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A COMPUTATIONALLY EFFICIENT PROCEDURE FOR TUNING OF SHIP TRANSFER FUNCTIONS

Raphaël E. G. Mounet [1] [2], Ulrik D. Nielsen [1] [2], Astrid H. Brodtkorb [2]

ABSTRACT

The analysis of wave-ship interactions is highly relevant for the safety – as well as the energy efficiency – of the maritime operations. One application is the onboard estimation of the ship's responses in incoming seaways, which requires regular and accurate updates of the vessel's seakeeping model, accounting for possible changes in the operational conditions. This paper presents a simple approach for fast estimation of the wave-to-motion transfer functions of vessels. Prior information of the wave spectrum, characterizing the sea state, and ship motion measurements, i.e. time series sequences from onboard sensors, are supposed to be available. Semi-empirical closed-form expressions derived for a box-shaped vessel define Parameterized Response Amplitude Operators (P-RAOs) for the heave and pitch motions. The five input parameters, namely the ship speed, length, breadth, draught and block coefficient, are regarded as optimization variables. An optimization problem is established to minimize the spectral discrepancy between, on one hand, the *measured responses, and, on the other hand, the theoretical responses computed with the wave spectrum and P-RAOs. Numerical simulations of measured motions in a predefined long-crested sea state are carried out in the frequency domain for two different ships, a small research vessel and a container ship, using a set of RAOs obtained by a commercial potential flow code. The simulated measurements are considered as the ground truth. Tuning of the P-RAOs is carried out, and the results show a fairly good agreement between the tuned P-RAOs and the true RAOs over a wide portion of the frequency range. Moreover, the normalized error between the true and estimated response spectra is significantly decreased after tuning the P-RAOs.*

KEY WORDS

Measured ship motions; Parameterized Response Amplitude Operators; Wave spectrum; Transfer function tuning; Seakeeping model; Closed-form expressions.

INTRODUCTION

The information extracted from a vessel's response during operations at sea is essential for ensuring the safety of the ship, crew and cargo, as well as to evaluate the energy efficiency of the operations. Historically, the theory of the linear response of a marine vessel sailing in ocean waves was to a large extent inherited from the theory of linear superposition, which at first was conceptualized in electronics and communication (Rice 1944; Lee 1960). St. Denis and Pierson (1953) resolved many of the problems inherent to the application of the principle of superposition to marine systems, including the fact that a vessel is typically moving with non-zero speed within the seaway. Modelling the behavior of vessels in moderate seas for analysis of wave-ship interactions, the so-called *seakeeping* problem, relies typically on accurate complex-valued linear transfer functions (TRFs) which describe the relation in the frequency domain between the energy spectra of the first-order wave-induced ship motions and the wave spectrum. In the literature, the amplitude of the TRFs is commonly related to the *Response Amplitude Operator* (RAO). The RAOs are estimated from the knowledge of the ship geometry, and additionally depend on the loading conditions and forward speed. They are functions of the wave direction and the encountered wave frequency. They can be computed by solving the equations of motions for the ship, on the basis of potential flow theory. Some numerical methods to estimate the transfer functions make use of hydrodynamic models (radiation-diffraction codes) based on 3D panel code or strip theory (e.g. Salvesen, Tuck and Faltinsen 1970). These methods can have varying degrees of accuracy and computational costs, depending on the methodology of calculation. In the reality of ship operations, there is a significant degree of uncertainty in the experienced operational conditions, including the uncertain knowledge of the ship speed and loading conditions. For these reasons, it is quite unpractical to directly use precomputed linear transfer functions for the purpose of real-time estimation (or prediction) of the wave-induced responses of vessels during their operations at sea. Research has been conducted to solve this issue, employing various *system identification* techniques to estimate the important parameters (center of gravity, inertia terms, etc.) upon which the hydrodynamic vessel model is determined, e.g. Han (2021). Studies have also been conducted on estimating the hydrodynamic coefficients - i.e. added mass, damping, stiffness coefficients, and wave excitation forces - based on relevant vessel data, estimates of the directional

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wave spectrum, and measurements of the wave-induced response (Yuan *et al.* 2016; Kaasen *et al.* 2020; Skandali *et al.* 2020).
Jensen *et al.* (2004) developed closed-form solutions for the RAOs for the vertical motions, roll, relative motions and vertical wave induced bending moment. The derived closed-form expressions (CFEs) for the heave and pitch RAOs simplify the geometry of the hull to that of a box-shaped vessel. This is an advantage for onboard applications, as the simplified vessel model requires only a limited number of parameters. However, the simplifications can introduce many errors in the estimation of the RAOs, depending on the geometry of the vessel and the sea state. This motivates the need to tune the CFEs before use in a seakeeping analysis. Nielsen *et al.* (2021) proposed an optimization algorithm for direct tuning of RAOs, using these CFEs, in addition to vessel response measurements and ERA5 2D wave spectra. The main inconvenience was that the algorithm is only able to tune RAOs at the observed wave directions and frequencies. The present paper proposes an alternative computationally efficient tuning method, considering the CFEs as *parameterized RAOs* (P-RAOs). The input parameters of the P-RAOs are optimized to better match the vessel's seakeeping behavior in given operational conditions. In other words, one tries to identify an (hydrodynamic-) equivalent box experiencing the same motions as the actual ship.

THEORY

Fundamental Assumptions and Definitions for the Seakeeping Study

A floating vessel at sea is excited by wave loads, in addition to loads from wind and sea current which are disregarded in the present study. The wave loads are related to the environmental parameters describing the sea state, including significant wave height H_s, wave spectral peak period T_p, mean wave direction μ, and others. The wave process is assumed to be stationary, ergodic and Gaussian over the observation period. This enables to perform a linear steady-state approach in the frequency domain. The wave-induced vessel motions can be estimated by using linear transfer functions and a wave spectrum.

Long-crested waves are assumed, meaning that the wave system is uniquely described by a 1-D wave spectrum $E(\omega_0)$, where ω_0 is the (intrinsic) wave frequency. The wave direction β is defined relatively to the ship heading, as shown in Figure 1, and it follows that $\beta = 0°$ is a following sea, $\beta = 180°$ is head sea, and $\beta = 90°$ or $270°$ is beam sea.

