method, Monte Carlo method, vector loop model, and Unified Jacobian-torsor model. Each method was applied to an open-loop assembly, allowing for comparison in performances and outcomes.

2. Methods and materials

In this research study, four tolerancing methods were compared: tolerance chart method [16], Monte Carlo method, vector-loop model [15], and Unified Jacobian-torsor (see Section 2.1). Each method is described, along with the steps necessary to apply it. Those include tolerancing problem identification, mathematical modelling, and calculation procedures.

2.1 Used tolerance analysis methods

Tolerance chart method is the most frequently used tolerance analysis method in the industry [16], mostly due to its simplicity. It is widely used for solving problems concerned with dimensional tolerances, although the recent improvements enabled its application to geometric tolerances [15]. The method is one-dimensional; in order to apply it to the multi-dimensional geometric tolerances, they must be converted to 1D space [15].

Tolerance chart method can be performed on both the part and assembly level. For assembly level, parts included in the tolerance chain represent one of the tolerance end-points (maximum or minimum). Each part is placed against its mating part in one of its tolerance end-points. As a

result, the worst-case tolerance chart method illustrates the minimal and maximal variation of a functional requirement based on the values in the tolerance chain [9].

When performing the tolerance chart method analysis, the first step is to set a goal by labelling the chain starting and ending points [16]. The starting point is selected on one edge and the ending point on the opposite edge of the analysed feature (see Fig. 1). The chain indicator is placed to determine the direction of the dimension vector and is either positive or negative [17]. The vector pointing toward the chain end-point is marked "⊕", and the vector pointing opposite of the end-point is marked "⊖". The indicator shows whether to add or subtract dimensions and tolerances during the stack calculation. Additionally, it simplifies the interpretation of tolerance chart results [16]. The resulting dimension chain is the shortest possible and consists only of known dimensions - dimensions set by designers.

Tolerance chart method was used in recent studies [5, 6, 9, 15-17], mostly as a reference for the comparison of advanced tolerance analysis methods. Its most important advantage is simplicity; no computational tools are needed as it can be carried out by hand. The downside is that the user has to keep in mind all the standard rules [2, 8] for creating the stacks, making the process error-prone. Besides, the tolerance chart method creates stacks in one direction and ignores the contributions of others, possibly providing unsatisfactory results.
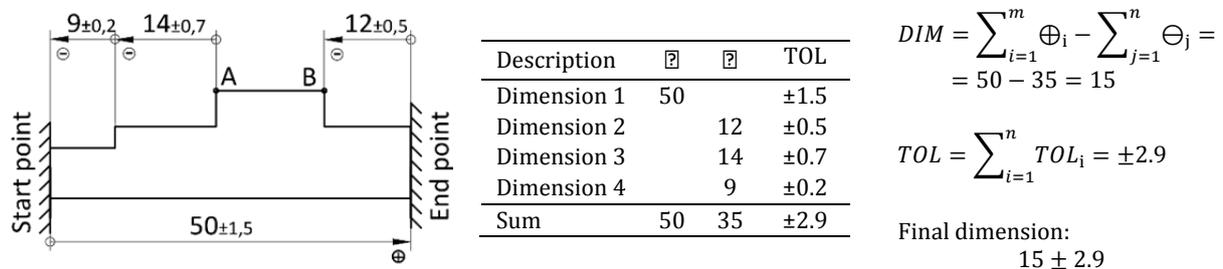
Description		TOL
Dimension 1	50	±1.5
Dimension 2	12	±0.5
Dimension 3	14	±0.7
Dimension 4	9	±0.2
Sum	50 35	±2.9

$$DIM = \sum_{i=1}^{m} \oplus_i - \sum_{j=1}^{n} \ominus_j =$$
$$= 50 - 35 = 15$$

$$TOL = \sum_{i=1}^{n} TOL_i = \pm 2.9$$

Final dimension:
$$15 \pm 2.9$$

Fig. 1 Tolerance chart method

A plethora of statistical approaches was introduced to conduct non-linear statistical tolerance analysis. A typical example is the ***Monte Carlo simulation (MCS)*** based on the algorithm of the same name. It utilises random sampling input values to calculate the output results. For a given input vector $\boldsymbol{x}$, the number of sampling values n is determined $\{x_1, x_2, \dots, x_n\}$. By using the mathematical model (transfer function) $\boldsymbol{y} = f(\boldsymbol{x})$ new output vector of same length is found $\{y_1, y_2, \dots, y_n\}$. Finally, the output results $\boldsymbol{y}$ are analysed by calculating statistical data such as mean, standard deviation, or range.

Monte Carlo simulation (MCS) is a beneficial tool for tolerance analysis of mechanical assemblies. Its main advantage is flexibility and ability to use various non-normal input or output distributions [18]. A large set of sample parts is created by randomly assigning a tolerance value to each nominal dimension. Values are selected within the tolerance interval to simulate the manufacturing variation [18]. The process is repeated until enough output data is acquired to enable the use of statistical techniques. It allows the calculation of the mean value, standard deviation, range, upper and lower specification limit, and share of rejected samples [19].
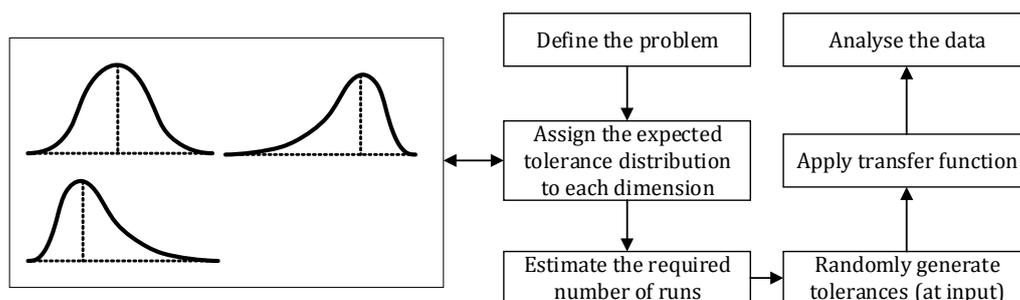
Fig. 2 Monte Carlo simulation for implicit assembly constraints

In this article, MCS is applied as an extension to the Tolerance chart method. A modified form of MCS (McCATS) accounting for the implicit assembly variations was used, as suggested in [20]. In the modified simulation, the random parts are sent to the assembly function, which iteratively solves the tolerance chart equations for the dependent assembly variations [20]. The process is repeated until a sample of a suitable size to produce the assembly histogram is created. The steps necessary to carry out the tolerance analysis using the MCS are shown in Fig. 2.

