DTU

Distributionally Robust Co-Optimization of Energy and Reserve Dispatch of Integrated Electricity and Heat System

Mikhail Skalyga, Qiuwei Wu

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1. Introduction

2. District Heating Network (DHN) Model

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1. Introduction

- Wind power has grown strongly in recent years over the world. In Denmark, the amounts of renewables is expected to increase significantly over the next years.

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- Combined operation of integrated electricity and heating systems (IEHSs) and flexibility from the district heating system (DHS) is seen as prominent solution to handle the uncertainty of the RESs.

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- Combined operation of integrated electricity and heating systems (IEHSs) and flexibility from the district heating system (DHS) is seen as prominent solution to handle the uncertainty of the RESs.

- In this paper, we formulate distributionally robust optimization of energy and reserves dispatch model for the IEHS with a moment based ambiguity set to handle the uncertainty of the wind power

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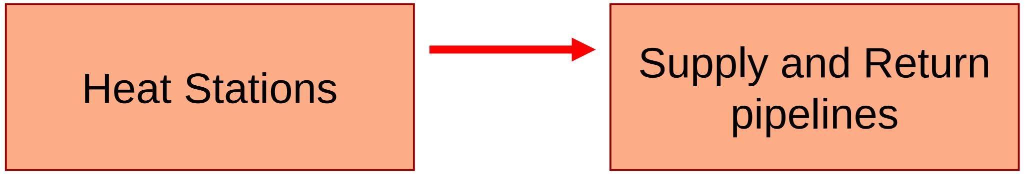
2. District Heating Network

Heat Stations

2. District Heating Network



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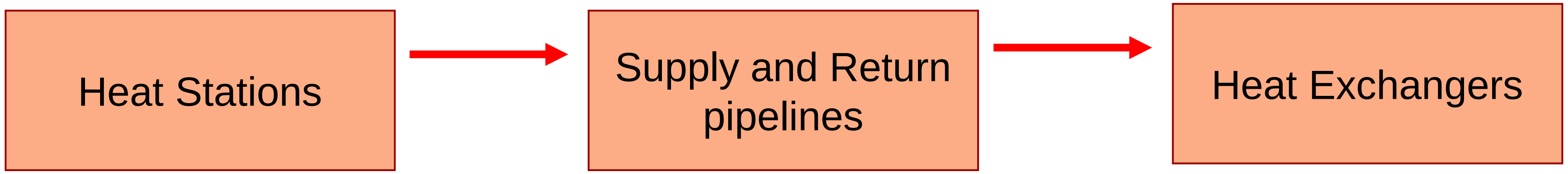

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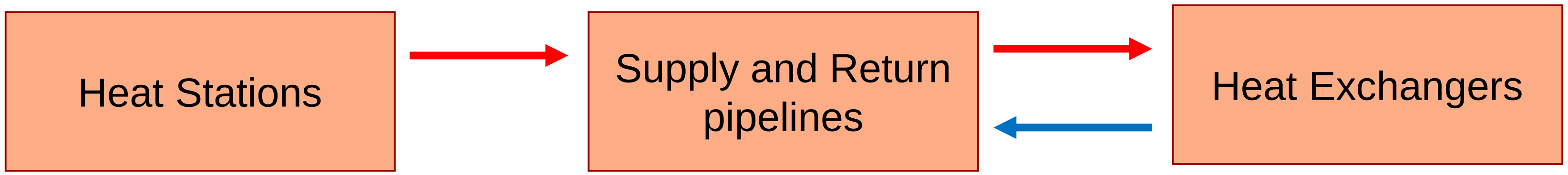

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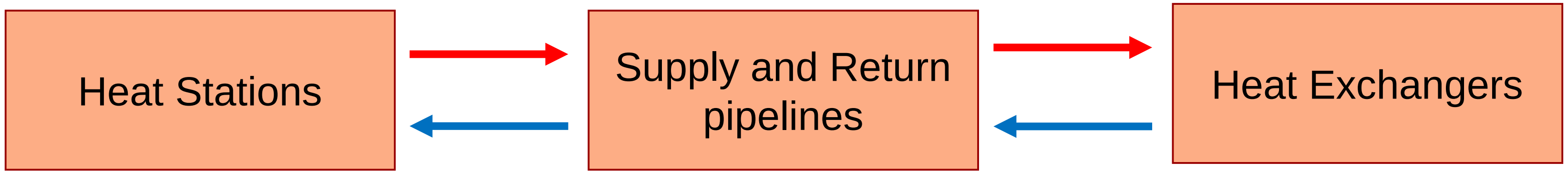

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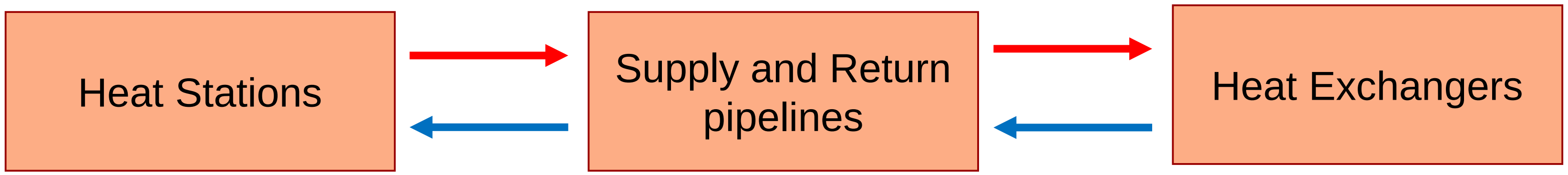
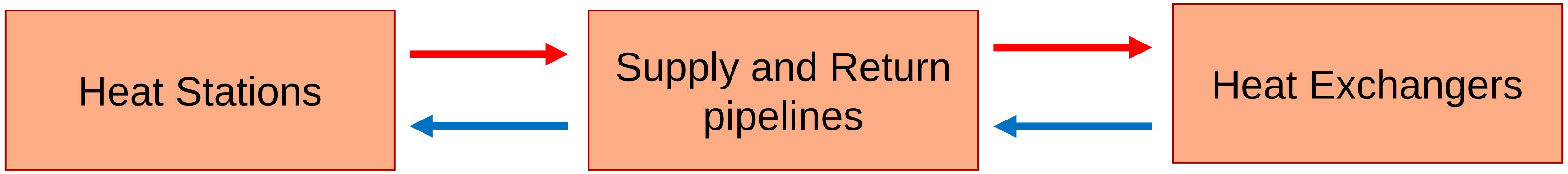


