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## Towards an Energy-based Indicator of Track Quality in Turnouts

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**Abstract:** This paper investigates track vibration energy as a potential novel indicator of turnout's track quality. Exploiting measurements of train-induced track vertical accelerations at different sections of a turnout, the track vibration energy is estimated and its variation over time analysed through the creation of statistical empirical distributions. A clear increase in vibration energy can be observed over a period of two years. An analysis of the turnout track geometry through a standard indicator adopted by the railway industry is then performed, and an increase in longitudinal level over the same investigation period clearly indicates track degradation due to cumulative loading. Last, a correlation analysis is performed between the estimated vibration energy and the indicator of track quality based on geometry data. Such analysis shows a significant correlation between the two indexes, thereby addressing the possibility of developing a novel condition monitoring tool for track quality based on track vibration energy. The whole investigation is based on full-scale measurements of track vertical acceleration and track geometry performed over a period of two years in a turnout of the Danish railway infrastructure.

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*Keywords:* Track quality estimation, Data fusion, Statistical methods for FDI, Time series modeling, Condition monitoring.

### 1. INTRODUCTION

Railways are surging among other transport modes thanks to the very low emissions per passenger per kilometer travelled. This growing interest has generated major investments across Europe to modernize the overall railway infrastructure, transitioning towards high-speed railway connectivity. Railway infrastructure managers are challenged by both pushing development of new lines as well as maximizing the capacity of the existing ones, which is paramount to guarantee profitability as well as customer satisfaction. The latter demands continuous effort to secure high reliability of the network, which is currently achieved through preventive maintenance of the assets. Turnouts are a critical element in railway networks towards the maximization of capacity, since they enable flexible train operations. Hence their dependability concerns railway infrastructure managers, which devote a significant share of the yearly operation and maintenance (O&M) budget for the inspection, repair and renewal of turnouts. The Danish railway infrastructure manager, Banedanmark, estimates that each year one third of the total track O&M cost is due to turnouts (Juul Andersen, 2012).

Railway asset monitoring is currently performed based on the periodic evaluation of track geometry parameters, which are measured by a track geometry recording car. The European standard EN 13848-6 (CEN, 2014) defines the geometry parameters to be monitored as well as the safety limits for planning of maintenance actions. For instance, tamping actions performed to pack the ballast layer below the sleepers, thereby restoring track stiffness, are

scheduled based on the evaluation of standard deviation of the track longitudinal level, which determine the track quality class (Mishra et al., 2017; Nielsen and Li, 2018). Track geometry measurements campaigns are carried out quarterly or more seldom depending on the specific track utilization. The slow development of track degradation and the large cost associated with deploying the measurement vehicle are the main drivers to determine the frequency of the measurement campaigns. However, track quality degradation occurs non-uniformly and it progresses at various rates in different sites due to several factors such as actual track quality, cumulative loading, maintenance actions, subsoil and weather conditions. Increasing the frequency of the measurements campaigns may provide additional insight into degradation process, yet this seems unfeasible with current methods due to associated cost.

The research effort has focused on the development of indirect approaches to evaluate track quality based on measuring or estimating track stiffness, which is largely considered a key parameter for understanding, modelling and monitoring track degradation. The rolling stiffness measurement vehicle (Berggren, 2009), the track loading vehicle (Thompson and Li, 2002) and the moving rail car (Norman et al., 2006) are specialized measurement vehicles to measure track stiffness. Track-side measurement systems have been recently adopted to explore the potential of train induced track vibrations for the development of continuous track monitoring. Le Pen et al. (2016) combined Fourier analysis with a mathematical model of a beam on an elastic foundation to estimate

track modulus from measurements of sleepers' motion. Barkhordari and Galeazzi (2018) developed an algorithm for the estimation of the first and second track resonance frequencies from measurements of track vertical acceleration. Asadzadeh and Galeazzi (2019b) devised a method to predict track degradation building a statistical mapping between changes in track geometry and features of the power spectral density of track vertical acceleration.

Another key parameter influencing track degradation is track damping, which determines the ability of dissipating the vibration energy induced by train passages. Track damping is the result of the interaction and quality of different components, such as ballast, sleepers, and subgrade, as well as the presence of elastic elements at the interface between components. Reduced track damping resulting from fouled ballast, degradation of elastic components, and variation in subgrade stiffness results in high dynamic stresses in the substructure (Sasaoka, 2006; Navaratnaram, 2017). Monitoring changes of track damping may offer additional insight into track degradation, and an opportunity to predict more accurately degradation rates, which are paramount to facilitate the transition towards predictive maintenance strategies.

