DTU Library

Self-Compacting Concrete (SCC)
Quantifying and modelling the rheology of fresh Concrete as a Bingham-material

Nielsen, Lauge Fuglsang

[Link back to DTU Orbit](#)

Lauge Fuglsang Nielsen

Self-Compacting Concrete (SCC)
Quantifying and modelling the rheology of Fresh Concrete as a Bingham-material

BYG DTU

DANMARKS TEKNISKE UNIVERSITET

Rotation Rheometer

R_O
R_I

Kernel (fixed)
Mantle (rotating)
Sample

h

r

Rheometer used		
R_I(m)	R_O(m)	h(m)
0.1	0.15	0.2

TORQUE VERSUS REV/SEC

TORQUE (Nm)

N (rev/s)

- EXPERIM
- Nplug
- Mplug
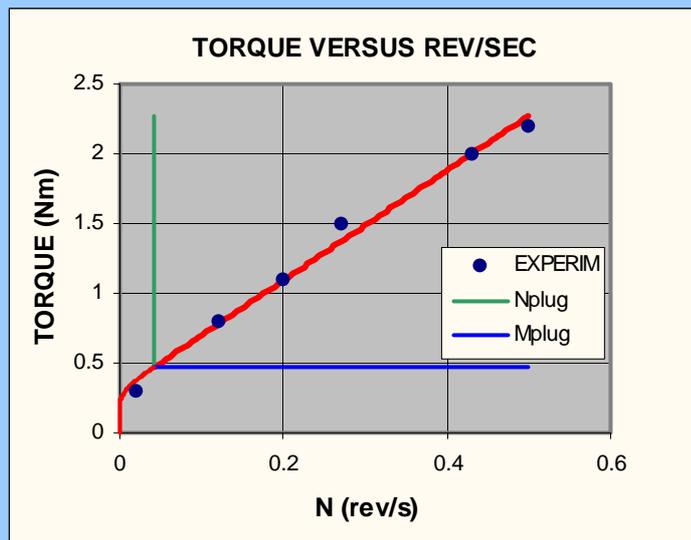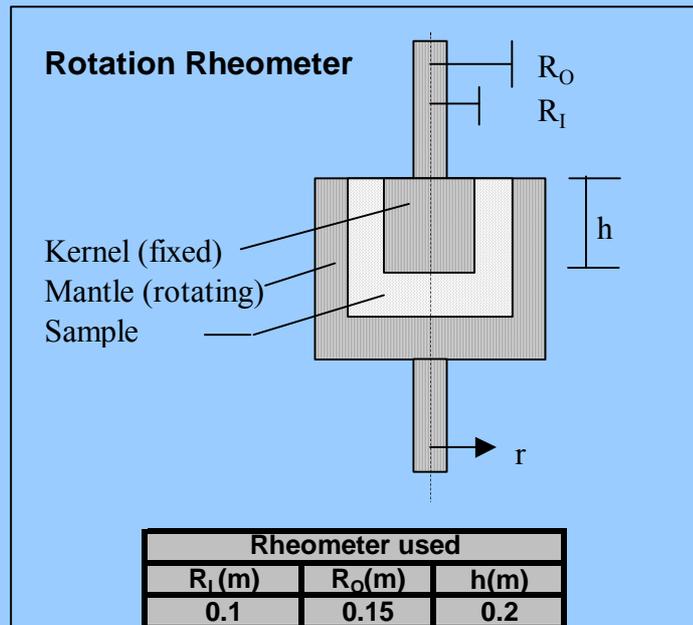
Introduction

This report is prepared for students in materials mechanics - and other persons - who show interest in exploring the mechanical behaviour of fresh Self-Compacting Concrete (SCC). The report consists of the following two papers written by the author:

Nielsen, L. Fuglsang: 'The behaviour of Bingham materials in a rotation rheometer', *A text note prepared for students in material mechanics, September 2002.*

The paper explains how concrete (and other materials), modelled as a Bingham material (explained in the paper), behaves when tested in a so-called rotation rheometer (described in the paper). Some results are well known – others are new, developed for research purposes.

Software, RHEOTEST[1], is attached for fast applications of the theory: Experimental quantification of fresh concrete as a Bingham material.

Nielsen, L. Fuglsang: 'Generalized Bingham description of fresh concrete'. *A paper presented at the XVIII Symposium on Nordic Concrete Research in Helsingør, Denmark, 12-14 June 2002. Proceedings, Danish Concrete Society 2002.*

The paper presents a method by which the well-known Bingham description of flow in homogeneous liquids with yield stress can be generalised to apply also for composite fluids. In the present context such fluids are defined as traditional Bingham fluids mixed with very stiff particles of known shapes and size distributions. In practice the composite aspects of the generalised Bingham description is a major advantage. Only a few geometrical parameters for the particles and two material properties for the fluid matrix are required in order to describe the Bingham behaviour of any composition of the composite fluid considered. The Bingham method normally used needs experimental calibration for any new composition.

Software, SCC[1], is attached for fast applications of the theory: Prediction of the influence of shapes and volume concentrations of aggregates on the rheological behaviour of concrete.

---

[1] To be downloaded from http://www.byg.dtu.dk/publicering/software_d.htm. It is much recommended to 'play around' with this software while reading the report.

A note on the behaviour of

A Bingham-material in a rotation rheometer

Lauge Fuglsang Nielsen   22. July 2002 (rev. Sept, 2002)

Introduction

This note is prepared for a lecture on the deformations occurring in a rotation viscosi-meter (see Figure 1) filled with a Bingham-material. The text of the note is rather brief. For a full understanding of the note it is required that the student is present at the lecture – or on her own has obtained some basic knowledge of the theory of elasticity, and knows about the analogy between elasticity and viscosity. It is assumed that the student knows about the stress-strain relation of a Bingham model, see Equation 1.