The vector loop model is a stack-up technique used to extend the stack analysis to two and three-dimensional assemblies [1]. The idea of the vector loop method is to use vectors to describe the dimensions and associated tolerances. Vectors are arranged in loops to determine the assembly deviations. Tolerance analysis problems are solved using the kinematic concept; contact points are set as kinematic joints. A number of possible motions is defined for each joint (i.e. degrees of freedom), along with the local datum reference plane. Three types of variations are described in vector loop model: dimensional variations (lengths and angles), kinematic variations (small adjustments between mating points, joints) and geometric/feature variations (position, roundness, angularity) [1].

Dimensional and geometric tolerances are described as additional degrees of freedom on the kinematic joints [1]. Kinematic simplification is required to represent geometric tolerances in such way. Thus, in the vector loop model, geometric tolerances are included only at mating points, in the direction defined by the type of kinematic joint [1]. They are described as additional translational and rotational transformations (displacement vectors, rotation matrices) – as gaps with zero-length nominal dimension vectors.

The assembly graph is a diagram that represents the analysed assembly, including its parts, dimensions, mating conditions, functional elements, and functional requirements. The graph is used to represent any linear dimension in the assembly as a vector (see Fig. 3). Vectors are connected and form chains or loops, reflecting how assembly parts stack-up together. The associated tolerance is included as a small kinematic adjustment of such a vector (gap) [1, 12]. Such representation allows us to determine the functional requirements of an assembly. Stack-up functions are built by including the vector variations involved in each chain into implicit kinematic equations. As such, they can then be solved using various mathematical approaches [1, 6].

For each part in the tolerance chain, a local datum reference frame (DRF) is added to identify the relevant features of a part for tolerance analysis. DRFs are then connected using datum paths representing geometric layouts, which define the direction and orientation of vectors forming the loop [1]. They are created by stacking and chaining the dimensions that locate the contact point between two parts. After creating datum paths, the vector loops can be created by connecting datums. Loops can be open or closed, depending on the functional requirement of the tolerance analysis. The number of closed loops is calculated as $L = J - P + 1$, where J is the number of the mating points, and P the number of parts.
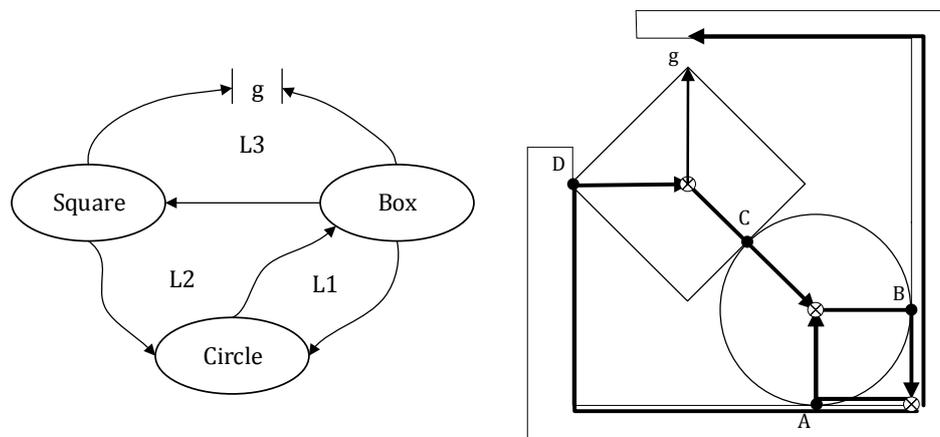

Fig. 3 Assembly graph and the example of vector loops

After defining the vector loops, the calculation is carried out [1, 11]. When considering the closed-loop problem, the equations are often non-linear; they must be linearized using direct linearization method [1, 11], producing approximate results. Thus, vector loop, when using direct linearization, is unable to generate true worst-case results [4, 11]. When the open-loop problem is considered, deviations are calculated directly using explicit equations [11].

Unified Jacobian-torsor (JT) method [21] is a 3D tolerance analysis method. It uses the Jacobian matrix to relate the functional requirement (FR) and virtual joints displacements. JT advances the punctual small-displacement variables of the Jacobian formulation to represent tolerance zones using the torsor model and interval arithmetic. It offers more output information on the FR, reducing the size of the analysed model since it is no longer point-based [21].

Torsor model uses small displacement screws to establish tolerance zones of points, curves, and surfaces [21, 22]. Each real surface is modelled by a substitution surface defined by a set of screw parameters that are modelling the deviations from nominal geometry [23]. Screw parameters are arranged in torsors containing translational components of a point (u, v, w) and α, β, δ as rotational components with respect to the nominal geometry:

$$T = \begin{Bmatrix} \alpha & u \\ \beta & v \\ \delta & w \end{Bmatrix}_R , \qquad (1)$$

where R is DRF used to evaluate the screw components. Torsor model can fully define the tolerance zones due to its ability to shape spatial volumes within which the surfaces are deviating [10].

The procedure of Unified Jacobian-torsor method consists of 4 steps [24]. The first step is to identify all functional elements (FEs) affecting the FR by distinguishing kinematic chains involving the functional condition or part under study. Functional element can be any point, curve or surface of a part and creates internal or kinematic pairs [21]. The second step is to associate a torsor or screw parameter to each element (surface, axis) of the kinematic chain. Torsors express the degrees of freedom and the allowable element displacements and their bounds. Small displacements are applied to parts' geometrical features affecting the FR [21], after which the Jacobian matrix is used to determine relative positions and orientations of torsors within the chosen kinematic chain (step three) [22]. The final step is to combine torsor and Jacobian to provide a matrix equation. Solving a resulting matrix using interval algebra provides the functional condition bounds.