### 1.1 Main contribution

Exploiting continuous measurements of track vertical acceleration over a period of two years at multiple locations of a turnout located in the Danish railway network, as well as data from six track geometry measurement campaigns occurred within the same period, the paper investigates track vibration energy as novel indicator of track quality. In particular, changes in levels of vibration energy are interpreted as the result of variations in the damping characteristics of the track. The correlation analysis between a standard indicator of track quality computed from geometry data and the vibration energy shows that a significant correlation exists between these two indexes. Therefore, it is concluded that the vibration energy induced by the train passage may be utilized as an indicator of track quality.

## 2. DESIGN OF EXPERIMENT

### 2.1 Data collection and data type

Two types of data are utilized for this study: the measured track vertical acceleration induced by the train passage and the track geometry data. Both type of data are available for a period of two years, namely from August 2016 to August 2018, however their time resolution differs since acceleration measurements were collected daily while geometry data were gathered quarterly.

The track vertical acceleration is measured through a distributed sensor network deployed on a turnout of the Danish railway infrastructure. Twelve 2-axis accelerometers measuring along the vertical and transverse directions with a measurement range of  $\pm 500g$  have been installed at the locations shown in the schematics of Fig. 1 covering both the straight and diverging tracks. The sensors are connected to a data acquisition system where signals are conditioned and data temporary stored. Wheel detectors located at the entry points of the turnout are employed to

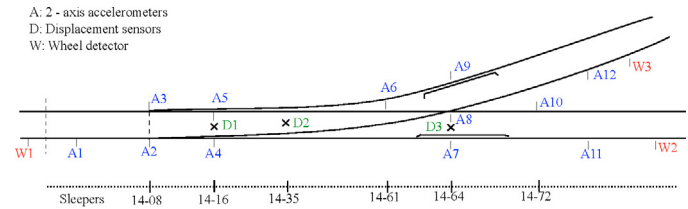


Fig. 1. Layout of the sensors location along the turnout at Tommerup station (Fyn - Denmark).



(a) Overview of the turnout close to Tommerup station.

(b) Accelerometer magnetically connected to the rail web.

Fig. 2. Pictures of the instrumented S&C at Tommerup station (Fyn - Denmark).

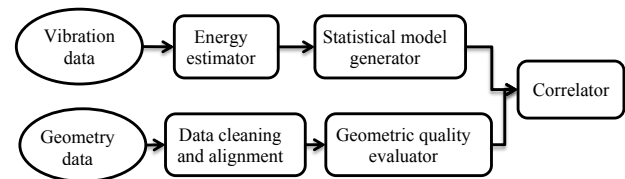


Fig. 3. Architecture for data analysis.

awake the data collection system whenever a train passes through. Data are collected with a sampling frequency  $F_s = 20$  kHz. Figure 2 shows an overview of the turnout and sensor installation.

The data utilized in this study are the vertical accelerations measured by accelerometer A1 near the insulation joints and 6 meters before the tip of switch blade, accelerometer A2 on the switch panel, accelerometer A7 on the tip of crossing, and accelerometer A11 located 13 meters after the tip of crossing.

The track geometry data are collected by the Danish railway infrastructure manager Banedanmark using the universal measuring vehicle UFM120. Parameters commonly measured are the longitudinal level, horizontal alignment, twist, gauge, and cant. Measurements are performed every 25 cm along the track.

### 2.2 Proposed methodology for data analysis

To investigate the track vibration energy as potential measure of track quality, the architecture sketched in Fig. 3 is proposed.

In the top branch each observation of vertical acceleration is first processed to compute an estimate of the associated vibration energy. The dynamic track response to train passage excitation is related to both train-specific characteristics as well as track-specific characteristics. Train-specific characteristics are wheels type and their quality,

train speed, wheel set static load, wheel's unsprung mass, etc. Track-specific characteristics are geometric deviations from the original design, stiffness and damping characteristics of superstructure and substructure components. As the focus of the study is on evaluating track quality, an effort is made to filter out the effects of vehicle-related characteristics from the estimation of track vibration energy. Therefore, a homogeneous set of trains running at a speed in the range 150-165 km/h is considered. These trains are of the same type (IC3), and have the same number of wheel sets (5 wheel sets) to minimize the effect of static loads and unsprung masses. We define this set as

$$\mathcal{T} \triangleq \{T \mid 150 \leq v_T \leq 165 \text{ km/h} \wedge t_T = \text{IC3} \wedge w_T = 5\} \quad (1)$$

where for a given train  $T$ ,  $v_T$  is the train speed,  $t_T$  is the train type, and  $w_T$  is the number of wheel sets.

