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A theoretical approach

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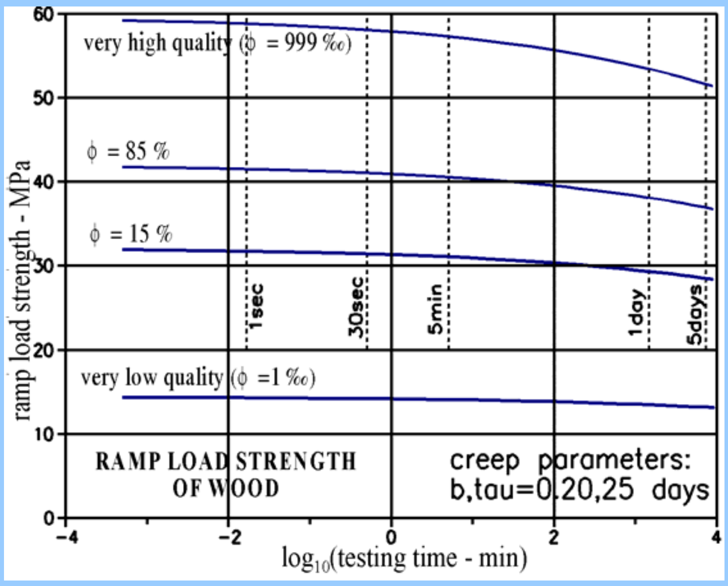

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Lauge Fuglsang Nielsen

Abstract

Strength of wood is normally measured in ramp load experiments. Experience shows that strength increases with increasing rate of testing. This feature is considered theoretically in this note. It is shown that the influence of testing rate is a phenomenon which depends on the quality of the wood considered. Low quality wood shows the lesser influence of testing rate. This observation agrees with experimental ramp load experience – experience which is consistent with the well-known statement made by Borg Madsen that weak wood subjected to a constant load has a longer lifetime than strong wood.

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1. INTRODUCTION

The DVM-theory (Damaged Viscoelastic Material) has been developed in [1,2,3] to predict strength of wood subjected to static and variable loads. Because of the non-dimensional formulation of the theory it applies for a number of loading modes such as tension and bending for example [1]. The quality of DVM-predictions has often been shown to be quite good. Examples are shown in Figures 1 and 2 (reproduced from [3,4]) with experimental data reproduced from Hoffmeyer's and Bach's unique works on 'duration of load' [5,6] and fatigue [7] respectively.

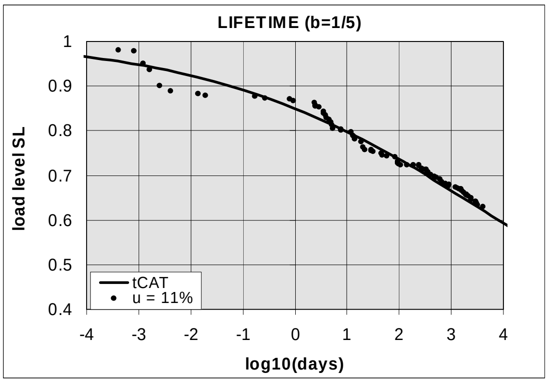

Figure 1. *Lifetime of Spruce lumber in dead load bending. FL = 0.25, τ = 25 days. Experimental data: P. Hoffmeyer [5,6].*

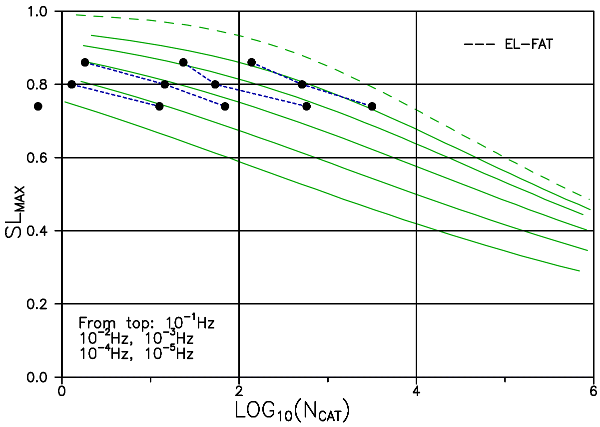

Figure 2. *Fatigue of Spruce subjected to square wave compressive loading, SL = 0-SL_{MAX}, parallel to grain. FL = 0.4, b=0.25, τ = 1 day. Experimental data from [7]. 'El-Fat' indicated is predicted lifetime at very high frequencies.*

The DVM-theory is based on the mechanics of cracks (damages) expanding in wood as a viscoelastic material. The basic crack model is the one of Dugdale's, illustrated in Figure 3 and further explained in [1].