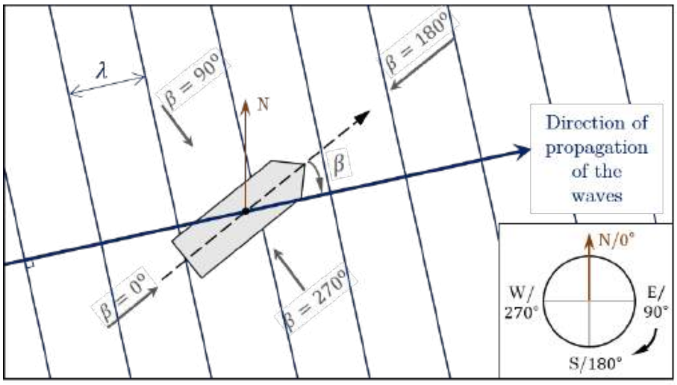


Figure 1: Definition of the heading angles.

For an advancing vessel, the waves are encountered at a frequency that is different from the (intrinsic) wave frequency, due to the Doppler Effect. Typically, the frequency is increased in head waves and decreased in following waves. The so-called *encounter frequency* is calculated by Equation (1), which is found in e.g. Nielsen (2010):

$$\omega_e = \omega_0 - \omega_0^2 \frac{U}{g} \cos(\beta) \tag{1}$$

where U is the ship forward speed.

The transition of the wave spectrum from the intrinsic wave frequency domain to the encounter frequency domain is essentially a transformation of the frequency coordinate system such that the wave energy is preserved. The encountered wave spectrum for long-crested waves may be written (Lewis 1989) as in Equation (2):

$$E_e(\omega_e; \beta, U) = E(\omega_0) \frac{g}{|g - 2\omega_0 U \cos\beta|} \tag{2}$$

Computation of Linear Wave-To-Motion Transfer Functions for Monohull Ships

The linear wave-to-motion transfer functions are denoted Φ_R for a given response R. Those are complex-valued functions of the wave frequency and direction. They depend on the geometry of the vessel and the operational conditions, especially the ship speed. In the present paper, the amplitude of the transfer function is referred to as the *Response Amplitude Operator*

(RAO). There exist many methods to estimate the RAOs of vessels; in particular, strip theory or panel element codes relying on potential flow theory can be used. Alternatively, *closed-form expressions* (CFEs) for the RAOs have been found for monohull ships, and are given in Jensen *et al.* (2004). For the vertical ship motions, namely heave (w) and pitch (θ), the equations of motion (EoMs) are solved analytically for a homogeneously loaded box-shaped vessel using linear strip theory. The coupling terms between heave and pitch are neglected. The required information for the procedure is restricted to the main dimensions of the box – length L, breadth B, draught T, block coefficient C_B – together with the forward speed U and relative wave heading β. The simplified EoMs in regular waves with amplitude a are given in Equations (3) and (4):

$$2\frac{kT}{\omega^2}\,\ddot{w} + \frac{A^2}{kBC_B\delta^3\omega}\dot{w} + w = aF\cos(\omega_e t) \qquad [3]$$

$$2\frac{kT}{\omega^2}\,\ddot{\theta} + \frac{A^2}{kBC_B\delta^3\omega}\dot{\theta} + \theta = aG\sin(\omega_e t) \qquad [4]$$

where $k = \omega^2/g$ is the wave number, and differentiation with respect to time t is denoted with a dot. Parameters A and δ are related to the sectional hydrodynamic damping and Doppler shift, respectively, while F and G are the forcing functions; all four can be approximated using the aforementioned input information, see Jensen *et al.* (2004) for more details. Note that the closed-form expressions for pitch motion always predict zero amplitude in beam seas ($\beta = 90°$ or $270°$), because of the bow-stern symmetry of the box-shaped vessel.

Estimation of Response Spectra in Long-crested Waves

The so-called *response spectrum* for the individual rigid-body motion component R can be estimated theoretically by filtering the wave energy spectrum with the appropriate motion transfer function. In long-crested waves, this is achieved by multiplying each ordinate of the encountered wave spectrum $E_e(\omega_e; U)$ by the RAO at the corresponding encounter frequency (Lewis 1989):

$$\hat{S}_{RR}(\omega_e) \overset{\text{def}}{=} \Phi_R(\omega_e,\beta;U)\cdot\overline{\Phi_R(\omega_e,\beta;U)}\cdot E_e(\omega_e;\beta,U) = |\Phi_R(\omega_e,\beta;U)|^2\cdot E_e(\omega_e;\beta,U) \qquad [5]$$

where the overbar indicates the complex conjugate.

Strictly speaking, Equation (5) is only valid in head or bow seas, that is $90° \leq \beta \leq 270°$. In quartering or following seas, i.e. $\beta \leq 90°$ or $\beta \geq 270°$, the transformation between encounter and wave frequency is not a unique mapping, meaning that a given encounter frequency can be mapped into up to three wave frequencies, located in three different regions of the frequency domain. Thus, the encountered response spectrum is formed by summation of contributions from these three regions:

$$\hat{S}_{RR}(\omega_e) = |\Phi_R^{I}(\omega_e,\beta;U)|^2\cdot E_e^{I}(\omega_e;\beta,U) + |\Phi_R^{II}(\omega_e,\beta;U)|^2\cdot E_e^{II}(\omega_e;\beta,U) + |\Phi_R^{III}(\omega_e,\beta;U)|^2\cdot E_e^{III}(\omega_e;\beta,U) \qquad [6]$$

where, for example, $E_e^{I}(\omega_e;\beta,U)$ is the contribution to the wave encounter spectrum from Region I, and $|\Phi_R^{I}(\omega_e,\beta;U)|^2$ is the RAO associated with the wave frequencies of Region I (Lewis 1989). Similar can be said about $E_e^{II}(\omega_e;\beta,U)$ and $E_e^{III}(\omega_e;\beta,U)$ as the contributions from Regions II and III, respectively.