Nonetheless, small variations in speed, static load, and wheel quality as well as the stochastic nature of vehicle/track interaction cause the estimated energy to vary in a limited range. Therefore, estimates generated over a fix time interval are utilized to create a statistical model in the form of an empirical distribution. A bundle of distributions is then produced for the period of investigation, which should convey information about changes in vibration energy due to wear and tear of the turnout at a specific measurement location. This process is applied to the selected four locations along the S&C.

In the bottom branch multiple observations of track geometry data are first spatially aligned to ensure that the subsequent track quality analysis precisely locates infrastructure degradation along the turnout. The aligned time series of longitudinal level are then processed to compute a standard geometric quality index, which carry information about track degradation due to cumulative loading at different locations along the turnout.

Last, a statistical measure of each empirical distribution is correlated with the track quality index to verify if track vibration energy can be utilized as novel indicator of track quality to monitor degradation of the infrastructure.

### 3. TRACK ENERGY ESTIMATION AND STATISTICAL MODELLING

When the railway track has a greater vibration decay rate, more vibration energy is absorbed and less vibrations are transferred to the substructure, the ballast and the soil (Haladin et al., 2016). The dissipation of vibration energy has two sources: elastic superstructure components including rail pads and under-sleeper pads, and substructure components including ballast and the soil (Thompson, 2008). Elastic and plastic deformations are the consequence of the energy transferred to the sleepers and then to the ballast and the sub-ballast layers.

In this study, the vibration energy of the track when excited by passing trains is estimated from rail acceleration measurements. The frequency response of the elastic superstructure components lies in the range 500-1000 Hz and the response of ballast and sub-ballast ranges between 25-250 Hz (Barkhordari et al., 2017; Johansson et al., 2008; Lam and Wong, 2011; Kaewunruen and Remennikov, 2007; Asadzadeh and Galeazzi, 2019b). Accordingly, the first step to estimate vertical vibration energy is to

filter the rail acceleration measurements in the frequency band [10, 1000] Hz by means of a third order Butterworth filter.

Let  $\mathbf{x} = [x_1, x_2, \dots, x_N]^T$  be the filtered vertical acceleration of the track at a given measurement location along the turnout associated with a train passage. Then the mean vibration energy is calculated by

$$\bar{e}(\mathbf{x}) = \frac{1}{N} \sum_{n=1}^N |x_n|^2 \quad (2)$$

Let  $P$  be the length of a fixed time interval (e.g. a month) and  $M_T = |\mathcal{T}|_P$  the cardinality of the set  $\mathcal{T}$  over the time period  $P$ . Then the statistical model of the vibration energy for the time interval  $P$  is defined as

$$\mathcal{E}_P \triangleq \{\bar{e}_1(\mathbf{x}), \bar{e}_2(\mathbf{x}), \dots, \bar{e}_{M_T}(\mathbf{x})\}. \quad (3)$$

Figure 4 shows the Normal probability plots of the energy estimations in six time intervals during the period August 2016 to August 2018. Each data point corresponds to the average of vibration energy recordings for all the trains passing the turnout on the same day. The time intervals have been selected such that both the rail acceleration data and track geometry recordings are available. The length  $P$  of the time intervals is 3-4 weeks, and each period includes one geometry campaign date at its center. In the studied turnout, a track maintenance event (ballast tamping) has occurred in late September 2016. Such tamping event has determined some changes in the mean track vibration energy along the whole turnout; however the magnitude and direction of change are different at different locations based on the track condition prior to the tamping.