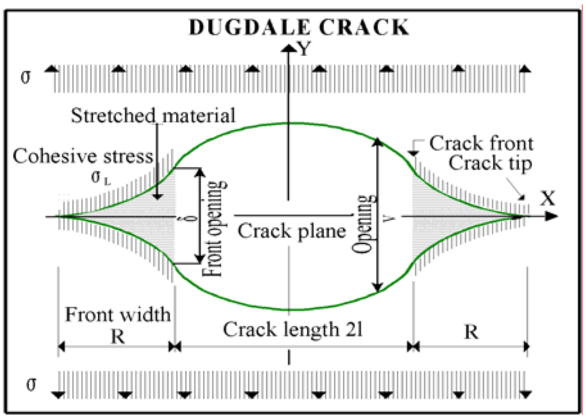

Figure 3. *Modified Dugdale crack loaded perpendicular to crack plane.*

In the DVM-theory a crack expands as outlined/explained in Figure 4: Two stages characterize strength degradation in damaged materials: 1) The initial cracks start propagating at time t = t_S; 2) Propagating cracks cause catastrophic failure at time t =

t_{CAT} where the rate of propagation becomes infinitely high.

The quality ($FL = \sigma_{CR}/\sigma_L$) of the wood considered is quantified as traditional strength relative to theoretical strength as predicted in Figure 5 by a modified Dugdale load capacity graph [2].

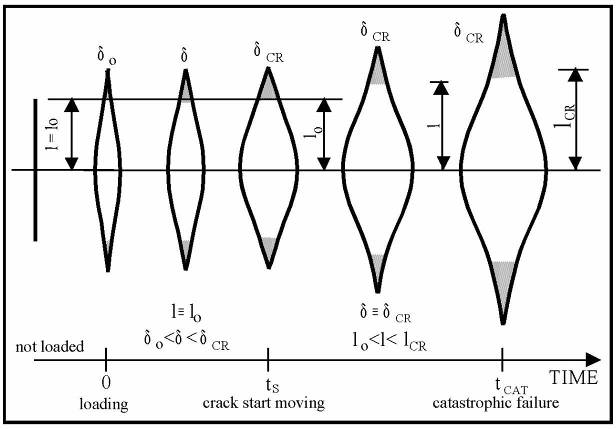

Figure 4. *Stages of crack propagation. In the initial stage, $t < t_S$, the crack has a constant length. Due to creep the thickness becomes thicker and thicker until the crack opening becomes critical. Then, with a constant crack opening, the crack starts propagating until its length becomes critical and the rate of propagation becomes infinite at $t = t_{CAT}$.*

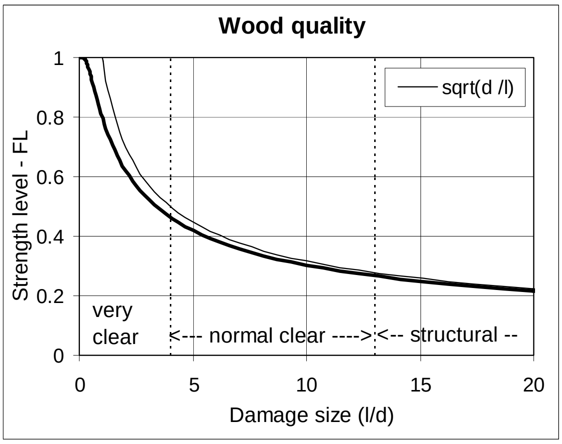

Figure 5. *Wood quality, FL, estimated from damage size, l, relative to the damage nucleus (inherent defect) d = 0.3 mm, see [2].*

We notice that a wood quality of FL = 0.25 estimated for the analysis of Hoffmeyer's data in Figure 1 corresponds to high quality structural wood.

Viscoelasticity in damaged wood areas is characterized by the so-called Power-Law creep function described in Equation 1 and further considered in [4]. This function is quantified by the creep power b and the relaxation time τ.

CREEP in damaged areas :

$$c(t) = \frac{1}{E}\left(1+\left(\frac{t}{\tau}\right)^b\right) \quad \text{creep function (power–law creep)} \Rightarrow \qquad (1)$$

$$C(t) = E * c(t) = 1+\left(\frac{t}{\tau}\right)^b \quad \text{normalized creep function}$$

Normally [2,3,4] b is approximately 0.2-0.3 and τ = 1 – 25 days depending on moisture content, loading mode (bending, compression, tension, perpendicular or parallel to grain). In this note we concentrate mainly on (b,τ) = (0.2,25 days) as estimated for the analysis of Hoffmeyer's data in Figure 1.

