Numerical analysis of an active magnetic regenerator with parallel wire geometry based on a 1D AMR Model

Zhang, Ruochen; Zhang, Xuexi; Qian, Mingfang; Bahl, Christian

Published in: International Journal of Refrigeration

Link to article, DOI: 10.1016/j.ijrefrig.2021.04.024

Publication date: 2021

Document Version Peer reviewed version

Numerical analysis of an active magnetic regenerator with parallel wire geometry based on a 1D AMR Model

Ruochen Zhang, Xuexi Zhang, Mingfang Qian, Christian R.H. Bahl

PII: S0140-7007(21)00149-3
DOI: https://doi.org/10.1016/j.ijrefrig.2021.04.024
Reference: JIJR 5118


Received date: 17 February 2021
Revised date: 31 March 2021
Accepted date: 17 April 2021

Please cite this article as: Ruochen Zhang, Xuexi Zhang, Mingfang Qian, Christian R.H. Bahl, Numerical analysis of an active magnetic regenerator with parallel wire geometry based on a 1D AMR Model, International Journal of Refrigeration (2021), doi: https://doi.org/10.1016/j.ijrefrig.2021.04.024

This is a PDF file of an article that has undergone enhancements after acceptance, such as the addition of a cover page and metadata, and formatting for readability, but it is not yet the definitive version of record. This version will undergo additional copyediting, typesetting and review before it is published in its final form, but we are providing this version to give early visibility of the article. Please note that, during the production process, errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

© 2021 Published by Elsevier Ltd.
Numerical analysis of an active magnetic regenerator with parallel wire geometry based on a 1D AMR Model

Ruochen Zhang¹, Xuexi Zhang¹*, Mingfang Qian¹, Christian R. H. Bahl²*

¹School of Materials Science and Engineering, Harbin Institute of Technology, Harbin 150001, China
²Department of Energy Conversion and Storage, Technical University of Denmark, Lyngby, Denmark

*Corresponding authors. Tel: 86-451-86415894, Fax: 86-451-8641392, E-mail: xxzhang@hit.edu.cn (XZ); Tel: 45-21325492, Fax: 45-45252525, E-mail: chrb@dtu.dk (CB).

Abstract

The numerical geometrical configuration and cooling performances of parallel wire geometry have been investigated in one-dimensional active magnetic regenerator. Based on the actual regenerator, the wires in geometries were primarily arranged in square and triangular array, and worked in cross and parallel flow patterns. Regenerators containing wire bundles with three geometries were evaluated by varying utilization at fixed temperature spans between hot and cold reservoirs. The effects of geometrical parameters on heat transfer performance and friction factor of regenerator were firstly discussed here. Two materials, gadolinium and La(Fe,Mn,Si)₁₃H₅, have been used for simulating the cooling performance of wire geometry. The numerical results showed that the arrangement of wires did not have influence on the cooling power greatly and the wire geometry could provide a higher coefficient of performance in parallel flow. After optimization, the desirable parallel wire geometry had a porosity of 0.40 and wire diameter of 0.15 mm. When the frequency was 2 Hz and temperature span was 10 K, the maximum cooling power could reach 158.40 W and 100.3 W for Gd and La(Fe,Mn,Si)₁₃H₅ wire geometries, whose coefficients of performance were as high as 13.46 and 14.96 respectively. Comparatively, the cooling powers of commonly used Gd and La(Fe,Mn,Si)₁₃H₅ packed bed were one third lower than that of optimized wire geometry and the coefficients of performance were lower half.
Keywords: magnetic refrigeration, active magnetic regenerator, modeling, parallel wire geometry, solid-state refrigeration

Nomenclature

Abbreviations

1D one-dimensional
2D two-dimensional
3D three-dimensional
AMR active magnetic regenerator
FOPT first-order phase transition
Gd gadolinium
MCE magnetocaloric effect
MCM magnetocaloric material
SOPT second-order phase transition

Variables

$\alpha_s$ specific surface area
$B$ magnetic field
$B_{\text{int}}$ internal magnetic field
$COP$ coefficient of performance
$COP_{\text{id}}$ ideal or reversible $COP$
$c_f$ specific heat capacity of heat transfer fluid
$c_H$ zero-field specific heat capacity of MCM
$D_h$ hydraulic diameter
$D_w$  wire diameter