At location A1 (Fig. 4a) after tamping (November 2016), the shape of the empirical distribution of the energy estimates differs from the one before tamping, since the heavy tail on the right is no longer present. In other words, the 90% quantile of the mean vibration energy is reduced from 1 to  $0.15g^2$ . At location A2 (Fig. 4b) an effect similar to A1 is observed, although the magnitude of the change is smaller. At locations A7 and A11 (Fig. 4c-d), the effect of tamping is opposite to the former two locations: the 90% quantile of the mean vibration energy raises from  $1 - 2g^2$  to  $1.7 - 4g^2$ . These different outcomes can be due to temporal effects of tamping and the track condition before tamping. On one hand, with increasing track settlement and compaction of the ballast layer, the ability to absorb energy in the substructure decreases resulting in an increase of the track vibration energy. In this case, tamping, by lifting the track and reshuffling the ballast layer, may partly restore track flexibility and its capability to absorb energy. On the other hand, in the case of unsupported sleepers due to ballast void, the entire superstructure bends and deflects according to its bending stiffness. In this case, the substructure loses its ability to absorb energy and the vibration energy is more visible in high frequencies (between 1500 – 2500 Hz) related to rail vibrations. In this case, tamping, by filling in the voids and providing full ballast support may restore the contribution of the substructure to absorbing energy, which results in an increase of track vibration energy in the frequency range 10 – 1000 Hz.

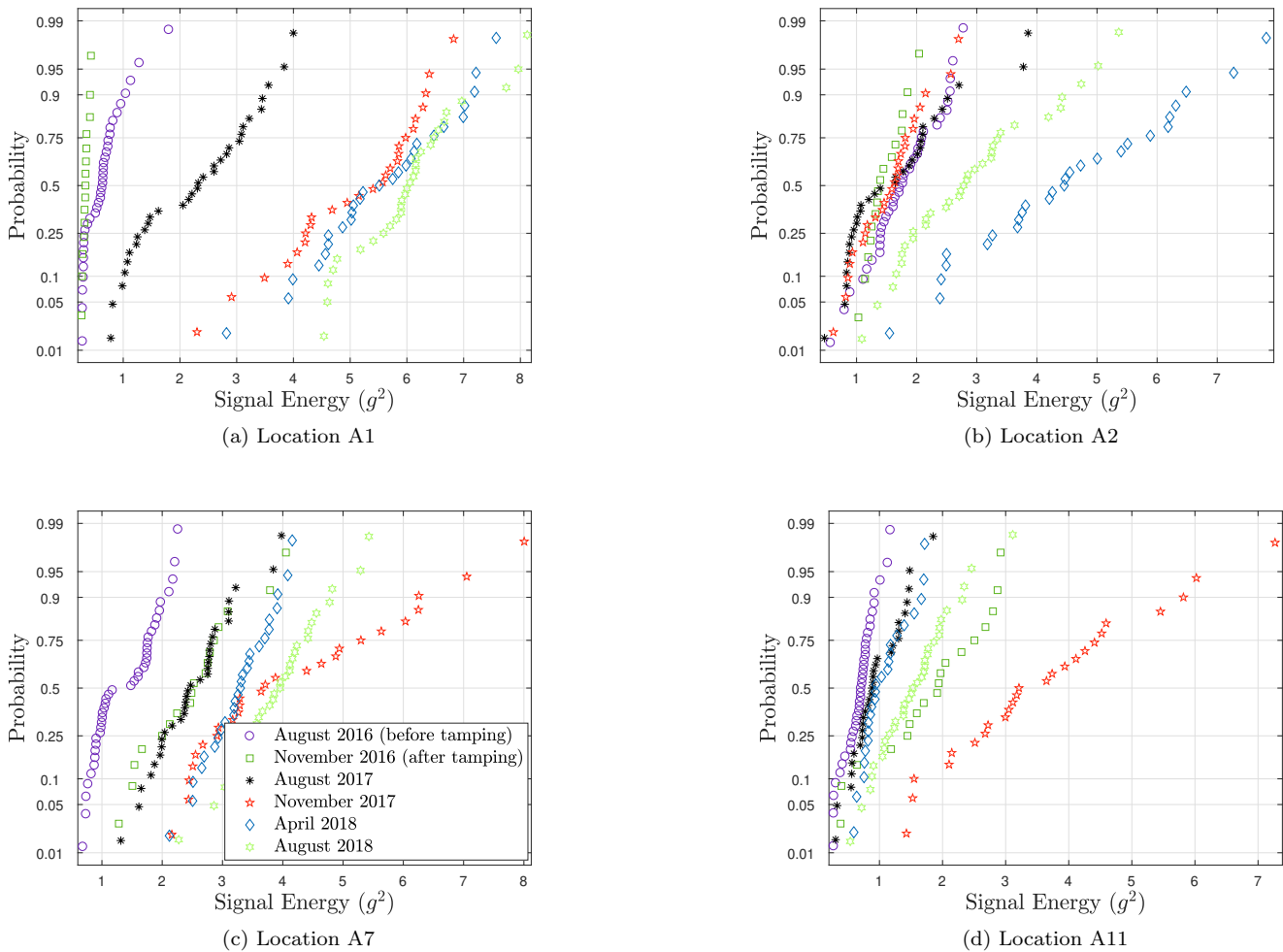


Fig. 4. The mean vibration energy estimates for six time periods along the turnout.