2.2 Assembly model for case study

The above-described methods were compared by analysing a 3D tolerancing problem. The assembly consisting of the cantilever and the rotating handle (open loop) was used as an example. Thus, both the dimensional and geometric tolerances were considered. The functional requirement deviations are assessed using each of the methods, while the results are compared in Section 3.5. The nominal dimension (DIM), upper deviation limit (UDL), and lower deviation limit (LDL) were calculated. The comparison is focused on the similarities and differences between results obtained by each method. Differences in procedures and calculation approaches are also observed.

A simple rotating handle assembly consisting of four parts was used to carry out the comparison between the methods (see Fig. 4). The pole (2) is fixed to the bottom plate (1), while the lever (3) is mounted onto the journal located on the pole. The handle (4) is installed into the bore located on the lever. Tolerances were assigned to all the dimensions apart from a distance between the handle (4) and the base plate (1), which is selected as a functional requirement).
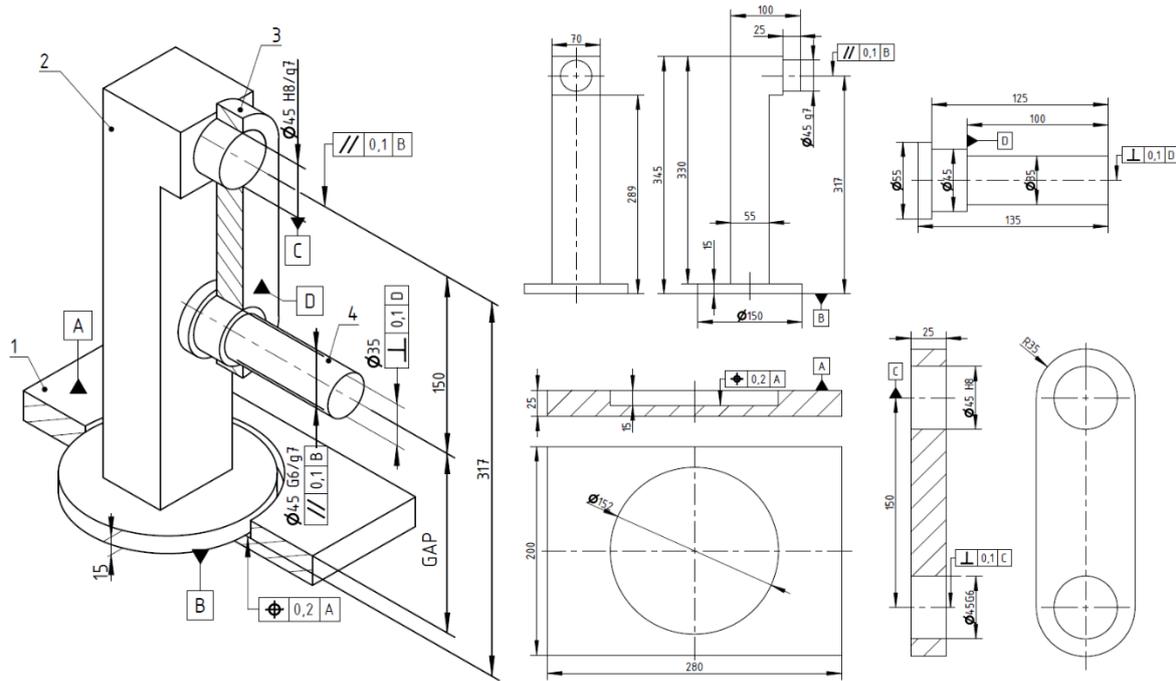
Fig. 4 Case study model

The positional tolerance between the top surface of the base and its cut-out was included. The contact between the base and the cylindrical base of the pole is considered ideal. The parallelism tolerance between the axis of the cylindrical pin located on the pole and the bottom surface of the pole was also included. The lever is mounted onto the pole pin (see Fig. 4) by clearance fit ⌀45 H8/g7. On the opposite side of the lever, the handle is mounted into the bore with a clearance fit ⌀45 G6/h7. Regarding the geometric tolerances, the parallelism between two lever bores and perpendicularity between the handle and the mounting sleeve wall were required.

Each method was then applied to the above-described open-loop assembly. The results were compared according to three criteria:

- identification of the contributing tolerances,
- calculation of the dependent dimension (functional requirement),
- analysis of calculation differences compared to the assemblies with closed loops.

An assembly graph was created for each of the methods except for the Monte Carlo simulation, as it is based on Tolerance chart method.

3. Results and discussion

3.1 Tolerance chart results

Tolerance chart method is mostly used for dimensional tolerances, even though the recent modifications have enabled the analysis of geometric tolerances as well [15]. The geometric tolerances are to be transformed into their dimensional counterparts. Yet, such transformation does not account for the angular surface deviation. In this article, the tolerance chart method is applied only to dimensional tolerances.

Tolerance chain consisting of base plate bore depth (a), length between the pole basis and pole journal axis (b), tolerance fit between pole journal and lever bore (c and d), distance between lever bore axes (e), and tolerance fit between lower lever bore and handle (f and g). As mentioned, the distance between the top surface of the base plate (part 1) and handle (4) is a functional requirement. Indices U and L were included to denote the upper and lower deviation limit, respectively.
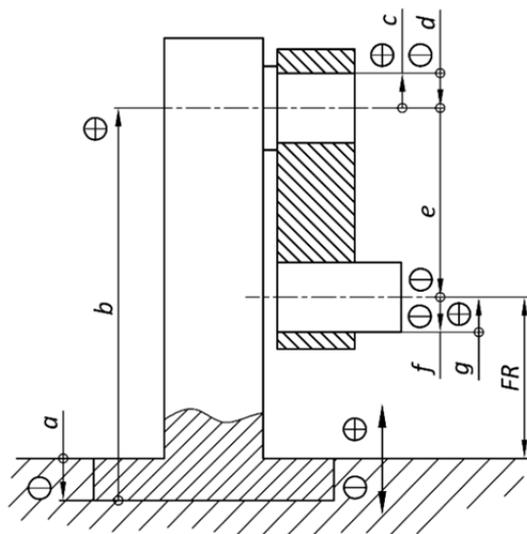
Dimension	+	−	TOL.
a		15	0
b	317		0
c	22.5		$c_{ut} = -0.009/2$; $c_{lt} = -0.034/2$
d		22.5	$d_{ut} = 0.039/2$; $d_{lt} = 0$
e		150	0
f		22.5	$f_{ut} = 0.025/2$; $d_{lt} = 0.006/2$
g	22.5		$g_{ut} = 0/2$; $g_{lt} = -0.025/2$
Sum	362	210	