Error Measure of Response Spectrum Estimate

In order to quantify the level of error ϵ_R in the spectrum estimate for a given response R, a difference metric between the true (or measured) response spectrum $S_{RR}(\omega_e)$ and an estimate $\hat{S}_{RR}(\omega_e)$ can be obtained as the normalized "area-deficit" encased between the two spectra:

$$\epsilon_R \overset{\text{def}}{=} \frac{\int_0^\infty |S_{RR}(\omega_e) - \hat{S}_{RR}(\omega_e)|\,d\omega_e}{m_{0,R}} \qquad [7]$$

where $m_{0,R} = \int_0^\infty S_{RR}(\omega_e)\,d\omega_e$ is the zeroth order spectral moment of the response spectrum.

METHODOLOGY

General overview of the procedure

The closed-form expressions (Jensen *et al.* 2004), already introduced in the "Theory" section, are considered as *Parameterized Response Amplitude Operators* (P-RAOs), taking five input parameters: the ship's forward speed U, length L, breadth B, draught T, and block coefficient C_B. Obviously, these are also functions of the relative wave heading and wave frequency. Three inputs are needed to tune the P-RAOs: a wave spectrum, the measured response spectra for the heave and pitch motions, and an initial guess of the input parameters for the P-RAOs. For the first guess, the P-RAOs are computed for the actual (physical) values of the aforementioned quantities, namely the logged speed-through-water and the ship's main particulars. It must be noted that both the wave spectrum and the measured response spectra are considered as the ground truth. The consideration of uncertainties in these quantities is out of the scope of the paper and left for future work.

Implementation of the RAO-tuning procedure

The RAO-tuning method solves an optimization problem that can be formulated as in Equation (8):

$$\begin{aligned} &\text{minimize } f(\boldsymbol{\alpha}) \\ &\text{subject to } \alpha_i \geq 0, \text{for each } i \in \{1,2,\dots,4\}, \\ &\text{and } 0.4 \leq \alpha_5 \leq 1 \end{aligned} \qquad [8]$$

The set of tuning parameters is composed of five quantities, which form the variables of optimization:

$$\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5] \propto [U, L, B, T, C_B] \qquad [9]$$

where the symbol “∝” indicates the physical quantities to which the tuning parameters relate in the P-RAOs. The reason for the imposed bounds on the tuning parameters in Equation (8) is that the values of the physical quantities must be positive and that the block coefficient must not exceed one. The cost function $f(\boldsymbol{\alpha})$ is computed as shown in Equation (10):

$$f(\boldsymbol{\alpha}) = \int_0^{\infty} \left[\left(\frac{|S_{zz}(\omega_e) - \hat{S}_{zz}(\omega_e; \boldsymbol{\alpha}, \beta, U)|}{m_{0,z}} \right)^2 + \left(\frac{|S_{\theta\theta}(\omega_e) - \hat{S}_{\theta\theta}(\omega_e; \boldsymbol{\alpha}, \beta, U)|}{m_{0,\theta}} \right)^2 \right] d\omega_e \qquad [10]$$

It expresses the squared error between the true response spectrum $S_{RR}(\omega_e)$ and the theoretical response spectrum $\hat{S}_{RR}(\omega_e; \boldsymbol{\alpha}, \beta, U)$, which is calculated using the tuning parameters in connection with the P-RAOs, as described in Equation (11):

$$\hat{S}_{RR}(\omega_e; \boldsymbol{\alpha}, \beta, U) = \left|\hat{\Phi}_{R,\text{P-RAO}}(\omega_e; \boldsymbol{\alpha}, \beta)\right|^2 \cdot E_e(\omega_e; \beta, U) \qquad [11]$$

where it is emphasized that the Doppler shift presented in Equations (1) and (2) must be calculated with the logged physical speed U, which will differ from tuning parameter α_1 after optimization.

The errors for the heave and pitch spectra are summed in the cost function, so that both responses take part into the tuning process. Normalization of the error is needed before summation in order to give equal weight to heave and pitch. The zeroth order moment of the measured response spectrum is used for the normalizing factor in Equation (10).

The range of encountered frequencies ω_e is chosen as $[0, 2\pi]$ rad/s, with a spacing of $\pi/100$ rad/s in the discretization, corresponding to 200 frequency components, which is supposed to be sufficient to perform the numerical integrations in Equation (10). The Trust-Region Constrained algorithm developed by Byrd *et al.* (1999) is exploited to solve the optimization problem, noticing that an implementation in Python 3 is available in the optimisation library of SciPy, under the function `minimize` with the method `'trust-constr'`.

Scenarios of simulation for the case study

A simulation study is performed with two vessels: the NTNU-owned research vessel Gunnerus (denoted “RV”) and a larger S175 container ship (denoted “S175”). Their main particulars are given in Table 1.

Table 1: Main particulars of the two studied vessels.

	RV	S175	
Length between perpendiculars, L_{pp}	28.9	175.0	[m]
Breadth middle, B	9.6	25.4	[m]
Draught, T	2.63	9.4	[m]
Displacement, Δ	418.06	24589.00	[t]
Block coefficient, C_B	0.559	0.570	[-]

The wave-induced motions of these two vessels are simulated in the frequency domain for one predefined sea state characterized by $H_s = 2$ m and $T_p = 10$ s. The (generalised) JONSWAP spectrum (Hasselmann *et al.*, 1973) is assumed with a peak shape parameter $\gamma = 1$, and the corresponding wave spectrum is plotted in Figure 2.