The basic results presented in the lecture agree with such obtained by Reiner (and Riwlin) (1,2). However, the methods used by the lecturer to develop these results deviate from those used by Reiner. Additional results are obtained in special sections of the note where the theoretical results are adapted to become useful expressions for experimental detection of material properties of a Bingham material – also when a so-called 'slip-effect' (later explained) appears.

The aspects of experimental determination of material properties are considered in details at the end of this note. Algorithms are presented which form the basis of the software, RHEOTEST, developed for this lecture. The software is demon-strated. It is 'constructed' for practice as well as for teaching purposes. It can be downloaded from http://www.byg.dtu.dk/publicering/software_d.htm.

$$\dot{\gamma} = \frac{d\gamma}{dt} = \frac{\tau - s}{\eta} \quad \left\{ \begin{array}{l} Bingham \quad material: \quad Newton\,fluid\,with \\ viscosity \quad \eta \quad and \quad threshold \quad stress \quad s \end{array} \right\} \qquad (1)$$
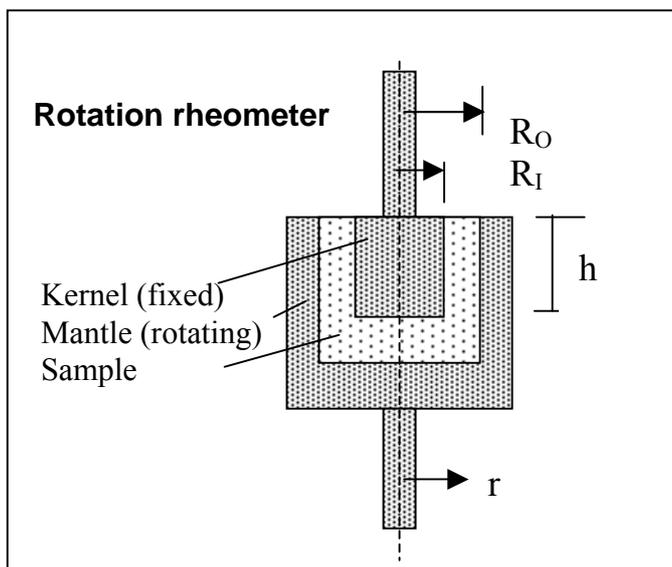

Figure 1. *Rotation rheometer: The mantle is rotated with N revolutions/sec, by which the kernel is affected with the torsional moment M.*

It should be mentioned that the RHEOTEST can be used also to determine the rheological properties of composite fluids. Fresh concrete such as young Self-Compacting Concrete (SCC) is an example of composite fluids.

Lists of symbols used in this note and relevant references are presented at the end of this note. The 'slip effect' previously mentioned is considered separately in an Appendix, also at the end of this note. The results already developed for Bingham materials are modified to include the special phenomenon that the Bingham material, as the result of being tested, might change to a fluid very close to the surface of the fixed kernel. The 'slip-effect' is considered in a separate software RHEOTEST(2) embedded in RHEOTEST.

Elastic solution

As an introduction to the Bingham problem we will look at the elastic counter problem outlined in Figure 2: We will determine the deflection v (or the rotation, $\Omega = v/r$) located at (r,θ) in a composite cylinder where the mantle material behaves conditionally elastic as described in Equation 2.
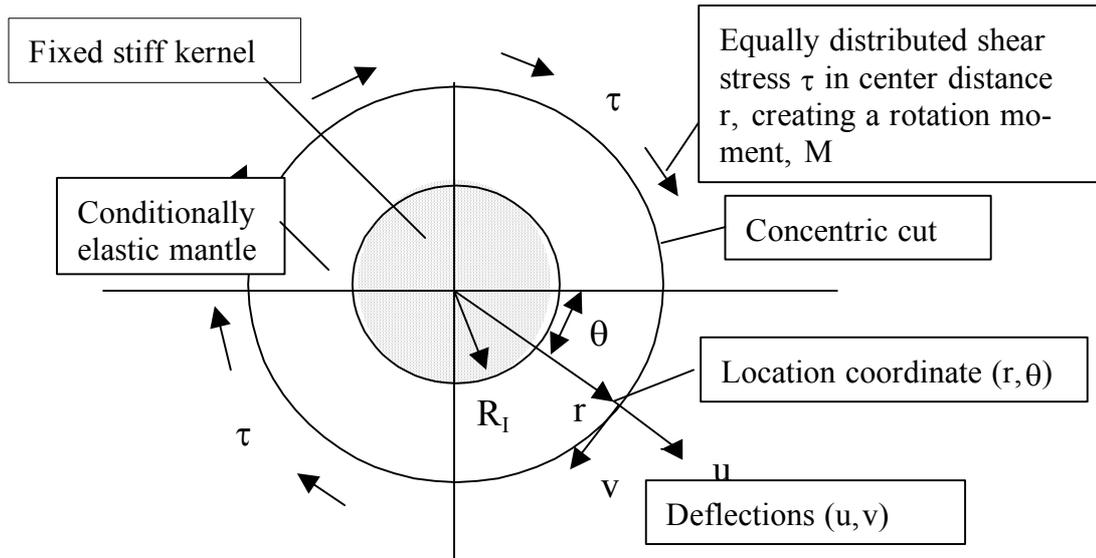
Figure 2. Concentric composite cylinder of height h with a fixed stiff kernel embedded in an infinite mantle of a conditionally elastic material, see Equation 2. The load is a 'far away' acting rotational moment, M, corresponding to equally distributed shear stresses τ acting in the mantle at a centre distance of r.

$$\gamma = \frac{\tau - \tau_{TH}}{G} \left\{ \begin{array}{l} Conditionally\ elastic\ material\ with\ shear\ \text{mod}ulus\ G \\ and\quad threshold\quad stress\quad \tau_{TH} \end{array} \right\} \quad (2)$$

Analysis

Strain in (r,θ) according to Timoschenko and Goodier (4, Equation 51 with $\delta u/\delta\theta = 0$):

$$\gamma = \gamma_{r\theta} = \frac{\delta v}{\delta r} - \frac{v}{r} \quad (change\ of\ angle)\quad which\ can\ also\ be\ written$$

$$\gamma = \gamma_{r\theta} = r\frac{\delta\Omega}{\delta r} \quad with\ rotation\ \ \Omega = \frac{v}{r} \quad (v = r\Omega)$$

(3)

(Notice that indices r,θ on τ and γ subsequently are implicitly understood.)