Nominal functional requirement dimension:
$$DIM_{TC} = -a + b + c - d - e - f + g = 152 \text{ mm}$$

Stack equation for the upper deviation limit:
$$UDM_{TC} = -a + b + c_{ut} - d_{lt} - e - f_{lt} + g_{ut} = -0.008 \text{ mm}$$

Stack equation for the lower deviation limit:
$$UDM_{TC} = -a + b + c_{lt} - d_{ut} - e - f_{ut} + g_{lt} = -0.062 \text{ mm}$$

Fig. 5 Application of tolerance chart method

The tolerance stack coordinate system is defined next; the starting point is set at the base plate surface (1). The upward dimension is shown in Fig. 5 is selected as positive and marked with the indicator "⊕", while the downward is negative and marked with "⊖". The direction of the tolerance chain is chosen arbitrarily, but it is important to respect the specified direction along the chain. Finally, the results are calculated by adding and subtracting values along the tolerance chain and shown in Fig. 5.

3.2 Monte Carlo simulation results

Monte Carlo simulation was applied following the procedure explained in Section 2.1. Determining the appropriate distribution to each of the tolerances was the crucial step, as it affects the results. The distribution of geometric tolerances along with the interval between the upper and lower deviation limit most frequently follows the normal distribution.

Tolerance fits are asymmetrical, requiring the use of the skewed distribution according to [19]. Distribution of input values is described using ± 3σ process range (6σ). Since data about the manufacturing process was not available, 3DCS CATS software was used to determine the distribution models of tolerance fits. According to 3DCS, tolerance fits have unimodal continuous probability distribution called Pearson 1. Since tolerance limits assigned to dimension c are negative, the distribution model is skewed left from the nominal dimension. The same can be concluded for the tolerance assigned to dimension g. On the other hand, tolerance limits assigned to dimensions d and f are positive, and distribution is right-skewed.

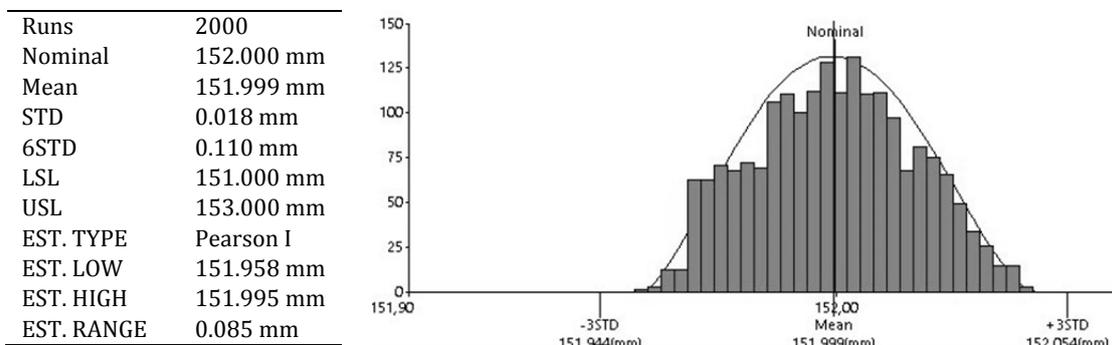|	
Runs	2000
Nominal	152.000 mm
Mean	151.999 mm
STD	0.018 mm
6STD	0.110 mm
LSL	151.000 mm
USL	153.000 mm
EST. TYPE	Pearson I
EST. LOW	151.958 mm
EST. HIGH	151.995 mm
EST. RANGE	0.085 mm

**Fig. 6** Monte Carlo assembly results and histogram

Next step is to define the variation model function. Since Monte Carlo is applied to tolerance chart method, tolerance stack equations are used to define it. After running the simulation for n = 2000 times with randomized input variables, an output model for FR was created. A variation analysis provides descriptive statistics, inferential statistics data, and a histogram (shown in Fig. 6). By adding and subtracting the input variables using the variation model functions, distribution of FR tolerances was found. The resulting functional requirement distribution is also Pearson 1. However, it is not as profoundly left- or right-skewed as are the input variables.

3.3 Vector-loop results

An assembly graph describing the open-loop of the assembly and its vector loop tolerance chain (or datum path) was created. It was used to identify of the number of vector chains and loops involved in the assembly (Fig. 4). Since each part is in contact with its two neighbouring parts only once, this assembly contains one open loop. Same can be seen on the assembly graph (Fig. 7) where each arrow is representing the contact between parts. Vector loop is open at the gap (noted g) between the Handle (Part 4) and Base (Part 1).

The datum path (Fig. 7, right) connects the point, surface, axis or DRF of a part with next part's point, surface axis or DRF. DRFs have been assigned to each part with respect to the origin coordinate system at the top of the Base (Part 1). All the DRFs have a horizontal x-axis and vertical z-axis. Origin coordinate system is set in such a way that positive direction of Z_0 axis corresponds with the positive direction of a tolerance chain in Tolerance chart method. This eases the tracking and method comparison.

