TAILORING NONLINEAR DYNAMICS OF MICROBEAM RESONATORS WITH ELECTROSTATIC ACTUATION

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Summary We have developed an efficient shape optimization procedure for tailoring the nonlinear dynamic performance of microbeam resonators with electrostatic excitation. By relatively simple modifications of the cross-sectional dimensions along the length of the beam we demonstrate a considerable degree of control over the characteristic cubic nonlinear coefficient, allowing e.g. for extending the linear range of operation for resonant MEMS devices, which is important for improving signal-to-noise and other performance metrics.

INTRODUCTION

Recently, we demonstrated in a numerical study [1] the possibility to significantly alter the nonlinear dynamic behavior of beams and frames by employing shape optimization of the cross-sectional beam dimensions based on direct finite element calculations of the cubic nonlinear coefficient. This was later supported by experimental observations for microbeams for MEMS applications [2]. Both an increase of more than a factor 3 and a decrease of almost a factor 3 in the characteristic cubic nonlinear coefficient was demonstrated for beams with optimized height profiles.

In the present paper the design procedure is extended to cover electrostatic excitation which is of major relevance for MEMS applications. Furthermore, it introduces an additional nonlinearity in the problem due to the electrostatic force that depends on the transverse displacements. In addition to the added complexity in modelling, the extra nonlinearity offers increased possibility for tailoring the behavior since two nonlinear effects counteract: the structural hardening nonlinearity (due to midplane stretching) and the electrostatic nonlinearity which is softening. Examples of shape optimization of electrostatically actuated microbeams have recently appeared (eg. [3]), however, with focus on control of pull-in voltage and stability. To the authors' best knowledge we here present the first study on structural optimization with the aim to control the nonlinear dynamics of electrostatically actuated microbeams.

COMPUTATIONAL MODEL

The basis for the optimization procedure is a model of a clamped-clamped beam including nonlinear effects from midplane stretching and from electrostatic actuation caused by a symmetric set of fixed actuators located at distance d from the undeformed beam midplane (Figure 1 left). The basic beam equation with a variable beam height $h = h(x)$ reads:

$$\rho h b \ddot{w} + \frac{Eb}{12}(h^3 w'')'' - \frac{Eb}{2L} w'' \int_0^L h(w')^2 dx = \frac{\varepsilon b V_{\mathrm{DC}}^2}{2}\left(\frac{1}{(d-w-\frac{h}{2})^2} - \frac{1}{(d+w-\frac{h}{2})^2}\right) \tag{1}$$

here, w is the transverse beam displacement, ρ is the mass density, E is Young's modulus, b is the out-of-plane thickness, L is the beam length, ε is the permittivity of air and V_{DC} is the constant DC voltage applied to both fixed electrodes (the time-dependent actuation part is here omitted). The r.h.s. represents a simplification of the electrostatic actuation force but has

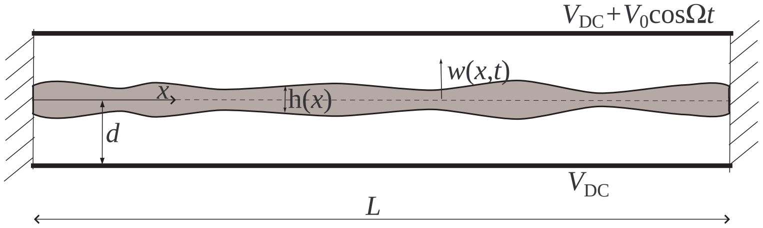

Figure 1: Left: Microbeam with variable height $h(x)$ with two fixed electrodes on each side at the nominal distance of d from the beam centerline. Deflections of the beam are given as $w(x, t)$. Right: Example of optimized beam profile.

*Corresponding author. Email: json@elektro.dtu.dk recently been experimentally supported for a similar configuration [5]. The model is discretized using Bernoulii-Euler beam finite elements as outlined in [1].

As the first step in the analysis the corresponding linear FE eigenvalue problem is solved and based on the fundamental linear mode $\mathbf{\Phi}_1$ we directly compute the cubic nonlinear coefficient using normal forms [1, 4] as

$$\alpha = \sum_{e=1}^{ne} \frac{Ebh_e}{8l} (\mathbf{\Phi}_1^T \mathbf{K}_g \mathbf{\Phi}_1)^2 - \sum_{e=1}^{ne} \frac{\varepsilon V_{dc}^2 b}{d_e^5} \int_0^l (\mathbf{N}^T \mathbf{\Phi}_1)^4 dx \tag{2}$$

where $\mathbf{N}^T$ is the shape function matrix and $\mathbf{K}_g$ the geometric stiffness matrix.

PARAMETRIZATION AND OPTIMIZATION PROCEDURE

We choose the height of each beam element to be our design variables, i.e. we set

$$h_{\min} < h_e < h_{\min} \tag{3}$$

where the minimum and maximum values of the beam height are chosen from fabrication tolerance (min) and minimum allowable distance to the electrodes (max).

The optimization problem is now defined as a minimization problem wrt. the cubic nonlinear coefficient:

$$\begin{aligned} \min_{\mathbf{h}} : \quad & \alpha(h_e) \\ \text{s.t.} \quad & \omega_1(h_e) \leq \omega^* \\ & f(h_e, \mathbf{\Phi}_1(h_e)) \geq f^* \end{aligned} \tag{4}$$

where we have introduced constraints in form of the maximum value of the first eigenfrequency ω^* and on the minimum value of the modal excitation force f^*. To solve the optimization problem we apply an iterative gradient-based approach with analytically computed sensitivities and use the robust mathematical programming tool MMA [6] to obtain design updates.

RESULTS

A preliminary beam design is shown in Figure 1 (right). The initial configuration has a softening nonlinearity due to the dominating electrostatic effect. However, by modifying the shape of the beam the magnitude of the nonlinear coefficient has been reduced by a factor of approximately 5. At the same time both the fundamental eigenfrequency as well as the modal excitation force is unchanged compared to the initial straight beam.

CONCLUSION

We have demonstrated that it is possible, by simple variations of the beam cross-sectional geometry along the beam, to significantly reduce out the effective nonlinearity while keeping the actuation level and linear dynamic unchanged. The results shows promising perspectives for enhancing the linear operation range for electrostatically actuated microbeams and also more complex microsized structures and devices. Further work include modifying also the shape of the electrostatic actuators as well as experimental validation of the results.

References

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