Physical condition and equilibrium:

$$Physical\ condition\quad \gamma = \frac{\tau - \tau_{TH}}{G}$$

$$Equilibrium\ at\ each\ r:\ M = 2\pi\,r^2 h\,\tau \quad \Rightarrow\ \tau = \frac{M}{2\pi h\,r^2} \Rightarrow$$

(4)

Resulting equation of movement

From Equations 3 and 4:

$$\frac{d\Omega}{\delta r} = \frac{\gamma}{r} = \frac{M}{2\pi hG\,r^3} - \frac{\tau_{TH}}{Gr} \Rightarrow$$

(5)

from which the following rotation solutions are obtained.

Rotation

$$\Omega = \frac{1}{G}\left(\begin{array}{ll} \frac{M}{4\pi h}\left[\frac{1}{R_i^2} - \frac{1}{r^2}\right] - \tau_{TH}\log_E\frac{r}{R_i} & for\quad r < r_o \\[3mm] \frac{M}{4\pi h}\left[\frac{1}{R_i^2} - \frac{1}{r_o^2}\right] - \tau_{TH}\log_E\frac{r_o}{R_i} & for\quad r \ge r_o \end{array}\right. \qquad with\ r_o = \sqrt{\frac{M}{2\pi h\tau_{TH}}}$$

(6)

r_o *is the so-called plug-radius* outside which the rotation is constant, meaning that material for $r > r_o$ moves as a stiff mantle. It can be shown by Equation 4 that stress $\tau = \tau_{TH}$ at $r = r_o$.

Deformation

The movement itself ($v = r\Omega$) becomes as expressed by Equation 7. It is noticed that the movement is proportional with the radius vector for $r > r_o$.

$$v = r * \Omega = \frac{r}{G}\left(\begin{array}{ll} \frac{M}{4\pi h}\left[\frac{1}{R_i^2} - \frac{1}{r^2}\right] - s\log_E\frac{r}{R_i} & for\quad r < r_o \\[3mm] \frac{M}{4\pi h}\left[\frac{1}{R_i^2} - \frac{1}{r_o^2}\right] - s\log_E\frac{r_o}{R_i} & for\quad r \ge r_o \end{array}\right.$$

(7)

Viscoelastic analysis

The elastic solutions just derived can be used to develop the counterpart velocity solutions for a Bingham material. We just have to use the elastic-viscous analogy. (Replace deformation, including rotation, with the corresponding velocities, and G with η). At the same we inter change the threshold stresses τ_{TH} and s.

Equation 3 becomes Equation 8. After this Equations 4-6 become Equations 9-11 respectively.

Strain rate

$$\dot{\gamma} = r\frac{\delta\dot{\Omega}}{\delta r} \; where \; rate \; of \; rotation \quad \dot{\Omega} = \frac{d\Omega}{dt} = \frac{\dot{v}}{r} \quad (\dot{v} = r\dot{\Omega}) \tag{8}$$

Physical condition and equilibrium

$$Physical \quad condition \quad \dot{\gamma} = \frac{d\gamma}{dt} = \frac{\tau - s}{\eta} \qquad (Bingham)$$

$$Equilibrium: \quad \tau = \frac{M}{2\pi h \, r^2} \; \Rightarrow \tag{9}$$

Resulting equation of movement

$$\frac{d\dot{\Omega}}{dr} = \frac{\dot{\gamma}}{r} = \frac{M}{2\pi h \eta \, r^3} - \frac{s}{\eta r} \tag{10}$$

Rotation velocity

$$\dot{\Omega} = \frac{1}{\eta}\left(\begin{array}{cc} \dfrac{M}{4\pi h}\left[\dfrac{1}{R_i^2} - \dfrac{1}{r^2}\right] - s\log_E \dfrac{r}{R_i} & for \quad r < r_o \\[3mm] \dfrac{M}{4\pi h}\left[\dfrac{1}{R_i^2} - \dfrac{1}{r_o^2}\right] - s\log_E \dfrac{r_o}{R_i} & for \quad r \geq r_o \end{array}\right. \qquad med \; r_o = \sqrt{\frac{M}{2\pi h s}} \tag{11}$$

where r_o *is the so-called plug-radius* outside which the rotation velocity is constant, see Figure 3, meaning the material moves as a stiff mantle for $r > r_o$. From Equation 9 is derived that the stress $\tau = s$ for $r = r_o$. (The similarities are noticed between the viscous Equation 11 and the elastic Equation 6).