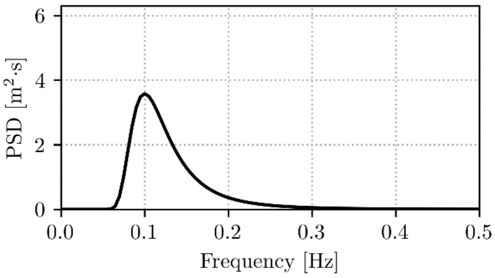
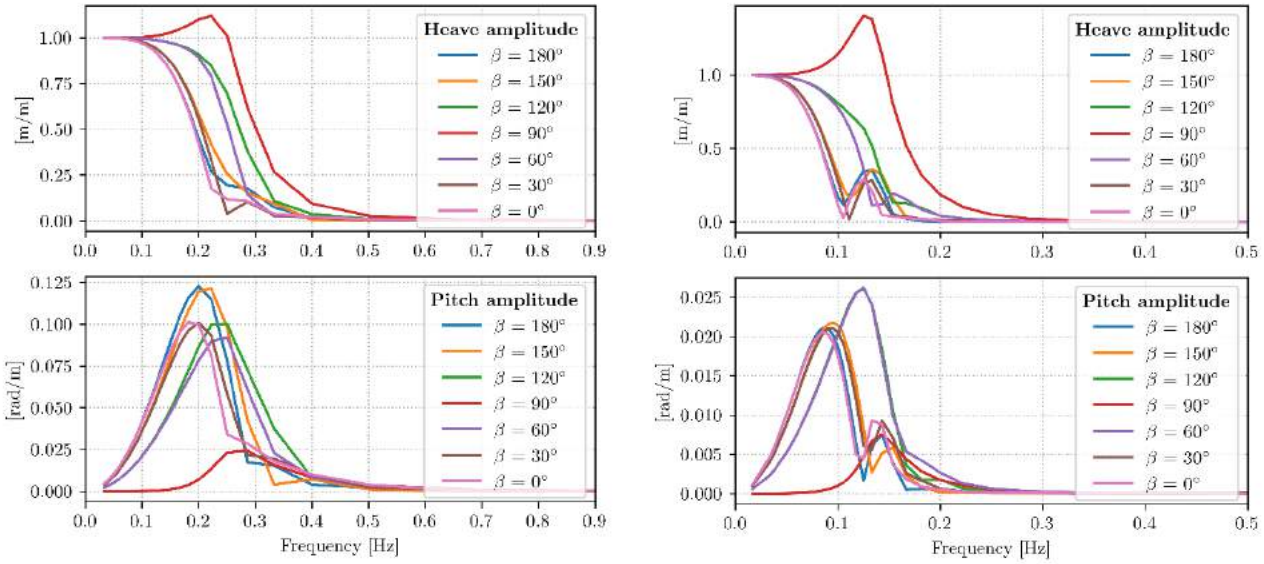


Figure 2: 1-D wave spectrum used for the case study. PSD stands for power spectral density.

Realism of the study is introduced by working with a set of theoretical RAOs purely used for generating the simulated wave-induced responses. To serve this purpose, the "ground true" heave and pitch RAOs of the vessels are obtained from the hydrodynamic code ShipX. It is emphasized that the tuning algorithm applies only on the P-RAOs, and not on the "true" ShipX RAOs. In this regard, the methodology for the simulations resembles the work of Mounet *et al.* (2022). The (true) RAOs of the two vessels are plotted in Figure 3 for various headings and zero forward speed. Although not represented here, the RAOs for the same vessels at forward speeds $U = 5$ and 10 knots are also available. As expected, the two vessels behave as low-pass filters with respect to the encountered waves. Overall, it is noted that S175 has a lower cut-off frequency than Gunnerus, which is explained by the larger dimensions of the former.



(a) RV (b) S175

Figure 3: Heave and pitch amplitude of the transfer functions at zero forward speed for the two ships selected for the case study. Note the different vertical and horizontal scales in the subplots.

In total, twenty-one scenarios are analysed for each of the two vessels, as listed in Table 2. Three different vessel speeds are considered, including cases with zero forward speed, and for each of them, simulations are carried out for seven relative wave headings covering head, bow quartering, beam, stern quartering, and following seas, which allows for a detailed sensitivity study. Due to the port-starboard symmetry of the vessels, there is no need to study the relative wave headings in the range $[180°, 360°]$.

Table 2: Scenarios of simulation, in terms of forward speed and relative wave heading.

Scenario number	U [knots]	β [deg]
A1, A2,…, A7	0	[180, 150, 120, 90, 60, 30, 0]
B1, B2,…, B7	5	[180, 150, 120, 90, 60, 30, 0]
C1, C2,…, C7	10	[180, 150, 120, 90, 60, 30, 0]

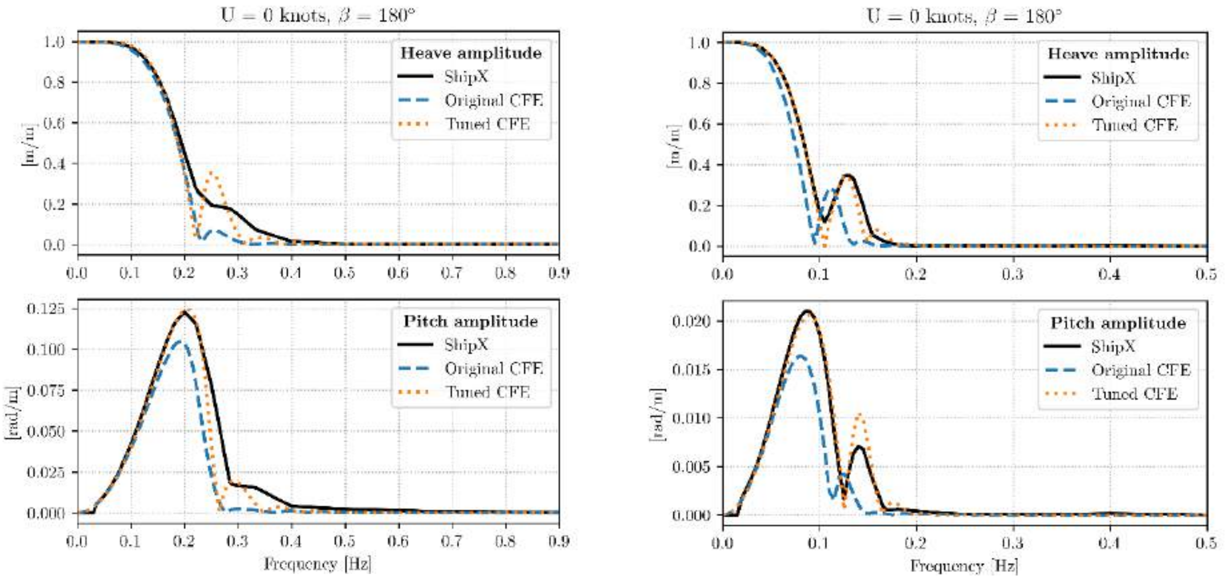
The initial guess $\boldsymbol{\alpha}^{\mathbf{0}}$ for the optimization variable $\boldsymbol{\alpha}$ has values taken:

- For U: from the particular scenario in Table 2.
- For $L, B, T,$ and C_B: from the ship's main particulars provided in Table 1.