Variables:

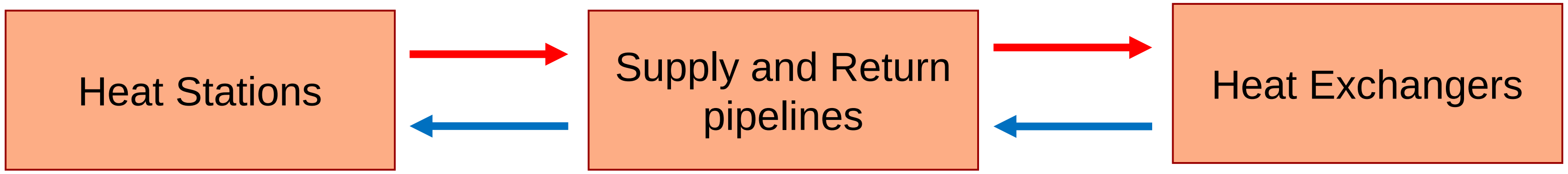
2. District Heating Network



Variables:

Heat Sources Power

2. District Heating Network

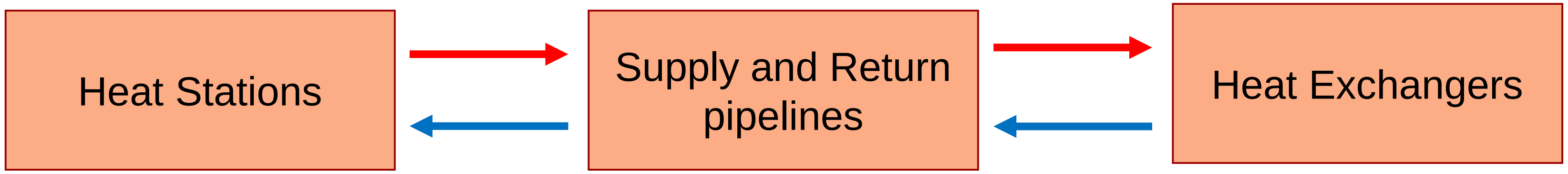


Variables:

Heat Sources Power

Power consumed by Water Pump

2. District Heating Network



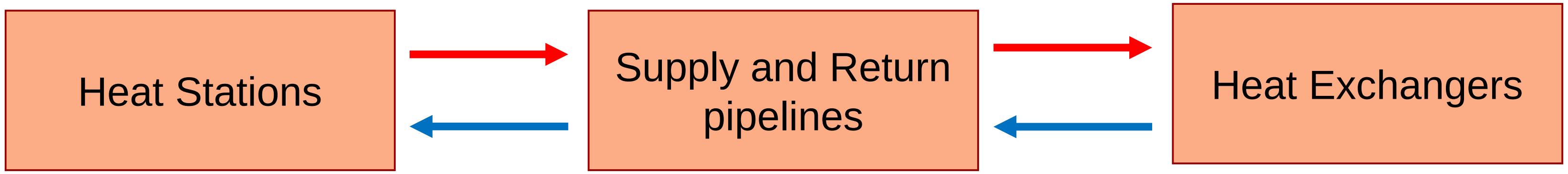
Variables:

Heat Sources Power

Power consumed by Water Pump

Mass Flow rates

2. District Heating Network



Variables:

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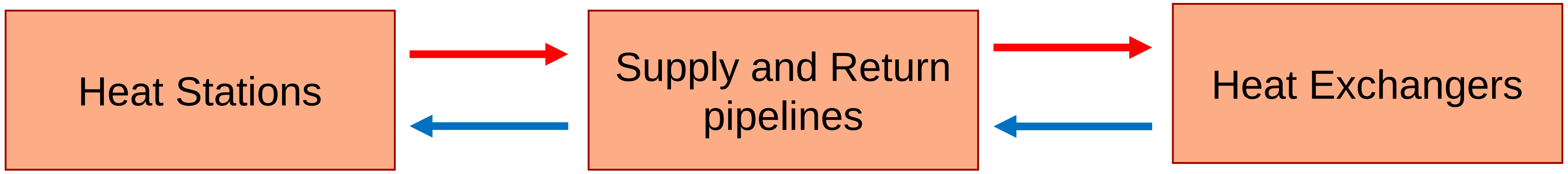
Mass Flow rates

Nodal Temperatures in Supply and Return pipelines

2. District Heating Network



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Starting and Ending Temperatures in Supply and Return pipelines

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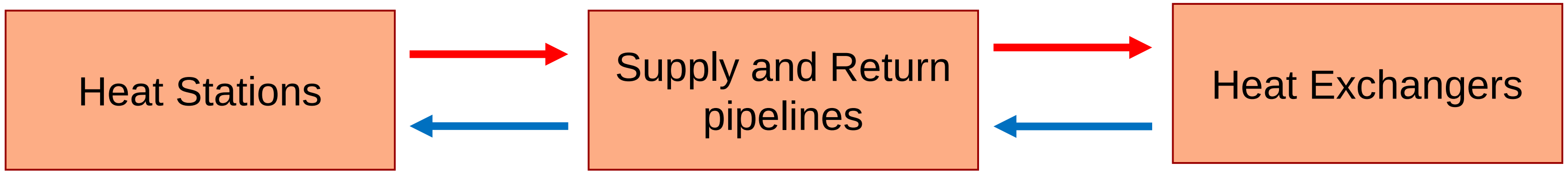

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Nodal Pressures

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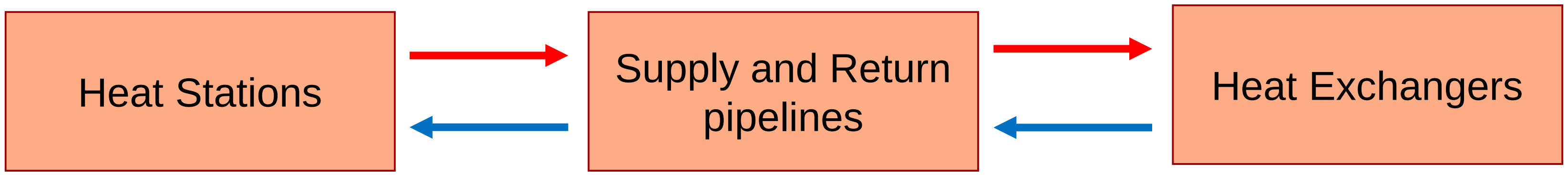