$D_{sp}$  spherical diameters

$f_F$  friction factor

$G_z$  Graetz number

$H$  height of regenerator

$k$  thermal conductivity

$L$  length of regenerator

$M$  magnetization at each point

$m_s$  mass of MCM

$\dot{m}$  mass flow rate

$\dot{m}_{\text{max}}$  maximum mass flow rate

$N_D$  demagnetizing factor of MCM

$\text{Nu}$  Nusselt number

$\Delta P$  pressure drop

$\frac{\partial P}{\partial x}$  pressure drop per unit length

$Q_{\text{cool}}$  cooling power

$Q_{\text{heat}}$  heating power

$\text{Re}$  Reynolds number

$s$  specific entropy

$S$  distance between wires

$\Delta S_{\text{iso}}$  magnetic entropy change
$T$  temperature

$T_{\text{amb}}$  ambient temperature

$T_{\text{span}}$  temperature span

$T_{\text{hot}}$  hot-end temperature

$T_{\text{cold}}$  cold-end temperature

$T_{\text{curie}}$  Curie temperature

$T_{\text{span}}$  temperature span

$t$  time

$\tau$  cycle time

$U$  utilization,  $U = \frac{m_f c_f}{m_r c_H}$

$v$  operational frequency

$W$  width of regenerator

$x$  axial position

Greek letters

$\varepsilon$  porosity

$\rho$  density

$\eta_{2\text{nd}}$  second law (Carnot) efficiency

Subscripts

$f$  fluid

$s$  solid MCM

$r$  regenerator
1. Introduction

Over the years, the magnetic cooling technology has moved closer to be viable in residential and industrial applications, especially in refrigeration area. Magnetic cooling technology has gained attention from environmental and efficiency perspectives, since it can mitigate the emissions of greenhouse gases and provide a similar theoretical cooling service at a higher efficiency (more than 60% of the Carnot efficiency) at the same time (Engelbrecht et al., 2007; Gschneidner and Pecharsky, 2008; Zimm et al., 1998). However, the best energy efficiency is only 40% of the Carnot efficiency for vapor compression (Lyubina, 2017). The technology utilizes a solid refrigerant and water-based heat transfer fluids with anticorrosion inhibitor, which are more environmentally friendly than the alternatives used in conventional vapor compression.

Magnetic cooling technology is based on the exploitation of magnetocaloric effect (MCE), which occurs in magnetocaloric material (MCM) under a varying magnetic field. The main part of magnetic cooling device is active magnetic regenerator (AMR), which executes the energy conversion function during the thermodynamic cycles. It consists essentially of MCM, working with other accessories controlling the device in a magnetic field source (Lyubina, 2017; Navickaitė et al., 2018). Permanent magnets are preferred as magnetic source. Multiple designs can be found in the review study in (Bjørk. et al., 2010; Li. et al., 2021). The heat transfer fluid is pushed back and forth by a pump with control valves so that the flow can be regulated. The MCM in the regenerator bed is shaped into a porous structure to allow the fluid flowing through it. Generally, a good regenerator geometry should ensure a sufficient heat transfer between fluid and porous solid MCM, a low pressure drop and low axial conduction. Until now, the well-studied geometries include packed bed (Lei et al., 2016; Li et al., 2019; Monfared, 2018; Navickaitė et al., 2018), parallel plates (Bahl et al., 2017; Engelbrecht et al., 2011; Li et al., 2019; Tušek et al., 2014), a bulk material with microchannel (Moore et al., 2013; Navickaitė et al., 2019) and packed screens (Lei et al., 2016; Li et al., 2019; Monfared, 2018; Navickaitė et al., 2018).
2014). Although these geometries are easily constructed and can provide a good heat transfer performance, they still have unavoidable drawbacks that may deteriorate their cooling performances greatly. The packed bed exhibits a relatively large pressure drop and viscous dissipation, so the regenerator efficiency is reduced greatly (Lei et al., 2017). The pressure drop of parallel plate structure is much lower than those of packed bed, while a comparative heat transfer performance is hard to achieve due to the limitations in fabricating small channels (Nielsen et al., 2013). Notwithstanding the microchannel structure can provide a large efficiency theoretically, the difficulties of fabrication and corrosion, including the maldistribution of flow in channels still exist (Lei et al., 2017). Compared with the former structures, the channels in packed screen bed are more structured, which can provide an equivalent heat transfer performance to packed bed but significantly lower the pressure drop (Lei et al., 2017). However, the main problem of this structure is the difficulty in processing MCMs like gadolinium (Gd), LaFeSi or ceramics into screens because of their hardness and poor ductility. Therefore, it is quite essential to design a new geometry for MCMs in regenerator, which can relieve these problems effectively and display a relatively large cooling performance.

Accordingly, parallel wire structure is a geometry that can possibly meet these requirements. Above all, a wire with large draw ratio can provide a large specific surface area and thus a strong heat transfer performance that is similar to particles. Secondly, the pressure drop and viscous dissipation of parallel wire geometry is lower than that of packed bed when the porosity and MCM diameters are same (Vuarnoz and Kawanami, 2014). Thirdly, the maldistribution situation can be avoided to a great extent (Bergman et al., 2011). Fourthly, MCMs like Gd- and LaFeSi-based alloys have previously been fabricated into wires successfully (Dong et al., 2014; Qin et al., 2013; Zhang et al., 2019), which implies the configuration of parallel wire geometry is reasonable. Finally, it has been confirmed that the small-sized material, like wire, can solve the issue of large hysteresis in material exhibiting first-order phase transition (FOPT), such as LaFeSi-based alloys, and hence avoid the hysteresis loss during cycling work (Dong et al., 2014).
However, the building of parallel wire geometry and its cooling performance in AMR are relatively unexplored, especially for the FOPT material like LaFeSi-based alloy. Until now, only the cooling performance of Gd wires have been simulated in AMR model, but the porosity of regenerator used for simulation is only 0.1, which is possibly too small for the actual work (Vuarnoz and Kawanami, 2012). According to the experiments of Nam and Jeong (2006), the minimum porosity of stacked wires is around 0.4. So in this paper we revise the porosity of wires to be 0.4 to make the simulation results more approachable to the practical results. In addition, compared with traditional packed bed and parallel plates, the difficulties of simulating wire geometry are not only considering all the possible arrangements of wires, but also the two flow patterns that wires may work in. As different arrangements may attribute to different pressure drop and different flow patterns may cause different magnetic field inside regenerator as well as heat transfer capacity (Drummond and Tahir, 1984), so it is a big challenge to consider all these factors altogether in the simulation work of parallel wire geometry. However, two arrangements which possibly exists in stack wires and various flow patterns were not considered in previous simulation (Vuarnoz and Kawanami, 2012), so the final simulation results might not be complete and reasonable enough. Therefore, more detailed studies of wire geometry should be executed.