Deformation velocity

Velocities are determined as.

$$\dot{v} = \frac{dv}{dt} = r * \dot{\Omega} = \frac{r}{\eta}\left(\begin{array}{cc} \dfrac{M}{4\pi h}\left[\dfrac{1}{R_i^2} - \dfrac{1}{r^2}\right] - s\log_E \dfrac{r}{R_i} & ;(r < r_o) \\[3mm] \dfrac{M}{4\pi h}\left[\dfrac{1}{R_i^2} - \dfrac{1}{r_o^2}\right] - s\log_E \dfrac{r_o}{R_i} & ;(r \geq r_o) \end{array}\right. \tag{12}$$

It is noticed that velocities are proportional with centre distance for $r > r_o$, see Figure 3.
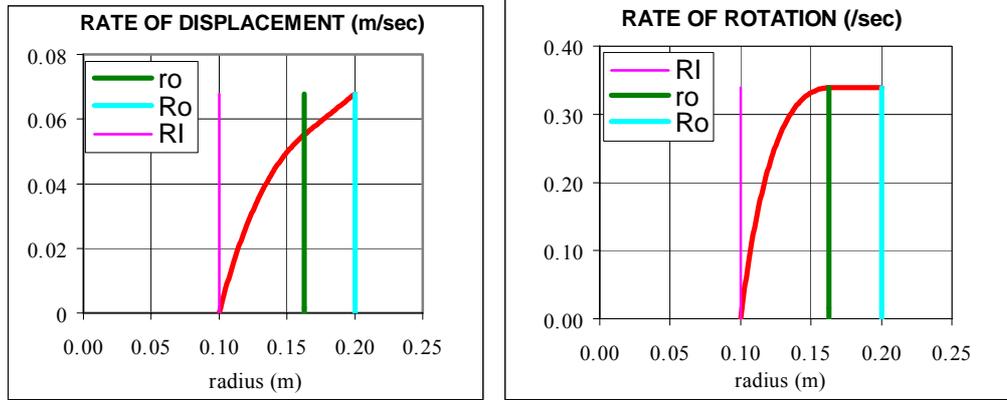
Figure 3. (R_I, R_O, h) = (0.1, 0.2, 0.2) m. M = 1 Nm. η = 30 Pa*sec, s = 30 Pa

Special solutions for a rotation rheometer

We will now think of the mantle material being very stiff outside a centre distance of $r = R_O$. In this way we have modelled the rotation rheometer outlined in Figure 1. The velocity of the stiff mantle becomes

$$\dot{v}(R_O) = \frac{R_O}{\eta} \left(\begin{array}{l} \dfrac{M}{4\pi h}\left[\dfrac{1}{R_I^2} - \dfrac{1}{R_O^2}\right] - s \log_E \dfrac{R_O}{R_I} \quad ;(R_O < r_o) \\[3ex] \dfrac{M}{4\pi h}\left[\dfrac{1}{R_I^2} - \dfrac{1}{r_o^2}\right] - s \log_E \dfrac{r_o}{R_I} \quad ;(R_O \geq r_o) \end{array} \right) \tag{13}$$

which can also be written as shown in Equation 14 with revolutions/sec introduced by $N = \dot{v}(R_O)/(2\pi R_o)$

$$N = \frac{1}{2\pi\eta} \left(\begin{array}{l} \dfrac{M}{4\pi h}\left[\dfrac{1}{R_I^2} - \dfrac{1}{R_O^2}\right] - s \log_E \dfrac{R_O}{R_I} \quad ;(R_O < r_o) \\[3ex] \dfrac{M}{4\pi h}\left[\dfrac{1}{R_I^2} - \dfrac{1}{r_o^2}\right] - s \log_E \dfrac{r_o}{R_I} \quad ;(R_O \geq r_o) \end{array} \right) \tag{14}$$

Potentials of theory in experimental parameter detections

It is obvious that the former expression in Equation 14 – together with measured N-M relations – is useful in determination of the basic material properties (s,η) of a Bingham material. From linear regression of experimental data we get

$$s = \frac{M_o(1/R_I^2 - 1/R_O^2)}{4\pi h \log_E(R_O/R_I)} \quad and \quad \eta = \frac{\alpha(1/R_I^2 - 1/R_O^2)}{8\pi^2 h} \tag{15}$$

where M_o and α are the intercept and slope respectively of the straight line obtained by graphically relating measured torsion moment (M) to measured revolutions/sec (N):

$$M = M_o + \alpha N \tag{16}$$

We notice that the method of regression has the following restrictions decided by the r_o-expression in Equation 11 ($r_o > R_O$) and the former expression in Equation 14.

$$M \geq M_{PLUG} = 2\pi h R_O^2 s \quad or$$
$$N \geq N_{PLUG} = \frac{s}{2\pi\eta}\left(\frac{R_O^2}{2}\left[\frac{1}{R_I^2} - \frac{1}{R_O^2}\right] - \log_E\frac{R_o}{R_I}\right) \tag{17}$$

For lower M and N the M-N relation has to be determined theoretically from the second expression in Equation 14, where we must remember that r_o is a function of M. To use this relation for parameter determination is significantly more difficult (not impossible) than using the linear regression method just considered with $R_o < r_o$.

The linear regression method forms the basis of the software, RHEOTEST, attached to this lecture note. An example of using this software is demonstrated in Figure 4. We notice that some experimental data set have to be excluded as a consequence of Equation 17. (OBS: 'Torque' is another word for torsional moment, M)
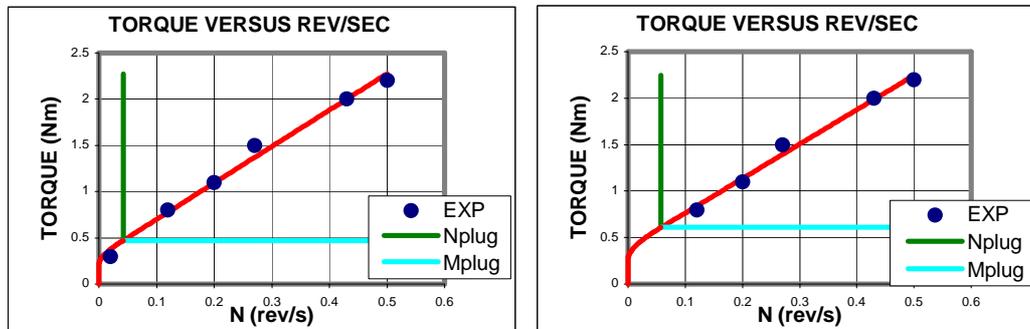
Figure 4. *Treatment of experimental data using the linear regression method, RHEO-TEST: The former figure shows that the first data set has to be excluded because N < N_{PLUG}. The final results are obtained by the latter figure: s = 21.5 Pa, and η = 13.04 Pa*sec. The rheometer applied has (R_I, R_O, h) = (0.1,0.15,0.2) m.*