1.1 Scope

The DVM theory just outlined is used in this note to develop strength results for wood subjected to ramp load, see Figure 6, which simulates very well strength determination in practice.

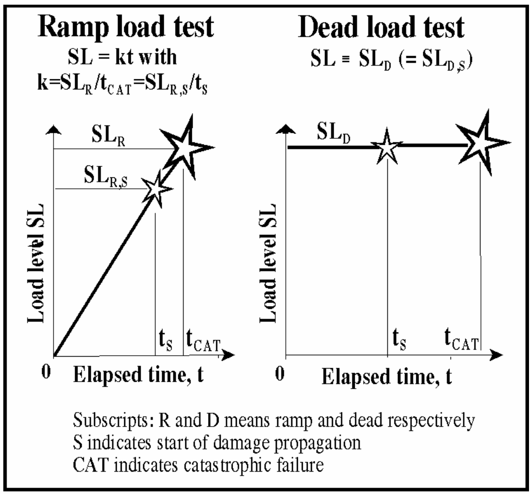

Figure 6. *Ramp load is load increasing proportional with time. Dead load is a constant load.*

The DVM-expressions needed for this analysis are summarized/developed in the Appendix presented at the end of this note together with a list of general notations. The more important notations are: Load level SL = σ/σ_{CR} is load (σ) relative to traditional strength σ_{CR}. Strength level FL = σ_{CR}/σ_L has already been defined. More specific notations defining the load histories considered are explained in Figure 6.

As previously indicated, ramp strength (SL_R and $SL_{R,S}$ in Figure 6) solutions are primarily considered. The influence of test rate, creep, and wood quality will be demonstrated. The solutions will be compared with dead load solutions previously developed by the author.

2. ANALYSIS

2.1 Time, t_S, to start of damage propagation

The following results on time to start (t_S) of damage propagation are reproduced from the appendix at the end of this note. It is noticed that t_S is independent of FL for both ramp- and dead load tests.

The influence of b (and τ) on time to start of damage propagation is demonstrated by non-dimensional graphs shown in Figures 6-8.

$$\frac{t_S}{\tau} = \begin{cases} \left(\dfrac{1}{SL_D^2} - 1\right)^{1/b} & \text{(deadload)} \\ \left(\dfrac{(2+b)!}{2b!}\right)^{1/b} \left(\dfrac{1}{SL_{R,S}^2} - 1\right)^{1/b} & \text{(rampload)} \end{cases} \qquad (2)$$

Remarks: We notice that dead load t_S and ramp load t_S are proportional. For the so-called Maxwell ramp load t_S is three times longer than the dead load t_S. It is also noticed that time to initiation of damage propagation (t_S) is not influenced by strength level FL.

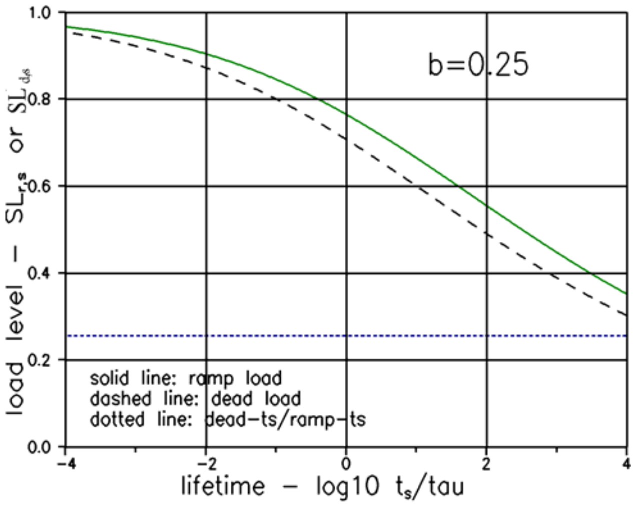

Figure 7. *Non dimensional time to start of damage propagation. Both ramp- and dead load situations are considered.*