Variables:

Heat Sources Power

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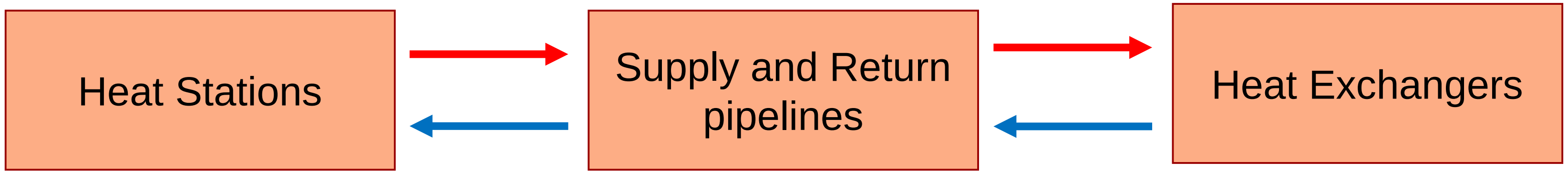
Nodal Temperatures in Supply and Return pipelines

Starting and Ending Temperatures in Supply and Return pipelines

Nodal Pressures

All variables are bounded within their operational limits

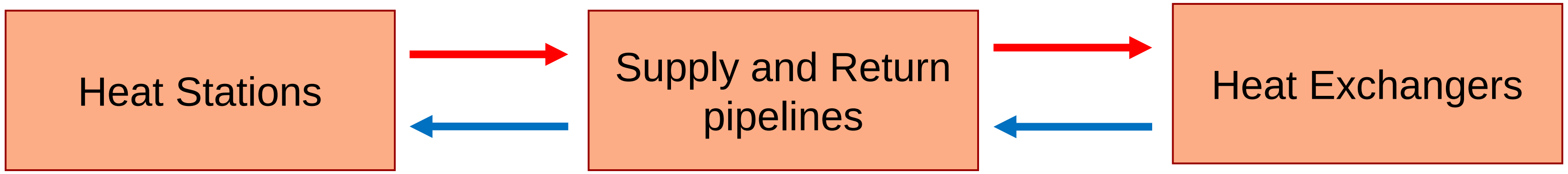
2. District Heating Network



$$H_s^{HS} = Cm_s^{HS}\left(T_n^S - T_n^R\right), \forall n \in \Lambda^N, \forall s \in \Theta_n^{HS}$$

Heat sources thermal power output

2. District Heating Network



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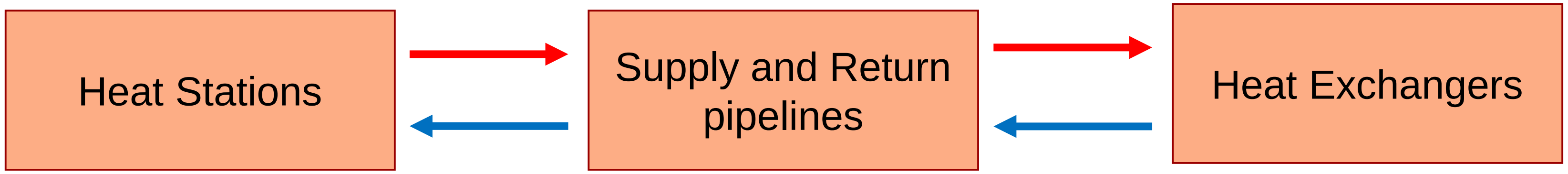
Heat sources thermal power output

$$P_s^{WP} = \frac{m_s^{HS}(pr_n^S - pr_n^R)}{\rho\eta^{WP}}, \forall n \in \Lambda^N, \forall s \in \Theta_n^{HS}$$

Water Pump power consumption

2. District Heating Network



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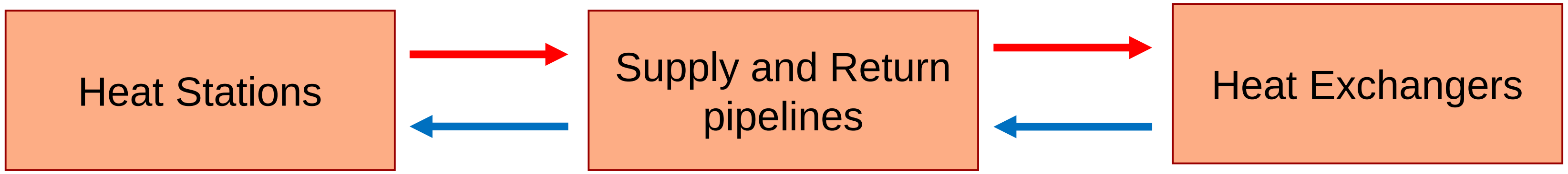
Water Pump power consumption

$$H_l^L = Cm_l^{HL}\left(T_n^S - T_n^R\right), \; \forall n \in \Lambda^N, \forall l \in \Theta_n^{HL}$$

Heat load power consumption

2. District Heating Network



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Water Pump power consumption

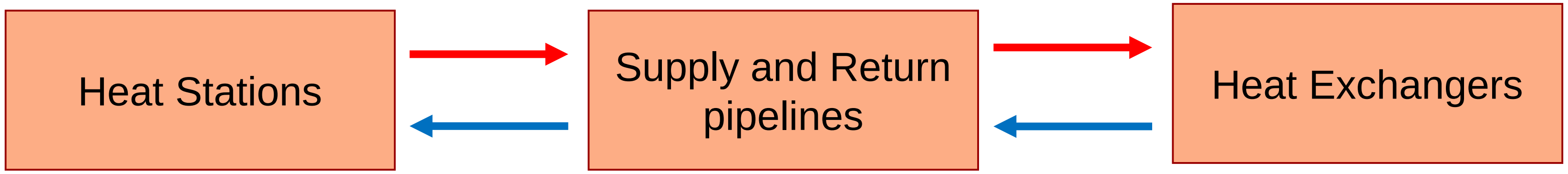
$$H_l^L = Cm_l^{HL}\left(T_n^S - T_n^R\right), \ \forall n \in \Lambda^N, \forall l \in \Theta_n^{HL}$$

Heat load power consumption

$$\sum_{p\in\Theta_n^{P,IN}} m_p^S T_p^{S,end} = T_n^S \sum_{p\in\Theta_n^{P,IN}} m_p^S, \ \forall n \in \Lambda^N$$
$$\sum_{p\in\Theta_n^{P,OUT}} m_p^R T_p^{R,end} = T_n^R \sum_{p\in\Theta_n^{P,OUT}} m_p^R, \ \forall n \in \Lambda^N$$