Plug flow

This term, used internationally, is explained indirectly as follows: In tests with N < N_{PLUG} parts of the material tested sticks to the mantle of the rheometer – and gets the same rotation velocity as the mantle (see Equation 11). Thus, 'plug flow' characterizes the state where the material tested is not stirred completely. The phenomenon of plug flow is considered in details in the software RHEOTEST previously referred to.

Remarks: Tests with N > N_{PLUG} are the most rational to use in parameter determi-

nation for Bingham-materials: Threshold stress, s, and viscosity, η. For practice we must expect that the determination becomes an iterative process, where some experimental data with $N < N_{PLUG}$ have to be excluded, see Figure 4.

Notations

R_O, R_I, h are dimensions of rheometer, see Figure 1

r is radius vector in polar coordinate system, see Figures 1 and 2

θ is angle in polar coordinate system, see Figure 2

M (or T) is torsional moment (torque) measured on the kernel of the rheometer

N is revolutions/sec of the rheometer mantle

Nplug is N under which parts of the test material sticks to rheometer mantle

Mplug (or Tplug) is torque at Nplug

r_o is 'plug' radius, outside which test material sticks to the rheometer mantle in tests with N < Nplug

τ (or 'tau') is shear stress

γ is change of angle

$d\gamma/dt$ is rate of change of angle

η (or 'eta') is viscosity of a Bingham-material, see Equation 1

s is threshold stress for a Bingham-material, see Equation 1

G is shear modulus of a material with conditionally elasticity, see Equation 2

τ_{TH} is threshold stress in a material with conditionally elasticity, see Equation 2

Literature

1. Reiner. M.: 'Lectures on theoretical rheology' (third edition*), North-Holland Publ. Comp., Amsterdam , 1960.*

2. Reiner, M. and Riwlin, R.: 'Über die Strömung einer elastische Flüssigkeit im Couette Apparat', *Kolloid-Z. 43(1927), 120.* 'Zur Hydrodynamik von Systemen veränderlicher Viskosität, *Kolloid-Z. 43(1927), 127.*

3. Schwedoff, T.: 'Recherches experimentales sur la cohésion des liquides', *J. de Phys., 9(1890), 116*

4. Timoschenko, S. and Goodier, J.N.: 'Theory of elasticity', second edition, *McGraw-hill Book Comp., New York, 1951.*

5. Nielsen, L. Fuglsang: "Rheology of extreme composites", *In "Papers in Structural Engineering and Materials - A Centenary Celebration", 179-187, Dept. of Struct. and Materials, Tech. Univ. Denmark, 2000.*

6. Idem: "Generalized Bingham description of fresh concrete", *XVIII Symposium on Nordic Concrete Research in Helsingør, Denmark, 12-14 June 2002. Proceedings, Danish Concrete Society 2002.*

7. Idem: "Rheology of some fluid extreme composites – such as fresh self-compacting concrete*", Nordic Concrete Research, 2(2001), 83 – 93.*

8. Brandl, M. and Thrane, L. Nyholm: "Rheological properties of Self Compacting Concrete (SCC*)", M.Sc. thesis, Department of Civil Engineering, Tech. Univ. Denmark, 2001.*

9. Geiker, M.R., Brandl, M., Thrane, L. Nyholm, and Nielsen, L. Fuglsang: "On the effect of coarse aggregate fraction and shape on the rheological properties of self-compacting concrete". ASTM, Cement, Concrete, and Aggregates, 24(2002), No.1.

10. Geiker, M.R., Brandl, M., Thrane, L. Nyholm, Bager, D.H., and Wallevik, O.: 'On the effect of measuring procedure on the apparent rheological properties of self-compacting concrete', *Cement and Concrete Research, 32(2002), 1791-1795.*

APPENDIX: Slip-layer

In practice some sort of smear effect can sometimes be observed between the test material and the kernel surface. Apparently the surface of the rheometer kernel can influence the test material such that a thin contact layer will exhibit a rotation velocity different from 0 as predicted by Equation 11.

Such behaviour can be counted for approximately by assuming that the test material in a transition zone (slip-layer) from $r = R_I$ to $r = R_{I2}$, see Figure 5, transforms to a plain Newton-fluid (defined in Equation 18).

Various slip-states can be described by varying the Newton viscosity (η_o) and/or the thickness of the slip layer. Although only very thin slip layers are expected, it is chosen, subsequently, to introduce slip layers of arbitrary thickness between 0 and R_{I2} - R_I. By doing so, the Bingham analysis originally developed in this note will include Newton materials.