Here, we not only numerically analyzed the flow and heat transfer properties of parallel wire geometries with two arrangements in two flow patterns, but also analyzed the effect of geometrical parameters on these properties. Moreover, the cooling performances of wire geometries in devices are simulated by building a one-dimensional (1D) AMR model and the geometrical parameters are optimized. Considering the effect of different magnetic transition characters and MCE on cooling performance, materials exhibiting second-order phase transition (SOPT) (Gd) and FOPT (La(Fe,Mn,Si)\textsubscript{13}H\textsubscript{y}) are used for simulation. Gd exhibits a small adiabatic temperature range, magnetic entropy change and specific heat capacity but an extremely wide transition temperature span. Inversely, La(Fe,Mn,Si)\textsubscript{13}H\textsubscript{y} shows a large adiabatic temperature range, magnetic entropy change and specific heat capacity while a narrow transition temperature range. Furthermore, combining with the present fabrication ability of wires, the diameters of wires chosen here are 0.05 mm, 0.15 mm, 0.3 mm, and the
Porosities of regenerators are 0.4. Besides, all the performances and properties are compared with two commonly used packed beds. Based on the practical application, the porosities of packed beds used here are 0.4, the particle diameters are 0.3 mm and 0.5 mm, respectively.

2. AMR Geometries

The performance of regenerator is strongly dependent on the arrangement of wires and the flow direction of heat transfer fluid. According to the previous research (Happel, 1959; Kirsch and Fuchs, 1967; Žukauskas, 1972), two geometrical configurations, square and triangular array, possibly exist in stack wires. For simplifying the analysis, here the triangular array are assumed as equilateral. The square and equilateral triangular arrays have been specifically discussed here, as presented in Fig. 1 (a) and (b), respectively. In addition, considering the actual working environment in regenerator, two flow directions, i.e. cross and parallel flow, have been chosen here and also been shown in Fig. 1 (a) and (b). Fig. 2 (a) and (b) illustrates the 3D diagram of the regenerator cassette which works with magnetic field source in parallel and cross flow patterns, respectively. They have revealed the different relative locations between the wires and field source in two flow patterns, which can make great effects on the cooling performance of wire geometry. The effects will be discussed in detail in next section. The size of cassette has also been presented in Fig. 2. It is based on the cassettes of MagQueen, a magnetocaloric heat pump developed in DTU Energy (Johra et al., 2019).

![Geometrical configuration of packed wires with two arrangements working in two flow patterns](image)

Fig. 1 Geometrical configuration of packed wires with two arrangements working in two flow patterns (cross flow and parallel flow). (a) Square array (b) Triangular array
Fig. 2 Sketch of the relative locations between the modeled regenerator cassette containing parallel wires and magnetic field source in two flow patterns. (a) Parallel flow (b) Cross flow

The geometrical parameters, including the $\varepsilon$, $D_n$ and $S$, of the parallel wire geometries used for simulation have been presented in Table 1. Three sets of geometrical parameters have been applied and the corresponding $S$ for square and triangular array have been calculated by Eq. (1) and Eq. (2), respectively. Besides, to describe the geometries of parallel wire structures more precisely, the $D_h$ and $a_s$ have also been calculated by Eq. (3) and Eq. (4) (Nam and Jeong, 2006), as presented in Table 1. From these results, it is revealed that both $S$ and $D_h$ increase with increasing $D_w$. However, $a_s$ shows an inverse trend. For contrast, the cooling performances of commonly used packed beds are also simulated. It is assumed that all spheres in packed bed are of the same size for simulation purposes. Therefore, $D_{sp}$ and $\varepsilon$ are sufficient to characterize the packed bed. Two sets of geometrical parameters of packed beds used here are $\varepsilon = 0.4$, $D_{sp} = 0.3\text{mm}$ and $\varepsilon = 0.4$, $D_{sp} = 0.5\text{mm}$, whose $D_h$ and $a_s$ are calculated by Eq. (S1) and Eq. (S2) in Supplementary Material.

$$S \text{ (square)} = D_n \sqrt{\frac{\pi}{4 (1 - \varepsilon)}}$$  \hspace{1cm} (1)

$$S \text{ (triangular)} = D_n \sqrt{\frac{3 \pi}{6 (1 - \varepsilon)}}$$  \hspace{1cm} (2)

$$D_h = D_n \times \frac{\varepsilon}{(1 - \varepsilon)}$$  \hspace{1cm} (3)
\[ a_s = \frac{4(1 - \varepsilon)}{D_u} \]  

(4)

Table 1 Geometrical parameters for characterizing different AMR geometries

<table>
<thead>
<tr>
<th>Geometry</th>
<th>( \varepsilon )</th>
<th>( D_u (D_{wp}) ), mm</th>
<th>( S ) (Square array), mm</th>
<th>( S ) (Triangular array), mm</th>
<th>( D_h ), mm</th>
<th>( a_s ), mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel wires</td>
<td>0.4</td>
<td>0.05</td>
<td>0.062</td>
<td>0.033</td>
<td>48.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.30</td>
<td>0.17</td>
<td>0.18</td>
<td>0.10</td>
<td>16.00</td>
<td></td>
</tr>
<tr>
<td>Packed bed</td>
<td>0.4</td>
<td>0.30</td>
<td>-</td>
<td>0.13</td>
<td>12.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.50</td>
<td>-</td>
<td>-</td>
<td>0.22</td>
<td>7.20</td>
<td></td>
</tr>
</tbody>
</table>

As discussed later, a 1D AMR model is applied for simulating the cooling performances of parallel wire geometry. Thus, some suitable parameters and equations need to be used to characterize the physical phenomena during work. For example, the \( \text{Nu} \) is usually used to describe the heat transfer in fluid, and the \( f_F \) is often used to describe the viscous dissipation.