Temperature mix equation

2. District Heating Network



$$H_s^{HS} = Cm_s^{HS}\left(T_n^S - T_n^R\right), \forall n \in \Lambda^N, \forall s \in \Theta_n^{HS}$$

$$P_s^{WP} = \frac{m_s^{HS}(pr_n^S - pr_n^R)}{\rho\eta^{WP}}, \forall n \in \Lambda^N, \forall s \in \Theta_n^{HS}$$

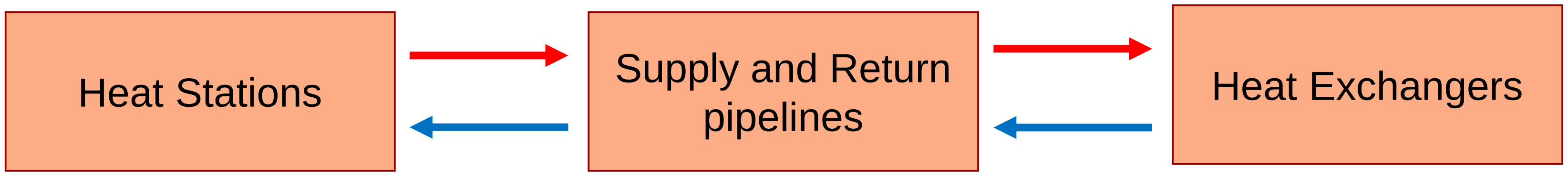
$$H_l^L = Cm_l^{HL}\left(T_n^S - T_n^R\right), \forall n \in \Lambda^N, \forall l \in \Theta_n^{HL}$$

$$\sum_{p \in \Theta_n^{P,IN}} m_p^S T_p^{S,end} = T_n^S \sum_{p \in \Theta_n^{P,IN}} m_p^S, \forall n \in \Lambda^N$$

$$\sum_{p \in \Theta_n^{P,OUT}} m_p^R T_p^{R,end} = T_n^R \sum_{p \in \Theta_n^{P,OUT}} m_p^R, \forall n \in \Lambda^N$$

Bilinear equality constraints

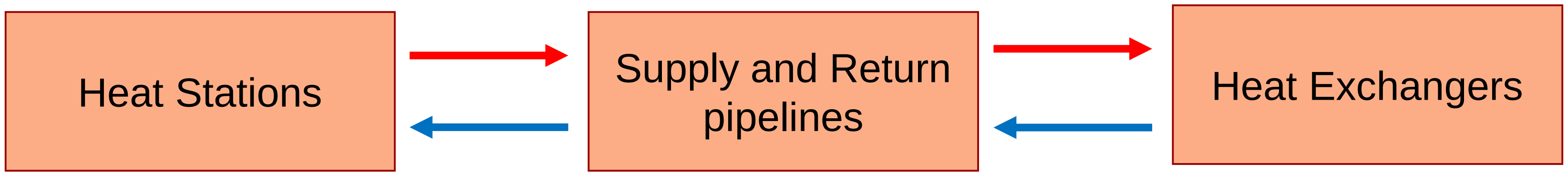
2. District Heating Network



$$pr^S_{n_1} - pr^S_{n_2} = K_p(m^S_p)^2,\ pr^R_{n_1} - pr^R_{n_2} = K_p(m^R_p)^2,$$
$$\forall p \in \Lambda^P, n_1 \in \Theta^{P,OUT}_n, n_2 \in \Theta^{P,IN}_n$$

Pressure drop along the pipes

2. District Heating Network



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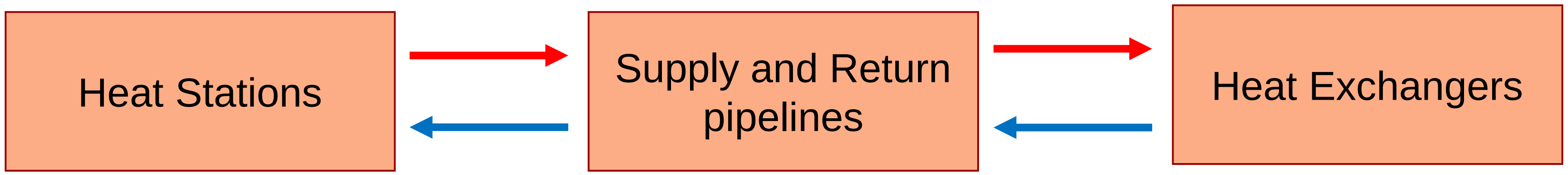
→ Quadratic equality constraint

2. District Heating Network

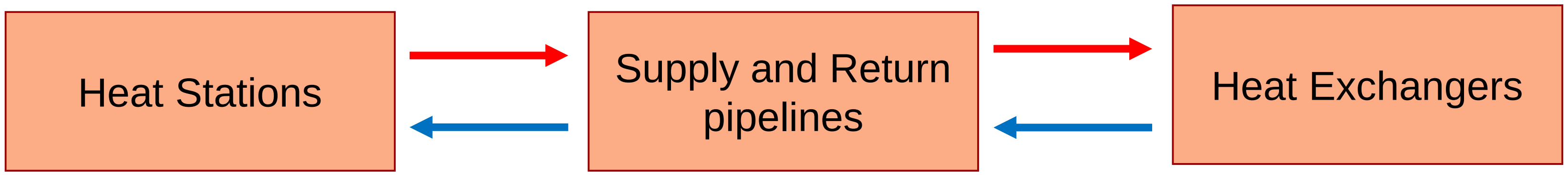


$$T_p^{S,end} - T_a = (T_p^{S,start} - T_a)e^{-\frac{\lambda L_p}{C m_p^S}}, \forall p \in \Lambda^P$$

$$T_p^{R,end} - T_a = (T_p^{R,start} - T_a)e^{-\frac{\lambda L_p}{C m_p^R}}, \forall p \in \Lambda^P$$