Remark: Presently, there is no convincing quantitative evidence on the physical nature of slip layers. Thus, at the present time, the subsequent analysis based on Newton layers must be considered as a pure hypothesises.
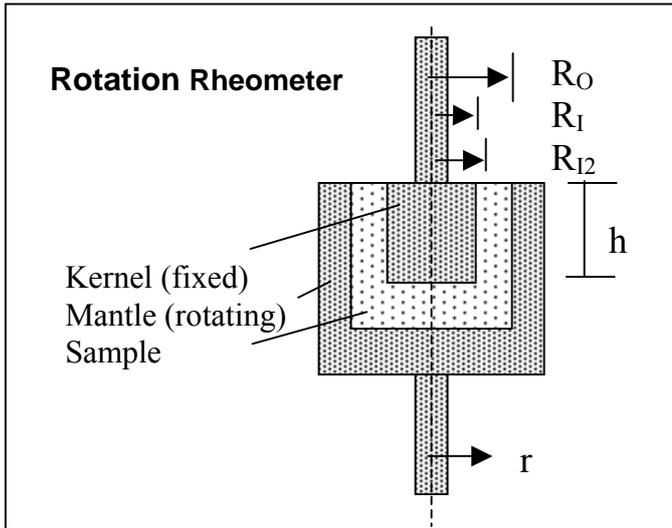
Figure 5. It is assumed that the Bingham material tested is influenced by the kernel surface such that a layer (of thickness R_{I2} - R_I) is created which behaves as a Newton liquid.

$$\dot{\gamma} = \frac{d\gamma}{dt} = \frac{\tau}{\eta_o} \quad \textit{Newton fluid with viscocity } \eta_o \qquad (18)$$

Analysis of slip layer effect

Locations are considered where $R_{I2} < r < r_o = \sqrt{(M/(2\pi hs))}$. Equat. 11 becomes

$$\dot{\Omega} = \frac{M}{4\pi h \eta_o}\left(\frac{1}{R_I^2} - \frac{1}{R_{I2}^2}\right) + \frac{1}{\eta}\left\{\frac{M}{4\pi h}\left[\frac{1}{R_{I2}^2} - \frac{1}{r^2}\right] - s\log_E \frac{r}{R_{I2}}\right\} \qquad (19)$$

where the former term is due to integration (Equation 10) through the slip layer.

The two latter terms stay as in Equation 11 – only that the original kernel radius R_I is replaced with the 'new kernel radius' R_{I2}. After this, the analysis proceeds in a similar way as the analysis previously used to obtain Equations 12-14. The properties of the Bingham material (outside the slip layer) become as expressed in Equation 20 after linear regression of experimental data - which must yield Equation 21.

$$
s = M_o \frac{(1/R_{I2}^2 - 1/R_O^2) + \eta/\eta_o (1/R_I^2 - 1/R_{I2}^2)}{4\pi h \log_E(R_O/R_{I2})} \quad \text{and}
$$

$$
\eta = \alpha \frac{(1/R_{I2}^2 - 1/R_O^2) + \eta/\eta_o (1/R_I^2 - 1/R_{I2}^2)}{8\pi^2 h}
$$

(20)

$$
M \geq M_{PLUG} = 2\pi h R_O^2 s
$$

$$
N \geq N_{PLUG} = \frac{s}{2\pi\eta}\left(\frac{R_O^2}{2}\left[\frac{1}{R_{I2}^2} - \frac{1}{R_O^2} + \frac{\eta}{\eta_o}\left(\frac{1}{R_I^2} - \frac{1}{R_{i2}^2}\right)\right] - \log_E \frac{R_O}{R_{I2}}\right)
$$

(21)

In principles, the determination of Bingham parameters (considering a slip layer) proceeds in a similar way as if no slip layer was present (RHEOTEST). Some modifications, however, have to be introduced respecting Equations 19-21. Such modifications are considered in software, RHEOTEST(2), included in RHEOTEST. An example of using RHEOTEST(2) is presented in Figure 6.
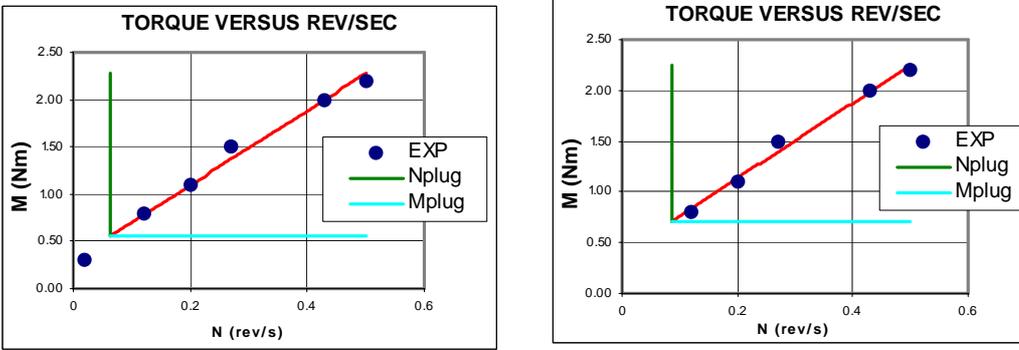
Figure 6. *Treatment of experimental data by RHEOTEST(2) assuming η_o/η = 0.1 and R_{I2} = 0.1005 m: The former figure shows that the first experimental data point must be excluded from analysis because $N < N_{PLUG}$. The final analysis is shown in the latter figure: s = 25.30 Pa, and η = 15.13 Pa*sec. The rheometer used is the same as defined in Figure 4.*

Generalized Bingham description of fresh concrete

A paper presented at the XVIII Symposium on Nordic Concrete Research in Helsingør, Denmark, 12-14 June 2002. Proceedings, Danish Concrete Society 2002.

Lauge Fuglsang Nielsen
Associate professor, Ph.D.
Department of Civil Engineering, Building 118
Technical University of Denmark
DK-2800 Lyngby, Denmark

Abstract

A method is presented by which the well-known Bingham description of flow in homogeneous liquids with yield stress can be generalised to apply also for composite fluids. In the present context such fluids are defined as traditional Bingham fluids mixed with very stiff particles of known shapes and size distributions. In practice the composite aspects of the generalised Bingham description is a major advantage. Only a few geometrical parameters for the particles and two material properties for the fluid matrix are required in order to describe the Bingham behaviour of any composition of the composite fluid considered. The Bingham method normally used needs experimental calibration for any new composition.

