An improved tensorial implementation of the incremental harmonic balance method for frequency-domain stability analysis

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Abstract. A tensorial implementation of the incremental harmonic balance method is improved to achieve an accurate stability analysis in the frequency domain. The improved implementation enables an accurate stability analysis with a small number of harmonics and therefore reduces the computational expense. The effectiveness of this improved implementation is demonstrated by a nonlinear finite element model of a clamped-clamped beam. This improved implementation may be also relevant in other implementations of the nonlinear harmonic balance methods.

Introduction

Recent years have witnessed a renewed academic interest in frequency-domain stability analysis of time-periodic response of dynamical systems [1, 2, 3]. Frequency-domain stability analysis is based on the Hill method, which combines the Floquet theory and the Fourier series expansion. One problem in frequency-domain stability analysis is linked to spurious eigenvalues. In [1], Lazarus and Thomas proposed a criterion to sort the most converged eigenvalues and used it in a framework where the harmonic-balance method (HBM) is combined with the asymptotic numerical method (ANM) for path-following or continuation of the response. However, even with a proper sorting criterion, Soykov and Ribeiro reported in [2] that an accurate frequency-domain stability analysis can require more harmonics than an accurate response analysis. This unaddressed problem has motivated the study in this work.

Results and discussion

This study is performed in a framework that combines the incremental harmonic balance (IHB) method and nonlinear finite element (FE) model, see e.g. [4]. The tensorial implementation of the IHB method was studied in [5, 6] for efficient structural optimization of nonlinear resonances and modes. This study applies the tensorial implementation to the stability analysis. Here a three-dimensional tensor is improved to accurately compute the frequency-domain counterpart of the tangent stiffness matrix. Fig. 1 displays a visualization of this improved three-dimensional tensor and the forced resonance of a doubly clamped micro-beam resonator [5]. The proposed improvement can reduce the number of harmonics required for an accurate stability analysis.

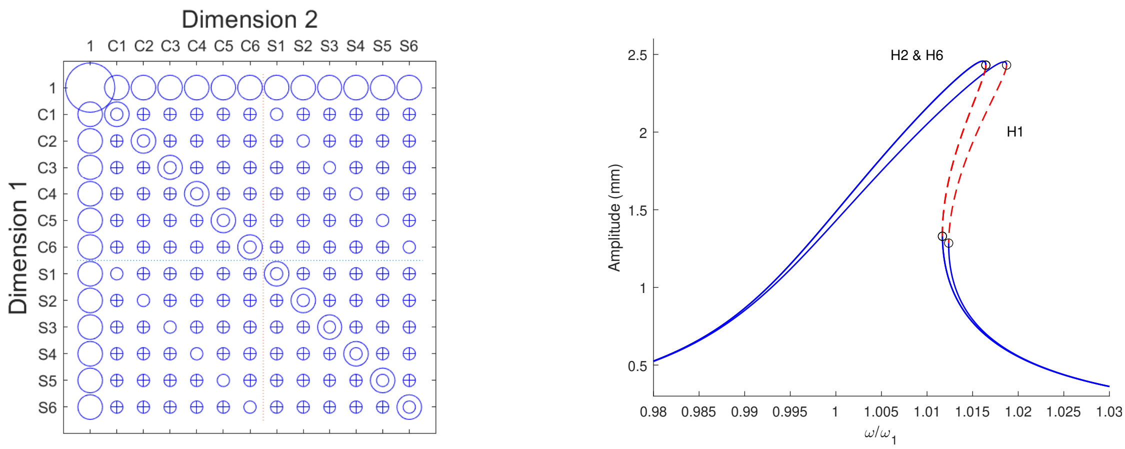

Figure 1: Left: visualization of the improved three-dimensional tensor corresponding to the tangent stiffness matrix. A circle or cross indicates there is a non-zero value along the third dimension. The size of the circle or cross is proportional to the magnitude of the value. $\mathtt{C}n$ and $\mathtt{S}n$ denote the n^{th} order cosine and sine function. Right: Nonlinear frequency response of a nonlinear finite element model of a clamped-clamped beam. The solid and the dashed line indicate stable and unstable response, respectively. The circles indicate the locations of bifurcations. The text $\mathtt{H}n$ indicates the number of harmonics.

References

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