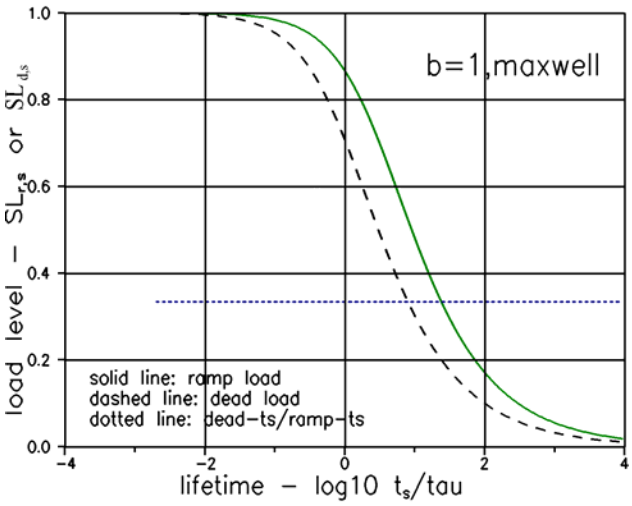

Figure 8. *Non dimensional time to start of damage propagation. Both ramp- and dead load situations are considered. For a Maxwell material the t_S is 3 times longer for ramp load than for dead load.*

2.2 Time, t_{CAT}, to catastrophic failure

As previously indicated a crack stops resting at $t = t_S$. Then the crack starts moving until its rate of expansion becomes infinitely high at $t = t_{CAT}$. The period of time under expansion ($t_{CAT} - t_S$) can be calculated by Equation 3 reproduced from the Appendix.

Examples of predicted total lifetime are shown in Figures 9 and 11. An example of a ramp load history is shown in Figure 10.

$$\frac{dt}{d\kappa} = \frac{8q\tau}{(\pi FL)^2} \frac{[1/(\kappa SL^2) - 1]^{1/b}}{\kappa SL^2} \quad \text{(lifetime expired when damage ratio } \kappa = \frac{1}{SL^2}\text{)}$$

$$\text{Numerically: } \Delta t = \frac{8q\tau}{(\pi FL)^2} \frac{[1/(\kappa SL^2) - 1]^{1/b}}{\kappa SL^2} \Delta\kappa \quad \text{with } \Delta\kappa = \left(\frac{1}{SL^2} - 1\right)/1000 \text{ (example)} \qquad (3)$$

Notice: For rampload the above calculation is made from $t = t_S$ with $SL = k * t$ where $k = SL_{R,S}/t_S$

Remark: It is noticed that wood quality (FL) does influence lifetime elapsed when damages are expanding. We re-call from the previous section that such influence does not apply for the time to start of crack propagation, t_S.

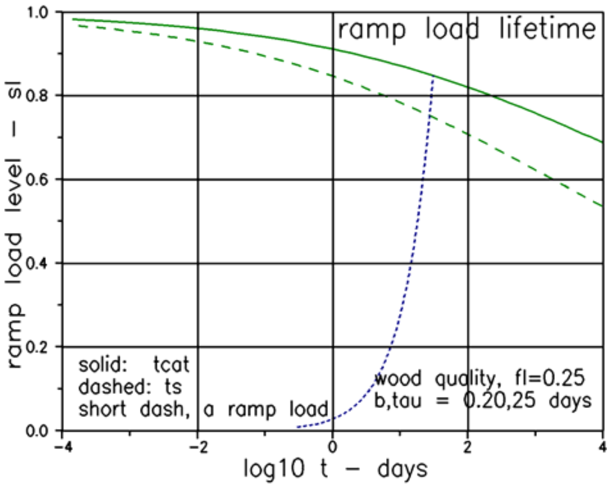

Figure 9. *Total lifetime of wood subjected to ramp load as described in Figure 10.*