Temperature drop equations

2. District Heating Network



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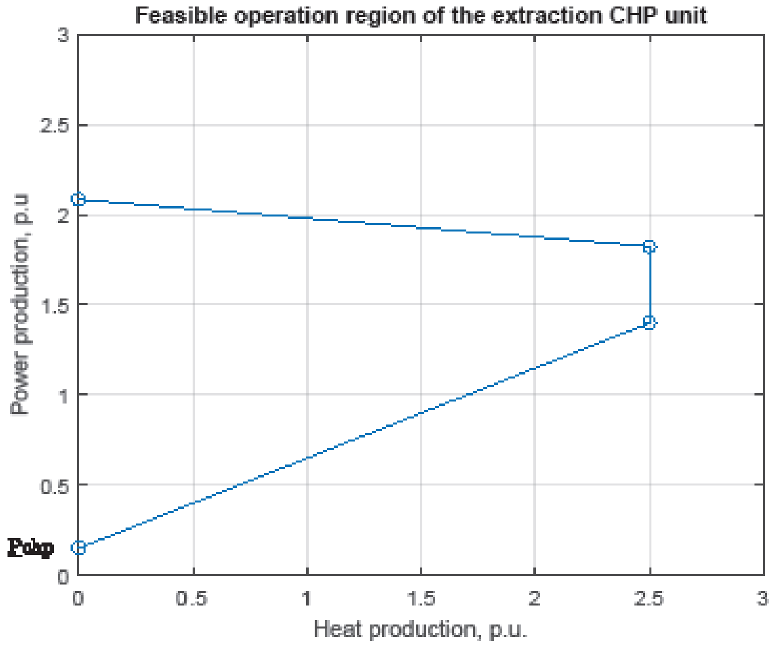
→ Nonlinear exponential function

2. District Heating Network

Linking units



$$
\begin{aligned}
&(P_s^{CHP} - R_s^{dc}) \geq r_s H_s^{HS} + \underline{P}_s^{CHP},\ \forall s \in \Theta_n^{CHP} \\
&0 \leq \rho_s^{E} (P_s^{CHP} + R_s^{uc}) + \rho_s^{H} H_s^{HS} \leq \overline{F},\ \forall s \in \Theta_n^{CHP}
\end{aligned}
\qquad (16)
$$

$$
H_s^{HS} = COP_s P_s^{HP},\ \forall s \in \Theta_n^{HP} \qquad (17)
$$

Fig. 1. Operation region of extraction CHP unit

2. District Heating Network

Linking units

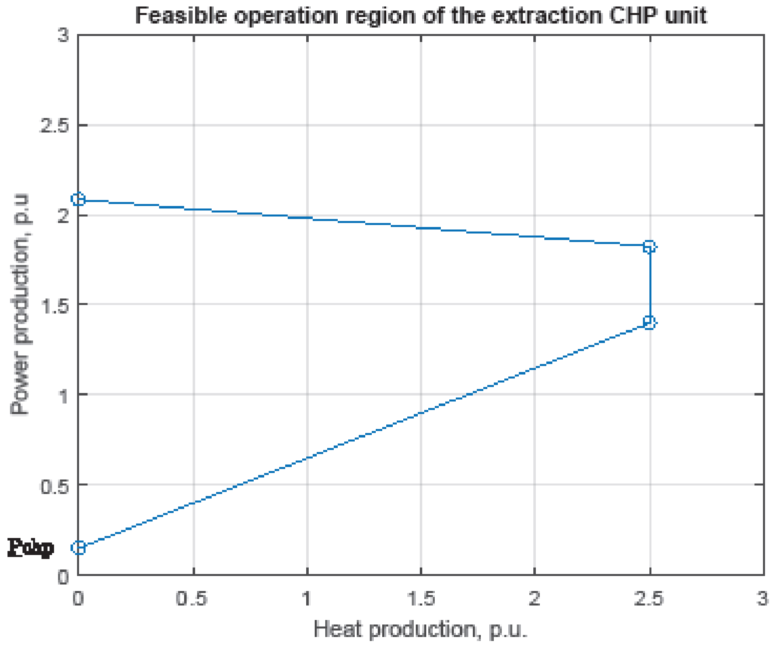


Fig. 1. Operation region of extraction CHP unit

Extraction CHP unit

$$(P_s^{CHP} - R_s^{dc}) \geq r_s H_s^{HS} + \underline{P}_s^{CHP},\ \forall s \in \Theta_n^{CHP}$$
$$0 \leq \rho_s^{E}(P_s^{CHP} + R_s^{uc}) + \rho_s^{H} H_s^{HS} \leq \overline{F},\ \forall s \in \Theta_n^{CHP} \quad (16)$$

Heat Pump

$$H_s^{HS} = COP_s P_s^{HP},\ \forall s \in \Theta_n^{HP} \quad (17)$$

Content

3. Electrical Power System Model

$$\mathbf{1}^T P^G + \mathbf{1}^T P^{CHP} + \mathbf{1}^T P^f = \mathbf{1}^T P^D + \mathbf{1}^T P^{WP} + \mathbf{1}^T P^{HP} \quad (18)$$

3. Electrical Power System Model

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Power balance

3. Electrical Power System Model

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Power balance

$$-P^l \leq A[C(P+R) + C_W(P^f + \Delta W) - C_D P^D - C_{WP} P^{WP} - C_{HP} P^{HP}] \leq P^l \quad (19)$$

Power limits along the line

3. Electrical Power System Model

$$\mathbf{1}^T P^G + \mathbf{1}^T P^{CHP} + \mathbf{1}^T P^f = \mathbf{1}^T P^D + \mathbf{1}^T P^{WP} + \mathbf{1}^T P^{HP} \quad (18)$$

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Power limits along the line

P^f - Wind power forecast vector

3. Electrical Power System Model

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Power limits along the line

P^f - Wind power forecast vector

ΔW - Vector of the forecast errors

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$$\mathbf{1}^T P^G + \mathbf{1}^T P^{CHP} + \mathbf{1}^T P^f = \mathbf{1}^T P^D + \mathbf{1}^T P^{WP} + \mathbf{1}^T P^{HP} \quad (18)$$

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Power limits along the line

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Power limits along the line

P^f - Wind power forecast vector

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$P = [P^G; P^{CHP}]$ Stacked column vector

3. Electrical Power System Model

$$\mathbf{1}^T P^G + \mathbf{1}^T P^{CHP} + \mathbf{1}^T P^f = \mathbf{1}^T P^D + \mathbf{1}^T P^{WP} + \mathbf{1}^T P^{HP} \quad (18)$$

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Power limits along the line

P^f - Wind power forecast vector

ΔW - Vector of the forecast errors ← Random variable

$P = [P^G; P^{CHP}]$ Stacked column vector

$$-RT^{dn} \leq R \leq RT^{up} \quad (20)$$

Reserve action limits

3. Electrical Power System Model

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Power balance

$$-P^l \leq A[C(P+R) + C_W(P^f + \Delta W) - C_D P^D - C_{WP} P^{WP} - C_{HP} P^{HP}] \leq P^l \quad (19)$$