All the equations used for calculating \( \text{Nu} \) and \( f_F \) have been described in detail in **Supplementary material**. The calculating results will be briefly summarized here.

The calculated \( \text{Nu} vs. \text{Re} \) curves for all parallel wire geometries and packed beds are shown in Fig. S2. One can see that the wires in parallel flow provide a much higher \( \text{Nu} \) than those in cross flow for all diameters, suggesting a better heat transfer performance of wires in parallel flow. Besides, the triangular array shows a larger \( \text{Nu} \) in both flow patterns, implying that triangular array can offer a higher heat transfer than square array. Moreover, the \( \text{Nu} \) of two packed beds are between those of all wire geometries in cross flow and parallel flow, which means the heat transfer performance of the commonly used packed bed is in the middle level.

The \( f_F \) is an important parameter to reflect the viscous dissipation during cyclic work. It is worthy to note that we make an assumption that the \( f_F \) calculated here is only related to the geometrical parameters, while the roughness of wire and wettability between wire and fluid are not considered here. After calculation, the dependence of \( f_F \) on \( \text{Re} \) for all parallel wire and
packed beds geometries have been shown in Fig. S3. Generally, the $f_F$ of the triangular array is larger than that of the square array both in cross and parallel flow, which is consistent with the previous results (Bergman et al., 2011). Besides, the wire geometries in parallel flow show extremely low $f_F$ compared with those in cross flow and are thus predicted to have a smaller viscous dissipation. The reason of smaller $f_F$ and $\Delta P$ in parallel flow is related to the less turbulence which can occur in the fluid when the fluid flowing around wires, which has been explained in detail in Supplementary material. Moreover, from the figures, the $f_F$ of wire geometries with $D_w$ of 0.15 mm and 0.3 mm are found to be smaller than those of packed beds, which implies these wire geometries may produce less viscous dissipation and thus higher efficiency during work when the diameters of wires and particles are similar. However, the $f_F$ of wire geometries are larger than those of packed beds when $D_w = 0.05$ mm, which is possibly attributed to the big difference of the diameter between these wires and packed bed.

Since a good regenerator is supposed to provide a high heat transfer and low friction loss, therefore the ratio between Nu and $f_F$ (Nu / $f_F$) vs. Re curves have been presented in Fig. S4. All the wire geometries exhibit the best Nu / $f_F$ ratio in parallel flow and square array shows a larger Nu / $f_F$ ratio than that triangular array due to the smaller $f_F$. Compared with the wire geometries, the Nu / $f_F$ ratio of the packed bed with $\varepsilon = 0.4$, $D_{sp} = 0.3$ mm is much lower than those of all wire geometries. The reason is possibly ascribed to the higher $f_F$ of packed bed with $\varepsilon = 0.4$, $D_{sp} = 0.3$ mm in flow. Nevertheless, the packed bed with $\varepsilon = 0.4$, $D_{sp} = 0.5$ mm shows a higher Nu / $f_F$ value than all wire geometries in cross flow. It is possibly because of its higher Nu making up its high $f_F$. Overall, from the perspective of flow pattern, the parallel flow can offer a higher heat transfer performance and lower viscous dissipation than cross flow. Similarly, when the wires are arranged in square array, the heat transfer capacity will be higher and the viscous dissipation will be smaller.

3. One-dimensional AMR model

Until now, many approaches have been reported in literature to build AMR models (Nielsen et al., 2011), including 1D (Aprea et al., 2012; Tušek et al., 2011), two-dimensional (2D) (Aprea et al., 2015; Guo et al., 2019; Liu and Yu, 2011) and three-dimensional (3D)
(Bouchard et al., 2009) models. No matter which approach is used, all AMR models must
calculate the heat transfer in the solid MCM matrix, and the MCE generated by varying
magnetic field which is coupled with the convective heat transfer in the fluid. Therefore, the
governing equations are given in Eq. (5) and (6) (Lei, 2016) and the equations for calculating
$k_{stat}$ and $k_{disp}$, i.e. Eq. (S15) and Eq. (S16) are described in Supplementary material.

$$
\frac{\partial}{\partial x} \left( k_{stat} A_s \frac{\partial T_s}{\partial x} \right) + \frac{Nuk_f}{D_h} a_s A_s (T_f - T_s) = A_s (1 - \varepsilon) \rho_f \left[ \frac{\partial T_s}{\partial t} + T_s \left( \frac{\partial x}{\partial B_{mz}} \right) \frac{\partial B_{mz}}{\partial t} \right] 
$$

$$
\frac{\partial}{\partial x} \left( k_{disp} A_s \frac{\partial T_f}{\partial x} \right) - m c_f \frac{\partial T_f}{\partial x} - \frac{Nuk_f}{D_h} a_s A_s (T_f - T_s) + \frac{\partial \rho_f}{\partial t} = A_s \varepsilon \rho_f \frac{\partial T_f}{\partial t} 
$$

The full description of model, including the applied modifications and the discretization
of governing equations have been given in Lei, (2016). The first term on the left side of Eq.
(5) describes the thermal conduction through the AMR bed and the second term expresses the
heat transfer between the fluid and MCM. The energy storage and magnetic work of MCM
are defined by the term on right side. As for Eq. (6), the first term on the left side represent
the thermal conduction through the fluid, the second term is enthalpy flow, the third term is
the heat transfer between the fluid and MCM, and the fourth term is the viscous dissipation.
Correspondingly, the term on the right side is the energy storage.