The geometric tolerances were also accounted for. Each tolerance was represented as an additional vector of magnitude equal to $\pm\, t/2$, where t is the width of corresponding tolerance field (see Fig. 4; 0.2 for the positional, and 0.1 for parallelism and perpendicularity tolerances). The additional vectors represent gaps between parts contacting points and were denoted based on the corresponding nominal dimension. The position tolerance on the Base cut-out with respect to the datum A (a_{pos}) is represented as a translation vector of the surface in the z-direction (Fig. 7). The parallelisms applied to the Pole's pin (b_{par}), and Lever holes (e_{par}) with respect to the datum B were also represented as translation vector along the z-axis [1]. Perpendicularity applied to the horizontal axis of the Handle (g_{per}) with respect to the datum D can be described as a translation along x-axis [1]. According to the assembly graph there are $J = 3$ contacting points and $P = 4$ parts, resulting in 0 closed loops (Eq. 1 was used). There is also one open-loop functional requirement.
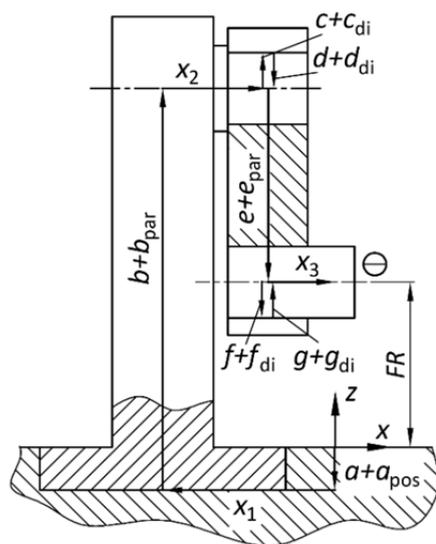
Var.	Tol. description	Gap vector length
a_{pos}	Position (Base top to cutout)	±0.1
b_{par}	Parallelism (Pole's to bottom)	±0.05
c_d	Dimensional tol. (Pole's pin)	$c_{du} = -0.009$; $c_{dl} = -0.034$
d_d	Dimensional tol. (Lever's upper hole)	$d_{du} = 0.039$; $d_{dl} = 0$
e_{par}	Parallelism (Lever hole axes)	±0.05
f_d	Dimensional tol. (Lever's lower hole)	$f_{du} = 0.025$; $f_{dl} = 0.009$
g_d	Dimensional tol. (Handle)	$f_{du} = 0.0$; $f_{dl} = -0.025$
h_{par}	Perpendicularity (Handle axis)	±0.05

Uppervalue of functional requirement:

$$FR_u = -(a - a_{pos}) + (b + b_{par}) + \frac{c + c_{du}}{2} - \frac{d + d_{dl}}{2} - (e - e_{par})$$
$$- \frac{f - f_{dl}}{2} + \frac{g + g_{du}}{2} + h_{par} = 152.241 \text{ mm}$$

Lower value of functional requirement:

$$FR_l = -(a + a_{pos}) + (b - b_{par}) + \frac{c + c_{dl}}{2} - \frac{d + d_{du}}{2} - (e + e_{par})$$
$$- \frac{f + f_{du}}{2} + \frac{g - g_{dl}}{2} - h_{par} = 151.680 \text{ mm}$$

Fig. 7 Vector loop assembly graph and results

3.4 Unified Jacobian-torsor results

Before creating the assembly graph (Fig. 8), it was necessary to identify the functional elements (*FE*) and functional requirements (*FR*). Also, it was required to differentiate between the internal and kinematic pairs. For the assembly at hand, there are four internal and one kinematic pair. First internal pair (FE_{0-1}) is located on the Base (Part 1), as the positional tolerance defined between its top and cut-out surface corresponds to functional surfaces 0 and 1 on the assembly. The parallelism tolerances define FE_{2-3} and FE_{4-5}. Internal pair FE_{6-7} is defined by the perpendicularity tolerance set on the Handle (Part 4).

Only kinematic pair (FE_{1-2}) is set between the Base cut-out and Pole's bottom. However, the contact is assumed to be ideal so that it will not impact the analysis. Two more contacts defined by tolerance fits (between Pole and Lever, and between Lever and Handle) were not set as kinematic pairs even though they are in physical contact. This means they are defined as important conditions to be satisfied between two *FE*s. So, according to [13], they are then defined as functional requirements that will be taken into account in the analysis as kinematic pairs.
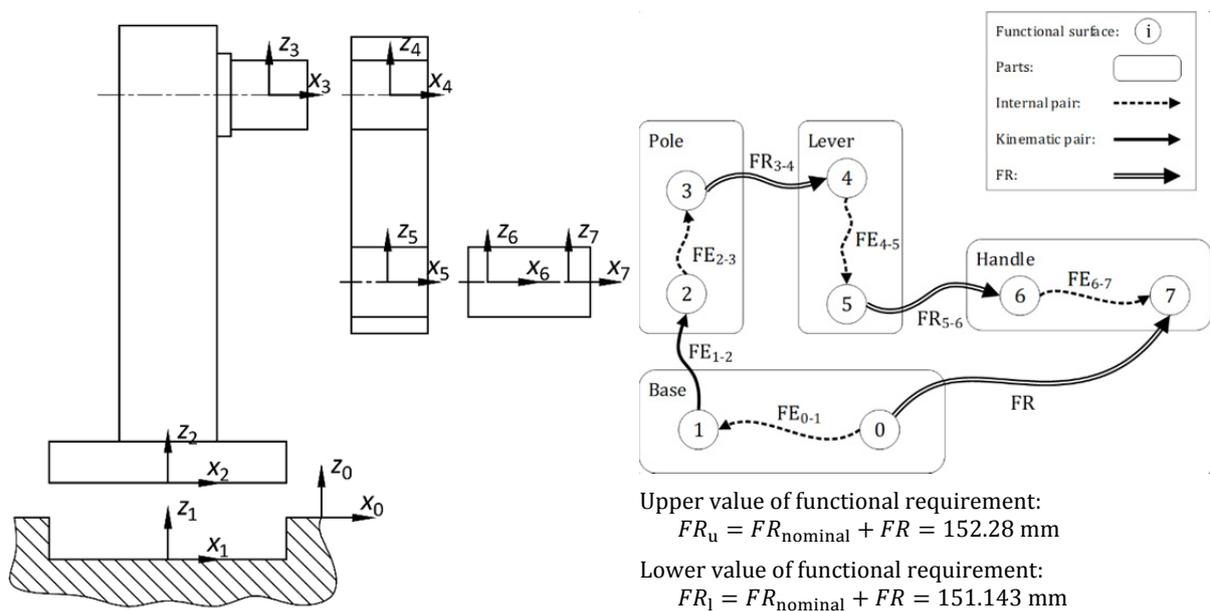