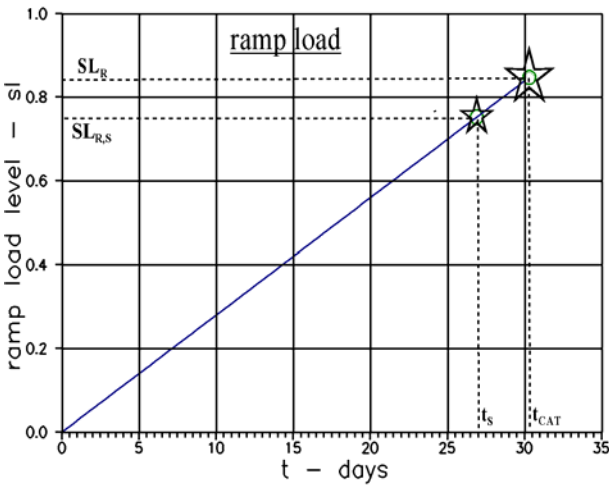

Figure 10. *Ramp load history as indicated in Figure 9.*

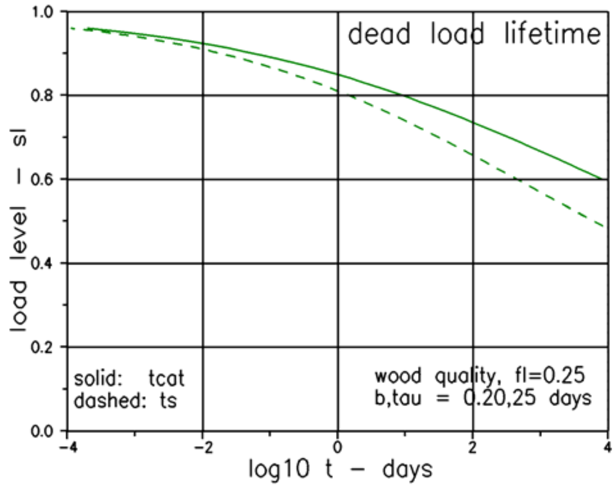

Figure 11 *Total lifetime of wood subjected to dead load.*

2.3 Ramp strength versus strength level (wood quality)

In practice it is of interest to know the influence of testing time on the strength measured. Estimates on this feature can be made by the analysis performed in this note. Some results are summarized in Figures 12 and 13.

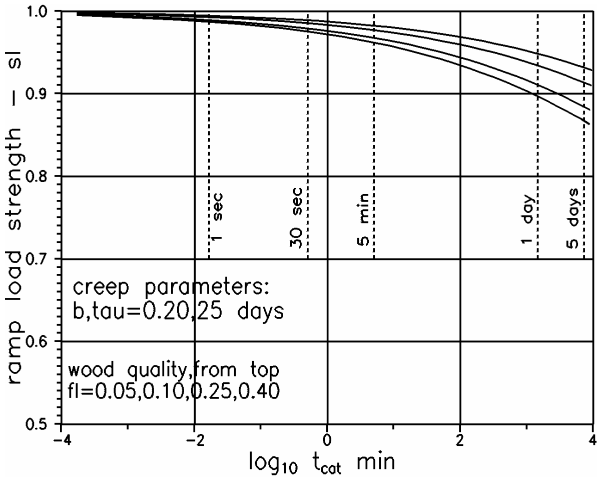

Figure 12. Strength obtained by ramp load tests. Influence of wood quality and time used in experiment.

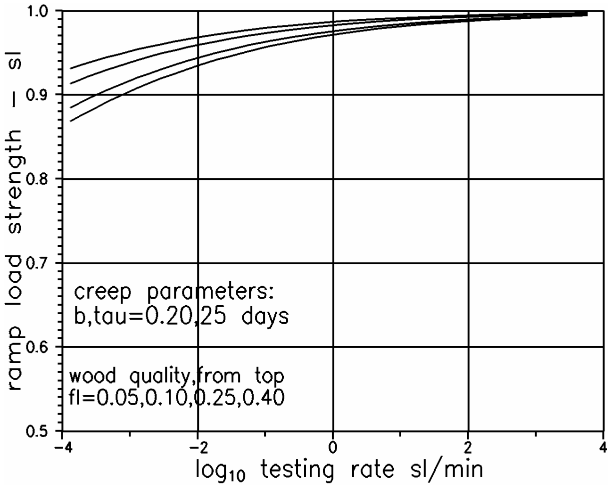

Figure 13. Strength obtained by ramp load tests. Influence of wood quality and rate of loading.

2.4 Intermediate conclusion

For the strength levels considered it is seen that results obtained in 5 minutes tests (which are commonly used) deviate only little (< 3%) from the 'real strength'. If a test period of 100 minutes, however, is used we may get results which are up to 6 % 'wrong' – and larger, increasing the testing time. The deviation from real strength depends on wood quality. Increasing quality promotes larger deviations. Thus, the results obtained from ramp load tests on structural wood are closer to 'real strength' than similar results obtained from ramp load tests on clear wood.

3. RAMP STRENGTH VERSUS NORMAL STRENGTH DISTRIBUTION

The theoretical solutions presented above with various strength levels (FL) can be related to real strength and strength distributions as shown in Equation 4 with only one reference strength level. In order to reflect the most genuine (true, creep independent) strength properties the distribution function (σ_{CR}) must be based on fast experiments.