One year after tamping, the track vibration energy is quite different from the time interval right after tamping. In general, as seen in Fig. 4, the distribution of track vibration energy is shifting toward higher values as more accumulated tonnage is operated from August 2017 to August 2018. However, the distribution of track vibration energy during November 2017 is quite different at locations A7 and A11. These distributions with a heavy tail on the right show that for some days in this month quite high track vibration energy is recorded, and the track was not able to damp the vibrations. Meteorological records from a weather station nearby the turnout show an average temperature of 5.2 degrees Celsius with several freezing hours in November 2017 as well as a high level of precipitation. Cold weather inducing frost and ice in the ballast can determine changes in the stiffness characteristics of the track Gonzales et al. (2013), which may result in a reduced capability to absorb energy.

As the study aims at assessing if track vibration energy can be used as an indicator of track quality, the effects of temperature and precipitation should be filtered out from the estimated energy to prevent factors other than track degradation biasing the subsequent correlation analysis with the track geometry indicators. However, the current study accepts these effects on track vibration energy as

exogenous random effects and establish the correlation in the face of such uncertainties. Future research may focus on incorporation of temperature and precipitation effects into track vibration energy modelling.

#### 4. STANDARD GEOMETRIC QUALITY INDICATOR

The quality of the railway track can be assessed with respect to its geometric condition. The most frequently used geometric parameters are longitudinal level (vertical profile), horizontal alignment, twist and gauge (Anderson, 2002; Berawi et al., 2010; Asadzadeh and Galeazzi, 2019a). The standard deviation of these parameters or their deviations from some acceptable levels are used to develop a quantified track quality indicator. Each geometric parameter can be more indicative of the quality of a particular track component than other parameters. In this sense, the track longitudinal level is more correlated with ballast differential settlement (Vale and Ribeiro, 2017), and its fractal analysis may be used to evaluate ballast condition (Landgraf and Hansmann, 2018).

The evaluation of track geometric quality in turnouts based on automated measurements require special considerations including accurate alignment of geometry measurements, correction of spatial misalignment, and selec-



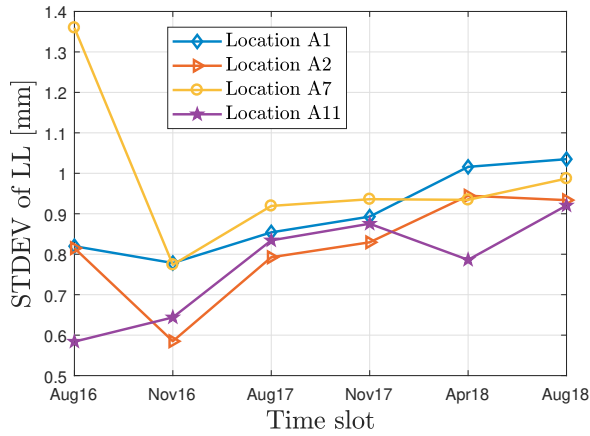


Fig. 5. The geometric track quality evaluated by the standard deviation of the track longitudinal level.

tion of the length of the section for which geometry quality is calculated. For more details on how to address these issues, interested readers are referred to (Asadzadeh and Galeazzi, 2019a). It should be noted that in this study the aligned and corrected geometry data by (Asadzadeh and Galeazzi, 2019a) is used.

To evaluate geometric quality at the location where each of the accelerometers are mounted, the standard deviation of the longitudinal level (STDEV-LL) signal in 10m neighbourhood of the sensor location is considered. This is to ensure that the geometry signal corresponding to one location has minimum overlap with geometry signal corresponding to other locations.

The results of track geometry evaluation are presented in Fig. 5. The geometric quality indicator is computed as the average of the standard deviation of the longitudinal variation for the left and right rails, at all locations A1, A2, A7, and A11, and in six time slots. The effect of tamping is observable by the decrease of the geometric index in November 2016 compared to August 2016. Afterward, a general increasing trend of the geometric index shows gradual deterioration of track geometry by accumulated tonnage.