Upper value of functional requirement:
$$FR_u = FR_{nominal} + FR = 152.28 \text{ mm}$$

Lower value of functional requirement:
$$FR_l = FR_{nominal} + FR = 151.143 \text{ mm}$$

Fig. 8 Jacobian-torsor method and results

Jacobian matrix and small-displacement torsor vector were calculated for each internal and kinematic pair. A small-displacement torsor vector was also calculated for each *FE* (based on torsor representing the tolerance zone [21]):

$$[FR]^T = \begin{bmatrix} \dfrac{u}{u} & \dfrac{v}{v} & \dfrac{w}{w} & \dfrac{\alpha}{\alpha} & \dfrac{\beta}{\beta} & \dfrac{\delta}{\delta} \end{bmatrix}_{FR}, [FEi]^T = \begin{bmatrix} \dfrac{u}{u} & \dfrac{v}{v} & \dfrac{w}{w} & \dfrac{\alpha}{\alpha} & \dfrac{\beta}{\beta} & \dfrac{\delta}{\delta} \end{bmatrix}_{FEi} \tag{2}$$

For each tolerance, translational and rotational components inside the tolerance zone were determined [21]. Since the direction of a functional requirement is along the z-axis and rotations that would influence functional requirement are around x and y-axis, w, α and β component must be calculated.

Contact between functional surfaces 0 and 1 in the FE_{0-1} is a planar contact with normal containing one translation component (w) and two rotational components (α, β) [12, 21]. Components are calculated using the equations for planar surface according to [21]. Internal pairs FE_{2-3}, FE_{4-5}, and FE_{6-7}, and functional requirements FR_{3-4} and FR_{5-6} are defined by the tolerance fits. They use translational components v and w and rotational components β and δ of a slipping pivot with the axis [21]. Below is a table containing displacements torsors for each internal and kinematic pair.

Table 1 Displacement torsors for each internal and kinematic pair

FE_{0-1}		FE_{2-3}		FR_{3-4}		FE_{4-5}		FR_{5-6}		FE_{6-7}	
$\underline{0}$	$\overline{0}$	$\underline{0}$	$\overline{0}$	$\underline{0}$	$\overline{0}$	$\underline{0}$	$\overline{0}$	$\underline{0}$	$\overline{0}$	$\underline{0}$	$\overline{0}$
$\underline{0}$	$\overline{0}$	$\underline{0.05}$	$\overline{-0.05}$	$\underline{0.00365}$	$\overline{0.0045}$	$\underline{0.05}$	$\overline{-0.05}$	$\underline{0.025}$	$\overline{0.045}$	$\underline{0.05}$	$\overline{-0.05}$
$\underline{0.1}$	$\overline{-0.1}$	$\underline{0.05}$	$\overline{-0.05}$	$\underline{0.00365}$	$\overline{0.0045}$	$\underline{0.05}$	$\overline{-0.05}$	$\underline{0.025}$	$\overline{0.045}$	$\underline{0.05}$	$\overline{-0.05}$
$\underline{0.0013}$	$\overline{-0.0013}$	$\underline{0}$	$\overline{0}$	$\underline{0}$	$\overline{0}$	$\underline{0}$	$\overline{0}$	$\underline{0}$	$\overline{0}$	$\underline{0}$	$\overline{0}$
$\underline{0.0013}$	$\overline{-0.0013}$	$\underline{0.004}$	$\overline{-0.004}$	$\underline{-0.0003}$	$\overline{0.0004}$	$\underline{0.004}$	$\overline{-0.004}$	$\underline{-0.002}$	$\overline{-0.0004}$	$\underline{0.001}$	$\overline{-0.001}$
$\underline{0}$	$\overline{0}$	$\underline{0.004}$	$\overline{-0.004}$	$\underline{-0.0003}$	$\overline{0.0004}$	$\underline{0.004}$	$\overline{-0.004}$	$\underline{-0.002}$	$\overline{-0.0004}$	$\underline{0.001}$	$\overline{-0.001}$

Jacobian matrix is calculated according to the procedure presented in [21]. Its purpose is to calculate the effect of the traditional torsor set for each functional element (*FE*) on the functional requirement (*FR*) of the assembly [21]. Finally, after calculating small-displacement torsor vectors and Jacobian matrices for each *FE*, the same can be done for *FR*:

$$FR = J \cdot FE = \begin{bmatrix} -0.058 & 0.239 & \mathbf{0.665} & 0.001 & 0.005 & 0.004 \\ -0.434 & 0.184 & -\mathbf{0.947} & -0.001 & -0.011 & -0.010 \end{bmatrix}^{\mathrm{T}} \tag{3}$$

3.5 Comparison of the results obtained by different methods

After analysing the assembly presented in Section 3.1 using the four methods, results are presented in Table 2. Abbreviations are used to ease the result disambiguation; TC was used for tolerance chain method, MC for Monte Carlo simulation, VL for the vector loop method, and JT for the Unified Jacobian-torsor method. Since it is not possible to include geometric tolerances in TC and MC analysis, two results were provided for VL and JT methods. The first batch of results included dimensional tolerances, while the second includes both. Besides the quantitative analysis results, qualitative properties such as the scope and perceived complexity of each method were assessed.

Proprietary CAT tools that are used in day-to-day work are often perceived as black boxes. That means that the users are frequently not familiar with the underlying processes and mathematical models. Besides, the tolerance analysis methods used in CAT tools are often not complying to the technical standards. For this reason, we have analysed underlying tolerance analysis methods, aiming to determine their advantages and shortcomings.