$$\sigma_{CR} = \bar{\sigma}_{CR} + \frac{s}{\pi}\log_E\left(\tan\left(\frac{\pi}{2}\varphi\right)\right) \qquad \underline{\text{strength distribution}}$$

$$\text{with} \begin{cases} \varphi & \text{acumulated distribution} \\ \bar{\sigma}_{CR} & \text{mean strength (at } \varphi = 0.5) \\ s & \text{standard deviation} \end{cases} \tag{4}$$

$$FL(\varphi) = FL(0.5)\frac{\sigma_{CR}(\varphi)}{\sigma_{CR}(0.5)} \qquad \underline{\text{strength level (quality)}}$$

$$\text{with reference strength level } FL(0.5) \text{ at } \varphi = 0.5$$

3.1 Example

For the purpose of demonstration we choose the strength distribution presented in Figures 14 and 15[1)]. Then the influence of quality and rate of loading on ramp strength for a whole wood population can be calculated as previously explained in Chapter 2. A reference strength level of FL(0.5) = 0.25 at $\varphi = 0.5$ has been assumed for the predictions illustrated in Figures 16 and 17.

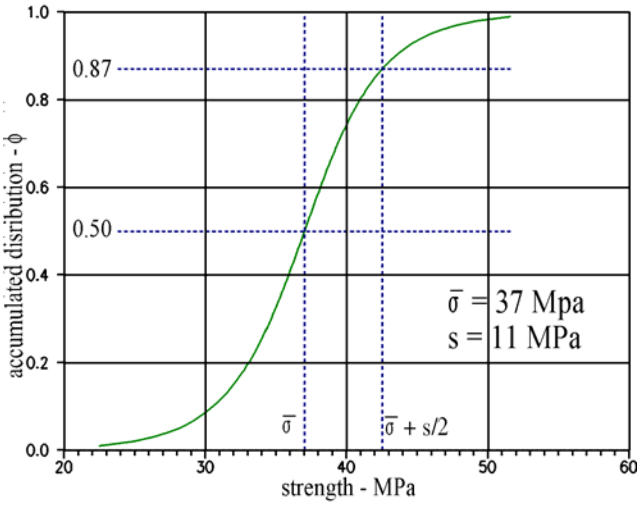

Figure 14. *Strength distribution according to Equation 4. It is indicated how mean strength and standard deviation can be estimated from experimental data.*

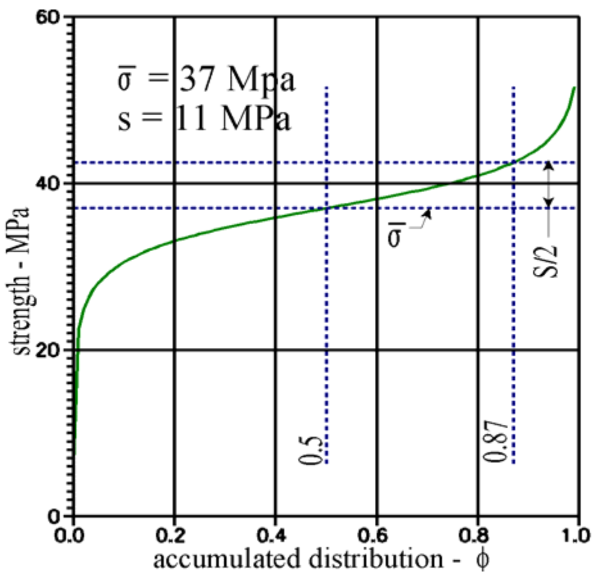

Figure 15. *Alternative representation of the strength distribution shown in Figure 14.*

4. CONCLUSION AND FINAL REMARKS

The observations made in this note are consistent with the overall conclusions made by Spencer [8] (commented by Borg Madsen [9]) from his ramp load bending experiments on Douglas-Fir lumber boards: '… the stronger boards show an increasing strength as the rate of stressing increases, but this effect becomes less pronounced for weaker boards'.

---

1) This distribution is an approximate description of the distribution applying for the wood population Q1 tested in the work [5] by Hoffmeyer previously referred to. The example presently considered keeps close to this work: We keep the creep parameters $(b,\tau) = (0.20, 25$ days$)$ and reference strength level (FL = 0.25) which were introduced in the lifetime analysis presented in Figure 1.

As a curiosum was noticed in [8,9] that very weak boards showed a decreasing strength with increasing rate of testing. In the present author's opinion such behavior is hard to believe. The statement might very well be the result of difficulties turning up when experiments are performed on very low quality wood – and reading the sensitive data from such tests.

The effect of increasing wood quality to decrease ramp strength is worthwhile noticing. Obviously, the theoretical reason for this phenomenon is the quality influence on the rate of crack propagation expressed by Equations 3 (and A2). The rate of crack propagation increases with increasing FL. Basically this observation is consistent with the well-known statement made by Borg Madsen, that weak wood subjected to constant loads has a longer lifetime than strong wood.