## 5. ENERGY AS INDICATOR OF TRACK QUALITY

A correlation analysis between the vertical track vibration energy and the track quality assessed by STDEV-LL is carried out. As the vibration energy data are represented in the form of empirical probability distributions, a 5% quantile of the distribution is selected to be correlated with the geometric quality indexes. The low quantile is selected to reflect mostly the change in vibration energy due to variations in track geometry. In fact, the upper quantiles of the energy distribution are more likely to be affected by train-specific parameters (e.g. speed, wheel quality, suspension stiffness, etc.), which for an equal track geometry give rise to larger vertical accelerations. Moreover, a comparison of the correlation between STDEV-LL and different quantiles of the energy distribution is presented in Table 1. As seen, the lower quantile shows the maximum correlation with the track geometry index.

Table 1. Correlation between STDEV-LL and different quantiles of the energy distribution.

Quantile [%]	5	25	50	75	95
Correlation [%]	77	72	71	70	72

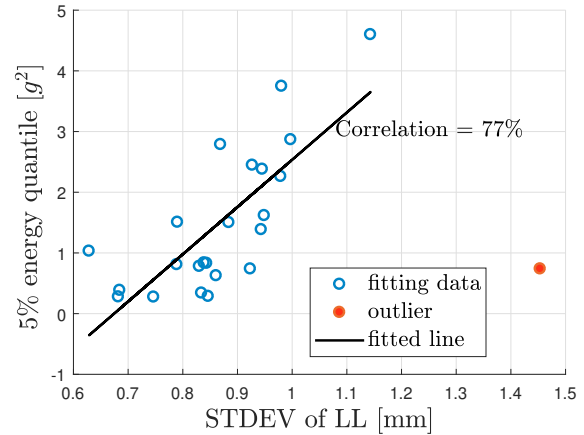


Fig. 6. The correlation of track vibration energy and geometric track quality.

Figure 6 presents the correlation of 5% energy quantile with the track geometry index. There is a data point corresponding to location A7 (nose of the crossing) in August 2016 (before tamping) for which the estimated vibration energy is low but the standard deviation of the longitudinal level is high. This data point is in a clear distance from the rest of the data points and for the purpose of correlation analysis, it is treated as an outlier. As seen, track vibration energy shows significant positive correlation with the track geometric quality. In particular, when geometric quality degrades, more vibration energy is recorded. This suggests that the track substructure loses its capability to dissipate energy when its geometry is not maintained in a good condition.

The results in Fig. 6 also suggest that track vibration energy could be regarded as an indicator for condition monitoring of track quality, and may be used to help maintenance decision making. This may be performed by assessment of track quality and setting alert and intervention limits, based on track vibration energy. For instance, should a track on high speed lines ( $> 160 \text{ Km/h}$ ) be of track quality class B, STDEV-LL needs to be below  $0.75 \text{ mm}$  (CEN, 2014). This class according to the fitted line in Fig. 6 may be defined by track energy below  $1 \text{ g}^2$ . Or, an alert limit for STDEV-LL for track on such speed class is  $1.2 \text{ mm}$ . This number leads to an alert limit of  $4 \text{ g}^2$  for track vibration energy. It is important to emphasize that such correlation model has been verified only on data concerning one turnout, therefore a greater validation embracing several turnouts at different locations is needed.

## 6. CONCLUSIONS

This study investigated the feasibility of evaluating track geometry condition from the measurements of train-induced track vertical acceleration at different sections of

a turnout. The study puts forward an integrated methodology for statistical analysis of track vibration energy data and track geometry data. Track vibration measurements during a time period of two years in a turnout were exploited to present the evolution of track vibration energy with accumulated tonnage. Particularly, the changes in track vibration energy around a tamping event were investigated and observation were made on how restoration of vertical track geometry affects track vibration energy. Synchronous measurements of track vibration and track geometry allowed for the analysis of the correlation between track vibration energy and track geometric quality. Results showed that there is a significant correlation between the two, and this suggested that track vibration energy could be regarded as an indicator for condition monitoring of track quality.

The assumptions made for establishing the energy-based indicator that contribute to the uncertainty in the results are: (i) although experiments were designed to minimize the influence of train-specific characteristics, these may still have minor effects; (ii) the energy measured in one location is associated with track geometry in its 10m neighbourhood; (iii) the measurement of track geometry is valid (i.e. geometric quality remains unchanged) for a period of 3-4 weeks around the time of measurement; (iv) the influence of weather conditions on track vibration energy was not incorporated. The inclusion of such effects remains for future research.

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