RESULTS AND DISCUSSIONS

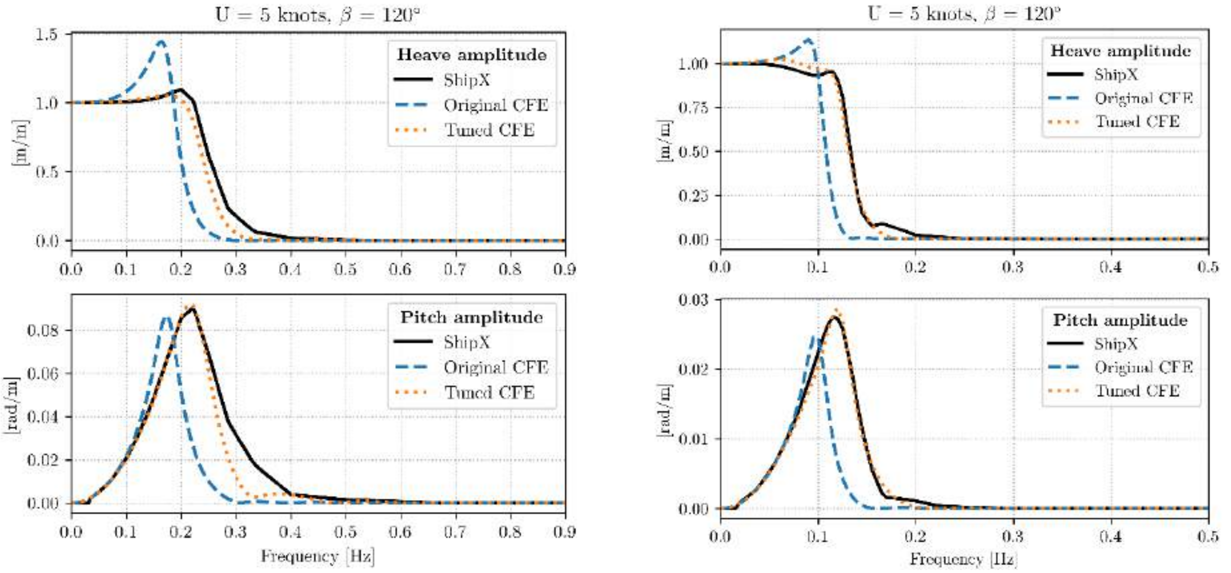
On average, it takes approximately 13-15 seconds to run the optimization procedure for one ship for one simulation scenario on the available laptop (CPU Intel(R) Core(TM) i7-10510U CPU @ 1.80 GHz, 16 GB memory). The computations are thus relatively fast.

The results of the tuning procedure are shown in Figure 4 for a few arbitrarily selected scenarios (A1, B3, and C5) for the two ships. The tuned P-RAOs are compared to the original P-RAOs and to the ShipX RAOs. Overall, it is seen that the tuned P-RAOs agree significantly better with the ShipX RAOs, than the original P-RAOs. The latter have a clear tendency to underestimate the amplitude, as a systematic error (bias) towards higher frequencies. This is quite successfully corrected by the tuning procedure in most cases. An important observation is that tuning the P-RAOs preserves their physical shape. However, for scenario C5, as well as scenarios C6 and C7 (results not shown here), the tuned P-RAOs present some oscillations of the heave and pitch amplitudes at higher frequencies, which do not correctly represent the true vessel



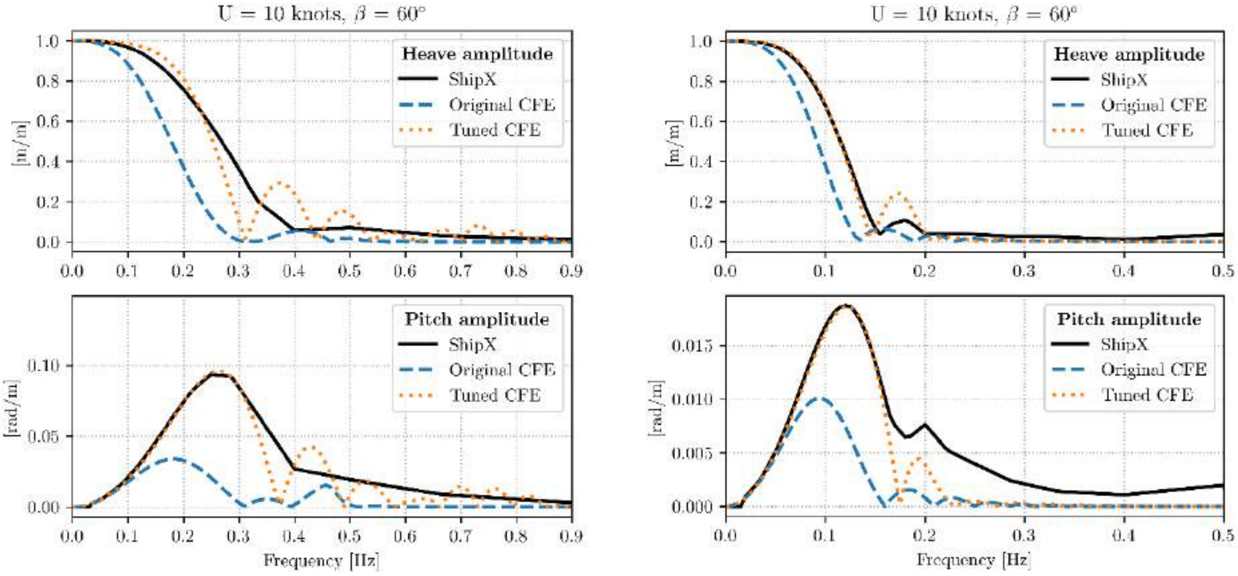
(a) RV, scenario A1

(b) S175, scenario A1



(c) RV, scenario B3

(d) S175, scenario B3



(e) RV, scenario C5

(f) S175, scenario C5

Figure 4: Some selected tuning results from the numerical simulations, at three different ship speeds and wave headings, for the two studied ships (RV and S175). Note the different horizontal and vertical scales in the subplots.

behavior. This affects RV even more than S175. Those scenarios correspond to stern quartering to following seas with a speed of 10 knots. Therefore, it is hypothesized that the oscillatory behavior is due to the handling of the Doppler shift in the closed-form expressions. The observed oscillations do not affect the theoretically computed response spectrum, because they happen at frequencies where the encountered wave spectrum has little energy. Nevertheless, extreme caution should be used when employing the tuned transfer functions to predict the response spectra in future sea states, because those might have energy at frequencies where the oscillations appear.