Due to the very strict space limits for papers to this conference the generalization method just outlined is presented as an operational summary of a detailed study on the rheology of fluid composites recently reported in [1][1]. In the present paper composites thought of are self-compacting concretes (SCC) modelled as aggregates in a fluid matrix of cement paste (or mortar)[2].

Key words: Composite fluid, Bingham, Composite Bingham, Self-compacting concrete (SCC)

Introduction and theoretical results

The prime scope of this paper is to look at possibilities of establishing a composite method of predicting the rheology of SCC, which may serve as an alternative to the semi-empirical method, suggested by deLarrard [2]. The theoretical basis of doing so has recently been developed in [1] from which the following presentation is summarized: The composite fluid considered is a mixture of very stiff particles (phase P) in a Bingham fluid (phase S). The shear stress (s) – shear strain (e) relation is expressed by Equation 2 where Equation 1 determines the volume concentration, c, of particles

$$c = \frac{V_P}{V_P + V_S} \quad Volume\ concentration\ of\ particles \tag{1}$$

in the composite fluid. V denotes volume. Subscripts P and S refer to particle phase

---

and matrix phase S respectively.

Formally the original- and the composite (or generalized) Bingham expression, see [3], look alike. The viscosity (η) of the material first becomes active when the matrix (fluid) stress exceeds the matrix yield stress S_S.

$$\frac{de}{dt} = \frac{s - S}{2\eta} \Rightarrow s = S + 2\eta \frac{de}{dt} \quad \textit{composite Bingham fluid with}$$

$$\textit{Yield stress}: \ S = S_S(1 + \gamma_\infty c) \ \textit{ and } \ \textit{Viscosity}: \ \eta = \eta_S \frac{1 + \gamma_\infty c}{1 - c} \tag{2}$$

Composite geometry

The composite geometry (particle shape and size distribution) is considered in Equation 2 by the so-called geometry function (Υ_∞) expressed by Equation 3 with shape functions (μ_P, μ_S), expressed by Equation 4 and illustrated in Figure 1.

Principal parameters for the description of geometry in this paper are aspect ratio (A = length/diameter) and the critical concentration c_S of ellipsoidal particles considered ($c_S \approx$ maximum packing density $\approx$ eigenpacking). Normally, we may expect improved quality of particle size distribution (smoothness and density) to be associated with higher c_S.

$$\gamma_\infty = \begin{cases} \dfrac{3}{2} \dfrac{\mu_P + \mu_S - 1}{\mu_S} & ; \quad c < c_S \\ \\ \infty & ; \quad c > c_S \end{cases} \qquad \textit{Geo-function} \tag{3}$$

The so-called shape factors, $\mu^o{}_P$, $\mu^o{}_S$, appearing in Equation 4 are determined by Equation 5.

$$\mu_S = \mu_S^o \left(1 - \frac{c}{c_S}\right)^M \ ; \ \mu_P = \mu_P^o \left(1 - \frac{c}{c_p}\right)^M \ \textit{with} \ \ c_p = -\frac{\mu_P^o}{\mu_S^o} c_S \ \ (c \leq c_S) \tag{4}$$

$$\mu_P^o = \begin{cases} \dfrac{3A}{A^2 + A + 1} & A \leq 1 \\ \\ 3 \dfrac{A^2 - A + 1}{4A^2 - 5A + 4} & A > 1 \end{cases} \ ; \ \mu_S^o = -\begin{cases} \mu_P^o & A \leq 1 \\ \\ 4\mu_P^o - 3 & A > 1 \end{cases} \quad \textit{Shape factor} \tag{5}$$

Normally an interaction power of M = 1 is used in composite analysis assuming a 'moderately' increasing state of interaction between aggregates at increasing concentration. Lower interaction and higher interaction can be described with M < 1 and M > 1 respectively. Unless otherwise stated, a moderate interaction with M = 1 is used in this paper. If otherwise stated M_S and M_V indicate interaction powers used in yield stress analysis and in viscosity analysis respectively.
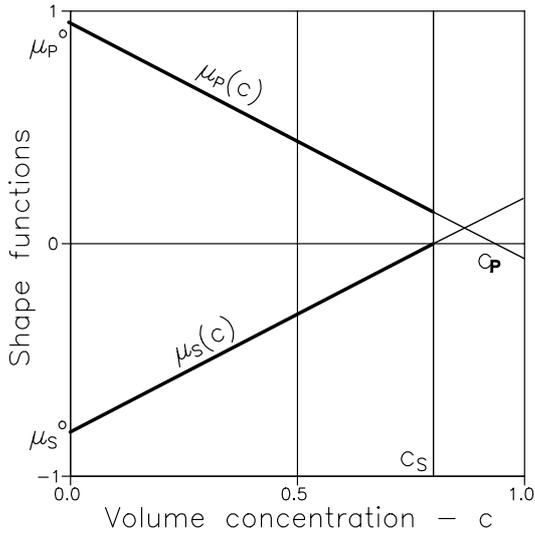
Figure 1. *Shape functions (μ_P,μ_S) with M = 1 and critical concentration c_S (concentration of solid phase in a pile of particles).*
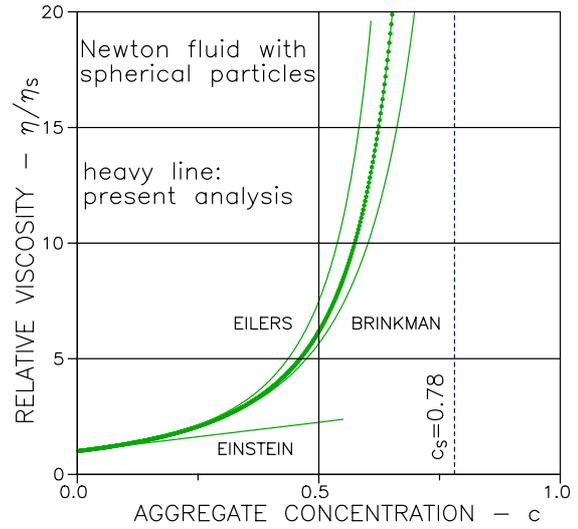
Figure 2. *Spherical particles (A = 1) in a viscous matrix. Present analysis and empirical descriptions by Eilers and Brinkman.*