Power limits along the line

P^f - Wind power forecast vector

ΔW - Vector of the forecast errors ← Random variable

$P = [P^G; P^{CHP}]$ Stacked column vector

$$-RT^{dn} \leq R \leq RT^{up} \quad (20)$$

Reserve action limits

$RT^{up} = [R^{up}; R^{uc}]$,

$RT^{dn} = [R^{dn}; R^{dc}]$

Stacked column vectors

3. Electrical Power System Model

$$P^{min} \leq P + R \leq P^{max} \qquad (21)$$

Generators and CHP limits

3. Electrical Power System Model

$$P^{min} \leq P + R \leq P^{max} \quad (21)$$

Generators and CHP limits

$$0 \leq R^{up} \leq \overline{R}^{G}, \quad 0 \leq R^{dn} \leq \underline{R}^{G}, \quad (22)$$

$$0 \leq R^{uc} \leq \overline{R}^{CHP}, \quad 0 \leq R^{dc} \leq \underline{R}^{CHP}, \quad (23)$$

Reserve capacity

3. Electrical Power System Model

$$P^{min} \leq P + R \leq P^{max} \tag{21}$$

Generators and CHP limits

$$0 \leq R^{up} \leq \overline{R}^{G}, \quad 0 \leq R^{dn} \leq \underline{R}^{G}, \tag{22}$$

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Reserve capacity

$Y \in \mathbb{R}^{(N_G + N_{CHP}) \times 1}$ - Vector of participation factors for controllable units

3. Electrical Power System Model

$$P^{min} \leq P + R \leq P^{max} \tag{21}$$

Generators and CHP limits

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$$R = -Y\sum_{w} \Delta W_w, \quad \sum_{i} Y = 1, \quad Y \geq \mathbf{0}, \quad Y \leq \mathbf{1} \tag{24}$$

Response rule in real-time

3. Electrical Power System Model

$$P^{min} \leq P + R \leq P^{max} \quad (21)$$

Generators and CHP limits

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Response rule in real-time

Linear response of the unit in real-time to the uncertain renewable power generation

3. Electrical Power System Model

Objective function

$$C^G(P^G) + C^{CHP}(P^{CHP}, H^{HS}) + C^R(R^{up}, R^{dn}, R^{uc}, R^{dc})$$

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Objective function

$$C^G(P^G) + C^{CHP}(P^{CHP}, H^{HS}) + C^R(R^{up}, R^{dn}, R^{uc}, R^{dc})$$

Generators operation cost

$$(P^G)^T c_2^T P^G + c_1^T P^G + c_0^T$$

3. Electrical Power System Model

Objective function

$$C^{G}(P^{G}) + C^{CHP}(P^{CHP}, H^{HS}) + C^{R}(R^{up}, R^{dn}, R^{uc}, R^{dc})$$

Generators operation cost

$$(P^{G})^{T} c_2^{T} P^{G} + c_1^{T} P^{G} + c_0^{T}$$

CHP operation cost

$$c_e^{T} P^{CHP} + c_h^{T} H^{CHP}$$

3. Electrical Power System Model

Objective function

$$C^G(P^G) + C^{CHP}(P^{CHP}, H^{HS}) + C^R(R^{up}, R^{dn}, R^{uc}, R^{dc})$$

Generators operation cost

$$(P^G)^T c_2^T P^G + c_1^T P^G + c_0^T$$

CHP operation cost

$$c_e^T P^{CHP} + c_h^T H^{CHP}$$

Reserve cost

$$\overline{c}_G^T R^{up} + \underline{c}_G^T R^{dn} + \overline{c}_c^T R^{uc} + \underline{c}_c^T R^{dc}$$

Content

4. Distributionally Robust reformulation

Constraints that include reserves actions R vector can be written as chance-constraints and we can formulate DR chance-constraints as follows:

$$\inf_{\mathbb{P}_\xi \in \mathcal{D}_\xi} \mathbb{P}((A_i^x)^T \xi \leq b_i^x) \geq 1 - \epsilon_i, \forall i = 1 \ldots m$$

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m - number of chance-constraints

4. Distributionally Robust reformulation

Constraints that include reserves actions R vector can be written as chance-constraints and we can formulate DR chance-constraints as follows:

$$\inf_{\mathbb{P}_\xi \in \mathcal{D}_\xi} \mathbb{P}((A_i^x)^T \xi \leq b_i^x) \geq 1 - \epsilon_i, \forall i = 1 \dots m$$

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How do we build an ambiguity set?

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Build ambiguity set

$$\mathcal{D}_\xi := \{\mathbb{P}_\xi \in \mathcal{P}' : \mathbb{E}_{\mathbb{P}_\xi}[\xi] = \mu, \mathbb{E}_{\mathbb{P}_\xi}[(\xi - \mu)(\xi - \mu)^T] = \Sigma\}$$

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Then, in [1], the exact reformulation of DR chance-constraints is obtained

$$\sqrt{\left(\frac{1-\epsilon}{\epsilon}\right)(A_i^x)^T(\Sigma)A_i^x} \leq b_i^x - (A_i^x)^T \mu, \ \forall i = 1 \dots m$$

Content

5. Convex Relaxation and Linearization of DHN Equations

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Heat sources thermal power output

Water pump power consumption

Heat load power consumption

Temperature mix equation

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Temperature drop equation → In exponential term use constant with the mid-range of mass flows for each pipe

5. Convex Relaxation and Linearization of DHN Equations

The solutions to the relaxed problem could be infeasible for the original model and relaxed constraints can be violated. In order to obtain feasible solutions, we solve the original problem with constant mass flow rates obtained from the relaxed problem.