When considering the dimensional tolerances (DT) exclusively, TC, MC, and VL provide similar results, with FR_u being practically equal. Contrary to the upper value of the functional requirement, the deviations in lower (FR_l) are greater – TC and VL provide more conservative results when compared to MC. A significant deviation in FR_u was found when calculating it using JT. Unlike other analysed methods, in Jacobian-torsor method tolerance analysis is carried out using a tolerance zone as a basis (instead of points), causing the afore-mentioned variations in results. By using zones and surfaces instead of points, it is possible to create a more credible representation of a realistic case.

Table 2 Method comparison

	Scope	FR_u	FR_l	Δ	Applicability	Complexity
Tolerance chain method	DT	151.992	151.938	0.054	For simple 1D tasks	Simple, carried out by hand
Monte Carlo simulation method	DT	151.995	151.958	0.037	Simple tasks, statistical analysis	Statistical tools required
Vector Loop method	DT	151.991	151.930	0.061	Multi-dimensional	Carried by hand/more complex
	D&GT	152.243	151.143	1.100		
Unified Jacobian-torsor method	DT	152.369	151.946	0.423	Multi-dimensional, automation	Requires mathematical tools, enables automation
	D&GT	152.665	151.053	1.612		

The MC method is the least conservative due to its statistical approach – when carrying out the tolerance analysis using the MC method, most extreme cases are excluded from the analysis and counted as write-offs. The advantage of such an approach is that it reduces the cost of manufacturing equipment; it is less expensive to write-off a portion of parts, then to purchase more accurate manufacturing tools. Thus, the analysis method should be selected in accordance with the manufacturing process. Statistical approaches are suitable when analysed products are manufactured in large series, while prototypes and one-of-a-kind products warrant the use of more complex and conservative methods, such as JT.

The applicability of methods regarding the tolerancing problem dimensionality should be addressed next. As applied in this study, by using TC and MC only 1D problem containing dimensional tolerances can be solved. This drawback can be partially mitigated by converting the geometric tolerances into their dimensional counterparts; however, methods remain limited to 1D problems. In comparison, VL and JT were developed with having the 2D and 3D tolerancing problems in mind. Besides the dimensional tolerances, both methods can be used to analyse the geometric tolerances as well. However, there is a significant difference between VL and JT in terms of tolerance representation. The former, vector loop, observes a set of tolerances simultaneously, forgoing the possible interactions among them. The latter, Unified Jacobian-torsor, includes both the translational and rotational components, thus including different tolerances as complementary.

The procedure complexity of each method should also be considered. TC is by far the most straightforward method and can be carried by hand. It is suitable for simple tolerancing problems that engineers solve daily. Second is the VL, which requires an additional schema of the vector loop. By procedure complexity, MC comes next. It is a statistical method, meaning that it requires a large sample in order to provide significant results – experience with similar parts and their tolerances is necessary. The last method is JT, which is found to be the most complex. To carry it out, it requires the detection of functional elements, functional requirements, and kinematic pairs.

When comparing the method performance on open and closed-loop tolerancing, changes were detected only in VL. The vector-loop method is affected by the procedure system of open and closed loops. In cases where only open-loop tolerances are used, the vector loop method is reduced to explicit equations. This allows for a direct calculation of the functional requirement values. In other words, the VL method loses its advantage to TC.

The limitations of the study should also be considered. Each of the methods is carried out strictly according to the literature, without additional data manipulation (for example, geometric tolerances were not converted to dimensional ones). Additionally, when carrying out MC, it should be stressed that previous knowledge about the manufacturing process and manufacturing tool properties is necessary to enable satisfying approximation of tolerance distribution.

Lastly, during the planning of the product design process, in addition to tolerance analysis methods, engineers should also consider applying the tolerance optimisation methods. Several studies have been carried out on the subject, such as [25, 26]. Using optimisation algorithms to tolerancing problems allows us to find the optimal trade-off between the tolerances, manufacturing costs, and quality loss [25]. Such an approach would surely increase the design effectiveness, increasing its market success.

4. Conclusion

Simple tolerancing problem was used to assess the similarities and differences between four tolerance analysis methods: Tolerance chart ("Worst-case analysis"), Monte Carlo Simulation method, Vector-loop method, and Unified Jacobian-torsor. Open-loop assembly was used to illustrate the problem-solving process using each of the methods. Based on the results, the authors have concluded the following:

- Tolerance chart and Monte Carlo Simulation methods do not account for the geometrical tolerances. This results in overly optimistic results; however, both methods are only suitable for solving simple, 1D tolerancing problems.
- The unified Jacobian-torsor method was found to be most conservative (i.e. provided the most substantial deviations in functional requirement), followed by Vector-loop, Tolerance chart, and Monte Carlo Simulation, respectively.
- Tolerance chart is the simplest and thus suitable for solving many day-to-day tolerancing problems. Monte Carlo Simulation and Unified Jacobian-torsor require more detailed analysis and know-how and are suitable for more pressing problems. Vector loop can be considered the middle ground – it offers good results at the moderate complexity.
- When comparing the method performance in open-loop assemblies to closed-loop ones, differences are detected only in Vector-loop method.

The field of tolerance analysis is fruitful, and there is more work to be done. Following this study, the authors aim to analyse the performance of tolerancing methods by carrying out an industrial case study. The part deviations measured during the quality assurance are to be compared to the values provided by analysis methods, providing additional insight.

Acknowledgement

References

[1] Polini, W. (2011). Geometric tolerance analysis, In: Colosimo, B., Senin, N. (eds.), *Geometric tolerances: Impact on product design, quality inspection and statistical process monitoring,* Vol. 2, Springer, London, United Kingdom, 39-68, doi: 10.1007/978-1-84996-311-4_2.

[2] International organisation for standardisation (2011). ISO 8015-2011 – Geometrical product specifications (GPS) – Fundamentals – Concepts, principles and rules, ISO, Geneva, Switzerland.

