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Spectra of superradiant lasers

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Abstract:
The emission spectrum of a superradiant laser is calculated analytically using quantum Langevin equations. New sideband peaks and fine structure in the spectrum are predicted for lasers with strong relaxation oscillations.

1. Introduction

Nowadays there is great interest in *superradiant* (SR) lasers, which combine an active medium with a large gain and a bad cavity [1], so that *collective* spontaneous emission into the lasing mode is significant. SR lasers have been realized with cold alkaline earth atoms [2], Rubidium atoms [3], and with quantum dots [4] as the active medium. SR lasers are less sensitive to cavity-length fluctuations, which is important for atomic clocks [2]. Superradiance leads to interesting collective effects, such as excitation trapping [3] and superthermal photon statistics [4].

An analytical description of SR lasers is complicated by the fact that their quantum noise is not a perturbation, the equations are nonlinear, and the polarization of the active medium cannot be adiabatically eliminated. We address these issues using an analytical approach in order to understand SR lasers and their spectra. We consider an example of single mode homogeneously broaden SR laser with active medium of $N_0 \gg 1$ two-level emitters.

2. Approach and basic equations

We describe the laser by operators $\hat{a}$ of lasing mode, $\hat{v}$ of active medium polarization and $\hat{N}_e = N_e + \delta\hat{N}_e$ of population of upper lasing level with mean value N_e and fluctuations $\delta\hat{N}_e$. We begin with well-known Maxwell-Bloch equations (MBE) for two-level laser, see for example, [5]. As a novel part of our approach, we rewrite MBE for *quadrature components* $\hat{\alpha} = 2^{-1/2}(\hat{\alpha}_x + i\hat{\alpha}_p)$, for field and polarization operators $\hat{\alpha} = \{\hat{a}, \hat{v}\}$ and then introduce *symmetric* (S) $\hat{\alpha}_+$ and *anti-symmetric* (A) $\hat{\alpha}_-$ parts of quadratures $\hat{\alpha}_x = \hat{\alpha}_+ + \hat{\alpha}_-$ and $\hat{\alpha}_p = \hat{\alpha}_- - \hat{\alpha}_+$. We linearize the MBE above the lasing threshold and show that linear MBE can be used also near and below threshold, if population fluctuations are small. Finally we derive the basic equations of our approach

$$\begin{aligned}
\dot{\hat{a}}_\pm &= -\kappa\hat{a}_\pm \pm \Omega_0\hat{v}_\pm + \hat{F}_{a_\pm} \\
\dot{\hat{v}}_+ &= -(\gamma_\perp/2)\hat{v}_+ + \Omega_0(\hat{a}_+ N + 2\sqrt{n}\delta\hat{N}_e) + \hat{F}_{v_+} \\
\dot{\hat{v}}_- &= -(\gamma_\perp/2)\hat{v}_- - \Omega_0\hat{a}_- N + \hat{F}_{v_-} \\
\delta\dot{\hat{N}}_e &= -2\sqrt{n}(\Omega_0\hat{v}_+ + \kappa\hat{a}_+) - \gamma_\parallel(P+1)\delta\hat{N}_e + \hat{F}_{N_e}.
\end{aligned}$$

Here κ, $\gamma_\perp$, and $\gamma_\parallel$ are, respectively, decay rate of lasing mode, polarization and population of upper lasing level; $\gamma_\parallel P$ is pump rate; Ω_0 is the vacuum Rabi frequency; n and N are mean photon number and population inversion, $\hat{F}_\alpha$, $\alpha = \{a_\pm, v_\pm, N_e\}$ are $\delta-$correlated Langevin forces $\langle \hat{F}_\alpha(t)\hat{F}_\beta(t')\rangle = 2D_{\alpha\beta}\delta(t-t')$, $\alpha, \beta = \{a_\pm, v_\pm, N_e\}$, with diffusion coefficients $2D_{a_\pm a_\pm} = \kappa/2$, $2D_{v_\pm v_\pm} = N_0\gamma_\perp/4$ and $2D_{N_eN_e} = (\gamma_\parallel/2)[P(N_0 - N) + N_0 + N]$. We show that equations for A-parts $\hat{a}_-$ and $\hat{v}_-$ describe phase fluctuations, equations for S-parts $\hat{a}_+$, $\hat{v}_+$ and for $\delta\hat{N}_e$ describe amplitude fluctuations of laser field and polarization.

The basic set of equations is linear and solved by Fourier-transform. We find Fourier-components $\hat{a}_\pm(\omega)$, spectra $n_\pm(\omega)$, $\langle \hat{a}_\pm(\omega)\hat{a}_\pm(\omega')\rangle = n_\pm(\omega)\delta(\omega+\omega')$ of ampliude and phase fluctuations, mean $n_\pm = (2\pi)^{-1}\int_{-\infty}^{\infty} n_\pm(\omega)d\omega$ and population inversion N determined from the energy conservation law $n = (\gamma_\parallel/4\kappa)[P(N_0 - N) - N_0 - N]$ with $n \equiv n_-(N) + n_+(N) - 1/2$.