The theoretical response spectra – both before and after the RAO-tuning – are plotted in Figure 5 in the encounter frequency domain, for the same scenarios as in Figure 4, and the true response spectrum is represented as well for comparison. It is seen that in all the presented cases, there is a good agreement between the true spectrum and the estimated spectrum after tuning. The original P-RAOs did not enable such a good match. Consequently, the tuned P-RAOs represent an improvement with regards to a seakeeping analysis focused on the estimation of ship motion responses. Note, for scenario C5, the true response spectrum of RV in pitch has a quite distorted shape, with a discontinuity at a frequency $f_e = 0.155$ Hz. This is due to how the Doppler shift is handled in frequency-domain simulations. Such a discontinuity would not be observed if one had generated a set of time series realizations of the motion response and computed the associated average spectrum. More insights into this technical issue can be found in Nielsen (2017). Nonetheless, it is interesting to notice that the tuning procedure is also able to correct – to some extent – the RAOs at unobserved frequencies, that is at frequencies where there is zero measured response energy. This is visible for scenario C5 for RV, for instance, where the tuned P-RAOs have also been successfully corrected at frequencies over 0.155 Hz, up to 0.25 Hz and 0.30Hz in pitch and heave, respectively.

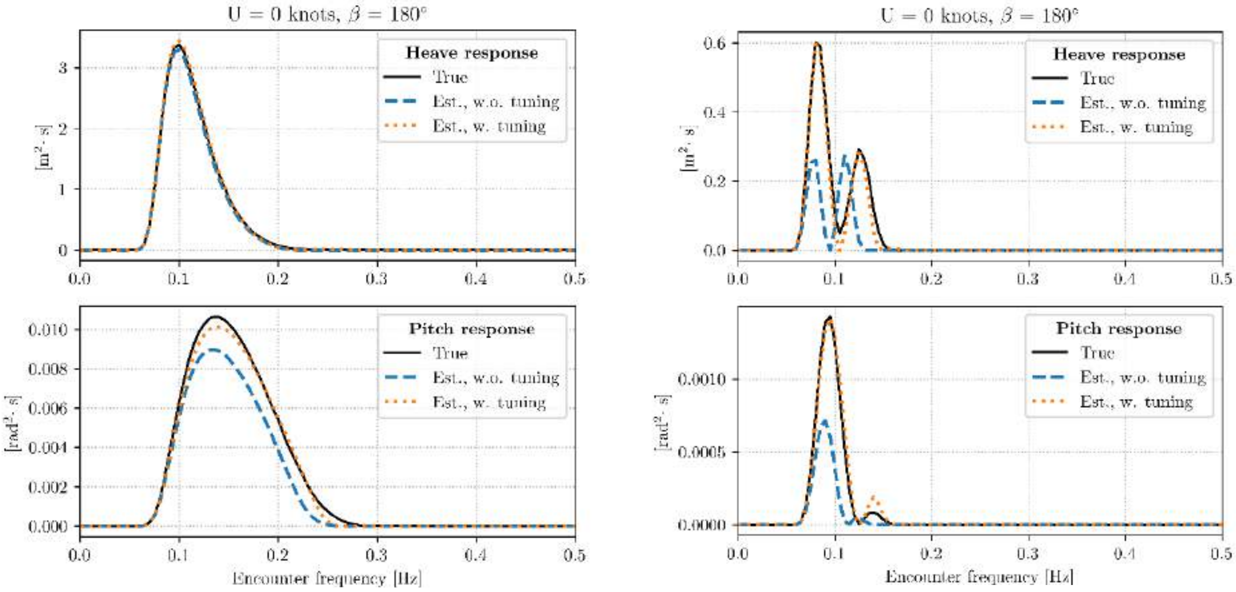
The tuning effect at unobserved wave headings was also investigated, by computing the full set of P-RAOs using the optimized tuning parameters $\boldsymbol{\alpha}^*$ for a given scenario. Contour lines of the error between the true ShipX RAOs and the P-RAOs are shown in a polar diagram in Figure 6, i.e. as a function of frequency and heading, for scenario B3 with the two ships. The error was normalized by the maximum amplitude of the ShipX transfer functions, and the dotted line represents the direction of tuning. In the presented scenario, it is remarkable that tuning reduces the error – compared to the original P-RAOs – not only in the vicinity of the tuning direction, but also in a wider portion of the angular domain. This must be contrasted by the fact that the tuning parameters are not optimal for all headings, and a closer look at the values of the optimized tuning parameters (not presented here) shows that they vary quite drastically between scenarios. Therefore, $\boldsymbol{\alpha}^*$ must rather be considered as a function of the logged ship speed and heading. This should be kept in mind when developing an extension of the proposed methodology in short-crested seas, which is left for future work.