Theory and experiments

As can be seen from Figure 2, reproduced from [1], the relative viscosity predicted by Equation 2 agrees with a solution developed by Einstein [4] in his study of the viscosity of dilute sugar solutions. The expression also agrees with data obtained from experiments on mixtures made of fluids with finite particle concentrations. Two empirical descriptions (Eilers and Brinkman), reported in [5,6,1] for such data are also shown in Figure 2. At the Technical University of Denmark an experimental study has recently been made on the influence of coarse aggregates on the rheology of fresh concrete. The study is reported in [7] from which the results presented in Figures 3 and 4 are reproduced.
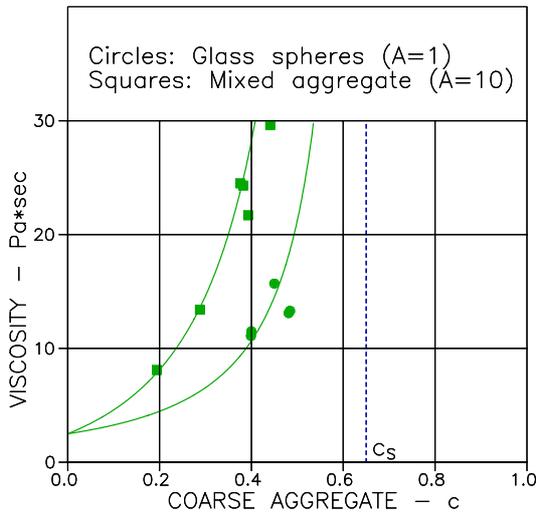
Figure 3. *Viscosity of concrete as related to volume fraction of coarse aggregates [7]. Solid lines are predicted with $M_V = 1$. Mortar viscosity is $\eta_S = 2.5$ Pa*sec. $c_S = 0.65$.*
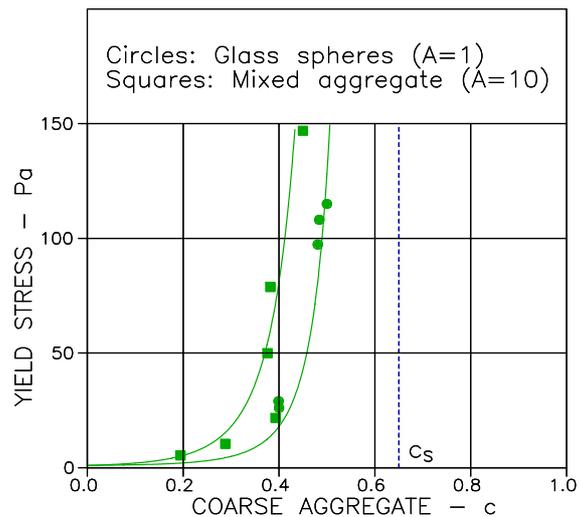
Figure 4. *Yield stress of concrete as related to volume fraction of coarse aggregates [7]. Solid lines are predicted with $M_S = 3.5$. Mortar yield stress is $S_S = 1$ Pa. $c_S = 0.65$.*

Conclusion

The well-known Bingham description of the rheology of homogeneous fluids has been generalised in this paper also to include the rheological description of composite fluids. The advantage of such generalisation is obvious: With a few parameters (shape factors, packing density c_S, and interaction power M) to describe the composite geometry, only two material properties (viscosity η_S, and yield stress S_S) of the fluid matrix are required to describe the generalised Bingham behaviour at any composition of the composite fluid considered. The traditional Bingham model needs experimental calibration for any new composition considered

References

1. Nielsen, L. Fuglsang: "Rheology of some fluid extreme composites – such as fresh self-compacting concrete", Nordic Concrete Research, 2(2001), 83 – 93.
2. deLarrard, F.: "Concrete Mixtures Proportioning: A Scientific Approach", Modern Concrete Technology Series, E&FN SPON, London 1999.
3. Bingham, E.C.: "An investigation of the laws of plastic flow", Bur. of Standards Bull., 13(1916), 309. 'Colloid types', Fifth Coll. Symp. Monograph, 1(1928), 219.
4. Einstein, A.: "Eine neue Bestimmung der Moleküldimensionen", An. Physik, 19(1906), 289 and 34(1911), 104.
5. Holliday, L. (ed): "Composite materials", Elsevier Publishing Company, New York, 1966, (p. 34, Chapter on Inclusions in a Viscous Matrix).
6. Eirich, R.F. (ed): "Rheology, Theory and Applications", Academic Press Inc., New York, 1958, (p. 363, Chapter on Rheological Properties of Asphalt).
7. Geiker, M.R., Brandl, M., Thrane, L. Nyholm, and Nielsen, L. Fuglsang: "Effect of coarse aggregate fraction and shape on the rheological properties of self-compacting concrete". Cement, Concrete, and Aggregates, Vol. 24, No. 1, June 2002.