Furthermore, the 1D AMR equations were developed based on the following
assumptions: (1) The MCM is distributed evenly in AMR bed; (2) No flow leakage or bypass
situation occurs; (3) The MCM displays a uniform magnetization and demagnetization; (4)
Fluid flows into the AMR at a uniform bulk temperature and a velocity profile across the
cross section of an AMR; (5) The fluid is incompressible; (6) There is no phase change in the
fluid; (7) The radiation heat transfer is negligible.

The number of space and time nodes used here are 40 and 4000, respectively. It was
discovered that the accuracy of the numerical solution remains unvaried when these two
values increased by 25 % and 50 %. When the numerical tolerance of $10^{-5}$ was reached and
the number of iterations of 500 was exceeded, the model was considered to reach
convergence. For simulation, the frequency is chosen to be 2 Hz. The applied $B$ and
$m / m_{max}$
are shown in Fig. 3, where the positive $\dot{m} / \dot{m}_{\text{max}}$ means the hot-to-cold blow and vice versa. The MCM used for simulation of the single-layer AMR are Gd and La(Fe,Mn,Si)$_{13}$H$_y$, exhibiting SOPT and FOPT respectively. The aim is to study the cooling performance of wire geometries fabricated by two commonly used kinds of materials.

![Fig. 3 Applied magnetic field $B$ and nominal mass flow rate $\dot{m} / \dot{m}_{\text{max}}$ as functions of nominal time $t/\tau$.](image)

4. Results and discussion

4.1 1D AMR packed with Gd wires

The material properties and boundary conditions used for numerical analysis of a 1D AMR with Gd parallel wire geometry have been presented in Table 2. These conditions are all based on the actual working conditions of magnetic refrigerator, so that the simulating results are more close to the actual cooling performance. Besides, the $\Delta S_{\text{iso}} - T$ and $c_H - T$ curves of Gd are shown in Fig. S5 (a) and (b), respectively. For a clearer understanding and optimization of the performance of regenerators, the numerical results are presented in plots with $COP$ as a function of $Q_{\text{cool}}$, as presented in Fig. 4 (a) - (c) and Fig. 5 (a) - (c). It is discovered that two values of $COP$ can usually be obtained for one $Q_{\text{cool}}$ value. The reason has been explicated in Navickaitė et al. (2019) and will not be explained in detail here. For most cooling applications, a high $COP$ at a desired $Q_{\text{cool}}$ is usually expected (Navickaitė et al., 2019). Therefore, the variation of system performance is needed, which is realized by varying $\dot{m}$, $v$ and thus $U$. 
Table 2 Material properties and boundary conditions for simulating 1D AMR packed with Gd wires

<table>
<thead>
<tr>
<th>Parameters, Symbol (Unit)</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat transfer fluid</td>
<td>Aqueous solution with 20vol.% ethylene glycol</td>
</tr>
<tr>
<td>Material</td>
<td>Gd</td>
</tr>
<tr>
<td>$T_{\text{amb}}$ (K)</td>
<td>300</td>
</tr>
<tr>
<td>$T_{\text{span}}$ (K)</td>
<td>10</td>
</tr>
<tr>
<td>$T_{\text{hot}}$ (K)</td>
<td>300</td>
</tr>
<tr>
<td>$T_{\text{cold}}$ (K)</td>
<td>290</td>
</tr>
<tr>
<td>$T_{\text{curie}}$ (K)</td>
<td>292</td>
</tr>
<tr>
<td>$v$ (Hz)</td>
<td>0.5-2</td>
</tr>
<tr>
<td>$\tau$ (s)</td>
<td>0.5-2</td>
</tr>
<tr>
<td>Number of bed</td>
<td>1</td>
</tr>
<tr>
<td>$U$</td>
<td>0.2-1.5</td>
</tr>
<tr>
<td>$B$ (T)</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Normally, $COP$ is a parameter that evaluates the performance of a cooling cycle and is mostly accepted in industry. While for the scientific community, $\eta_{2\text{nd}}$ is more common, as it allows to take the irreversibility of the process into consideration and evaluate the performance of system simultaneously. The $COP$ can be calculated as shown in Eq. (7), where $Q_{\text{heat}} - Q_{\text{cool}}$ is the total of pump work ($W_{\text{pump}}$) and magnetic work ($W_{\text{mag}}$). The $COP_{\text{id}}$ is calculated by Eq. (8). Finally, the $\eta_{2\text{nd}}$ is the ratio between the actual $COP$ and $COP_{\text{id}}$, as given in Eq. (9).

$$COP = \frac{Q_{\text{cool}}}{Q_{\text{heat}} - Q_{\text{cool}}}$$  \hspace{1cm} (7)

$$COP_{\text{id}} = \frac{T_{\text{cold}}}{T_{\text{hot}} - T_{\text{cold}}}$$  \hspace{1cm} (8)
\[ \eta_{2n} = \frac{COP}{COP_{\text{ref}}} \] \hfill (9)