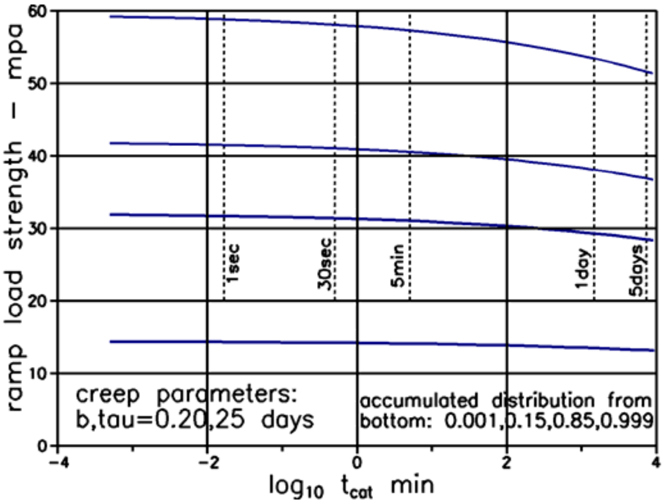

__Figure 16.__ Ramp strength as related to testing time. A wood population is considered with the traditional strength distribution described in Figure 14.

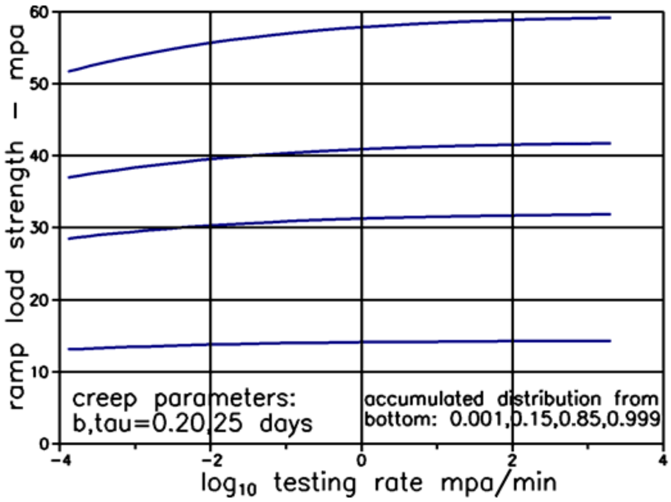

__Figure 17.__ Ramp strength as related to rate of loading. A wood population is considered with the traditional strength distribution described in Figure 14.

5. LIST OF SYMBOLS

The notations most frequently used in this note are listed below. Some times the sub/superscripts indicated are not used - only, however, when the proper meaning is obvious from the text associated.

Sub/superscripts	
RAMP	Ramp load (increasing prop. with time)
DEAD	Dead load (constant)
General	
Theoretical strength	σ_L
Real strength (at time 0)	σ_{CR}
Strength level (Materials quality)	$FL = \sigma_{CR}/\sigma_L$
Load	σ
Load level	$SL = \sigma/\sigma_{CR}$
Young's modulus	E
Creep (in damaged area)	
Time in general	t
Creep power	b
Relaxation time	τ
Time shift parameter	$q = [(1+b)(2+b)/2]^{1/b}$
Defects	
Critical damage opening	δ_{CR}
Damage size (half crack length)	$\ell = \ell(t)$
Damage size (at t = 0)	ℓ_o
Damage ratio	$\kappa = \ell/\ell_o$
Time to start of damage propagation	t_S
Time to catastrophic failure	t_{CAT}

APPENDIX: Elements of a lifetime analysis

A lifetime analysis of a cracked material must consider two phases, see Figure 4: 1) A period of time, t_S, where the crack has its original size, $\ell \equiv \ell_o$, until it starts propagating, and 2) a period of time, $t_{CAT} - t_S$, while the crack propagates until its rate of propagation becomes infinite at $t = t_{CAT}$. Total lifetime is the sum of these two contributions.

Appropriate lifetime expressions for the present analysis are presented below. Except for the ramp load solution for t_S in Equation A1 they are all reproduced from works previously presented by the author [e.g. 2,3]. Symbols are explained in a list of symbols at the end of this note.

The exception mentioned above is developed using LaPlace transformation technique and the so-called e-v-analogy (elastic viscoelastic analogy) explained in [10]. The auxiliary functions H(t) and δ(t) are the so-called Heaviside's function and the Dirac's delta function respectively.