The linewidth of the phase fluctuation spectrum $n_-(\omega)$ leads to the well-known laser linewidth above threshold, $\gamma_{las} = (1/2)[2\kappa\gamma_\perp/(2\kappa+\gamma_\perp)]^2 N_{\rm sp} h\omega_0/W_{out}$, $N_{sp} = (N_0+N)/2N_{th}$, $N_{th} = \kappa\gamma_\perp/2\Omega_0^2$; ω_0 is lasing frequency and W_{out} is the output power. Neglecting population fluctuations, we recover another well-known result for laser linewidth below threshold differing by a factor of 2 from the above-threshold expression. The field power spectrum is

$$n(\omega) = n_-(\omega) + n_+(\omega) - n_{+-}(\omega),$$

where $n_\pm(\omega)$ comes from commutation relations $\langle[\hat{a}_+(\omega), \hat{a}_-(\omega')]\rangle = n_{+-}(\omega)\delta(\omega+\omega')$.

3. Lasing spectra

We take parameters for photonic crystal lasers with quantum dots: wavelength of lasing transition $\lambda_0 = 1.55\ \mu$m, background refractive index $n_r = 3.3$, cavity mode volume $V_c = 10(\lambda_0/n_r)^3$ with $N_0 = 100$ emitters; population relaxation rate $\gamma_{\|} = 10^9$ 1/s; $\Omega_0 = (d/n_r)[\omega_0/(h\varepsilon_0 V_c)]^{1/2}$ with dipole momentum of lasing transition $d = 10^{-28}$ Cm so $\Omega_0 = 34\gamma_{\|}$; cavity quality factor $Q = 1.2 \cdot 10^4$ so $2\kappa = 100\gamma_{\|}$. Figure show spectra of SR laser (a,c) and non-SR laser (b,d) for large (a,b) and for weak (c,d) pump. For large P we found sideband peaks in the spectrum of SR

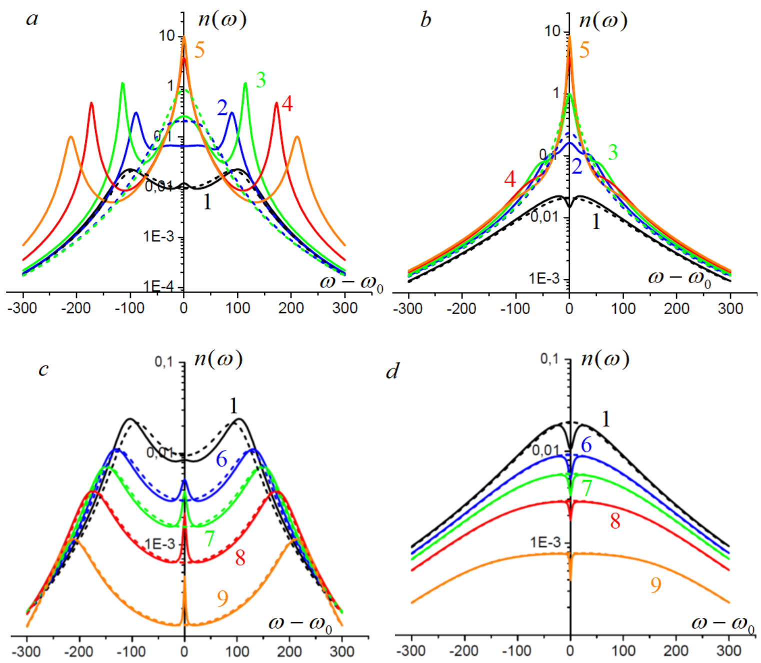

Fig. 1. Spectra of SR laser (a,c) with $\gamma_\perp = 5 \cdot 10^{10}$ 1/s and $2\kappa/\gamma_\perp = 2$ and non-SR laser (b,d) with $\gamma_\perp = 5 \cdot 10^{11}$ 1/s and $2\kappa/\gamma_\perp = 0.2$ for $P = 2$ (curves 1); 8 (2); 16 (3); 28 (4), 40 (5), 1.12 (6), 0.8 (7), 0.48 (8) and 0.16 (9). Dashed curves, found without population fluctuations, have no sideband peaks at large pump and structures in the center at small pump.

laser caused by large population fluctuations and relaxation oscillations. Such sideband peaks are not resolved in spectra of non-SR laser, where population fluctuations are weak. For small P we find a peak in the center of SR laser spectra exibiting collective Rabi spliting [5] and a dip for non-SR laser. This feature is due to interference in the linear and nonlinear (dependent on population fluctuations) parts of the polarization.

4. Conclusion

We present analytical quantum theory of spectra and fluctuations of superradiant (SR) and other lasers, above, near and below threshold retaining the polarization dynamics and collective effects. We reproduce well-known results for laser linewidth and predict new structures in the spectrum of SR lasers caused by population fluctuations.

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