The dependences of COP and \( \eta_{2nd} \) on \( Q_{\text{cool}} \) for all Gd wire geometries by varying \( \dot{m} \) are presented in Fig. 4 (a) - (c). The summaries of maximum \( Q_{\text{cool}} \), including the corresponding COP, \( \eta_{2nd} \), \( U \), and \( \Delta P \), are listed in Table S1. As the cooling performance of wire geometry is dependent on the MCE generated during cycling work, so the \( B_{\text{int}} \) of MCM is an important parameter that should be considered here. \( B_{\text{int}} \) at each point of MCM can be calculated by \( B_{\text{int}} \) (Chikazumi and Charap, 1964; Lipsø et al., 2011; Nielsen et al., 2012). As we know, the flow pattern of wire geometry has a great effect on \( N_D \), namely \( N_D \) is negligible when the direction of magnetic field is along the axial direction of wires (cross flow pattern, as shown in Fig. 2 (b)) while is 0.5 when the direction of magnetic field is along the diameter direction of wires (parallel flow pattern, as shown in Fig. 2 (a)). As a result, \( B_{\text{int}} \) can be influenced by the arrangement of wires significantly, so the average ratio between \( B_{\text{int}} \) and \( B \) for each regenerator is also listed in Table S1.
Fig. 4 COP as a function of $Q_{\text{cool}}$ for Gd wire geometries with porosities of 0.4 and wire diameters of 0.05 mm, 0.15 mm, 0.3 mm compared with Gd packed beds with porosities of 0.4 and particle diameters of 0.3 mm and 0.5 mm.

According to Fig. 4 (a) - (c), the maximum $Q_{\text{cool}}$ of square and triangular array are similar in both flow patterns in all geometries, as Table S1 shows, suggesting the different arrangements and flow patterns do not make too much effect on the $Q_{\text{cool}}$ for these geometries.
The reason can be deducted as below: for the wires in cross flow, although the $B_{int}$ is 1.3 times larger than that in parallel flow, the $\Delta P$ in parallel flow is several times smaller than that in cross flow conversely. Therefore, the positive and negative effects of both flow patterns can be “neutralized” to some degree and their $Q_{cool}$ are similar. However, the wire geometry in parallel flow shows a larger $COP$ ($\eta_{2nd}$) for both arrangements due to the extremely lower $\Delta P$. For example, for the wire geometry with $\varepsilon = 0.4$, $D_w = 0.15$ mm, the maximum $Q_{cool}$ of square array in cross flow is 164.81 W, which is quite similar to that in parallel flow (158.40 W). However, the corresponding $COP$ ($\eta_{2nd}$) is only 7.99 (27.55) in cross flow, which is much smaller than that in parallel flow, 13.46 (46.41). The same situation can also be found in staggered array and other geometries. Therefore, parallel flow is a more desirable flow pattern where the wire geometry can exhibit a high $Q_{cool}$ and $COP$ ($\eta_{2nd}$) at the same time. Similarly, when wire geometry works in cross flow, the $COP$ ($\eta_{2nd}$) of the square array is larger than that of the triangular array for all geometries for the same reason. Nevertheless, when the parallel wires work in parallel flow, the $COP$ ($\eta_{2nd}$) of the square and triangular array exhibit few difference, which may be related to the similar heat transfer and viscous loss between square and triangular array when working in the parallel flow pattern. Besides, the values of $COP$ can also be reflected by pump work ($W_{pump}$) and magnetic work ($W_{mag}$) when the regenerator is working, the comparison of $W_{pump}$ and $W_{mag}$ corresponding to the maximum $Q_{cool}$ among the Gd wire geometries have been presented in Fig. S6 (a) – (c). From the figures, one can see that $W_{pump}$ decreases with increasing wire diameter, which means that the electric power needed to drive the fluid becomes lower. It is possibly related to the fact that the turbulent flow around wires becomes more difficult with larger wire diameter. So, the resistance that the fluid meet during flow motion gets smaller and thus $W_{pump}$ goes down. Similarly, the $W_{pump}$ of square array is smaller than that of staggered array for all geometries and $W_{pump}$ of all wire geometries in parallel flow are smaller than those in cross flow. Thereby, a higher $COP$ can be obtained in wire geometries with square array and in parallel flow pattern. Comparatively, the $W_{mag}$ are basically the same for all geometries in parallel flow; however, $W_{mag}$ of wire geometries in cross flow shows increasing trend with decreasing wire diameter. It implies that a wire geometry with smaller wire diameter may impose extra burden on the motor. Similarly, the $W_{mag}$ of square array is
smaller than that of triangular array in cross flow, which is possibly also correlated to the higher COP of square array.

Furthermore, the maximum $Q_{cool}$ decreases with the increasing $D_w$, which may be correlated to the weakened heat transfer performance due to the reduced $a_s$, as Table 1 shows. Simultaneously, $\Delta P$ also shows a decreasing trend with the increasing $D_w$. It is reasonable because $S$ also becomes larger and hence it will be much easier for fluid flowing through the wires. Furthermore, the COP ($\eta_{2nd}$) corresponding to maximum $Q_{cool}$ primarily exhibits an increasing and then a decreasing trend in both flow patterns with an increase of $D_w$, and reaches a maximum at $D_w = 0.15$ mm. For example, the COP value corresponding to maximum $Q_{cool}$ of square array in parallel flow increases from 11.35 ($D_w = 0.05$ mm) to 13.46 ($D_w = 0.15$ mm), and then decreases to 12.74 ($D_w = 0.3$ mm). Additionally, a high maximum $Q_{cool}$ can also be obtained in parallel wires with $\varepsilon = 0.4$, $D_w = 0.15$ mm, which is as high as 158.40 W and 161.30 W for square and triangular arrays, respectively. Besides, this wire geometry display a modest $W_{pump}$ and $W_{mag}$, as shown in Fig. S6 (b). Therefore, it suggests that $\varepsilon = 0.4$, $D_w = 0.15$ mm are optimized geometrical parameters of parallel wire geometry. In contrast, the packed bed only shows a maximum $Q_{cool}$ of 107.50 W for $\varepsilon = 0.4$, $D_{sp} = 0.3$ mm and 70.03 W for $\varepsilon = 0.4$, $D_{sp} = 0.5$ mm. The corresponding COP are only 5.09 and 7.64, respectively. Therefore, wire geometry displays a better cooling performance than packed bed.