The normalized error between the theoretical response spectrum in heave and pitch – before as well as after tuning – and the true spectrum is computed for all scenarios as per Equation (7), and shown in Figure 7. In almost all scenarios, there is a much better agreement between the true spectrum and the theoretical spectrum obtained via the use of the tuned P-RAOs, compared to the original P-RAOs. Nonetheless, for scenarios in beam sea conditions, namely A4, B4 and C4, tuning does not improve the results in pitch. This is due to the fact that pitch energy vanishes in beam seas in the formulation of the P-RAOs, as previously mentioned in the 'Theory' section. A way to mitigate this issue could be to force some pitch amplitude in the beam sea cases by using, say, $\beta = 95°$, but keeping all weight on heave in the tuning process in the particular cases. Anyway, the fact that the tuned pitch P-RAO remains zero in beam seas is a minor concern for practical applications, simply because the measured pitch variance is significantly lower (actually, more than 95% lower) at a heading of $\beta = 90°$, compared to the neighboring heading ($\beta = 120°$). On the other hand, the tuning in heave for the corresponding scenarios is successful. Besides the special case of beam sea, the tuning results are in general similarly good for both responses (heave and pitch), which indicates that the normalization in the cost function of Equation (10) is made appropriately to tune both responses with equal weight.

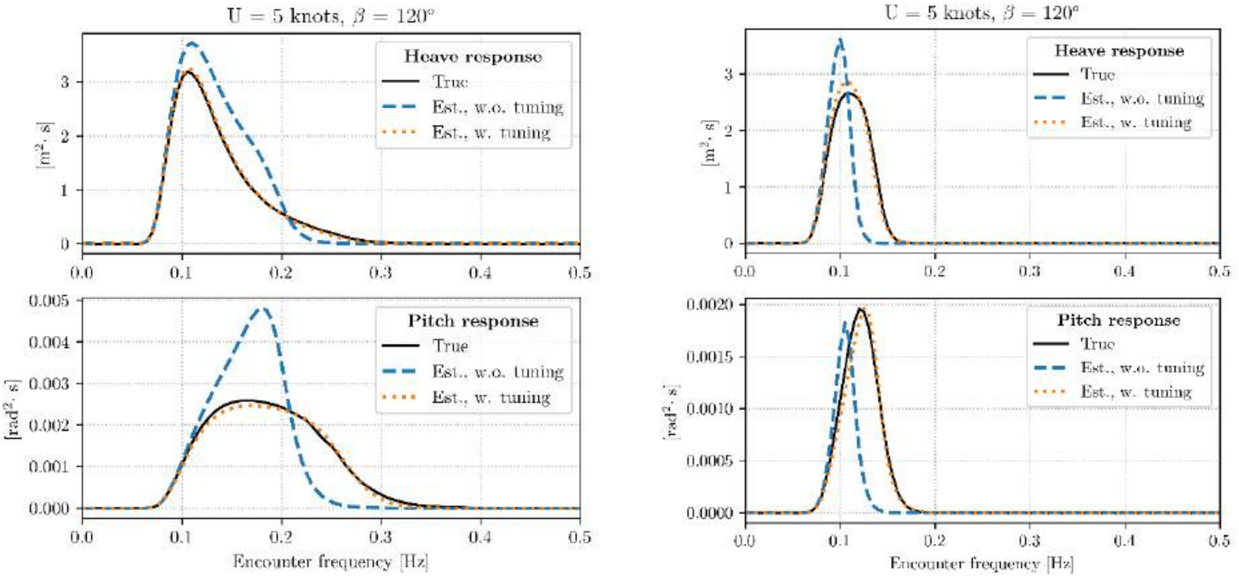
CONCLUSIONS

This paper presented a simple method to tune in an efficient way the closed-form expressions, developed by Jensen *et al.* (2004), for the response amplitude operators in heave and pitch motions of monohull ships. Those semi-empirical expressions are regarded as parameterized formulas, for which the input parameters are optimized so that the theoretically estimated motions closely match the measured ones. The optimization is carried out in a combined manner for the heave and pitch responses.

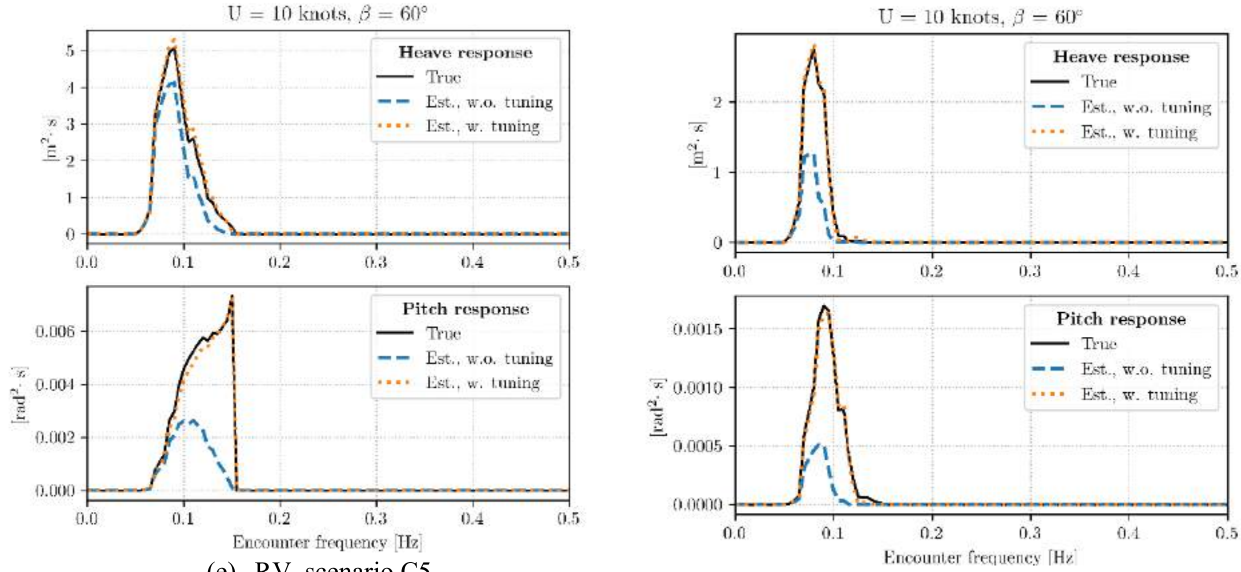
The procedure was tested via numerical simulations of motion responses in long-crested waves, for two ships of very different dimensions. In total, 42 scenarios were analyzed for various ship speeds and relative wave headings. In all cases, the error between the estimated and true response spectra is greatly reduced by tuning for both heave and pitch. Moreover, the tuned P-RAOs agree well with the true RAOs that were used for the simulations of vessel responses. In fact, the agreement is fairly reasonable over an extended portion of the frequency domain, even at frequencies where zero energy is found in the response spectra. The use of the optimized tuning parameters for RAO estimation at unobserved headings is touched upon, but it is argued that better results can be found by defining heading-dependent tuning parameters.