Time to start of crack propagation, t_S, where $l \equiv l_o$:

$\delta = \frac{\pi\sigma^2}{E\sigma_L}\ell_o$ (elastic crack opening) ; $\delta_{CR} = \frac{\pi\sigma_{CR}^2}{E\sigma_L}\ell_o$ critical crack opening

laPlace transformed viscoelastic opening with analogy Yoyng's modulus E^A :

$\bar{\delta} = \frac{\pi\overline{\sigma^2}}{E^A\sigma_L}\ell_o$ or $\frac{\bar{\delta}}{\delta_{CR}} = \frac{\overline{\sigma^2}}{\sigma_{CR}^2}\frac{E}{E_A}$ (e–v–analogy)

For wood with $E_A = E\frac{(\tau s)^b}{b! + (\tau s)^b}$ we get $\frac{\bar{\delta}}{\delta_{CR}} = \frac{\overline{\sigma^2}}{\sigma_{CR}^2}\frac{b! + (\tau s)^b}{(\tau s)^b}$

Dead load: $\sigma = \sigma_D H(t)$ (Heviside's unit function) $\Rightarrow \overline{\sigma^2} = \frac{\sigma_D^2}{s}$ (A1)

$$\frac{\bar{\delta}}{\delta_{CR}} = \left(\frac{\sigma_D}{\sigma_{CR}}\right)^2 \frac{1}{s}\frac{b! + (\tau s)^b}{(\tau s)^b} = SL_D^2\left(\frac{1}{s} + \frac{b!}{\tau^b s^{1+b}}\right) \Rightarrow \frac{\delta}{\delta_{CR}} = SL_D^2\left(1 + \frac{b!\,t^b}{\tau^b b!}\right) = SL_D^2\left(1 + \left(\frac{t}{\tau}\right)^b\right)$$

As $t \to t_S, \delta \to \delta_{CR} \Rightarrow$ $$\frac{t_S}{\tau} = \left(\frac{1}{SL_D^2} - 1\right)^{1/b}$$

Ramp load: $\sigma = k * t \Rightarrow \overline{\sigma^2} = \frac{2k^2}{s^3}$ (k = constant)

$$\frac{\bar{\delta}}{\delta_{CR}} = \frac{2k^2}{\sigma_{CR}^2 s^3}\frac{b! + (\tau s)^b}{(\tau s)^b} = \frac{2k^2}{\sigma_{CR}^2}\left(\frac{1}{s^3} + \frac{b!}{\tau^b s^{3+b}}\right) \Rightarrow \frac{\delta}{\delta_{CR}} = \frac{2k^2}{\sigma_{CR}^2}\left(\frac{t^2}{2} + \frac{b!}{\tau^b}\frac{t^{2+b}}{(2+b)!}\right)$$

As $t \to t_S, \delta \to \delta_{CR} \Rightarrow 1 = \frac{(kt_S)^2}{\sigma_{CR}^2}\left(1 + \frac{2b!}{(2+b)!}\frac{t_S^b}{\tau^b}\right) = SL_{R,S}^2\left(1 + \frac{2b!}{(2+b)!}\frac{t_S^b}{\tau^b}\right)$ with $SL_{R,S} = \frac{\sigma_{R,S}}{\sigma_{CR}}$

$$\frac{t_S}{\tau} = \left(\frac{(2+b)!}{2b!}\right)^{1/b}\left(\frac{1}{SL_{R,S}^2} - 1\right)^{1/b}$$

Time while crack is propagating, $t_{CAT} - t_S$:

$$\frac{d\kappa}{dt} = \frac{(\pi FL)^2}{8q\tau}\frac{\kappa SL^2}{[1/(\kappa SL^2) - 1]^{1/b}} \quad \text{or} \quad \frac{dt}{d\kappa} = \frac{8q\tau}{(\pi FL)^2}\frac{[1/(\kappa SL^2) - 1]^{1/b}}{\kappa SL^2} \qquad q = \left(\frac{(1+b)(2+b)}{2}\right)^{1/b}$$

as long as positive $\frac{d\kappa}{dt}$ is predicted; lifetime is obtained at $\kappa = \frac{1}{SL^2}$

Numerical solutions : (A2)

$$\Delta t = \frac{8q\tau}{(\pi FL)^2}\frac{[1/(\kappa SL^2) - 1]^{1/b}}{\kappa SL^2}\Delta\kappa \quad \text{with} \quad \Delta\kappa = \left(\frac{1}{SL_{MIN}^2} - 1\right)/1000 \text{ (example)}$$

Notice: For rampload the above calculation is made from $t = t_{S,RAMP}$ with $SL = k * t$ where $k = SL_{R,S} / t_{S,RAMP}$

6. LITERATURE

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