Finally, another major factor reflecting the motion of fluid that can make great effects on the cooling performance of wire geometry is the frequency. The above studies are executed by varying the mass flow to control the content of fluid that was pushed into the regenerator during the cycle, but the speed at which the fluid flow through the regenerator also decides the heat transfer ability and cooling performance of wire geometry. Therefore, the heat transfer abilities and cooling performances of all wire geometries at a frequency of 0.5 Hz and 1 Hz are also studied here. At first, the heat transfer abilities of Gd wire geometries at different frequencies have been studied through the number of thermal transfer units ($NTU$), which is calculated by Eq. (S17). The results of all wire geometries at working frequencies of 0.5 Hz, 1 Hz, 2 Hz have been illustrated in Fig. S7 – S9, respectively. From the figures, one
can see that the $NTU$ displays an increasing trend when the frequency goes down. Since a lower frequency will cause a longer heat transfer time between fluid and wires, so the heat transfer between fluid and wires is more sufficient and thus $NTU$ is higher. Then the cooling performances of Gd wire geometries whose parameters are $\varepsilon = 0.4$, $D_w = 0.05$ mm, 0.15 mm, 0.3 mm at a working frequency of 0.5 Hz, 1 Hz have been presented in Fig. S10 and Fig. S11, respectively. Compared with the performance of all wire geometries at a working frequency of 2 Hz (see Fig. 4), the $Q_{cool}$ shows a downtrend with a decreasing frequency, the reason of which is quite related to the decreased number of cycles at a lower frequency. Although the heat transfer between fluid and wires is more sufficient at a low frequency, the fluid is pushed back and forward for less times and thus less heat can be transferred. Therefore, the $Q_{cool}$ becomes smaller at a lower frequency. However, the $COP$ barely varies with frequency because the pump work is actually related to the frequency: a higher frequency implies that the pump does more work to push the fluid back and forward more quickly. Namely, the pump power is higher when the frequency is high. Therefore, $COP$ does not vary with the frequency basically.

4.2 1D AMR packed with LaFeMnSiH wires

Another material used for numerical simulation is La(Fe,Mn,Si)$_{13}$H$_y$ exhibiting FOPT. In contrast with Gd, the main distinctions of La(Fe,Mn,Si)$_{13}$H$_y$ are the narrower transition temperature range, higher $\Delta S_{iso}$ and larger $c_H$ peak, which all have effects on the cooling performance of parallel wire regenerator. The properties and boundary conditions of La(Fe,Mn,Si)$_{13}$H$_y$ used for simulation have been shown in Table 3. Unlike the Gd material whose transition temperature range is over 30 K, the transition temperature range of La(Fe,Mn,Si)$_{13}$H$_y$ is less than 10 K as shown in Fig. S5 (a) and (b) and the temperature range will shift with the increasing magnetic field (Piazzi et al., 2016). Therefore, $T_{hot}$ is an important parameter that decides the cooling performance. After a series of tentative simulation (as presented in Fig. S11 (a) - (d)), it is found that the optimized $T_{hot}$ is 304 K. The dependence of $COP$ and $\eta_{2nd}$ on $Q_{cool}$ for all La(Fe,Mn,Si)$_{13}$H$_y$ wire regenerators have been presented in Fig. 5 (a) - (c) and the values of maximum $Q_{cool}$ and the corresponding $COP$,
η_{2nd}, ΔP, average B_{int}/B, U are summarized in Table S2. The correlations of maximum $Q_{cool}$, and the corresponding COP, η_{2nd}, ΔP, U with geometries and flow patterns in La(Fe,Mn,Sn)_{13}H_y wire regenerators are similar to those in Gd wire regenerators, which will not be repeated here. After all, the most desirable geometrical parameters are still $ε = 0.4$, $D_w = 0.15$ mm, whose $Q_{cool}$ and COP are both larger than those of packed bed. The maximum $Q_{cool}$ of La(Fe,Mn,Sn)_{13}H_y wire geometry with $ε = 0.4$, $D_w = 0.15$ mm in parallel flow is 100.30 W and the corresponding COP is 14.96 when the wires are arranged in triangular array. In contrast, the maximum $Q_{cool}$ of La(Fe,Mn,Sn)_{13}H_y packed bed is only 57.08 W ($ε = 0.4$, $D_{sp} = 0.3$ mm) and 31.43 W ($ε = 0.4$, $D_{sp} = 0.5$ mm). Nevertheless, compared with Gd parallel wire regenerator, the maximum $Q_{cool}$ and COP of La(Fe,Mn,Sn)_{13}H_y parallel wire regenerator are slightly lower. It may be attributed to the narrower phase transition temperature range of La(Fe,Mn,Sn)_{13}H_y and the shifting of Δ$S_{iso}$ peak with increasing magnetic field.