Content

6. Case Study

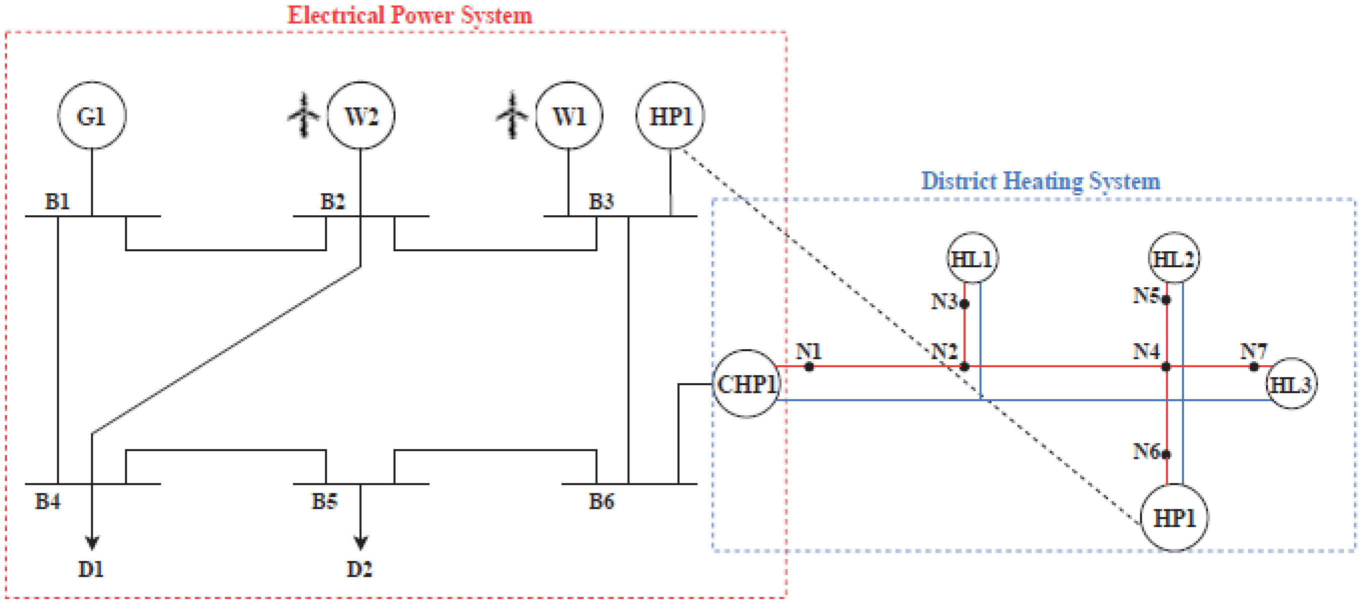

Fig. 2. Configuration of the six-bus and seven-node integrated system

6. Case Study

Table I. Cost Performance and CPU time averaged over 10 simulations

		Gaussian		DRO	
	$1-\epsilon$	95%	90%	95%	90%
Cost ($)	Relaxed	5350.2	5161.8	8803.5	6983.5
	Feasible	5975.9	5787.3	9429.8	7609.6
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- Under the assumption, that uncertainty follows Gaussian distribution, the problem can be reformulated as a convex problem.

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- Gaussian approximation obtains less cost than the distributional robust reformulation.
- The system cost grows with increasing confidence level

6. Case Study

Table II. Empirical Violation Probabilities

	$1-\epsilon$	Gaussian		DRO	
		95%	90%	95%	90%
Line vio.	max	0	0	0	0
G vio.	max	0	0	0	0
CHP vio.	avg	5.40%	8.41%	0.12%	1.21%
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- Superior results are seen for the 5% case with maximum violation probability of the CHP operation region 2.42%.

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- Generator operation limits and lines limits are not violated

6. Case Study

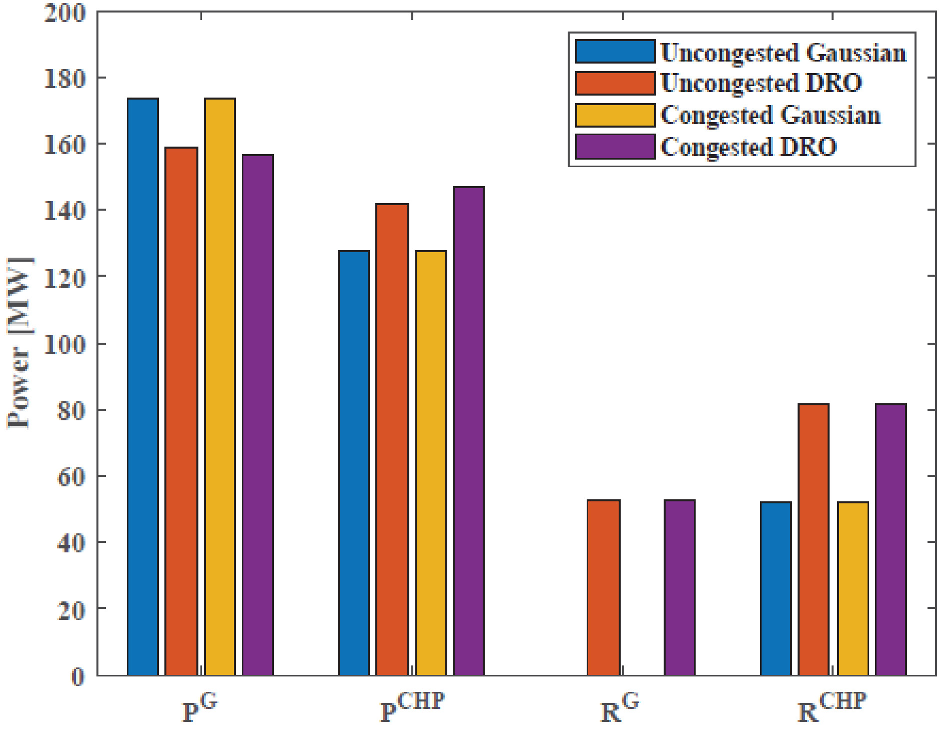
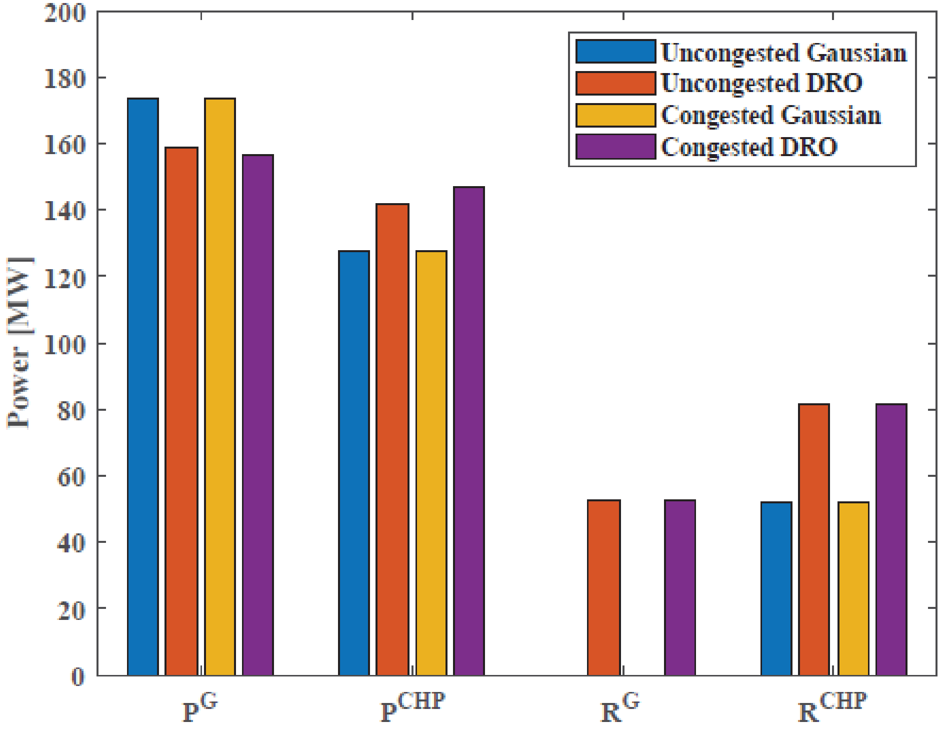