[3] Sigurdarson, N., Eifler, T., Ebro, M. (2018). The applicability of CAT tools in industry – Boundaries and challenges in tolerance engineering practice observed in a medical device company, *Procedia CIRP*, Vol. 75, 261-266, doi: 10.1016/j.procir.2018.04.066.

[4] Corrado, A., Polini, W. (2017). Manufacturing signature in jacobian and torsor models for tolerance analysis of rigid parts, *Robotics and Computer-Integrated Manufacturing*, Vol. 46, 15-24, doi: 10.1016/j.rcim.2016.11.004.

[5] Ramnath, S., Haghighi, P., Chitale, A., Davidson, J.K., Shah, J.J. (2018). Comparative study of tolerance analysis methods applied to a complex assembly, *Procedia CIRP*, Vol. 75, 208-213, doi: 10.1016/j.procir.2018.04.073.

[6] Chen, H., Jin, S., Li, Z., Lai, X. (2014). A comprehensive study of three dimensional tolerance analysis methods, *Computer-Aided Design*, Vol. 53, 1-13, doi: 10.1016/j.cad.2014.02.014.

[7] Morse, E.P., Shakarji, C.M., Srinivasan, V. (2018). A brief analysis of recent ISO tolerancing standards and their potential impact on digitalization of manufacturing, *Procedia CIRP,* Vol. 75, 11-18, doi: 10.1016/j.procir.2018.04.080.

[8] American Society of Mechanical Engineers (2004). ASME Y14.5 – Mathematical definition of dimensioning and tolerancing principles, ASME, New York, USA, 1-15.

[9] Shen, Z., Ameta, G., Shah, J.J., Davidson, J.K. (2005). A comparative study of tolerance analysis methods, *Journal of Computing and Information Science in Engineering*, Vol. 5, No. 3, 247-256, doi: 10.1115/1.1979509.

[10] Marziale, M., Polini, W. (2011). A review of two models for tolerance analysis of an assembly: Jacobian and torsor, *International Journal of Computing Integrated Manufacturing*, Vol. 24, No. 1, 74-86, doi: 10.1080/0951192X.2010.531286.

[11] Chase, K.W., Magleby, S.P., Gao, J. (2004). Tolerance analysis of 2-D and 3-D mechanical assemblies with small kinematic adjustment, *Advanced Tolerancing Techniques*, Vol. 218, 1869-1873.

[12] Ghie, W. (2009). Statistical analysis tolerance using jacobian torsor model based on uncertainty propagation method, *The International Journal of Multyphysics*, Vol. 3, No. 1, 11-30, doi: 10.1260/175095409787924472.

[13] Ghie, W., Laperrière, L., Desrochers, A. (2010). Statistical tolerance analysis using the unified Jacobian-Torsor model, *International Journal of Production Research*, Vol. 48, No. 15, 4609-4630, doi: 10.1080/00207540902824982.

[14] Wang, Y. (2008). Closed-loop analysis in semantic tolerance modeling, *Journal of Mechanical Design,* Vol. 130, No. 6, Article No. 061701, doi: 10.1115/1.2900715.

[15] Schleich, B., Wartzack, S. (2016). A quantitative comparison of tolerance analysis approaches for rigid mechanical assemblies, *Procedia CIRP*, Vol. 43, 172-177, doi: 10.1016/j.procir.2016.02.013.

[16] Magadum, R., Allurkar, B.S. (2015). A comparative study of fasteners tolerance analysis methods, *International Journal of Scientific & Engineering Research*, Vol. 6, No. 6, 700-705.

[17] Fischer, B.R. (2015). *Mechanical Tolerance Stackup and Analysis*, CRC Press, Boca Raton, USA.

[18] Yan, H., Wu, X., Yang, J. (2015). Application of Monte Carlo method in tolerance analysis, *Procedia CIRP*, Vol. 27, 281-285, doi: 10.1016/j.procir.2015.04.079.

[19] Schenkelberg, F. (2016). *Statistical tolerance analysis – Basic introduction*, FMS Reliability Publishing, Los Gatos, USA.

[20] Chase, K.W., Gao, J., Magleby, S.P. (1995). General 2-D tolerance analysis of mechanical assemblies with small kinematic adjustments, *Journal of Design and Manufacturing Automation*, Vol. 5, 263-274.

[21] Desrochers, A., Ghie, W., Laperrière, L. (2003). Application of unified jacobian-torsor model for tolerance analysis, *Journal of Computing and Information Science in Engineering*, Vol. 3, No. 1, 2-14, doi: 10.1115/1.1573235.

[22] Ghie, W. (2012). Tolerance analysis using Jacobian-Torsor model: Statistical and deterministic applications, In: Cakaj, S. (ed.), *Modelling simulation and optimization – Tolerance and optimal control*, InTech, Rijeka, Croatia, 147-160, doi: 10.5772/9043.

[23] Desrochers, A., Delbart, O. (1998). Determination of part position uncertainty within mechanical assembly using screw parameters, In: ElMaraghy H.A. (ed.), *Geometric design tolerancing: Theories, standards and applications*, Springer, Boston, USA, 185-196, doi: 10.1007/978-1-4615-5797-5_14.

[24] Peng, H., Lu, W. (2017). Three-dimensional assembly tolerance analysis based on the Jacobian-Torsor statistical model, In: *Proceedings of 3rd International Conference on Mechatronics and Mechanical Engineering (ICMME 2016)*, Les Ulis, France, doi: 10.1051/matecconf/20179507007.

[25] Siva Kumar, M., Kannan, S.M., Jayabalan, V. (2009). A new algorithm for optimum tolerance allocation of complex assemblies with alternative processes selection, *The International Journal of Advanced Manufacturing Technology*, Vol. 40, No. 7-8, 819-836, doi: 10.1007/s00170-008-1389-5.

[26] Ramesh Kumar, L., Padmanaban, K.P., Balamurugan, C. (2016). Optimal tolerance allocation in a complex assembly using evolutionary algorithms, *International Journal of Simulation Modelling*, Vol. 15, No. 1, 121-132, doi: 10.2507/IJSIMM15(1)10.331.