Table 3 Material properties and boundary conditions for simulating 1D AMR packed with La(Fe,Mn,Sn)_{13}H_y wires

<table>
<thead>
<tr>
<th>Parameters, Symbol (Unit)</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat transfer fluid</td>
<td>Aqueous solution with 20vol.% ethylene glycol</td>
</tr>
<tr>
<td>Material</td>
<td>La(Fe,Mn,Sn)_{13}H_y</td>
</tr>
<tr>
<td>$T_{amb}$ (K)</td>
<td>300</td>
</tr>
<tr>
<td>$T_{span}$ (K)</td>
<td>10</td>
</tr>
<tr>
<td>$T_{hot}$ (K)</td>
<td>304</td>
</tr>
<tr>
<td>$T_{cold}$ (K)</td>
<td>294</td>
</tr>
<tr>
<td>$T_{curie}$ (K)</td>
<td>297</td>
</tr>
<tr>
<td>v (Hz)</td>
<td>2</td>
</tr>
<tr>
<td>τ (s)</td>
<td>0.5</td>
</tr>
<tr>
<td>Number of bed</td>
<td>1</td>
</tr>
<tr>
<td>U</td>
<td>0.2-1.5</td>
</tr>
<tr>
<td>$B$(T)</td>
<td>1.4</td>
</tr>
</tbody>
</table>
Fig. 5 COP as a function of $Q_{cool}$ for La(Fe,Mn,Si)$_{13}$H$_y$ wire geometries with porosities of 0.4 and wire diameters of 0.05 mm, 0.15 mm, 0.3 mm compared with La(Fe,Mn,Si)$_{13}$H$_y$ packed beds with porosities of 0.4 and particle diameters of 0.3 mm and 0.5 mm.
5. Conclusion

The heat transfer capability, viscous dissipation and cooling performance of parallel wire geometries with square and triangular array, when working in cross and parallel flow, have been studied in this paper. The wire geometry exhibited the strongest heat transfer performance when working in parallel flow while this ability was weakest when the wires worked in cross flow. The $f_F$ of the wire geometry in cross flow was much larger than those in parallel flow, and the triangular array exhibited a larger $f_F$ than the square array, especially in cross flow.

Two materials, Gd and La(Fe,Mn, Si)$_{13}$H$_x$, have been used for numerical analysis. For both materials, the difference in the maximum $Q_{cool}$ between square and triangular array was small while the COP of square array was higher than that of triangular array. In addition, the $Q_{cool}$ of wire geometry in two flow patterns were similar but the COP in parallel flow was higher than that in cross flow. Moreover, although the heat transfer ability of wire geometry was enhanced by reducing the frequency but the $Q_{cool}$ still went down. The optimized wire geometry was with a porosity of 0.4 and wire diameter of 0.15 mm. Compared with parallel wires, the packed beds both showed a medium heat transfer performance and high $f_F$. As a result, the cooling performance of packed bed was not superior when compared with the optimized wire geometries.

In summary, the parallel wire geometry exhibits a high heat transfer performance, small viscous dissipation and excellent cooling performance especially in parallel flow. Consequently, the parallel wire geometry is predicted to be a promising geometry in a real cooling device and the optimized geometrical parameters can provide useful instructions to fabricate parallel wires regenerators. In order to validate the numerical results, an experimental investigation on AMRs with parallel wires flow structure is pending.

Declaration of interest statement

We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work, there is no professional or other personal
interest of any nature or kind in any product, service and/or company that could be construed as influencing the position presented in, or the review of, the manuscript entitled.

Acknowledgements

RCZ thanks China Scholarship Council for supporting her study at Technical University of Denmark. XXZ acknowledges funding from Heilongjiang Touyan Team. MFQ acknowledges funding from National Natural Science Foundation of China (NSFC, grant No. 51701052). CB acknowledges funding from Independent Research Fund Denmark – Technologies and Productions Sciences (grant No. 7017-00034B).

References


Lei, T., 2016. Modeling of active magnetic regenerators and experimental investigation of passive regenerators with oscillating flow, Department of Energy Conversion and Storage. Technical University of Denmark, Risø, Denmark.


Piazzi, M., Bennati, C., Curcio, C., Kuepferling, M., Basso, V., 2016. Modeling specific heat and entropy change in La(Fe,Mn,Si)\textsubscript{13}H compounds. Journal of Magnetism and Magnetic Materials 400, 349-355.


Table and figure captions

Fig. 1 Geometrical configuration of packed wires with two arrangements working in two flow patterns (cross flow and parallel flow). (a) Square array (b) Triangular array

Fig. 2 Sketch of the relative locations between the modeled regenerator cassette containing parallel wires and magnetic field source in two flow patterns. (a) Parallel flow (b) Cross flow

Fig. 3 Applied magnetic field $B$ and nominal mass flow rate $m_{\text{max}}/m_{\text{av}}$ as functions of nominal time $t/\tau$

Fig. 4 $COP$ as a function of $Q_{\text{cool}}$ for Gd wire geometries with porosities of 0.4 and wire diameters of 0.05 mm, 0.15 mm, 0.3 mm compared with Gd packed beds with porosities of 0.4 and particle diameters of 0.3 mm and 0.5 mm.

Fig. 5 $COP$ as a function of $Q_{\text{cool}}$ for La(Fe,Mn,Si)$_{13}H_y$ wire geometries with porosities of 0.4 and wire diameters of 0.05 mm, 0.15 mm, 0.3 mm compared with La(Fe,Mn,Si)$_{13}H_y$ packed beds with porosities of 0.4 and particle diameters of 0.3 mm and 0.5 mm.

Table 1 Geometrical parameters for characterizing different AMR geometries

Table 2 Material properties and boundary conditions for simulating 1D AMR packed with Gd wires

Table 3 Material properties and boundary conditions for simulating 1D AMR packed with La(Fe,Mn,Si)$_{13}H_y$ wires