Fig. 3. Optimal reserve procurement and generation dispatch with violation probability 5%.

6. Case Study



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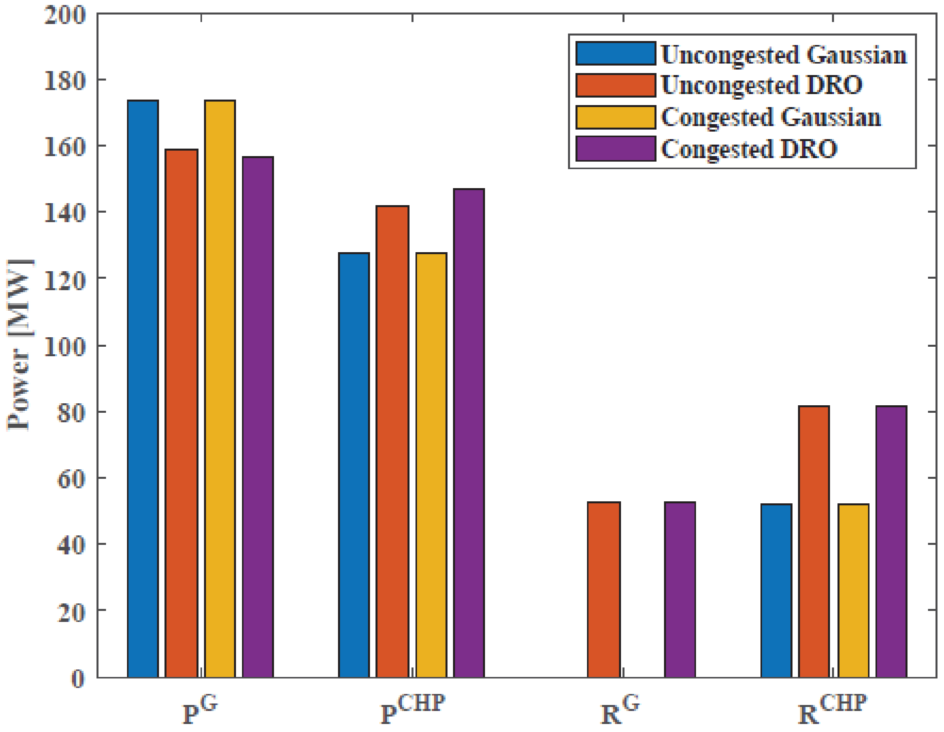

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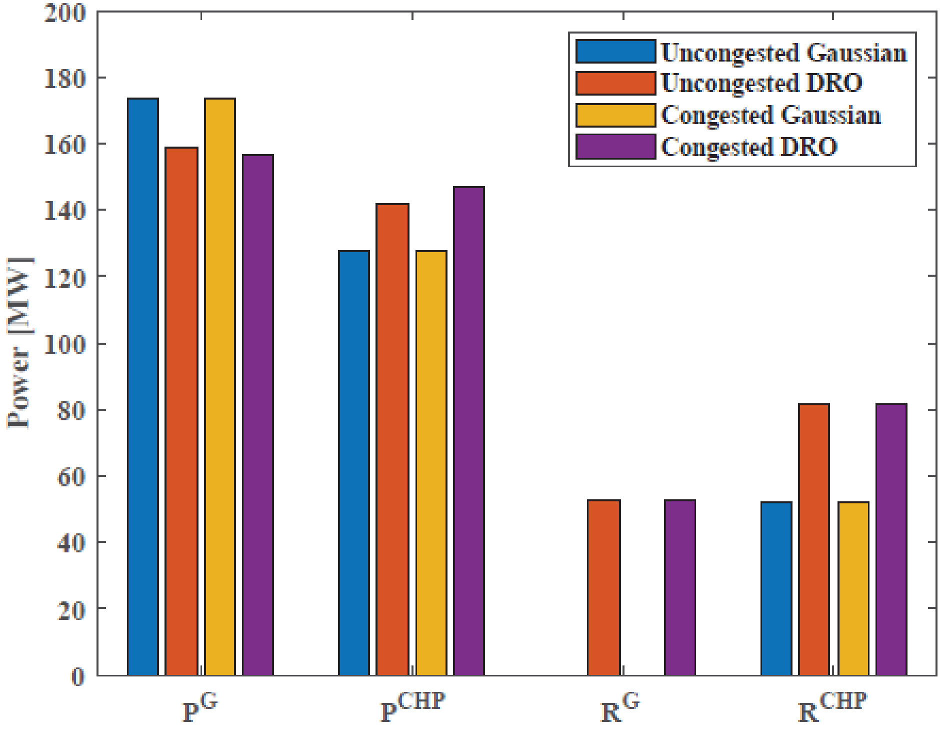

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Content

7. Conclusion

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The article is under revision and coming out soon!

Acknowledgment

- This work was supported in part by the EUDP programme through the SMART MLA project (grant no 64018-0302) and in part by DTU through the Alliance PhD scholarship.

References

[1] M. R. Wagner, “Stochastic 0–1 linear programming under limited distributional information,” Operations Research Letters, vol. 36, no. 2, pp. 150–156, 2008.

Thank you for your attention!