(a) RV, scenario A1 (b) S175, scenario A1

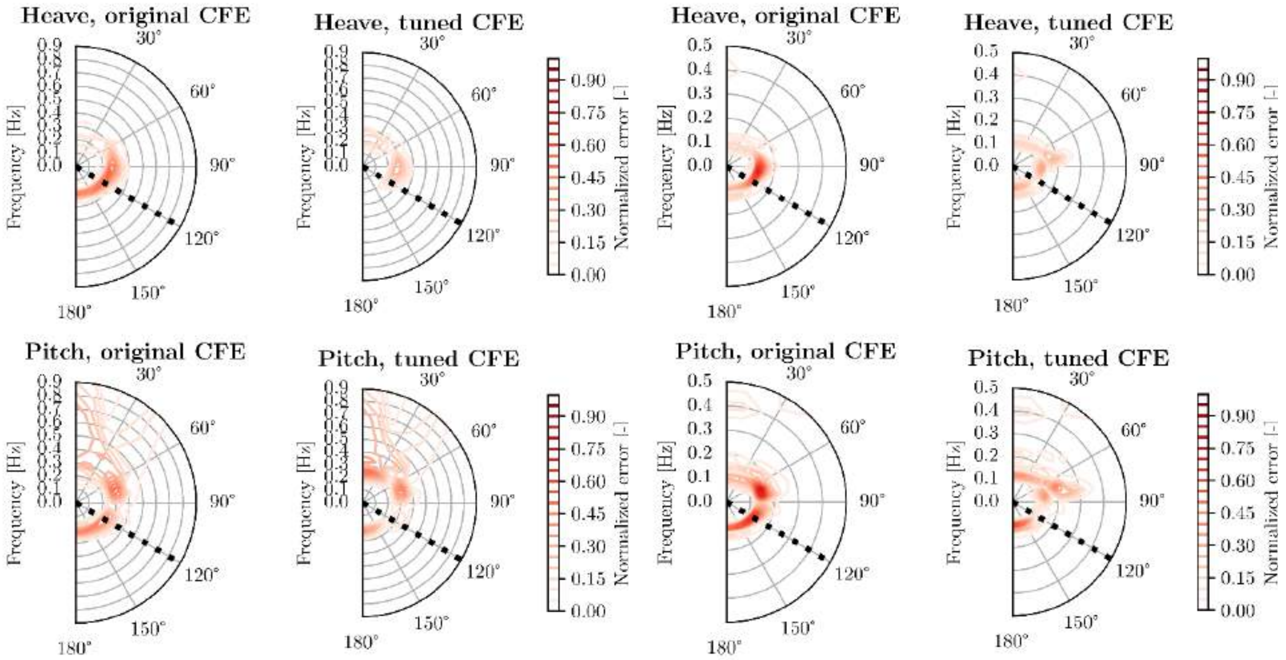


(c) RV, scenario B3 (d) S175, scenario B3



(e) RV, scenario C5 (f) S175, scenario C5

Figure 5: Estimated and true response spectra in heave and pitch at three different ship speeds and wave headings, for the two studied ships (RV and S175). Note the different vertical scales in the subplots.



(a) RV, scenario B3

(b) S175, scenario B3

Figure 6: Normalized error of the full set of original and tuned P-RAOs with respect to the ShipX RAOs, defined as: error = $\left|\hat{\boldsymbol{\Phi}}_{R,\text{P-RAO}} - \boldsymbol{\Phi}_{R,\text{ShipX}}\right| / \max_{\omega,\beta}(\boldsymbol{\Phi}_{R,\text{ShipX}})$. The dotted line indicates the direction of tuning.

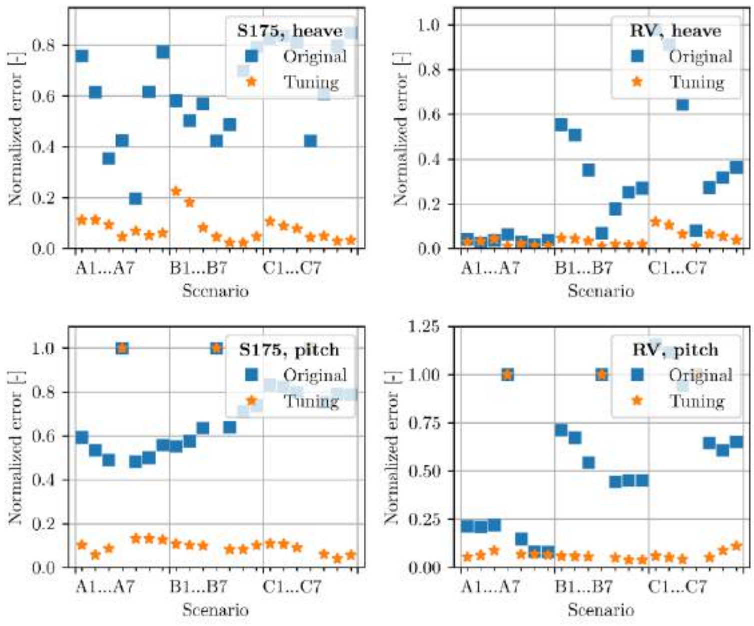


Figure 7: Overview of the normalized error between true and estimated response spectra in heave and pitch, for the two studied ships. 'Original' means that the original P-RAOs are used, while 'Tuning' uses the tuned P-RAOs. Note the different vertical scales in the subplots.

In future works, the procedure will be adapted to deal with scenarios in short-crested waves. Moreover, other responses could be included in the process, such as the roll motion or the wave-induced bending moment amidships. The ability to accurately predict, ahead of time, the response spectra based on tuned P-RAOs and sea state estimates could then be studied, in connection with real-life scenarios where in-service data collected onboard vessels are used. In this endeavor, quantification of the tuning uncertainties will require special attention.

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